Modeling phenomena based on Epidemic diseases

Sharif University of technology

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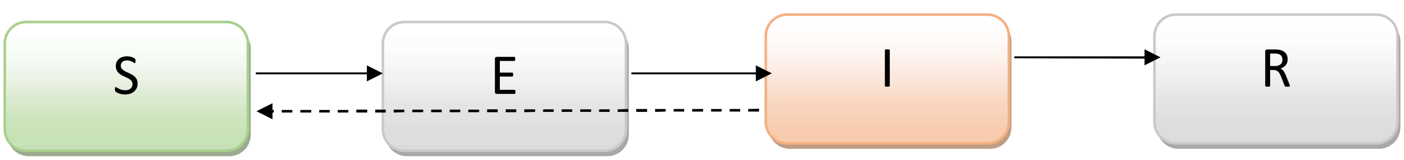
Homework7 Yasaman Asgari

Temporally Forced Models

In this Homework, we consider how seasonally varying parameters act as a forcing mechanism and examine their dynamical consequences. For the most part, we will use measles as a prototypical directly transmitted infectious disease. We demonstrate how such temporally forced models allow us to better capture the observed pattern of recurrent epidemics in contrast to unforced models, which predict oscillations that are damped toward equilibrium.

Dynamical Consequences of Seasonality: Harmonic and Sub harmonic Resonance

For this model, we assume 4 groups: S(susceptible) , E(exposed), I(infected) , R(recovered). We also have death and birth. So the model will be as below:



Equations are:

I. $\frac{dS}{dt} = \mu - \beta(t)I(t)S(t) - \mu S(t)$

II. $\frac{dE}{dt} = \beta(t)I(t)S(t) - \mu E(t) - \sigma E(t)$

III. $\frac{dI}{dt} = \sigma E(t) - \mu I(t) - \gamma I(t)$

IV. $\frac{dR}{dt} = \gamma I(t) - \mu R(t)$

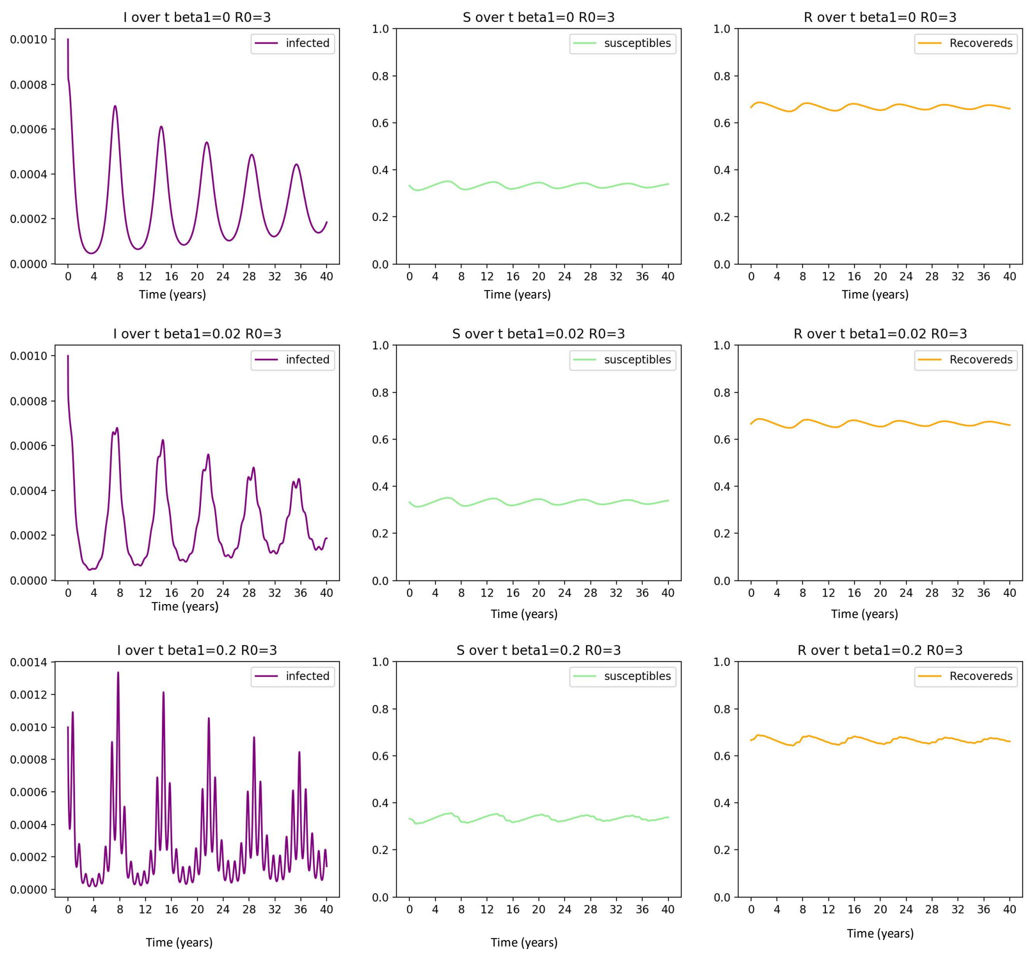
We assumed that σ is $\frac{1}{incubation\ period}$ and γ is recovery rate! Then we normalize the population to 1 and $\beta(t) = \beta_0(1 - \beta_1 \cos(\omega t))$.

