Modeling phenomena based on Epidemic diseases

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Multi-pathogen, Multi-host

In this homework we are going to introduce two models for more realization. In one hand, sometimes multiple pathogens attack host's body. But dependent to immunity caused by them, 4 scenarios may happen.

- Complete cross-immunity
- No cross-immunity
- Enhanced susceptibility
- Partial cross-immunity

On the other hand, a disease can attack multiple hosts and act differently by and within them. This is also an important model for cattle's diseases and Malaria.

Wild and resistant pathogens

Assume there are two types of a disease: Wild and resistant. As the names tell us that Wild one has biggerR_0. So:

$$R_0{}^W > R_0{}^r \qquad\qquad \beta_W > \beta_r$$

Notice that the wild type can be controlled by anti-biotic but the resistant cant. These are the differential equations in mean field view:

I. $\frac{dS}{dt} = \vartheta - S(\beta_W I_W + \beta_r \beta_r) - \mu S$

II. $\frac{dI_W}{dt} = \beta_W I_W S - (\gamma + T) I_W - \mu I_W$

III. $\frac{dI_r}{dt} = \beta_r \beta_r S - (\gamma) I_r - \mu I_r$

Important points: The critical point is when$R_0{}^W = R_0{}^r$.So we rewrite:

$$R_0{}^W = \frac{\beta_W}{\gamma + T + \mu} = \frac{\beta_r}{\gamma + \mu} = R_0{}^r$$

Then:

$$\beta_W(\gamma + \mu) = \beta_r(\gamma + T + \mu)$$

Treatment critical value is:

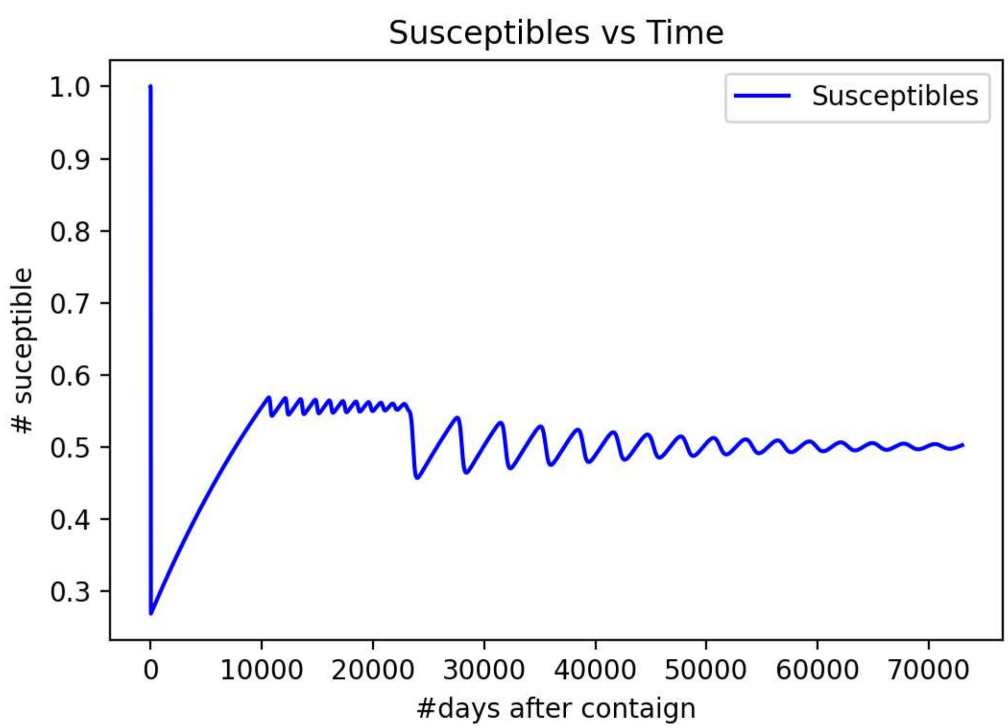
$$T = \gamma(\frac{\beta_W}{\beta_r} - 1)$$

