Modeling phenomena based on Epidemic diseases

Sharif University of technology

Winter/spring 2020

Homework8 Yasaman Asgari

Stochastic equations

Question1:

Five key features distinguish stochastic models from their deterministic counterparts. Although not evident for all forms of stochastic models, these features can have a profound impact on epidemiological dynamics and will be a consistent theme throughout this chapter. In brief, these distinguishing features are:

1. Variability between simulations
2. Variances and covariance
3. Increased transients
4. Stochastic resonance
5. Extinctions

Question2:

As a starting point, let us incorporate noise into only the transmission term. Let ξ(t) be a time series of random deviates derived from the normal distribution with mean zero and unit variance .The basic equations, assuming frequency dependent (mass-action) transmission, are transformed to:

I. $\frac{dX}{dt} = \nu N - [\beta \frac{XY}{N} + f(X,Y)\xi] - \mu X$

II. $\frac{dY}{dt} = [\beta XY/N + f(X,Y)\xi] - \gamma Y - \mu Y$

III. $\frac{dZ}{dt} = \gamma Y - \mu Z.$

For generality, a function $f(X,Y)$ has been included to scale the randomness in response to the current variable sizes. But now we have constant noise: $f(X,Y) = c$

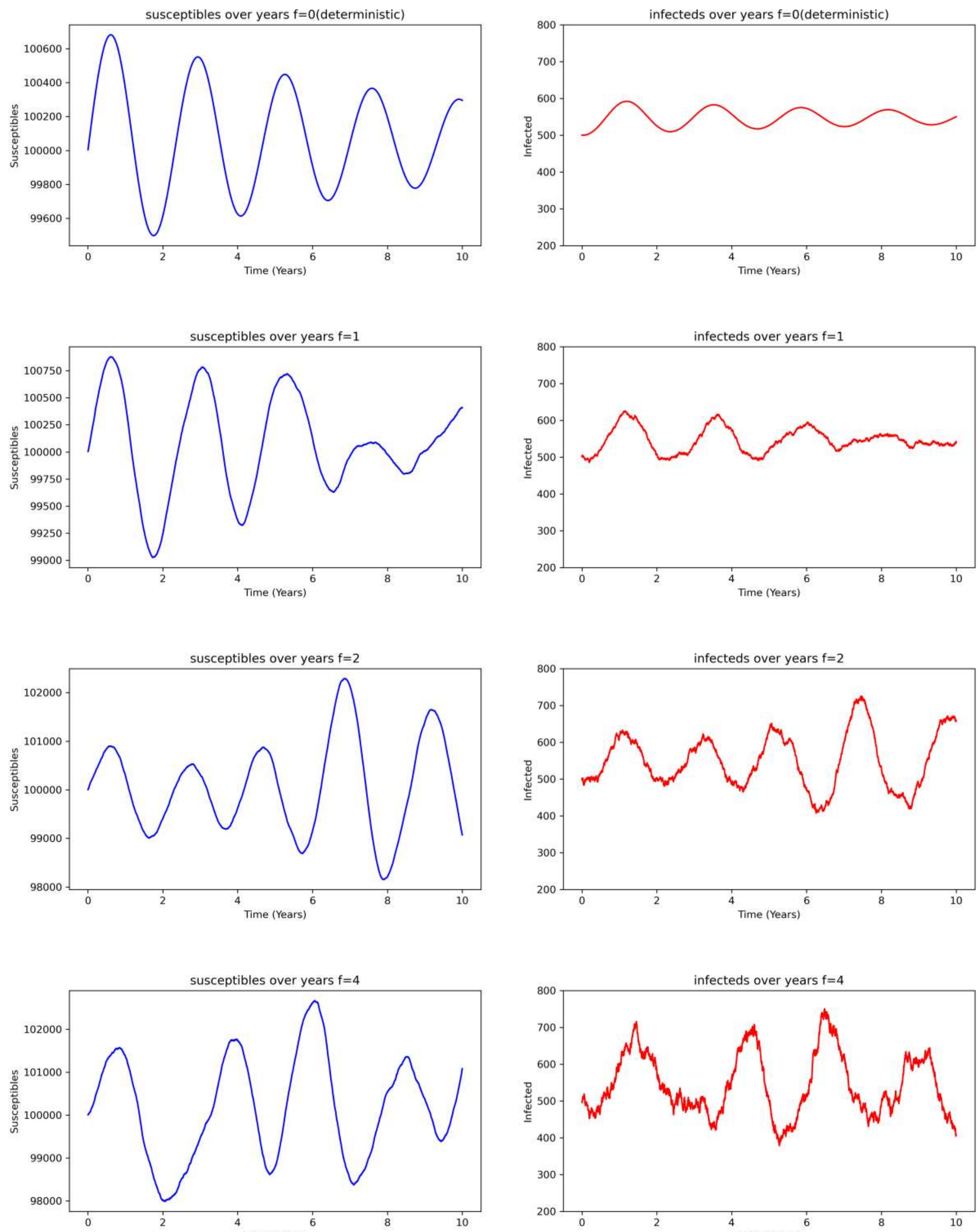
Now we solve for these initial condition:

