

Expectation Maximization

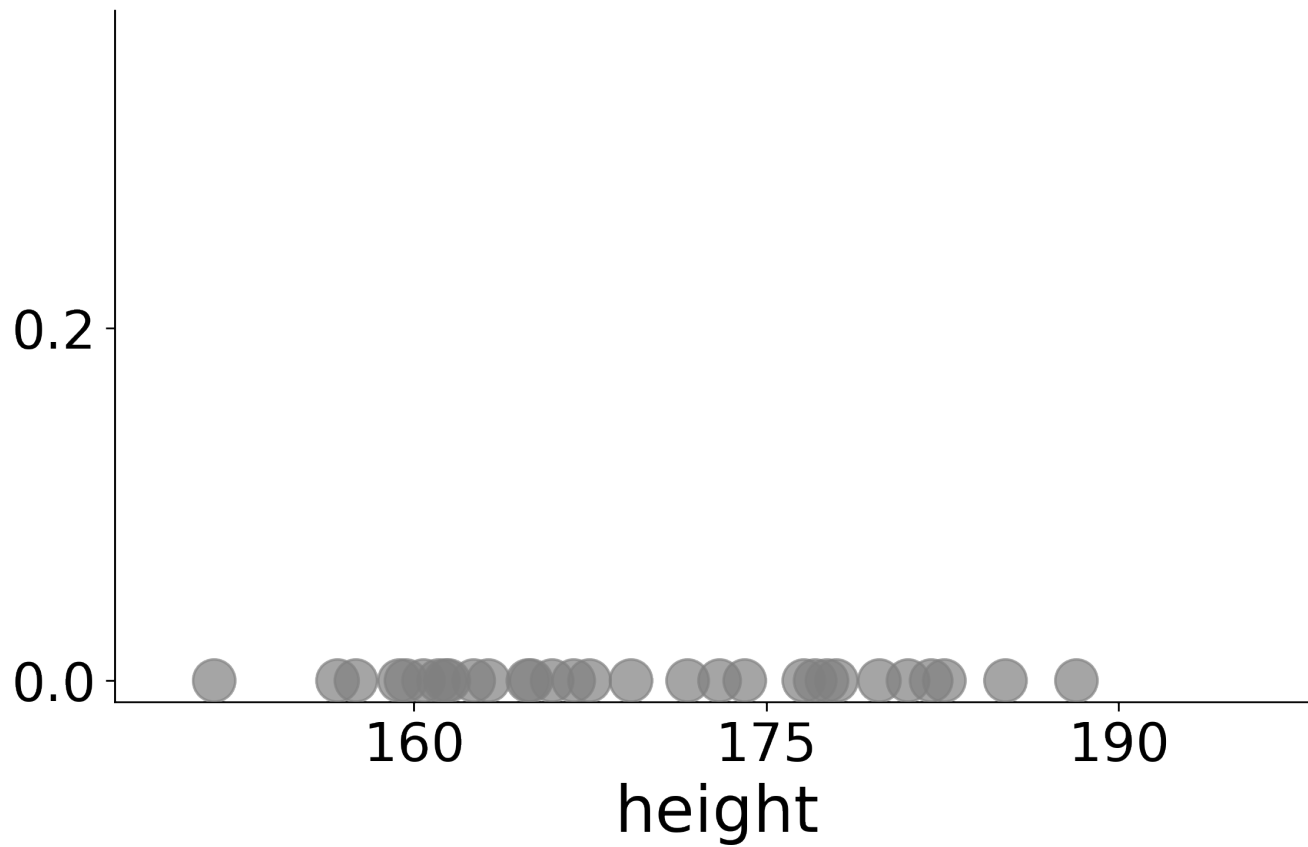
Chicken and egg problem

- Need Gaussian parameters to estimate sources
- Need sources to estimate Gaussian parameters