Initial condition

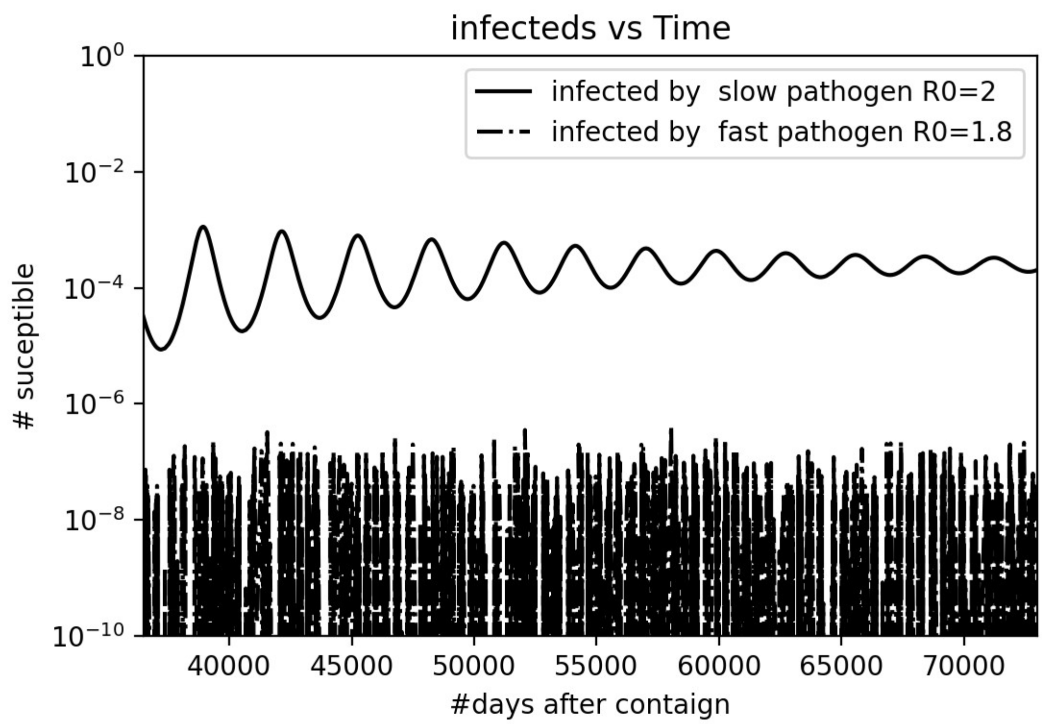
$$\nu = \mu = 5 \times 10^{-5}$$

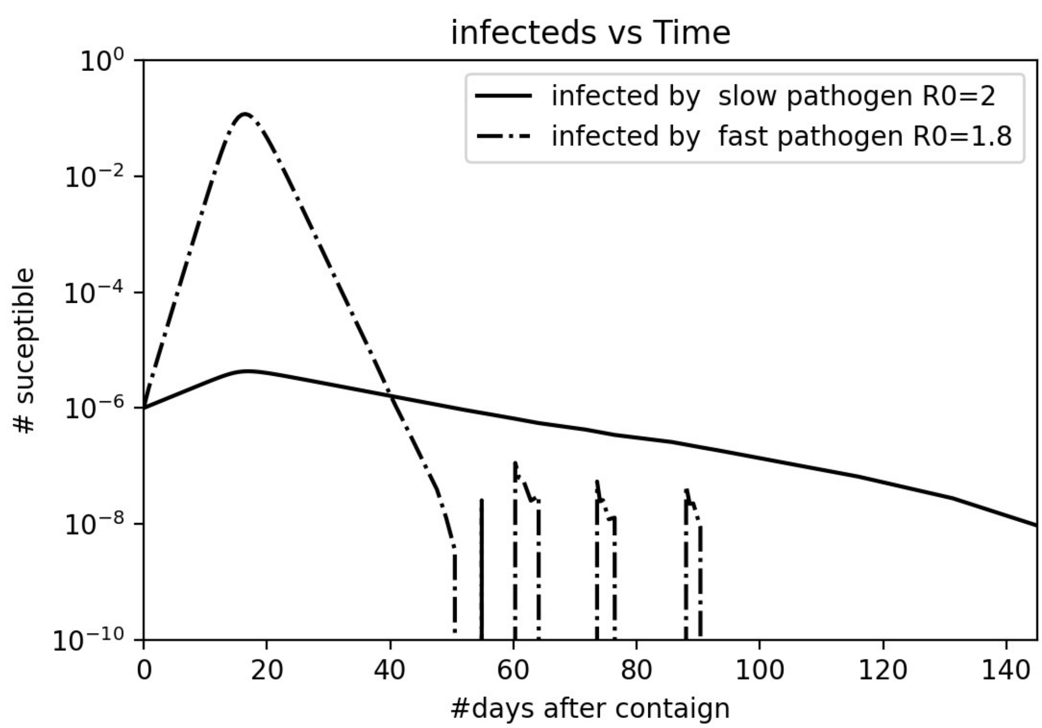
$$\beta_r = 0.2 \qquad \gamma_r = 0.1 \qquad R_0{}^r \approx 2 \qquad m_r = 0$$