$$(\mu = \nu = 0.02\, per\ year, R0 = 10, \frac{1}{\gamma} = 10\, days, N = 10^6 \ , X(0) = 10^5 \,, Y(0) = 500)$$

And

$$f = \{1\ ,2\ ,4\}$$

These are the results:



When we look at these diagrams, first of all we notice that the amplitude becomes bigger and bigger we have greater values of constant noise. And also we don't have any constant character for changes of susceptible diagrams over the time. When noise is small, behavior of infected over the time is near to natural frequency or deterministic SIR model. But when we increase f, infected fraction behavior during the years becomes noisy and there is no noise in the deterministic model so it's smooth.

Question 3:

I'll give a summary about the method and solve this for the initial conditions introduced above.

- Births occur at rate μN. Result: $X \rightarrow X + 1$.
- Transmission occurs at rate $\beta \frac{XY}{N}$. Result: $Y \rightarrow Y + 1$ and $X \rightarrow X - 1$.
- Recovery occurs at rate γY. Result: $Z \rightarrow Z + 1$ and $Y \rightarrow Y - 1$.
- Deaths of X, Y, or Z (three independent events) occur at rate μX, μY, and μZ. Result: $X \rightarrow X - 1$, $Y \rightarrow Y - 1$ or $Z \rightarrow Z - 1$.

