23/04

12.6.7

Experiment - 2.

Evaluation of Double Integral

1) $\int_0^1 \int_0^2 (x+y)\,dy\,dx$

$= \int_0^1 \left[xy + \frac{y^2}{2} \Big|_{y=0}^{y=2} \right] dx$

$= \int_0^1 2x+2 \; dx.$

$= \cancel{\frac{2x^2}{2} + 2x} \Big|_0^1 \Rightarrow \frac{2x^2}{2} + 2x \Big|_{0=x}^{1=x}$

$= \cancel{\frac{2(1)}{2} \; 2(1)} \Rightarrow \frac{2}{2} + 2$

$= \cancel{1}. \Rightarrow 8.$

2) $\int_0^{\pi/3} \int_0^{\pi/6} \cancel{\cos x \sin x \, dx} \, dy$

$= \int_0^{\pi/3} \int_0^{\pi/6} \frac{\sin 2x}{2} \, dx \, dy$

$= \frac{1}{4} \int_0^{\pi/3} \left[\cancel{-\cos 2x} \right]_0^{\pi/6} dy$

$= \frac{1}{4} \int_0^{\pi/3} \left[-\cos \cancel{2}\left(\frac{\pi}{\cancel{6}3}\right) + \cos(0) \right] dy$

$= \frac{1}{4} \int_0^{\pi/3} \left[-\frac{\sqrt{3}}{2} + 1 \right] dy.$

$= \frac{1}{4} \int_0^{\pi/3} \left[\frac{-\cancel{3}\sqrt{3} + 2}{2} \right] dy$

$$= \frac{1}{8} \int_0^{\pi/3} \left[-\sqrt{3} + 2 \right] dy.$$

$$= \frac{1}{8}.$$

$$=$$

(2) $\displaystyle 8 \int_0^{\pi/3} \int_0^{\pi/6} \cos x \; \sin x \; dx \, dy.$

$$= \int_0^{\pi/3} \left[\int_0^{\pi/6} \frac{\sin 2x}{2} dx \right] dy$$

$$= \int_0^{\pi/3} \frac{1}{2} \cdot \frac{-\cos 2x}{2} \Big|_0^{\pi/6} dy.$$

$$= \int_0^{\pi/3} \frac{-1}{4} \left[\cos\frac{\pi}{3} - \cos 0 \right]$$

$$= -\frac{1}{4} \int_0^{\pi/3} \frac{-1}{4} \left[\frac{1}{2} - 1 \right] dy.$$

$$= \int_0^{\pi/3} \frac{1}{8} \, dy.$$

$$= \frac{1}{8} \, y \Big|_0^{\pi/3}$$

$$= \frac{1}{8} \frac{\pi}{3} = \frac{\pi}{24}$$

(3) $\displaystyle \int_1^2 \int_3^4 (xy + e^y) \, dy \, dx.$

$$= \int_1^2 \left[\frac{xy^2}{2} + e^y \right]_{y=3}^{y=4} dx.$$

$$= \int_1^2 \left[(8x + e^4) - \left(\frac{9}{2} x + e^3 \right) \right] dx.$$

$$= \left[4x^2 + e^4 x - \frac{9}{2} \frac{x^2}{2} - e^3 x \right]_1^2$$

$$= 16 + 2e^4 - 9 - 2e^3 - \left[4 + e^4 - \frac{9}{4} - e^3\right]$$

$$= 3 + e^4 + \frac{9}{4} - e^3$$

$$= -e^3 + 21/4 + e^4$$

(4) $$\int_0^a \int_0^b (x^2 + y^2)\, dy\, dx.$$

$$= \int_0^a \left[x^2 y + \frac{y^3}{3} \right]_{y=0}^{y=b} dx$$

$$= \int_0^a x^2 b + \frac{b^3}{3}\, dx$$

$$= \left[\frac{x^3 b}{3} + \frac{b^3 x}{3} \right]_0^a$$

$$= \frac{a^3 b}{8} + \frac{ab^3}{3}.$$

*

(5) $\int_0^1 \int_0^1 x^2 y^2 \, dx \, dy$

$= \int_0^1 \left[\frac{x^3}{3} y^2 \Big|_0^1 \right] dy$

$= \int_0^1 \left[\frac{1}{3} y^2 \right] dy$

$= \frac{y^3}{9} \Big|_0^1$

$= \frac{1}{9}$

(6) $\int_1^2 \int_0^x x y^2 \, dy \, dx$

$= \int_1^2 \left[x \frac{y^3}{3} \Big|_0^x \right] dx.$

$= \int_1^2 \left[\frac{x^4}{3} \right] dx$

2x2 x
4 x4
4x2
2x16

$= \frac{x^5}{15} \Big|_1^2$

$= \frac{2^5}{15} = \frac{31}{15}$

7) $\int_0^3 \int_0^{\sqrt{9-x^2}} dy\, dx$

$\int \sqrt{a^2-x^2}\, dx$

$= \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

$= \int_0^3 \left[y \Big|_0^{\sqrt{9-x^2}} \right] dx$

$= \int_0^3 \sqrt{9-x^2}\, dx$

$= \frac{x}{2}\sqrt{3^2-x^2} + \frac{3^2}{2}\sin^{-1}\left(\frac{x}{3}\right) \Big|_0^3$