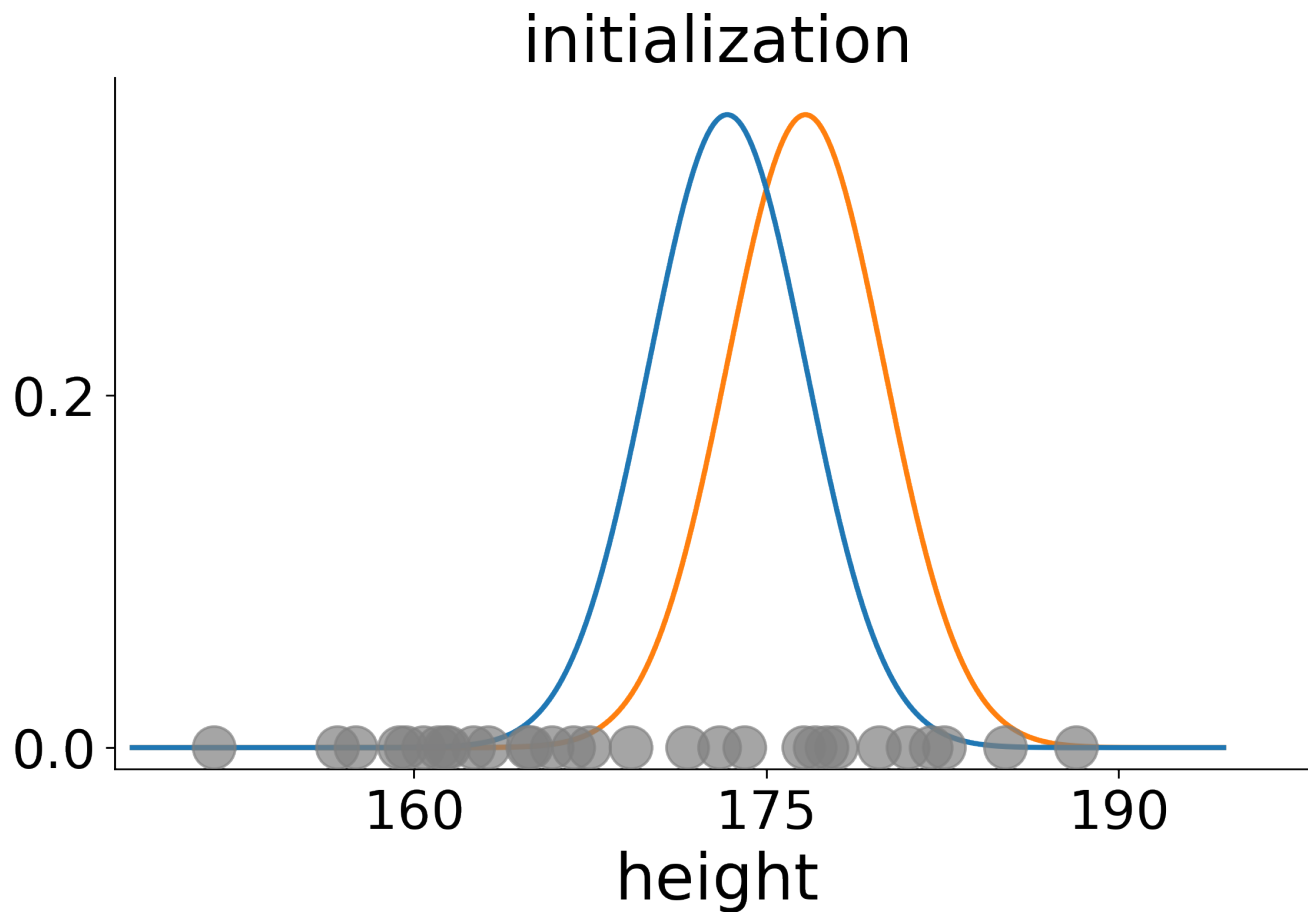
EM algorithm

1. Start with 2 randomly placed Gaussians parameters θ
2. Until convergence repeat:
 - a) For each point compute $p(t = c \mid x_i, \theta)$: does x_i look like it came from cluster c ?
 - b) Update Gaussian parameters θ to fit points assigned to them