Algorithm

1. Label all possible events $E1, \dots, En$.

2. For each event determine the rate at which it occurs, $R1, \dots, Rn$.

3. The rate at which any event occurs is $R_{total} = \sum_{m=1}^{n} R_m$.

4. The time until the next event is $\delta t = \frac{-1}{Rtotal} \; log(RAND1)$.

5. Generate a new random number, $RAND2$. Set $P = RAND2 \times Rtotal$.

6. Event p occurs if $\sum_{m=1}^{p-1} R_m < P \leq \sum_{m=1}^{p} R_m$

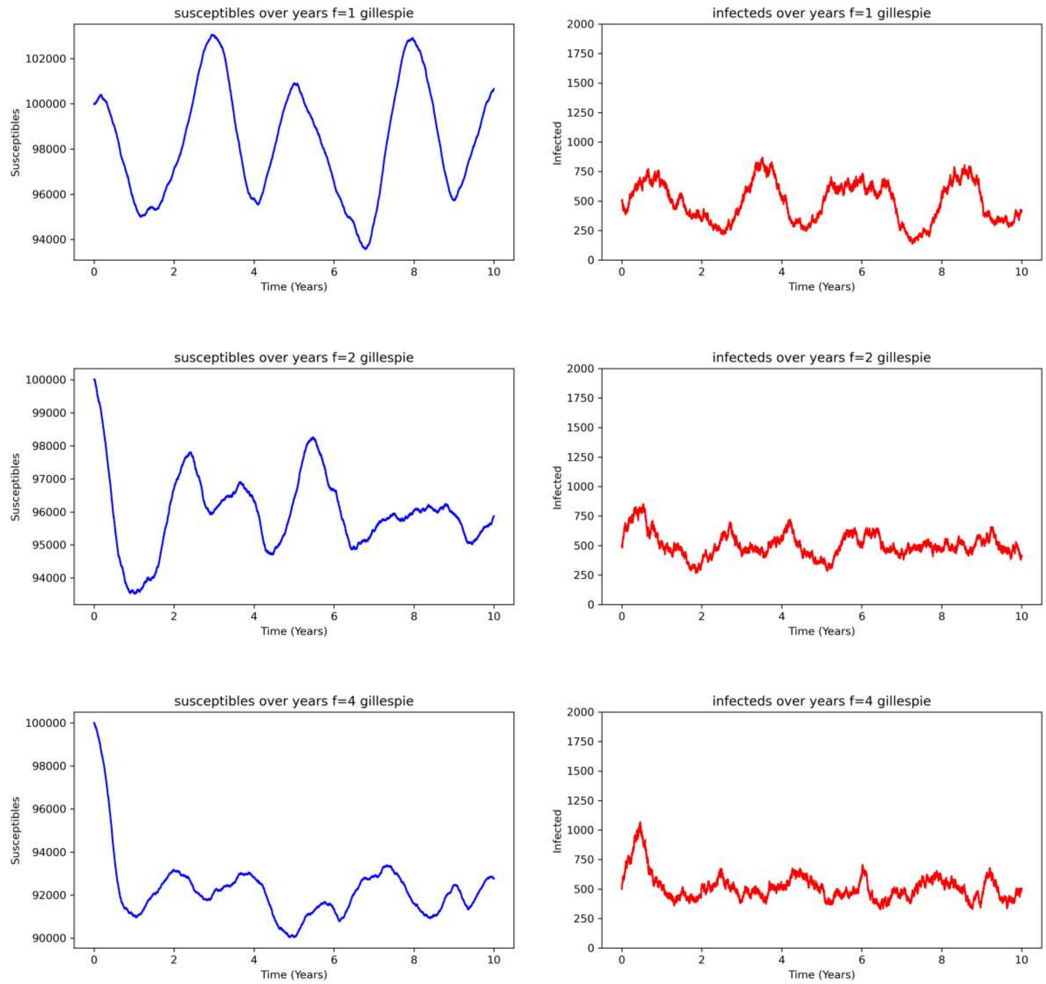
7. The time is now updated, $t \rightarrow t + \delta t$, and event p is performed.

8. Return to Step 2.

Initial conditions:

$$(\mu = \nu = 0.02 \; per \; year, R0 = 10, \frac{1}{\gamma} = 10 \; days, N = 10^6 \; , X(0) = 10^5 \, , Y(0) = 500)$$

Results



Differences:

I. Gillespie method is nosier that stochastic ODE s.

II. When $f = 1$ two methods are nearly similar to each other.

III. When we increase f in Gillespie method the amplitude decreases but in other method is different.

Question4:

First I'll solve this method for simple SIR then I add birth and death rates to the equations.

Simple SIR

I. $\frac{dX}{dt} = -\beta \frac{Y}{N} X$

II. $\frac{dY}{dt} = \beta \frac{Y}{N} X - \gamma Y$

III. $\frac{dZ}{dt} = \gamma Y$

So the master equation will be as below:

$$\frac{dp}{dt} = \left[\beta \frac{(Y-1)}{N}(X+1)\right] p_{X+1,Y-1} + [\gamma(Y+1)]p_{X,Y+1} - \left[\beta \frac{Y}{N} X + \gamma Y\right] p_{X,Y,N}$$

If we modify the equations with birth and death rates we have:

SIR with birth and death

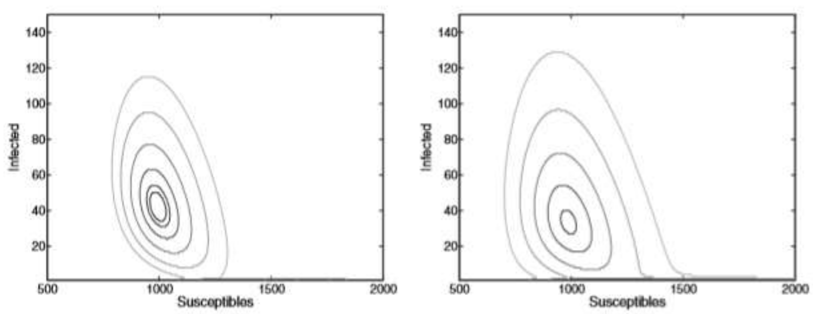
I. $\frac{dX}{dt} = \vartheta N - \beta \frac{Y}{N} X - mX$

