

# **Bayesian approach to statistics**



# Different approaches to statistics

Frequentist



Bayesian



# Uncertainty interpretation

Frequentist



Bayesian

Subjective

Objective



# Data and parameters

Frequentist



$\theta$  is random  
 $X$  is fixed

$\theta$  is fixed  
 $X$  is random

Bayesian



# Data and parameters

Frequentist



For any  $|X|$

$$|X| \gg |\theta|$$

Bayesian



# Training

Frequentist



Bayesian



Maximum Likelihood:  
 $\hat{\theta} = \arg \max_{\theta} P(X|\theta)$



# Training

Frequentist



Bayes theorem:

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Bayesian



# Classification

Training:

$$P(\theta|X_{\text{tr}}, y_{\text{tr}}) = \frac{P(y_{\text{tr}}|X_{\text{tr}}, \theta)P(\theta)}{P(y_{\text{tr}}|X_{\text{tr}})}$$

Prediction:


$$P(y_{\text{ts}}|X_{\text{ts}}, X_{\text{tr}}, y_{\text{tr}}) = \int P(y_{\text{ts}}|X_{\text{ts}}, \theta)P(\theta|X_{\text{tr}}, y_{\text{tr}})d\theta$$





# Regularization

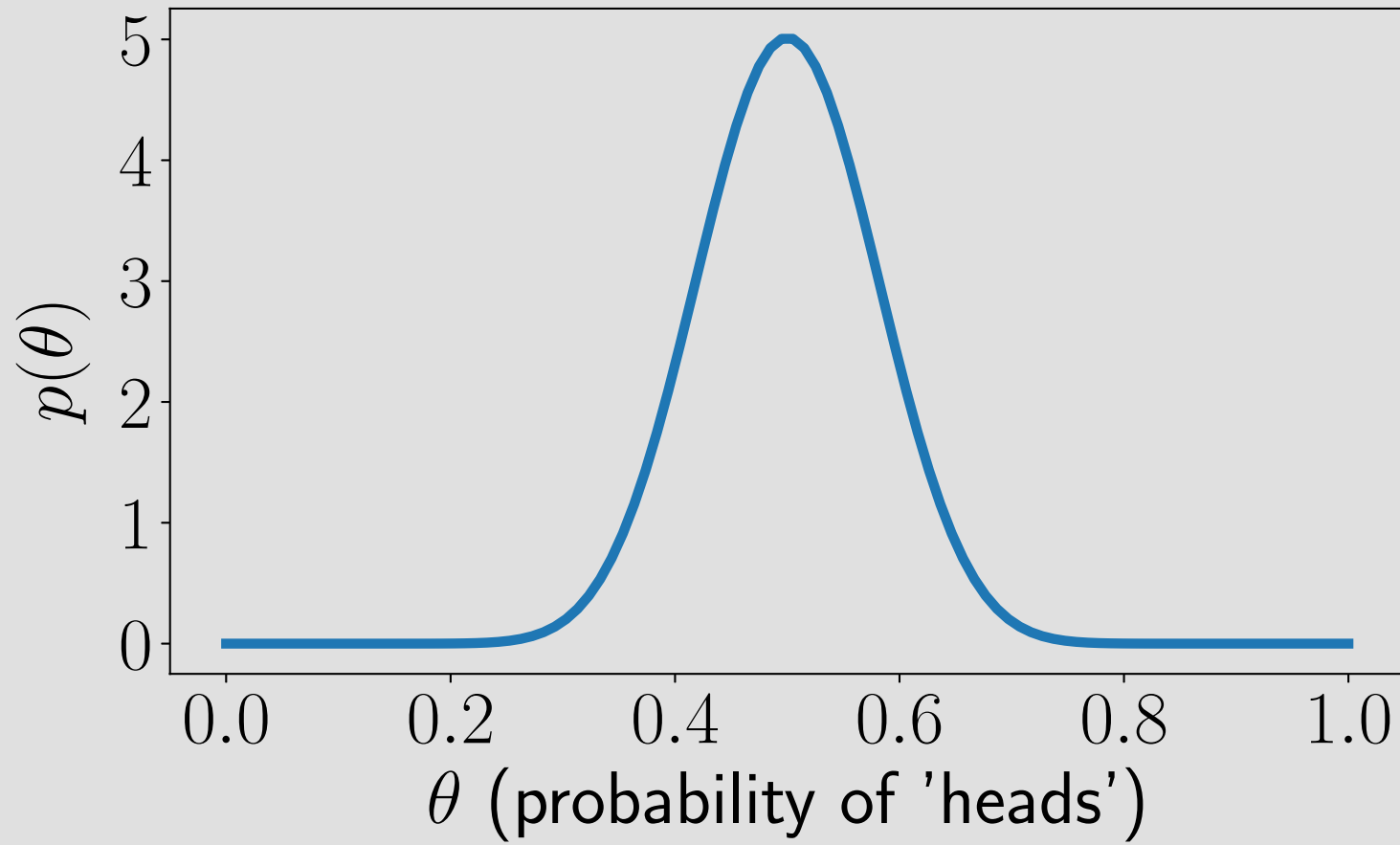
Regularizer



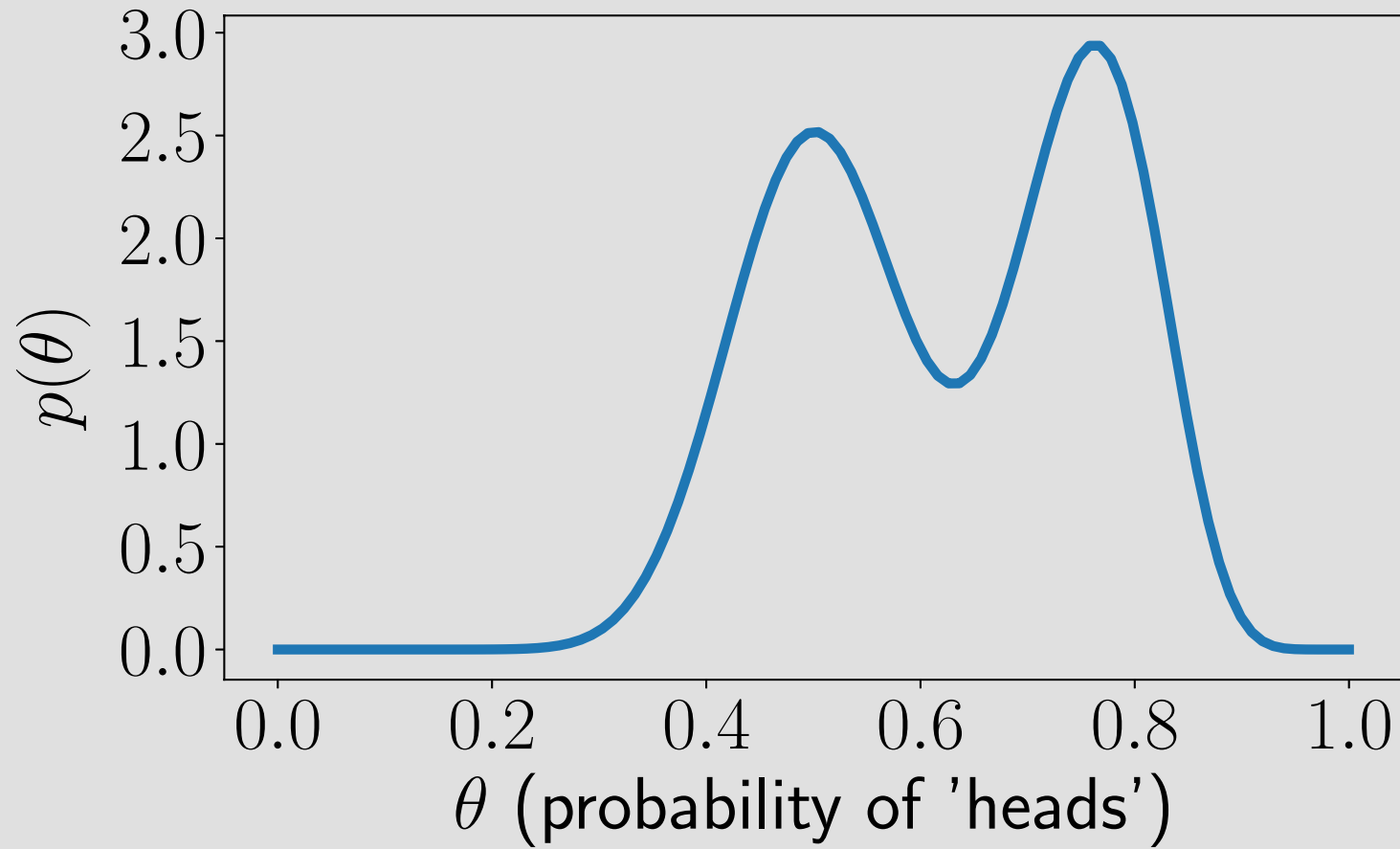
$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$