$$\beta_w = 1.8 \qquad \gamma_w = 1 \qquad R_0{}^r \approx 1.8 \qquad m_w = 0.$$



I wanted to have book's graph too:



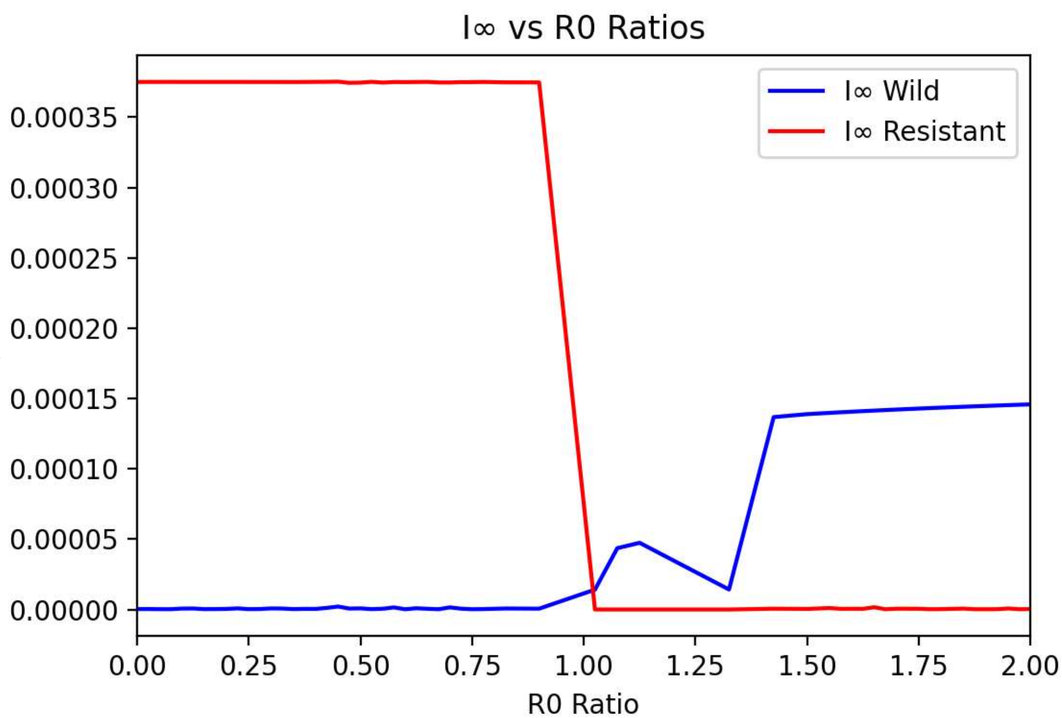


For the Initial condition(simulation time=300 years):

$$\nu = \mu = 5 \times 10^{-5}$$

$$\beta_r = 0.4 \qquad \gamma_r = 0.1 \qquad m_r = 0$$

$$\beta_w = i \qquad \gamma_w = 0.3 \qquad m_w = 0.$$



The graph tells us that if the ratio of $\frac{{R_0}^w}{{R_0}^r}$ is less than 1, the resistant will remain in society and if the ratio is greater than 1, the wild one will survive.

Note:

I don't think that this question is actually good enough to answer. If we want to understand which of these two types will win the game-if game is to better infect the group- , we should redefine the question. The Book tells us that the Wild type is one who has the moreβ. So if we change β from 0 zero to x for changingR_0 ratio, we've broken the law. So what we should do? If we want to find the winner we should give two diseases basic info for ***Complete Cross-immunity***.