Initial conditions are:

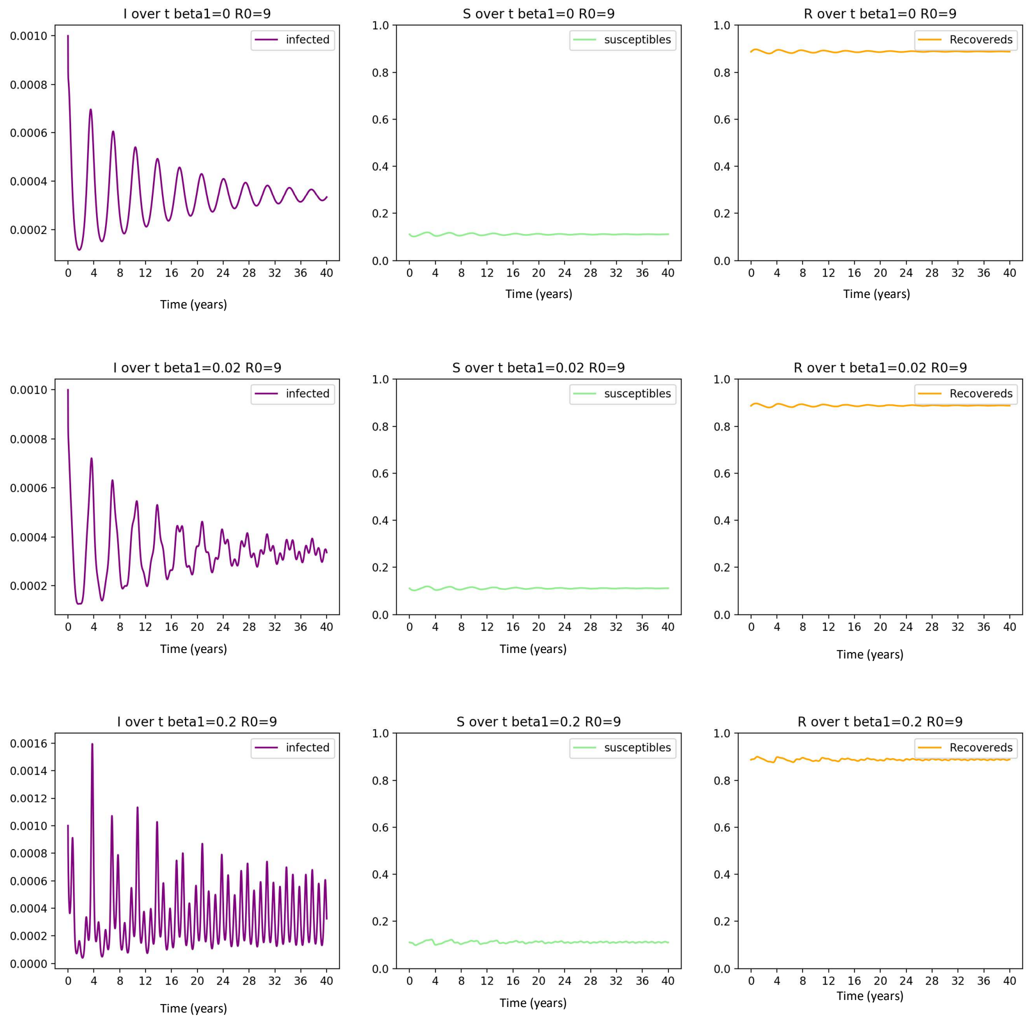
- $S(t = 0) = 6 \times 10^{-2}$
- $I(t = 0) = E(t = 0) = 10^{-3}$
- $\frac{1}{\sigma} = 10\ days$
- $\frac{1}{\gamma} = 7\ days$
- $\mu = 0.02\ per\ year$
- $\omega = 1 year$

Pay attention that the parameters should have same unit. Now we use the code!

