28/05/22
Saturday

# Experiment - 6

## Inverse Laplace Transform

If $F(s)$ is Laplace transform of $F(t)$ then $F(t)$ is called the inverse Laplace transform of $F(s)$. Denoted by $L^{-1}$

$$L(f(t)) = f(s)$$

$$f(t) = L^{-1}(F(s))$$

**Formula**

$F(s)$ $\quad L^{-1}(F(s))$

$$\frac{1}{s} = L^{-1}(1) \Rightarrow L^{-1}\left(\frac{1}{s}\right) = 1$$

$$L^{-1}\left(\frac{1}{s^2}\right) = t$$

$$L^{-1}\left(\frac{1}{s^3}\right) = \frac{t^2}{2!}$$

$$L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!}$$

$$L^{-1}\left(\frac{1}{s-k}\right) = e^{kt}$$

$$L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos(at)$$

$$L^{-1}\left(\frac{a}{s^2+a^2}\right) = \sin(at)$$

$$L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cosh(at)$$

$$L^{-1}\left(\frac{a}{s^2-a^2}\right) = \sinh(at)$$

$$\frac{a}{s^2-a^2} = \sinh(at)$$

$$\frac{1}{(s-k)^n} = \frac{e^{kt}\, t^{n-1}}{(n-1)!}$$

$$L^{-1} f(s-a) = e^{at} f(t)$$

1) $L^{-1}\left[\frac{1}{(s+3)^3}\right]$

$$\frac{1}{(s+k)^n} = e^{kt}\frac{t^{n-1}}{(n-1)!}$$

$$= e^{3t} \cdot \frac{t^2}{2!}$$

② $L^{-1}\left(\frac{3s^2+10s-6}{s^4}\right)$

2) $\Rightarrow L^{-1}\left[\frac{3}{s^2} + \frac{10}{s^3} - \frac{6}{s^4}\right]$

$$= 3t + \frac{10t^2}{2} - \frac{6t^3}{3!}$$

$$= 3t + \frac{10t^2}{2} - \frac{6t^3}{6}$$

$$= -t^3 + 10t^2 + 3t$$

③ $L^{-1}\left(\frac{s}{(s-3)^5}\right)$

$$= \frac{(s+3)-3}{(s-3)^5}$$

$$= L^{-1}\left[\frac{-3}{(s-3)^4}\right]$$

$$= \frac{e^{3t}\, t^3}{6}$$

(4) $L^{-1}\left[\dfrac{2s-11}{s^2+4s+8}\right)$

$= \dfrac{2s-11}{s^2+4s+4+4} = \dfrac{2s-11}{(s+2)^2+4}$

$= \dfrac{2s}{(s+2)^2+4} - \dfrac{11}{(s+2)^2+4}$

$= \dfrac{2s}{(s+2)^2+(2)^2} - \dfrac{11}{(s+2)^2-(2)^2}$

$= \dfrac{2(s+2)-4-11}{(s+2)^2+2^2}$

$= \dfrac{2(s+2)}{(s+2)^2+2^2} - \dfrac{15}{(s+1)^2+2^2}$

$= 2\cos 2t e^{-t} - \dfrac{15}{2}\dfrac{2}{(s+1)^2+2^2}$

$= 2\cos^2 t e^{-2t} - 15/2 \sin 2t e^{-2t}$.

(5) $L^{-1}\left[\dfrac{3s+7}{(s^2+2s-3)}\right]$

$= \dfrac{3(s+1)+9}{(s+1)^2-4} = \dfrac{3(s+1)}{(s+1)^2-2^2} + \dfrac{4}{(s+1)^2-2^2}$

$L^{-1}\left[\dfrac{3(s+1)}{(s+1)^2-5^2} + \dfrac{4}{(s+1)^2-2^2}\right]$

$= 3\cosh(2t)e^{-t} + 2\sinh 2t e^{t}$

$= e^{-t}\left[3\cosh(2t) + 2\sinh(2t)\right)]$

(6) $\dfrac{s}{s^2+s-2}$

$$\frac{s}{s^2+s-\frac{1}{4}+\frac{1}{4}-2} = \frac{s}{(s+\frac{1}{2})^2-\frac{9}{4}}$$

$$= \frac{s}{(s+\frac{1}{2})^2-(\frac{3}{2})^2}$$

$$= \mathcal{L}^{-1}\left[\frac{s}{(s+\frac{1}{2})^2-(\frac{3}{2})^2}\right]$$

$$= \cos at\, e^{-t/2}$$

$$= \cos \tfrac{3}{2}t\, e^{-t/2}.$$

(7) $\dfrac{1}{s^2(s^2+1)}$

$$\frac{1}{s^2(s^2+1)} = \frac{A}{s^2} + \frac{B}{(s^2+1)}$$

$$1 = A(s^2+1) + (s^2)B$$

$$B = -1$$

$$A = 1$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2} - \frac{1}{s^2+1}\right]$$

$$= t - \sin t.$$

Program

1) from sympy import *
from sympy.abc import t, s
print (inverse_laplace_transform (1/s+3) **3, s, t))

output

t**2* exp(3*t) * Heaviside (t)/2

2) print (inverse_laplace_transform (3*s**2+10*s-6)/
s**4, s, t))

output

t*(-t**2 + 3+t+3)* Heaviside (t)

3) print (inverse_laplace_transform (3/(s-3)**2, s, t))

output

(3*t ~~+1~~ )* exp (3*t)* heaviside (t)

4) print (inverse_laplace_transform ((2*s-11))/
(s** 2+4*s+8), s, t))

output

- (15* sin (2*t) - 4*cos (2*t))* exp (-2*t) *
heaviside (t)/2.