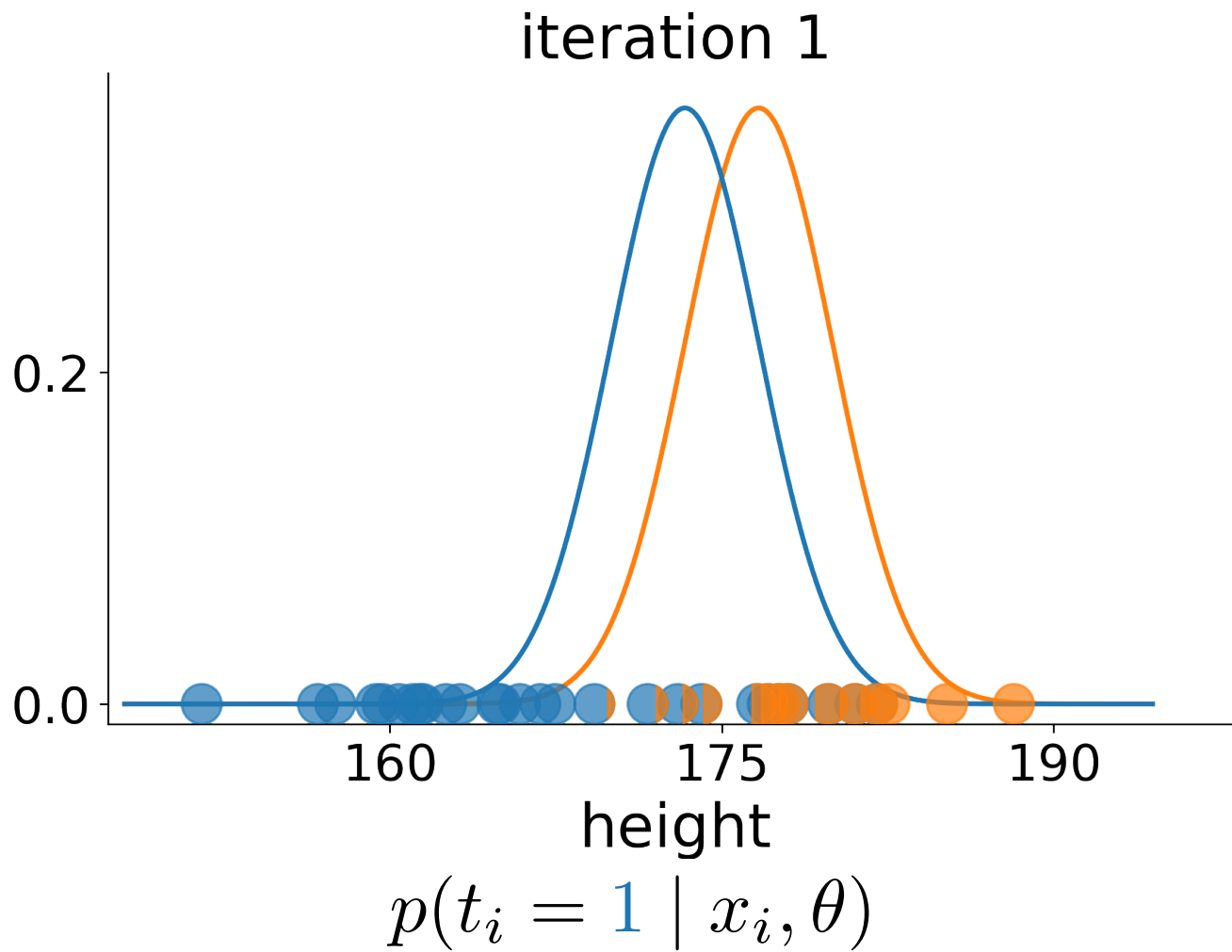
GMM EM example



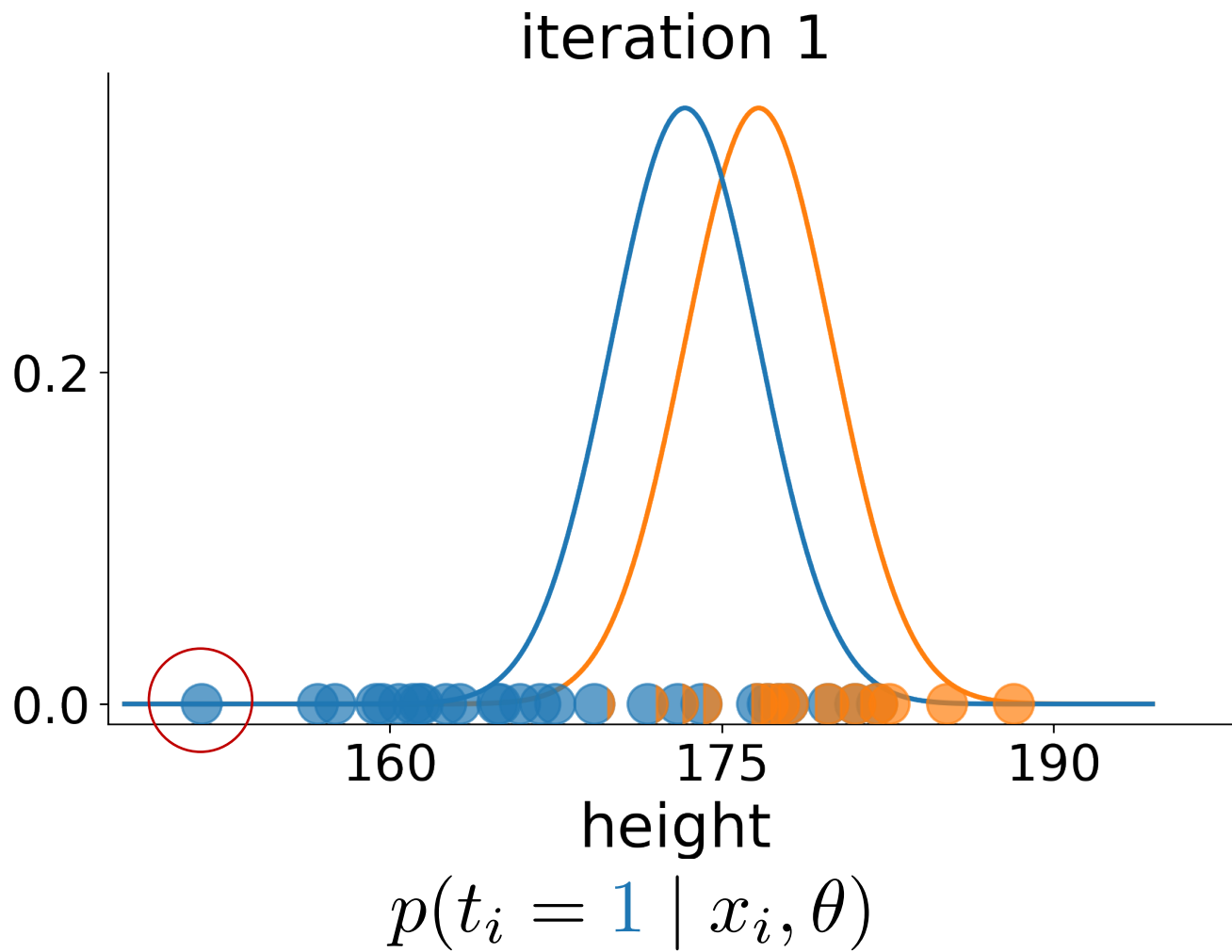
GMM EM example