Disease in Goat and Sheep societies

Assume letter G for goats and letter S for sheeps. These are the equations:

I. $\frac{dS_G}{dt} = \vartheta_G - \beta_{GG} S_G I_G - \beta_{GS} S_G I_S - \mu S_G$

II. $\frac{dS_S}{dt} = \vartheta_S - \beta_{SS} S_S I_S - \beta_{SG} S_S I_G - \mu S_S$

III. $\frac{dI_G}{dt} = \beta_{GG} S_G I_G + \beta_{GS} S_G I_S - \mu I_G - m_G I_G - \gamma_G I_G$

IV. $\frac{dI_S}{dt} = \beta_{SS} S_S I_S + \beta_{SG} S_S I_G - \mu I_S - m_G I_S - \gamma_S I_S$

V. $\frac{dR_G}{dt} = \gamma_G I_G - \mu I_G$

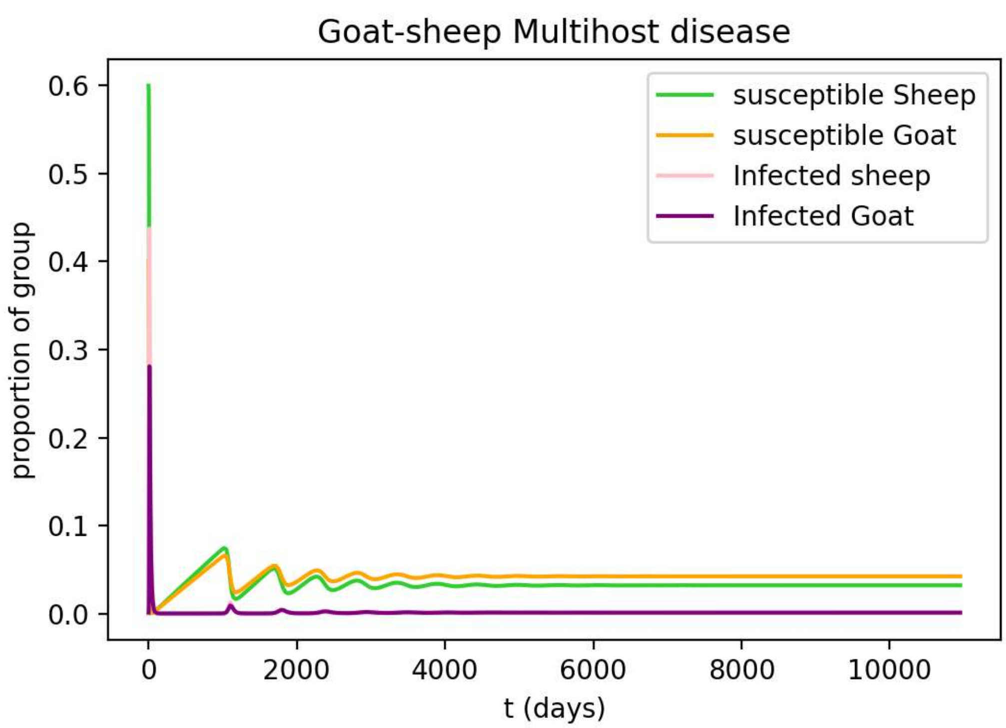
VI. $\frac{dR_S}{dt} = \gamma_S I_S - \mu I_S$

Initial conditions are:

$$\beta = \begin{pmatrix} 0.5 & 2 \\ 1 & 0.6 \end{pmatrix} \qquad n = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix} \qquad I = \begin{pmatrix} 0 \\ 0.001 \end{pmatrix} \qquad \vartheta = \begin{pmatrix} 8 \times 10^{-5} \\ 7 \times 10^{-5} \end{pmatrix} \qquad \mu = 5 \times 10^{-5}$$