II. $\frac{dY}{dt} = \beta \frac{Y}{N} X - \gamma Y - mY$

III. $\frac{dZ}{dt} = \gamma Y - mZ$

$$\begin{aligned}\frac{dp_{X,Y,N}}{dt} = &\left[\beta \frac{(Y-1)}{N}(X+1)\right] p_{X+1,Y-1,N} + [\gamma(Y+1)]p_{X,Y+1,N} + [\vartheta(N-1)]p_{X-1,Y,N-1} \\ &+ [m(X+1)]p_{X+1,Y,N+1} + [m(Y+1)]p_{X,Y+1,N+1} + [m(N+1-X-Y)]p_{X,Y,N+1} \\ &- [\beta \frac{Y}{N} X + \gamma Y + \vartheta N + mX + mY + m(N+1-X-Y)]p_{X,Y,N}\end{aligned}$$

Schematic view is hard to sketch so I use book diagrams.



Question5:

We have three species: plants, animal and humans. I'll write SIS equations for each group and I'll compare them.

Animals

I. $\frac{dX_i}{dt} = \vartheta_i - \beta_i X_i Y_i - \mu_i X_i + \sum_j m_{ij} X_j - \sum_j m_{ji} X_i + \gamma_i Y_i$

II. $\frac{dY_i}{dt} = \beta_i X_i Y_i - \gamma_i Y_i - \mu_i Y_i + \sum_j m_{ij} Y_j - \sum_j m_{ji} Y_i$

Humans

I. $\frac{dX_{ii}}{dt} = v_{ii} - \beta_i X_{ii} \frac{\sum_j Y_{ij}}{\sum_j N_{ij}} - \sum_j l_{ji} X_{ii} + \sum_j r_{ji} X_{ji} - \mu_{ii} X_{ii} + \gamma Y_{ii}$

II. $\frac{dX_{ij}}{dt} = v_{ij} - \beta_i X_{ij} \frac{\sum_j Y_{ij}}{\sum_j N_{ij}} + l_{ij} X_{jj} - r_{ij} X_{ij} - \mu_{ij} X_{ij} + \gamma Y_{ij}$

III. $\frac{dY_{ii}}{dt} = \beta_i X_{ii} \frac{\sum_j Y_{ij}}{\sum_j N_{ij}} - \sum_j l_{ji} Y_{ii} + \sum_j r_{ji} Y_{ji} - \mu_{ii} Y_{ii} - \gamma Y_{ii}$

IV. $\frac{dY_{ij}}{dt} = \beta_i X_{ij} \frac{\sum_j Y_{ij}}{\sum_j N_{ij}} + l_{ij} Y_{jj} - r_{ij} Y_{ij} - \mu_{ii} Y_{ii} - \gamma Y_{ij}$

V. $\frac{dN_{ii}}{dt} = v_{ii} - \sum_j l_{ji} N_{ii} + \sum_j r_{ji} N_{ji} - \mu_{ii} N_{ii}$

VI. $\frac{dN_{ij}}{dt} = v_{ij} + l_{ij} N_{jj} - r_{ij} N_{ij} - \mu_{ii} N_{ii}$

Plants

I. $\frac{dX_i}{dt} = \vartheta_i N_i - \lambda_i X_i - \mu_i X_i + \gamma_i Y_i$

II. $\frac{dY_i}{dt} = \lambda_i X_i - \gamma_i Y_i - \mu_i Y_i$

III. $\lambda_i = \beta_i \sum_j \rho_{ij} \frac{Y_i}{\sum_k N_k}$

Comparison

The most obvious defining feature of plants (from an epidemiological perspective) that separates them from other hosts is that they do not move. This means that any spatial transmission must be wind- or vector-borne. But in the animals and humans we have transmission and immigration. These three 3 Equation systems have death rates and birth rates but in humans birth is a little different. We assume births in other populations too.