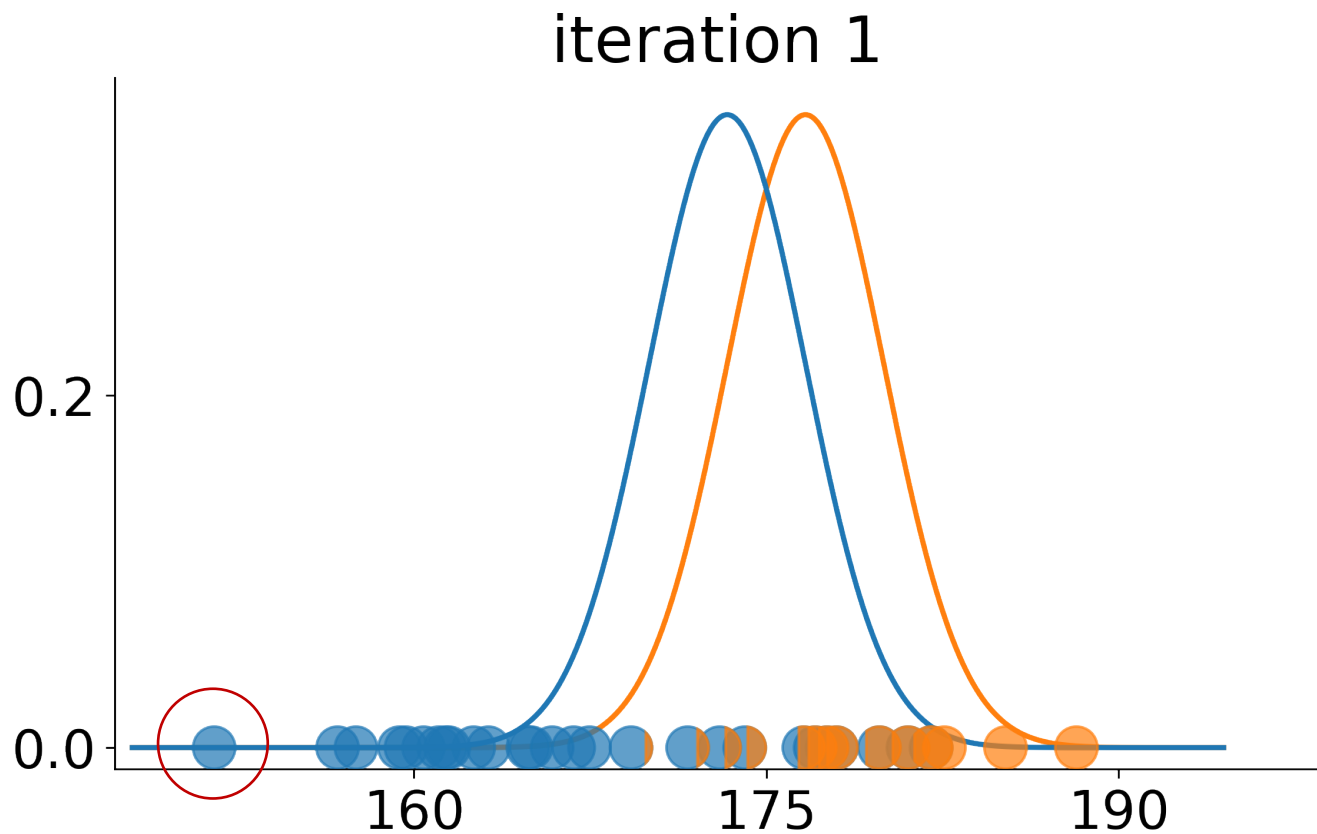
GMM EM example



GMM EM example

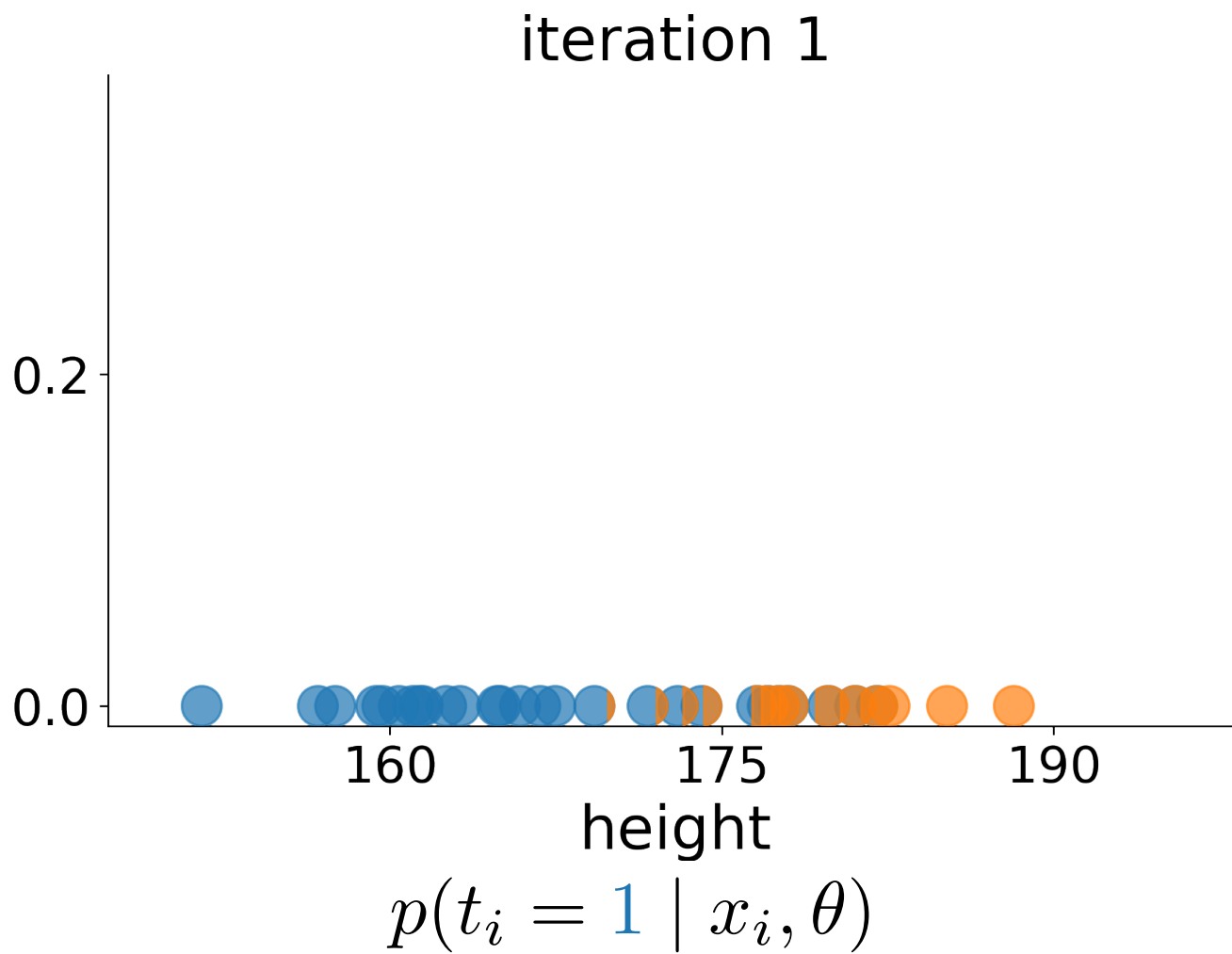


GMM EM example