$= \frac{3}{2}\sqrt{3^2-3^2} + \frac{3^2}{2}\sin^{-1}\left(\frac{3}{3}\right) - \frac{3^2}{2}\sin^{-1}(0)$

$= \frac{3^2}{2}\sin^{-1}(1) - \frac{3^2}{2}\sin^{-1}(0)$

$= \frac{9}{2}\left[\frac{\pi}{2} - 0\right]$

$= \frac{9\pi}{4}$

(8) $\int_0^1 \int_x^{x^2} (x^2+y)\, dy\, dx$

$= \int_0^1 \left[x^2 y + \frac{y^2}{2} \right]_x^{x^2} dx$

$= \int_0^1 \left(x^4 + \frac{x^4}{2} - \left(x^3 + \frac{x^2}{2}\right)\right) dx$

$= \int_0^1 \left[\frac{2x^4+x^4}{2} - \left(\frac{2x^3+x^2}{2}\right)\right] dx$

$= \frac{1}{2}\int_0^1 (3x^4 - 2x^3 - x^2)\, dx$

$$= \frac{1}{2}\left[\frac{3x^5}{5} - \frac{2x^4}{4} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2}\left[\frac{3}{5} - \frac{2}{4} - \frac{1}{3}\right]$$

$$= \frac{1}{2}\left[\frac{3}{5} - \frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{1}{2}\left[\frac{3}{5} - \frac{3-2}{6}\right]$$

$$= \frac{1}{2}\left[\frac{3}{5} - \frac{5}{6}\right]$$

$$= \frac{1}{2}\left[\frac{18-25}{30}\right]$$

$$= \frac{1}{2}\left[\frac{-7}{30}\right]$$

$$= \frac{-7}{60}.$$

(9) $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx.$

$$= \int_0^1 \left[\frac{y^3}{3}\Big|_0^{\sqrt{1-x^2}}\right] dx$$

$$= \int_0^1 \left[\frac{\sqrt{1-x^2}}{3}\right] dx.$$

$$= \int_0^1 \left(\frac{1}{3}(1-x^2)^{3/2}\right) dx$$

$$= \frac{1}{3}\int_0^{\pi/2} (1-\sin^2 t)^{3/2} \cos t \, dt \qquad x = \sin t, \quad dx = \cos t \, dt.$$

$$= \frac{1}{3}\int_0^{\pi/2} (\cos^2 t)^{3/2} \cos t \, dt. \qquad 1 = \sin \pi/2, \quad 0 = \sin 0.$$

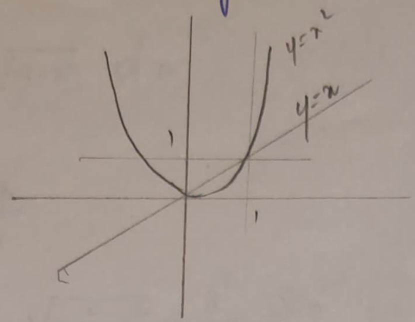
$$= \frac{1}{3} \cdot \int_0^{\pi/2} \cos^4 t \, dt$$

$$= \frac{1}{\cancel{3}}\left[\frac{\cancel{3}}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}\right]$$

$$= \frac{\pi}{16}$$

(10) $\iint_R xy(x+y)\,dx\,dy$

where R is the region bounded by the curves $y = x^2$ & $y = x$.



$$\int_0^1 \int_{x=y}^{\sqrt{y}} xy(x+y)\,dx\,dy$$

$$\int_0^1 \left[\frac{x^3y}{3} + \frac{x^2y^2}{2}\right]_y^{\sqrt{y}} dy$$

$$\int_0^1 \left(\frac{y^{5/2}}{3} + \frac{y^3}{2} - \frac{y^4}{3} + \frac{y^4}{2}\right) dy$$

$$= \left[\frac{2y^{7/2}}{21} + \frac{y^4}{8} - \frac{y^5}{15} + \frac{y^5}{10}\right]_0^1$$

$$= \frac{2}{21} + \frac{1}{8} - \frac{1}{15} + \frac{1}{10}$$

$$= 0.095 + 0.125 - 0.0066 + 0.11$$

$$= 0.1284$$