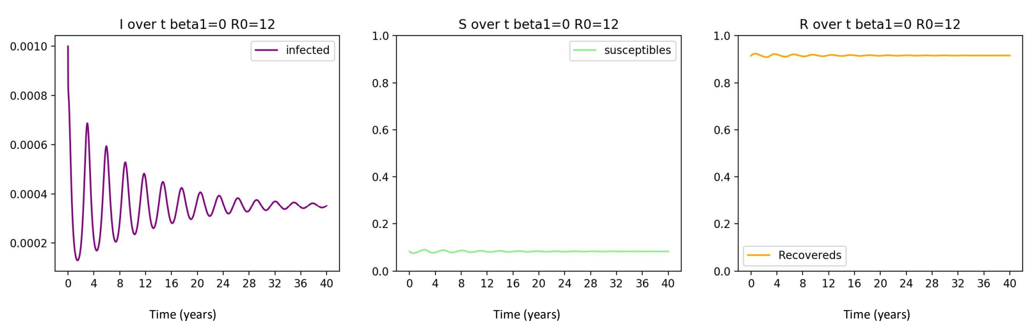
❖ Changing β_1 for $R_0 = 3$

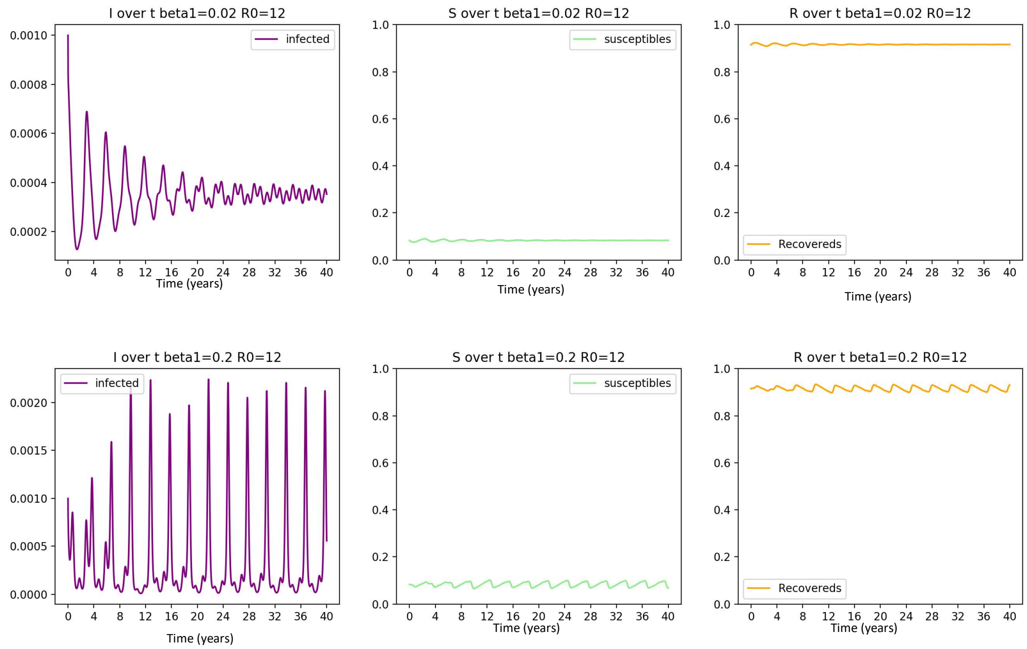


❖ Changing β_1 for $R_0 = 9$

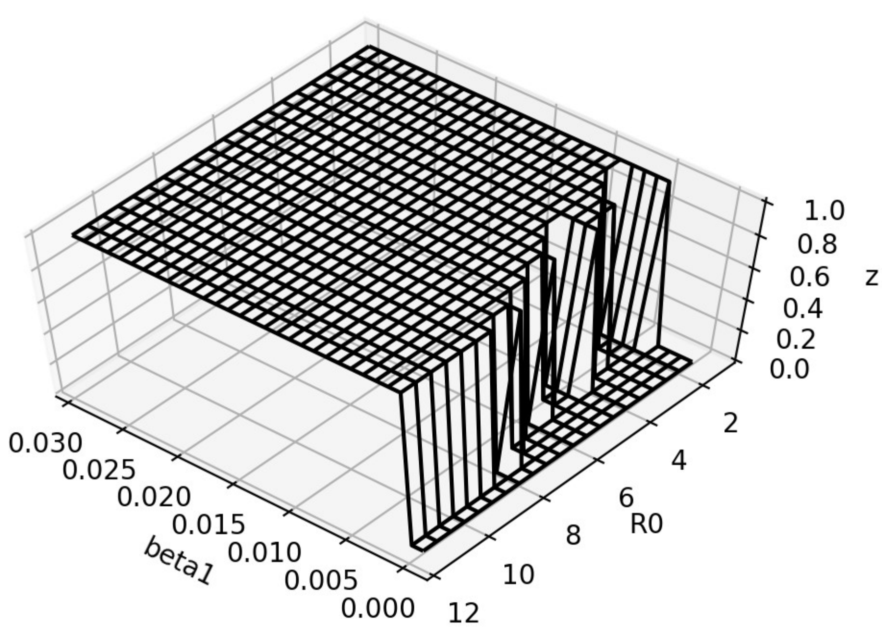


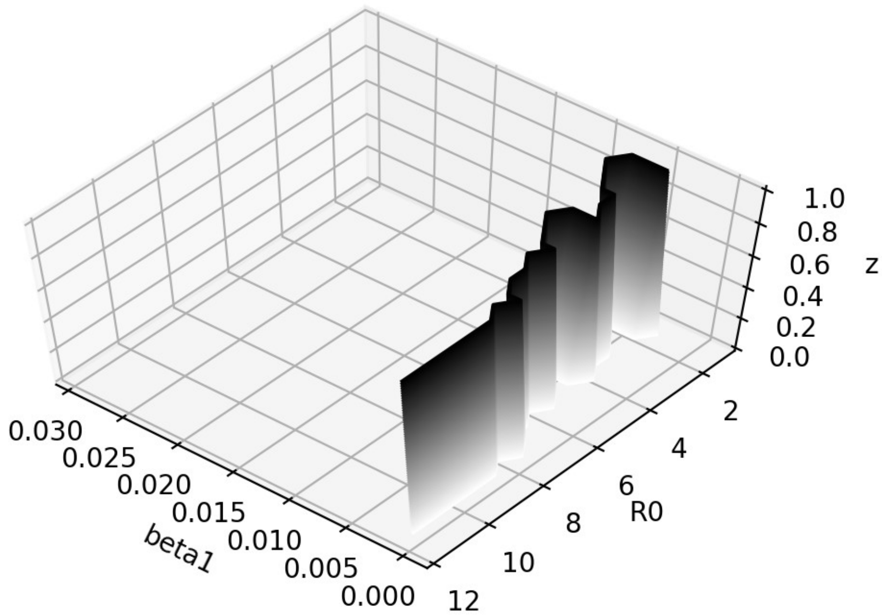
❖ Changing β_1 for $R_0 = 12$



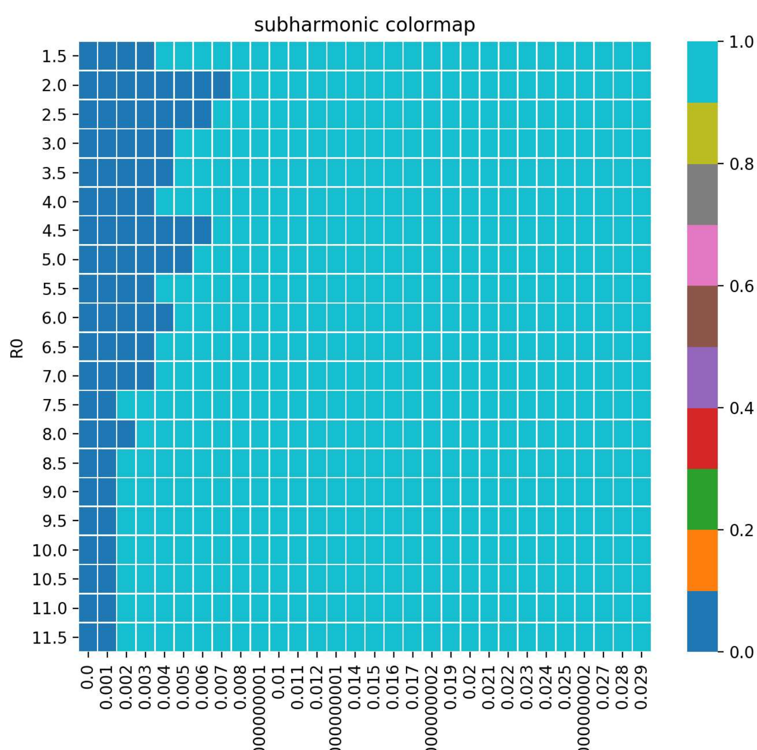


To find this answer we should figure out which has sub harmonic oscillations. I get all the local maximums then I know that when $\beta_1 = 0$ all the maximums are in descending order. But if we look at $\beta_1 \neq 0$ we understand that sometimes it goes down and sometimes goes up. So I just get 5 ordered peak and then check their order. If they are in descending order I put **zero** otherwise: **One**. Then I plot a 3D plot but changing β_1 from zero to 1/10 of β_0 and R_0 from 1 to 12 by steps 0.05! Then I project the plain into a 2D plot by restricting β_1 to the unit interval.





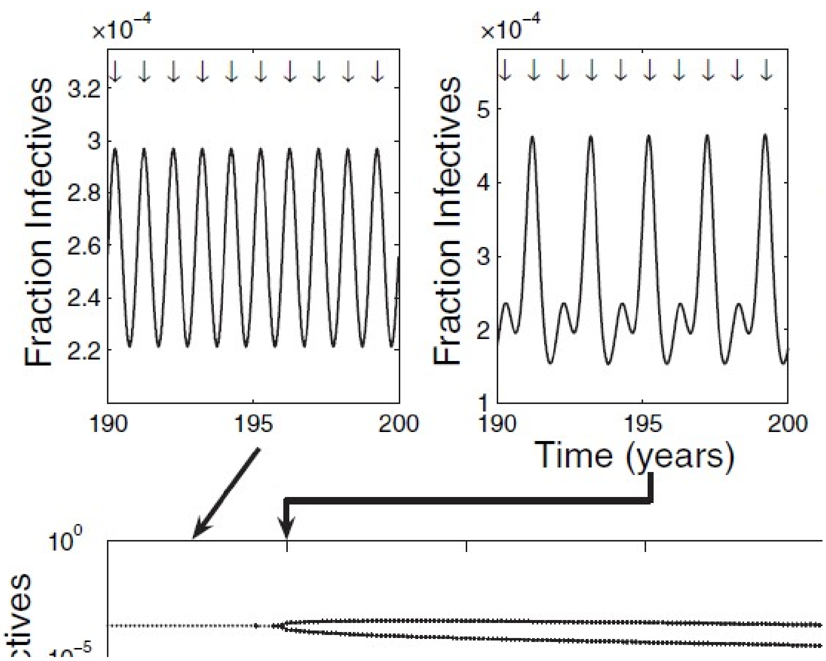
The critical points don't show specific character but it's correct!



If we look the graphs precisely, we will understand that there is no approximation for β_1 because the character of this parameter is complex related to others. But pay attention that we saw the sub harmonic resonance in linear graphs not logarithmic. It's better for the further research to see if there are any different or no! And also we know that ω and F are different and to get the book's diagrams we should put$\omega = F$ and F is related to other parameters. In this homework we put ω as a constant!