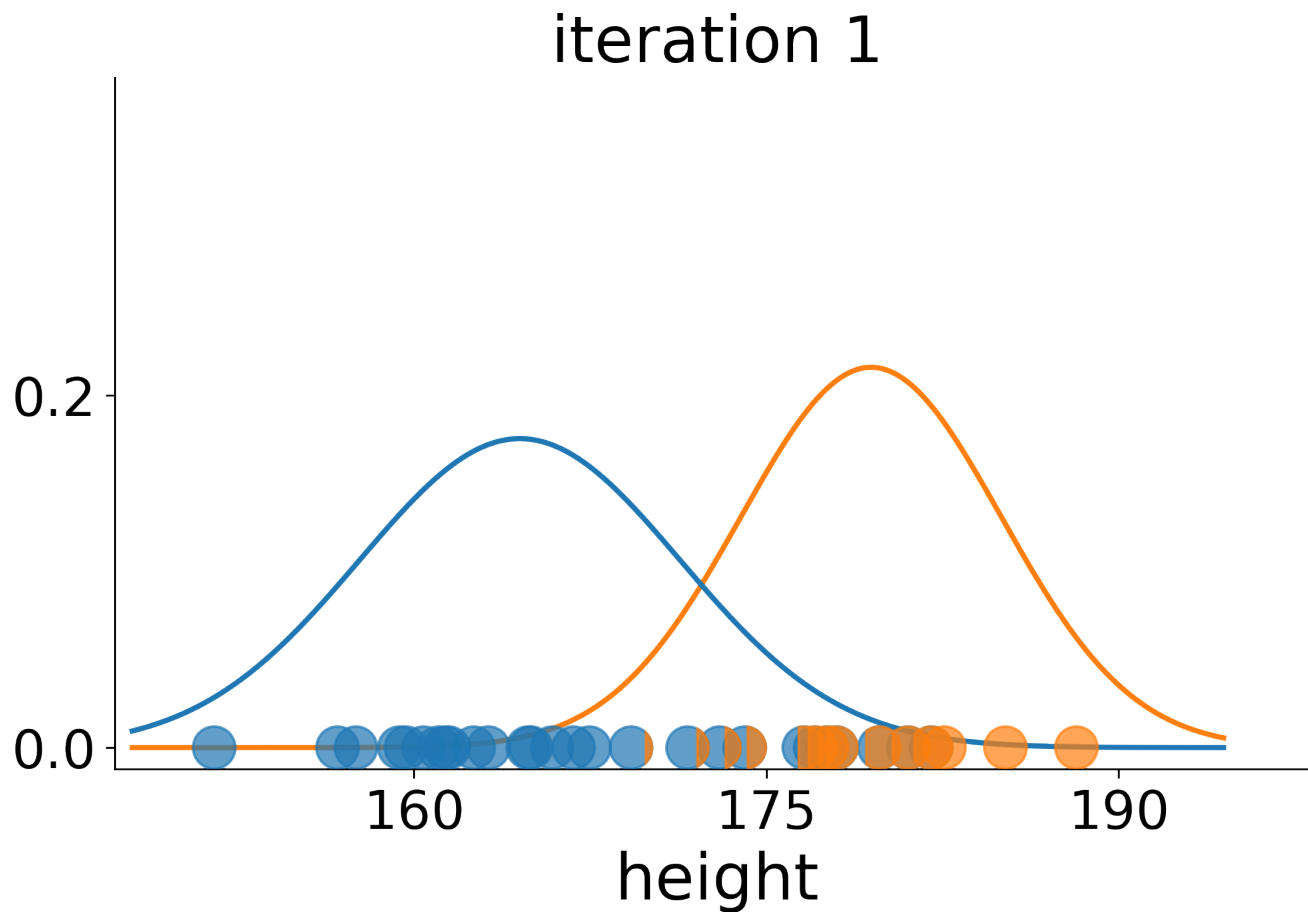


$$p(t_i = 1 \mid x_i, \theta) \approx \frac{0.01}{0.01 + 0.00002} \approx 1$$

