21/5/2 Experiment-5

Laplace Transformation of Function

①

Let $f(t)$ is defined with all values of t then the laplace transform of $f(t)$ denoted by $L[f(t)]$ is defined as $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$.

Where s is a parameter real or complex.

$L(1) = 1/s$

$L(t) = 1/s^2$

$L(t^n) = n!/s^{n+1}$

$L(e^{at}) = 1/s-a$

$L(e^{-at}) = 1/s+a$

$L(\cos(at)) = \frac{s}{s^2+a^2}$

$L(\sin(at)) = \frac{a}{s^2+a^2}$

$L(\cosh(at)) = \frac{s}{s^2-a^2}$

$L(\sinh(at)) = \frac{a}{s^2-a^2}$

the

] is

① $L(1+t)^3 = a^3+b^3+3a^2b+3ab^2$

$= L[1] + L\,3[t] + L\,3[t^2] + L[t^3]$

$= \frac{1}{s} + \frac{3}{s^2} + \frac{6}{s^3} + \frac{6}{s^4}$

$= \frac{s^3+3s^2+6s+6}{s^4}$

② $L(\cos^2(3t))$

$= \left(\frac{\cos 6t}{2}\right)$

$= \frac{1}{2} + (\cos 6t) + \frac{1}{2}L(1)$

$= \frac{1}{2} \times \frac{s}{s^2+36} + \frac{1}{2} \times \frac{1}{s}$

$= \frac{s}{2s^2-72} + \frac{1}{2s}$

$= \frac{2s^3+2s^2-72}{4s^3-144s}$

3) $L(2\sin(t))\cos(3t))$

$= \frac{L(\sin(4t)) + 2(\sin(-2t))}{2}$

$= \frac{1}{2}\left[L(\sin 4t) + 2(\sin(-2t))\right]$

$= \frac{1}{2}\left[\frac{4}{s^2+16} + \frac{(-2)}{s^2+4}\right]$

$= \frac{1}{2}\left[\frac{4}{s^2+16} - \frac{2}{s^2+4}\right]$

$= \frac{1}{2}\left[\frac{4(s^2+4) - 2(s^2+16)}{(s^2+16)(s^2+4)}\right]$

$= \frac{1}{2}\left[\frac{2s^2-16}{(s^2+16)(s^2+4)}\right]$

4) $\mathcal{L}(e^{-3t}(\sin(4))$

$$= \frac{4}{(s+3)^2+16}$$

$$= \frac{4}{s^3+27+9s^2+27s+16}$$

5) $\mathcal{L}(2\sin(t)\sin(3t))$

$$= \mathcal{L}(\cos(4t)-\cos(-2t))$$

$$= \mathcal{L}(\cos 4t)-\mathcal{L}(\cos(-2t))$$

$$= \frac{s}{s^2-16}-\frac{s}{s^2-4}$$

$$= \frac{s(s^2-4)-s(s^2-16)}{(s^2-16)(s^2-4)}$$

$$= \frac{\cancel{s^3}-4s-\cancel{s^3}+16s}{(s^2-16)(s^2-4)}$$

$$= \frac{12s}{(s^2-16)(s^2-4)}$$

6) $\mathcal{L}(t^3+3t^2-6t+8)$

$$= \mathcal{L}(t^3)+3\mathcal{L}(t^2)-6\mathcal{L}(t)+8\mathcal{L}(1)$$

$$= \frac{6}{s^4}+\frac{6}{s^3}-\frac{6}{s^2}+\frac{8 \cdot 1}{s}$$

$$= \frac{6+6s-6s^2+8s^3}{s^4}$$

7) $L(\sin^2(4t))$

$= \frac{1}{2} L(1 - \cos 2(4t))$

$= \frac{1}{2} \left[L[1] - L[\cos 8t] \right]$

$= \frac{1}{2} \left[\frac{1}{s} - \frac{s}{s^2-64} \right]$

$= \frac{1}{2} \left[\frac{(s^2-64) - s^2}{s(s^2-64)} \right]$

$= \frac{1}{2} \left[\frac{s^2-64-s^2}{s^3-64s} \right] \Rightarrow \frac{1}{\cancel{2}} \cdot \frac{-\cancel{64}^{32}}{s^3-64s} \Rightarrow \frac{-32}{s^3-64s}$

8) $L(e^{-2t} \cos(5t))$

$= \frac{s}{s^2+25}$

$= \frac{s}{(s+2)s+25}$

$= \frac{s}{s^3+4s+29}$

9) $L(\sin^3(2t))$

$= \frac{1}{4} L\left(3\sin(2t) - \sin(6t)\right)$

$= \frac{1}{4} \left[\frac{\cancel{6s^2} + 216 - \cancel{6s^2} - 24}{s^4+36s^2+4s^2+144} \right]$

$= \frac{1}{4} \left[\frac{\cancel{192s^2}\ 192}{s^4+40s^2+144} \right]$

10) $L(\cos^3(4t))$

$= \frac{1}{4} L\left[\cos(12t) + 3\cos(4t)\right]$

$= \frac{1}{4}\left[\frac{s}{s^2+144} + \frac{3s}{s^2+16}\right]$

$= \frac{1}{4}\left[\frac{s^3+16s+3x^2+432s}{s^4+16s^2+144s^2+2564}\right]$

11) $L(t^2e^{-3t})$

$= \frac{2}{s^3}\Big]_{s\to s+3}$

$= \frac{2}{(s+3)^3}$

12) $L((1+e^t)^2)$

$= L(1+e^2+2e^t)$

$= \frac{1}{s} + \frac{1}{s-2} + \frac{2}{s-1}$

13) $L(3\cosh(5t) - 4\sinh(5t))$

$= \frac{3s}{s^2-25} - \frac{5}{s^2-25}$

$= \frac{3s^3-75s-5s^2-125}{s^4-25s^2-25s^2+625}$

14) $= \cancel{L(e^4}\ L(4t)$

$= \frac{\log 4}{s^{1/2}}$

Program

```
from sympy.abc import s,t.
1) from sympy import *
t = symbols ("t")
f = 1+t** 3+3*t +3*t**2
d = laplace_transform (f,t, s)
print (d)
```

Ans → ((s** 3+3*s**2 +6*s+6)/(s** 4, 0, True)

```
2. f= 2* sin(t) * sin(3*t)
```

Ans: (12*s / ((s**2+4)*(s**2 +16)), 0, True)

```
3. f= exp(3*t) * sin(4*t)
```

Ans: (4/((s-3)** 2+16, 3, True)

```
4. f= cos(3*t)**2
```

Ans ((s** 2 +18)/(s*(s**2 +36)), 0, True)

```
5. f= 2* sin(t) * cos(3*t)
```

Ans (2* s**2 - 8)/((s**2 +4)*(s**2+16)), 0, True)

```
6) f= (t** 3) + (3* t**2) - (16*t) +8
```

Ans (2* (4* s**3 - 3* s**2+ 3* s +3)/ s*4, 0, True)

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