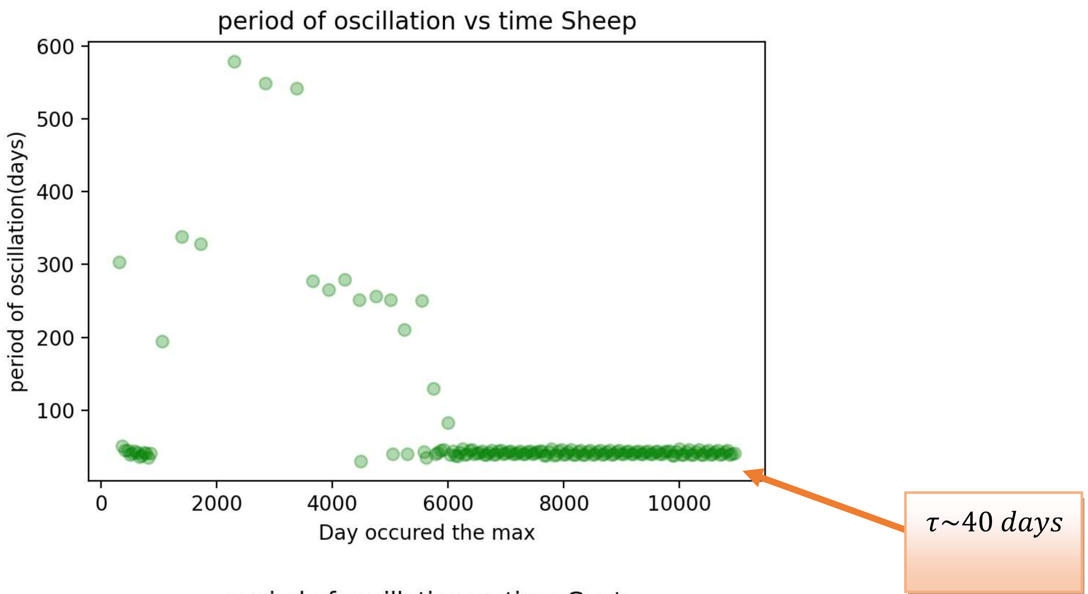
$$\gamma = 1 \qquad m = \begin{pmatrix} 2.3 \times 10^{-5} \\ 2 \times 10^{-5} \end{pmatrix} \qquad \tau = 30 years = 30 \times 365\ days$$

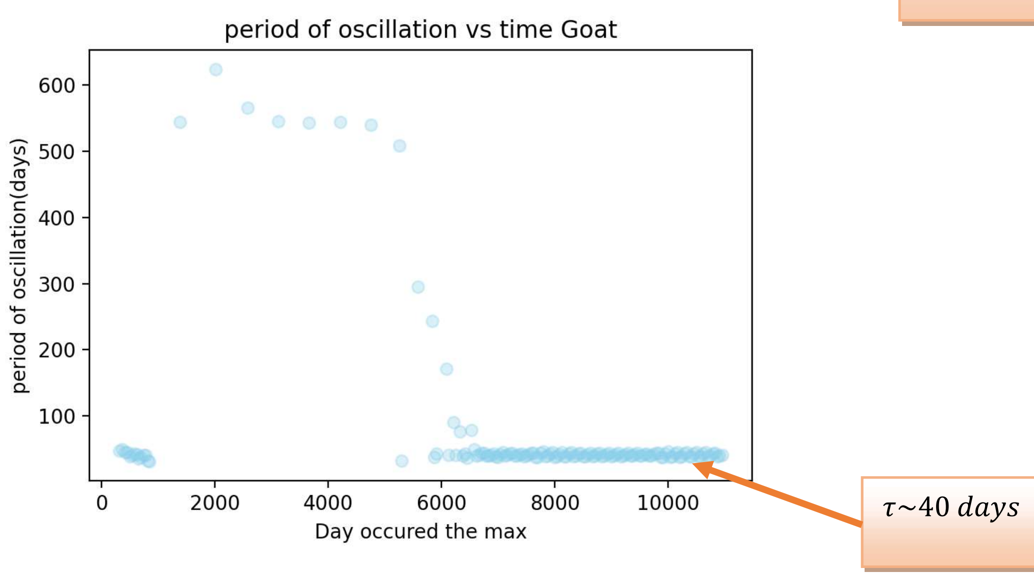
I will draw the graph I_G and I_r during this time:



Note: First of all if we use $\gamma = 1$, we don't have any oscillations. So γshould be less than 1(you can try the code).I use $\gamma = \frac{2}{30}$ .Because the code is really dependent to γ-remember results of chapter2-.We see oscillations such as over-damped oscillator in physics.

Period of oscillation: I used function "*argrelextrema*" from library *"scipy"*. It gives us all the maximums and minimums. Then I get the time between all the Max and then plot the period of oscillation over the time.





We see that after long time passed from start of epidemic, period of oscillation goes to special number! But pay attention that same as damped oscillation the period of oscillation varies by the time. Actually I get average of these last periods.