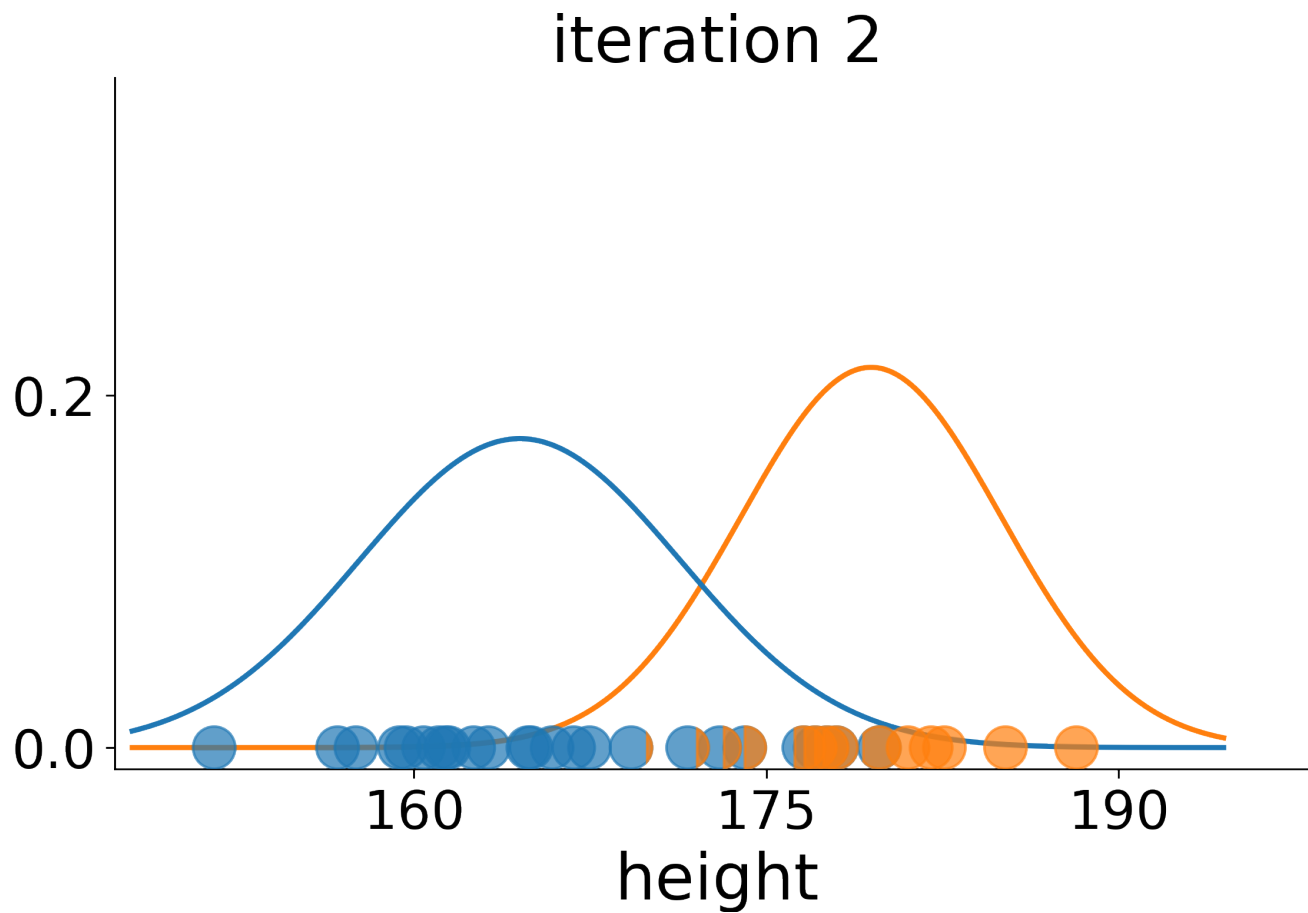
GMM EM example



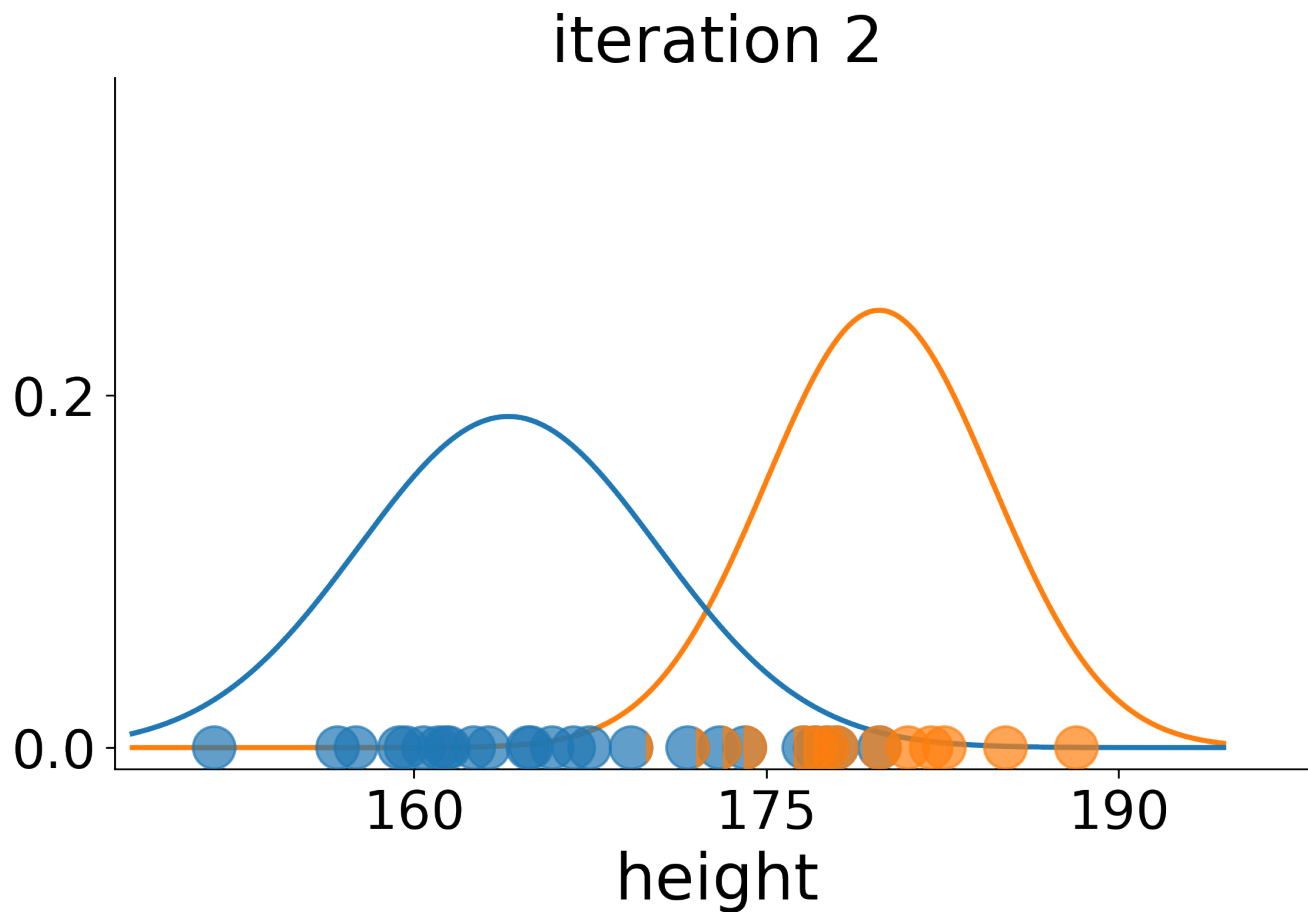
GMM EM example



