30/04/22 Evaluation of

121617.

Experiment - 3 TRIPLE Integral.

1) $\int_0^1\int_0^2\int_0^3 (x+y+z)\, dz\, dy\, dx$

$= \int_0^1\int_0^2 \left[xz+yz+\frac{z^2}{2} \Big|_0^3 \right] dy\, dx$

$= \int_0^1\int_0^2 3x+3y+\frac{3^2}{2}\; dy\, dx$

$= \int_0^1 \left[3xy + \frac{3y^2}{2} + \frac{9y}{2} \Big|_0^2 \right] dx$

$= \int_0^1 \left[6x + \frac{12}{2} + \frac{18}{2} \right] dx$

$= \left[\frac{6x^2}{2} + \frac{12x}{2} + \frac{18x}{2} \right]_0^1$

$= \frac{6}{2} + \frac{12}{2} + \frac{18}{2}$

$= \frac{36}{2} = 18$

2) $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\, dx\, dy\, dz$

$= \int_0^1\int_0^1 \left[e^{(x+y+z)} \right]$

$= \int_0^1\int_0^1 \left[[e^x]_0^1 e^y . e^z \right] dy\, dz$

$= \int_0^1\int_0^1 \left[e^y . e^z [e^1 - e^0] \right] dy\, dz$

$=$

$$= \int_0^1 \int_0^1 \left[e^y \cdot e^z e - e^y \cdot e^z \right] dy \cdot dz.$$

$$= \int_0^1 \left[e^z \cdot e [e^1 - e^0] - e^z [e^1 - e^0] \right] dz.$$

$$= \int_0^1 \left[e^z \cdot e^2 - e^z e - e^z e - e^z \right] dz. \rightarrow \left[e^z e^2 - 2e^z e + e^z \right] dz.$$

$$= \left[e^2 (e^1 - e^0) - 2e(e^1 - e^0) + (e^1 - e^0) \right]_0^1$$

$$= \left[e^3 - e^2 - 2e^2 + 2e - e - 1 \right]$$

$$= \left[e^3 - 3e^2 + e - 1 \right]$$

(3) $\int_0^1 \int_0^2 \int_1^2 x^2 y z \, dx \, dy \, dz.$

$$= \int_0^1 \int_0^2 \left[\frac{x^3}{3} yz \Big|_1^2 \right] dy \, dz.$$

$$= \int_0^1 \int_0^2 \left[\frac{8}{3} yz - \frac{4z}{3} \right] dy \, dz$$

$$= \int_0^1 \int \frac{8y^2 z}{6} - \frac{4^2 z}{6} \Big|_0^2 \Big] dz$$

$$= \int_0^1 \int \left[\frac{32z}{6} - \frac{4z}{6} \right] dz$$

$$= \left[\frac{32z^2}{12} - \frac{4z^2}{12} \Big|_0^1 \right]$$

$$= \frac{32}{12} - \frac{4}{12}$$

$$= \frac{7}{3}$$

$$= \int_0^1 \int \frac{}{69} \Big|_0 \, dx$$

$$= \int_0^1 \frac{x^2}{9}\, dx$$

$$= \frac{x^3}{27}\Big|_0^1$$

$$= \frac{1}{27}$$

(6) $$\int_0^1 \int_x^1 \int_0^{y-x} xz \, dz\, dy\, dx$$

$$= \int_0^1 \int_x^1 \frac{xz^2}{2}\Big|_0^{y-x} dy\, d$$

$$= \int_0^1 \int_x^1 \frac{x}{2}(y-x)^2 \, dy\, dx$$

$$= \int_0^1 \int_x^1 \frac{x}{2}\left[y^2 + x^2 - 2yx\right] dy\, dx$$

$$= \frac{1}{2}\int_0^1 \frac{xy^3}{3} + x^3 y - \frac{2x^2y^2}{2}\Big|_x^1 \, dx$$

$$= \frac{1}{2}\int_0^1 \frac{x}{3} + x^3 - x^2 - \frac{x^4}{3} - x^4 + x^4 \, dx$$

$$= \frac{1}{2}\left[\frac{-x^4}{12} + \frac{x^4}{4} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2}\left[-\frac{1}{12} - \frac{1}{4} - \frac{1}{3}\right]$$

$$= \frac{1}{120}$$

$$= \int_0^2 \int \left[\frac{x^9}{8} - \frac{x^5}{4} - \frac{x}{8} + \frac{x}{4}\right] dx$$

$$= \int \left[\frac{x^{10}}{80} - \frac{x^6}{24} - \frac{x^2}{16} + \frac{x^2}{8}\right]_0^2$$

$$= \int \left[\frac{1024}{80} - \frac{64}{24} - \frac{4}{16} + \frac{4}{8}\right]$$

$$= \frac{623}{60}.$$

(7) $\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x+y} xy \, dz \, dy \, dx.$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} xyz \Big|_0^{x+y} dy \, dx.$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} \left[x^2y + xy^2\right] dy \, dx$$

$$= \int_0^1 \left[\frac{x^2y^2}{2} + \frac{xy^3}{3}\right]_{x^2}^{\sqrt{x}} dx$$

$$= \int_0^1 \left[\frac{x^3}{2} + \frac{x(\sqrt{x})^3}{3} - \frac{x^6}{2} \ \frac{x^7}{3}\right] dx$$

$$= \int_0^1 \left[\frac{x^3}{2} + \frac{x^{5/2}}{3} - \frac{x^6}{2} - \frac{x^7}{3}\right] dx$$

$$= \frac{x^4}{8} + \frac{2x^{7/2}}{21} - \frac{x^7}{14} - \frac{x^8}{24} \Big|_0^1$$

$$= \frac{21 + 16 - 12 - 7}{168}$$

$$= \frac{37 - 19}{168} = \frac{18}{168}$$

$$= 3/28$$