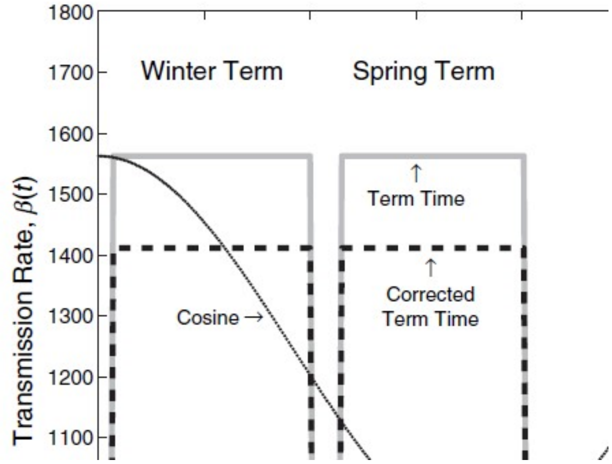
Bifurcation diagrams

This may be achieved by constructing bifurcation diagrams, where a bifurcation refers to a qualitative change in model dynamics as a **control parameter** is altered. For this diagram we controlled initial conditions andβ_1. For any specific parameter value, the model is started according to some specified initial conditions and integrated for a "reasonable" period of time, after which it is assumed the dynamics have reached their long-term (or asymptotic) state. Then, some measure of the population is plotted at one particular time-point each year for the subsequent n years.

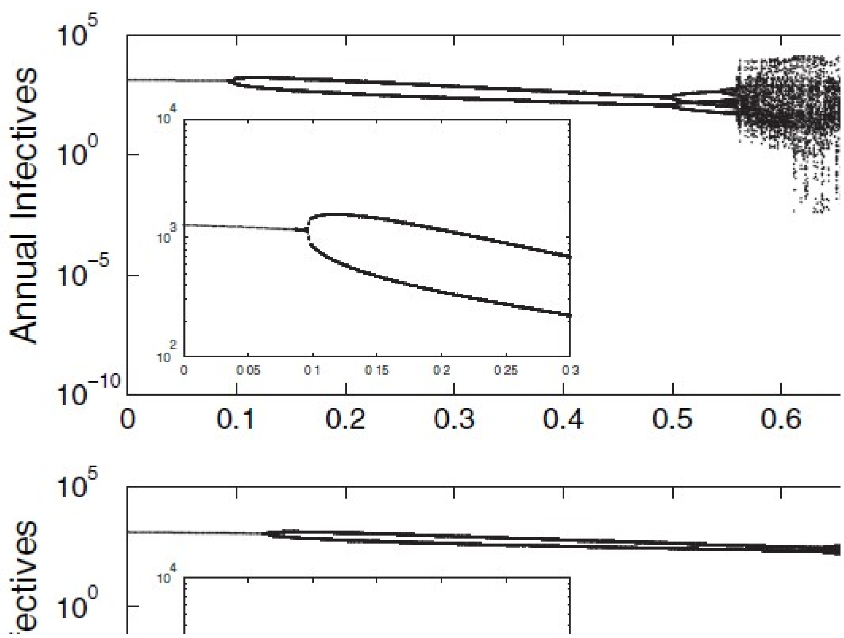


For specific models such as measles transmission rate is not as easy as sinusoidal functions. The diagram above is based on the$\beta(t) = \beta_0(1 - \beta_1 \cos(\omega t))$. But measles is a school year dependent infection. So the function is different!

$$\beta(t) = \frac{\beta_0}{1\ ((1+b_1)D \quad + (1-b_1)D\ )}$$



When we use these different transmission rate functions, the form of the diagrams are still similar to each other. Look at the graphs below!



But the appropriate β_1 where bifurcation appears is different. Second function is sharper and bifurcation happens with greater values of β_1.Chaotic windows are different in size and start point. But I don't have a reasonably proof for this.

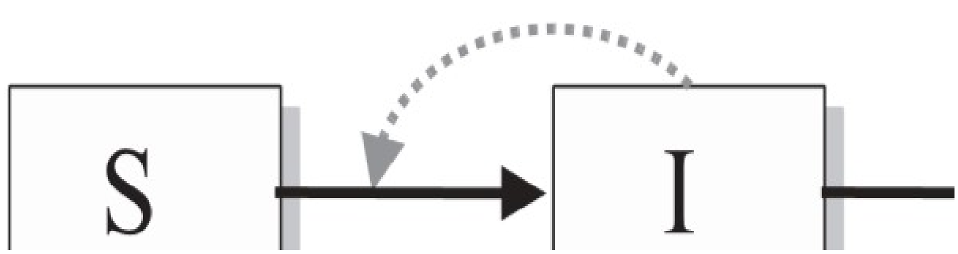
Seasonality in birth and transmission rate

Attention: The question is incorrect:α_1 and $\beta_1$don't have units. They are constants! But in the question there have units (per day) which are incorrect! Also the function for each of seasonality should be given because the results will be different!