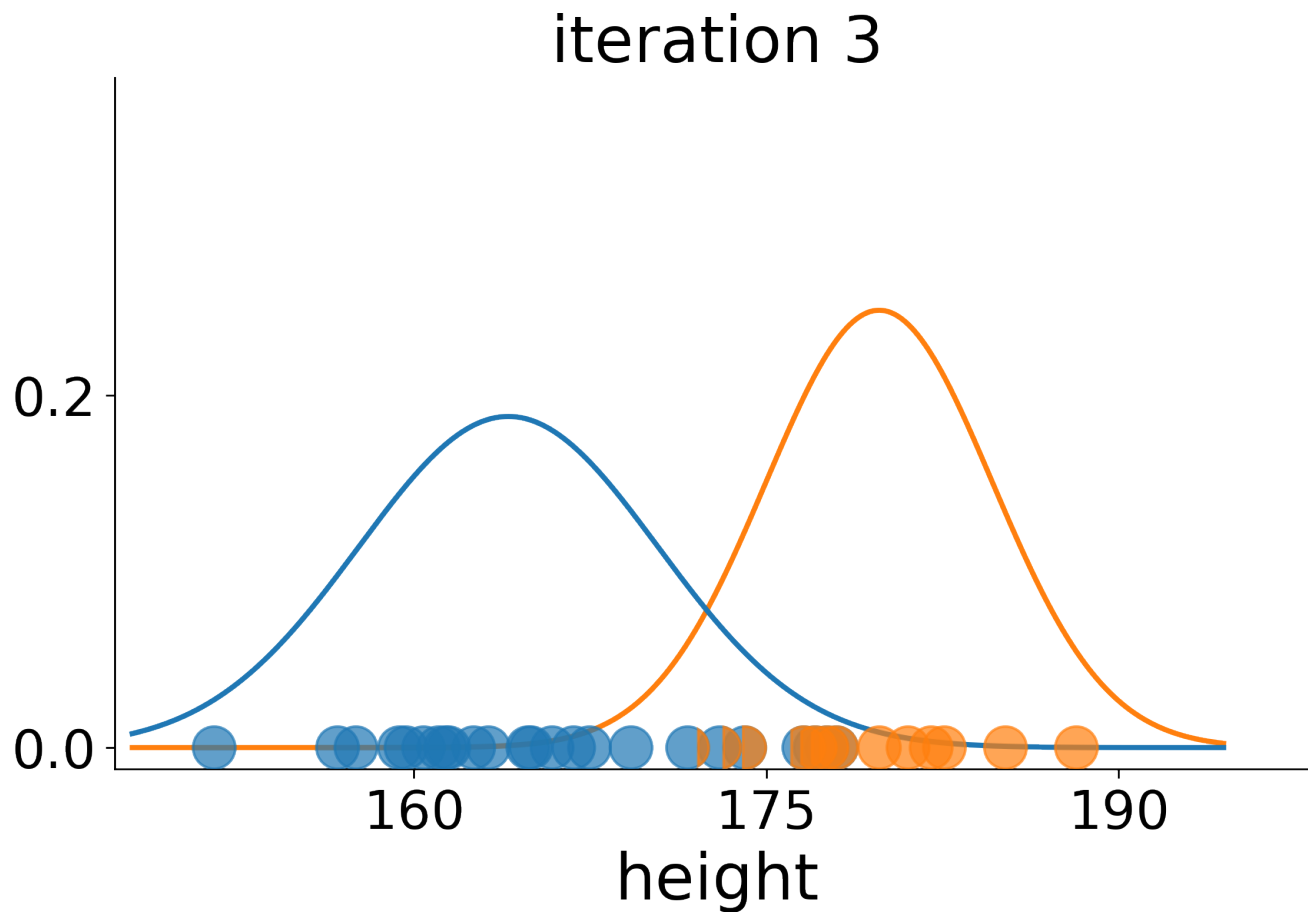
GMM EM example



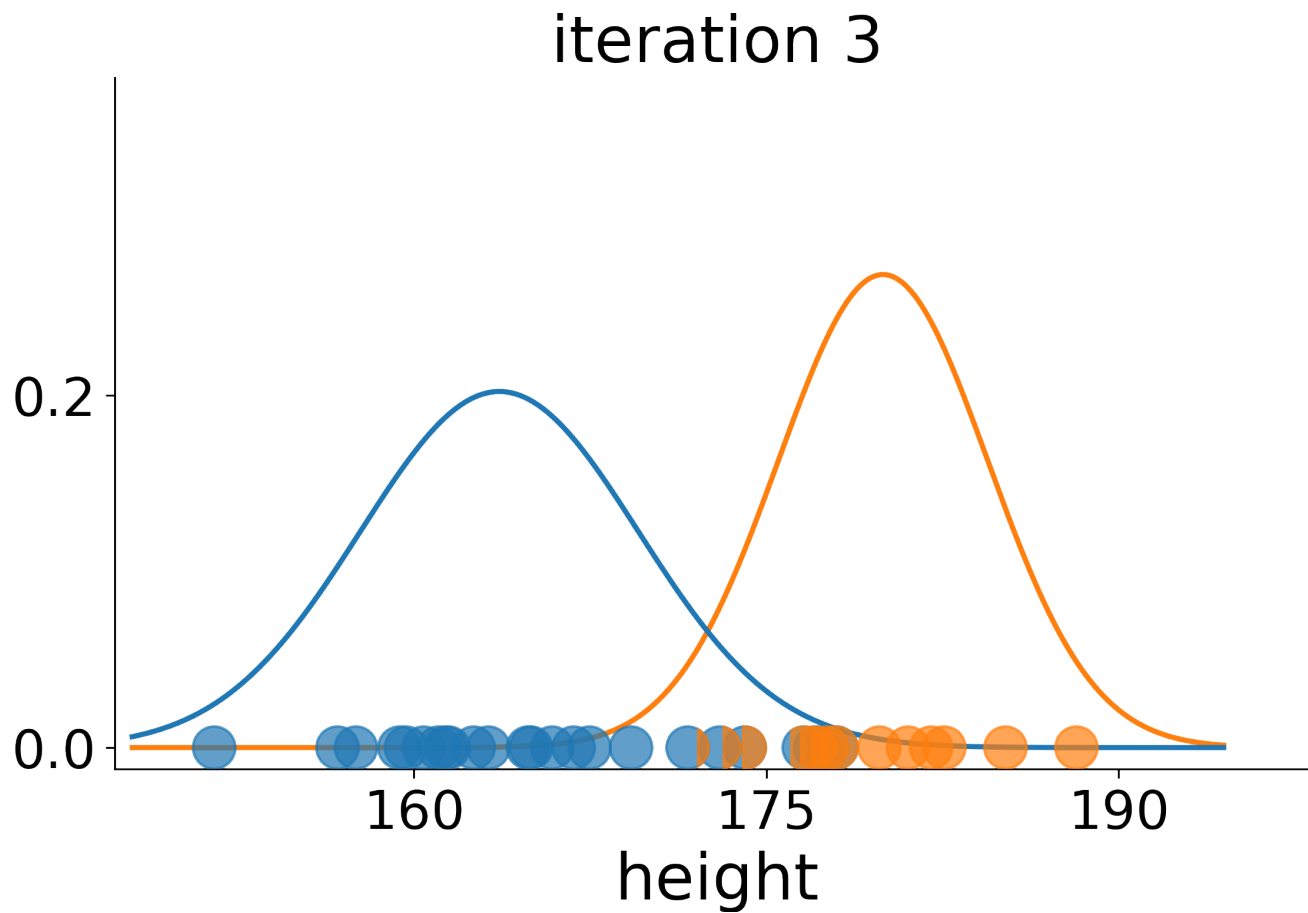
GMM EM example



GMM EM example