# Regularization



# Regularization



# On-line learning

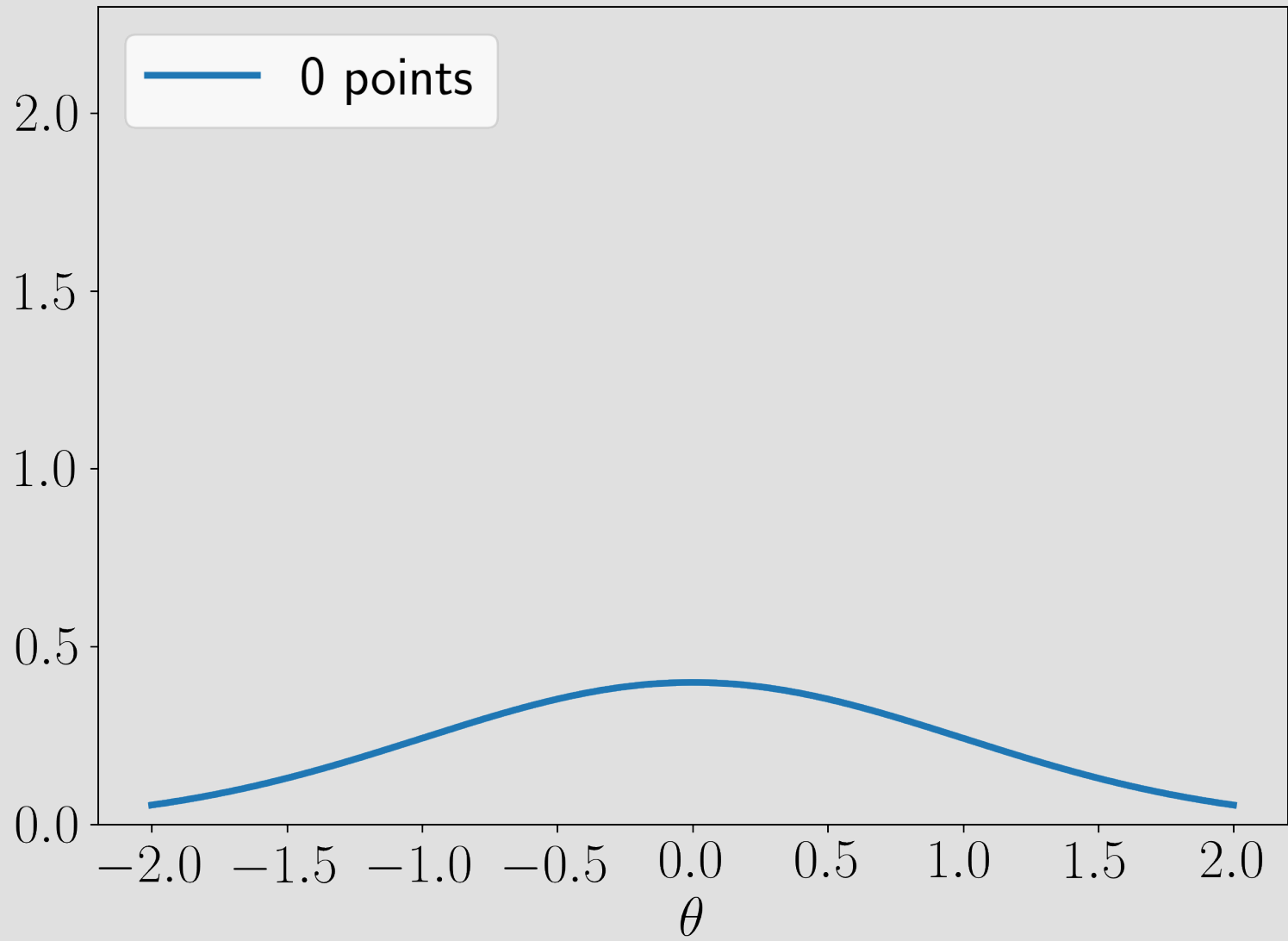
New prior      Likelihood      Prior