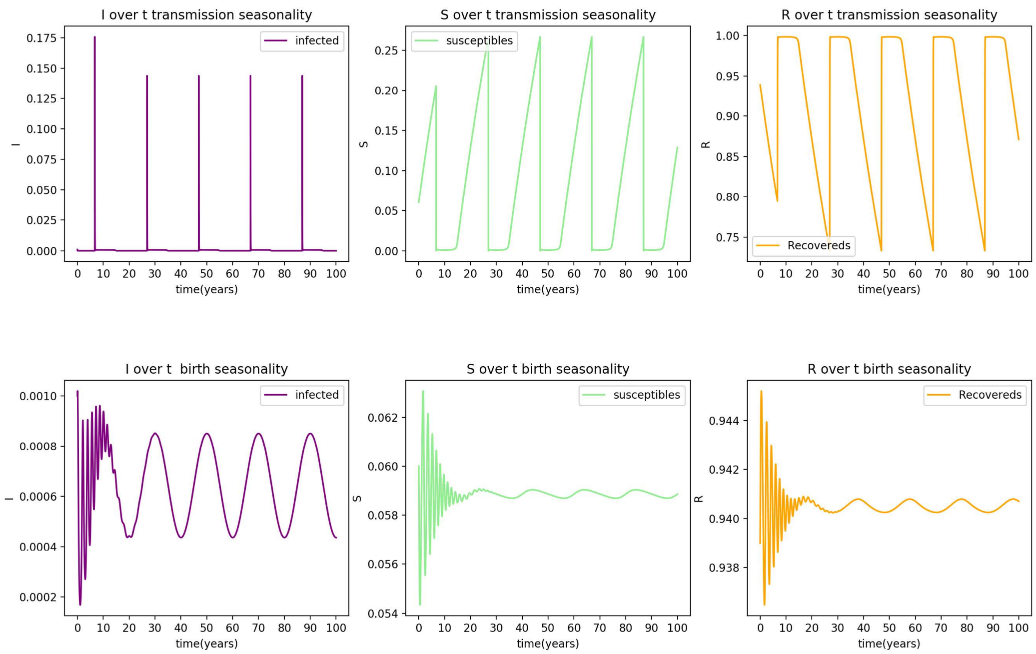
- $S(t=0) = 6 \times 10^{-2}$
- $I(t=0) = 10^{-3}$
- $\frac{1}{\gamma} = 10\ days$
- $\mu = \alpha_0 = 0.025\ per\ year$
- $\omega = \frac{2\pi}{20} year$
- $\alpha_1 = 0.3$
- $\beta_1 = 0.2$
- β_0=1.7 per day

Two models exist in this question

- First model:
  - I. $\frac{dS}{dt} = \mu - \beta(t)I(t)S(t) - \mu S(t)$
  - II. $\frac{dI}{dt} = \beta(t)I(t)S(t) - \mu I(t) - \gamma I(t)$
  - III. $\frac{dR}{dt} = \gamma I(t) - \mu R(t)$
  - IV. $\beta(t) = \beta_0(1 - \beta_1 \cos(\omega t))$
- Second model:
  - I. $\frac{dS}{dt} = \alpha(t) - \beta_0 I(t)S(t) - \mu S(t)$
  - II. $\frac{dI}{dt} = \beta_0 I(t)S(t) - \mu I(t) - \gamma I(t)$
  - III. $\frac{dR}{dt} = \gamma I(t) - \mu R(t)$
  - IV. $\alpha(t) = \alpha_0(1 - \alpha_1 \cos(\omega t))$



We just plot Infected, Susceptible and Recovered!



Oscillation period for birth seasonality: 20 years (code available)

Oscillation period for transmission seasonality: 20 years (coda available)

Differences

1. Birth seasonality infected plot is smooth but other is sharp!
2. Transmission seasonality's peaks are greater!
3. In transmission seasonality's diagram there are some near zero constant intervals!
4. Early time's amplitude of oscillation is smaller than equilibrium in transmission seasonality's suspected graph but in birth seasonality early time's amplitude of oscillation is greater than equilibrium!