$$P_k(\theta) = P(\theta|x_k) = \frac{P(x_k|\theta)P_{k-1}(\theta)}{P(x_k)}$$

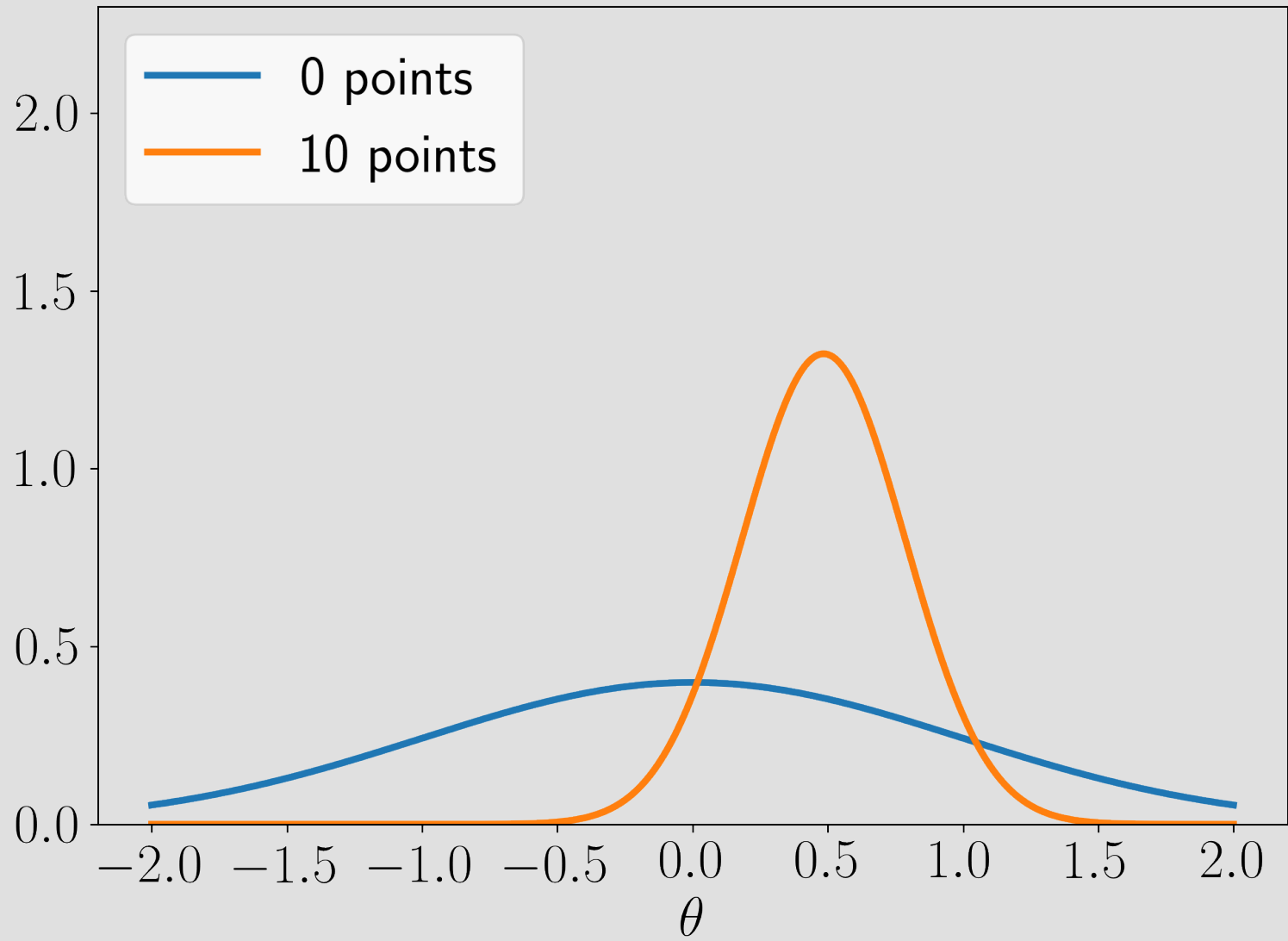
Posterior



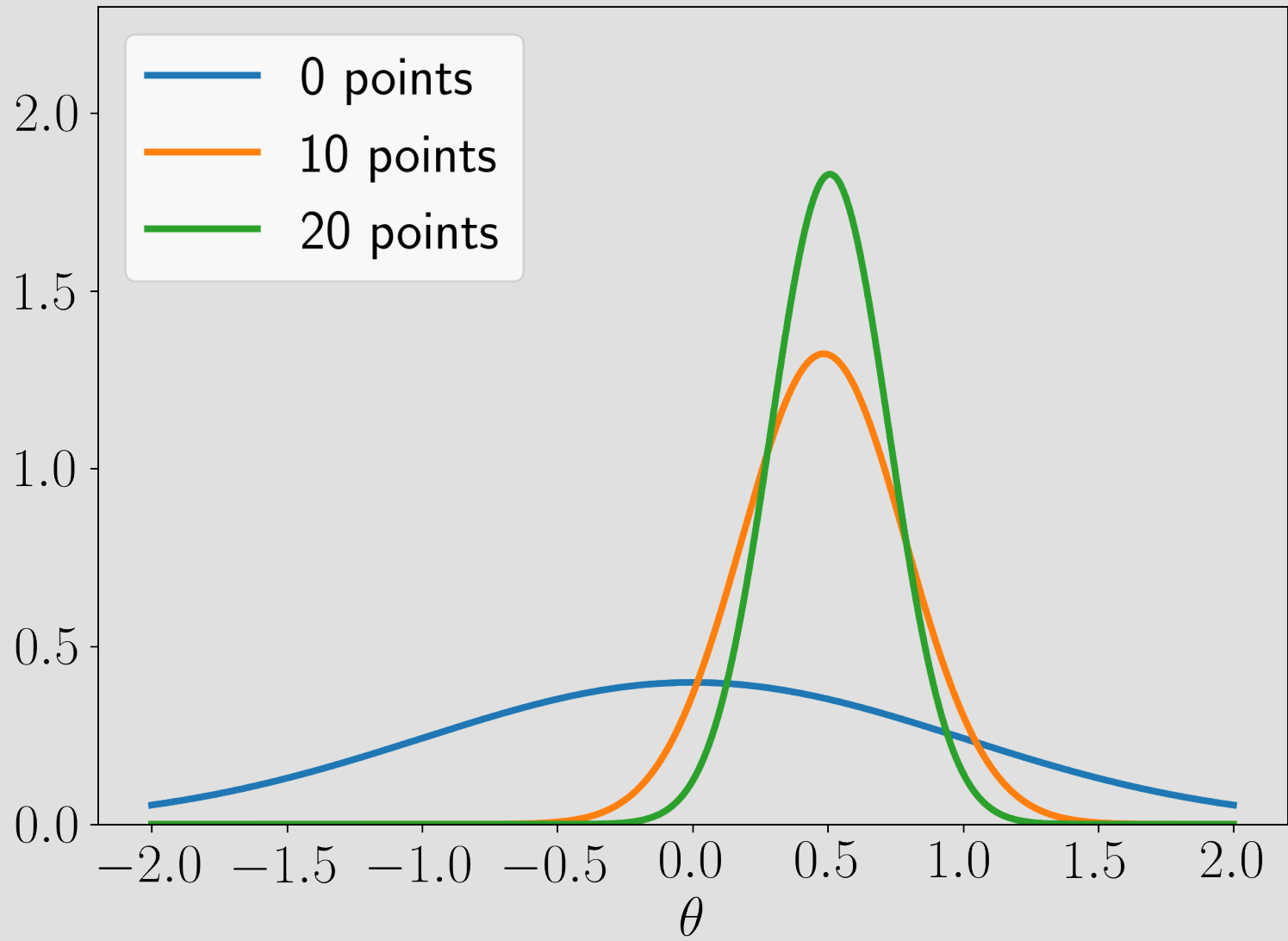
# On-line learning



# On-line learning



# On-line learning



# On-line learning