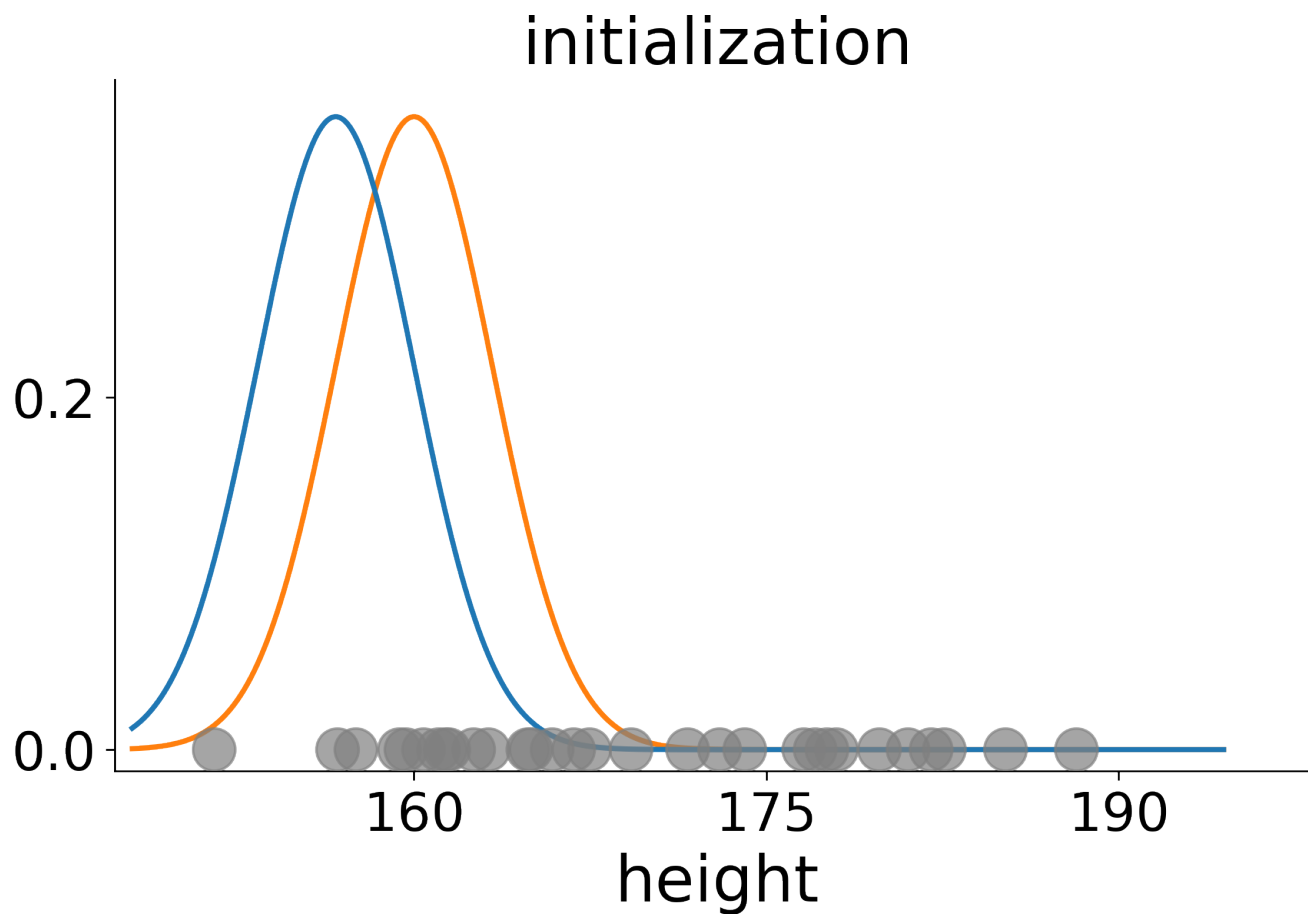


GMM EM example

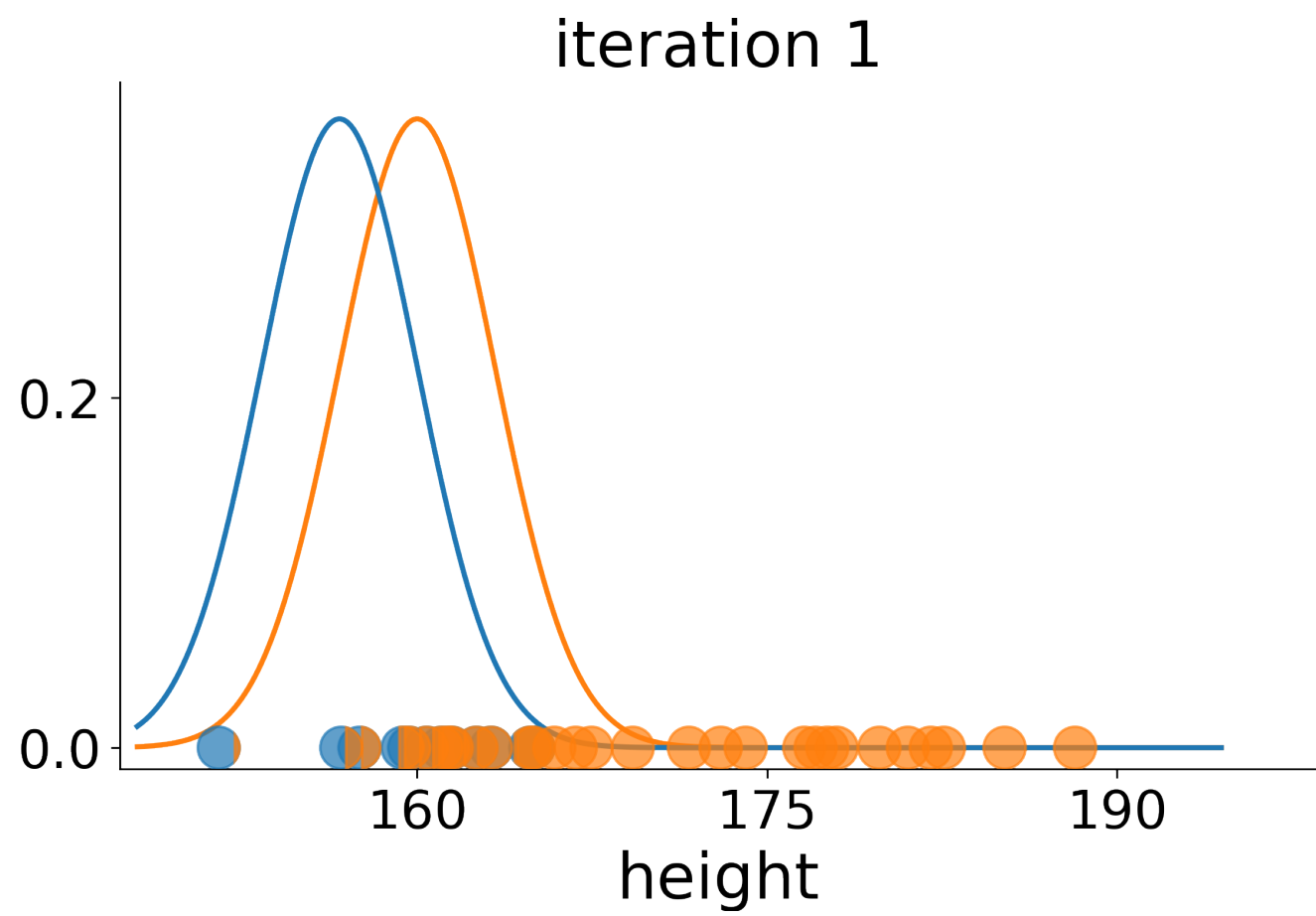


GMM EM local maximum example

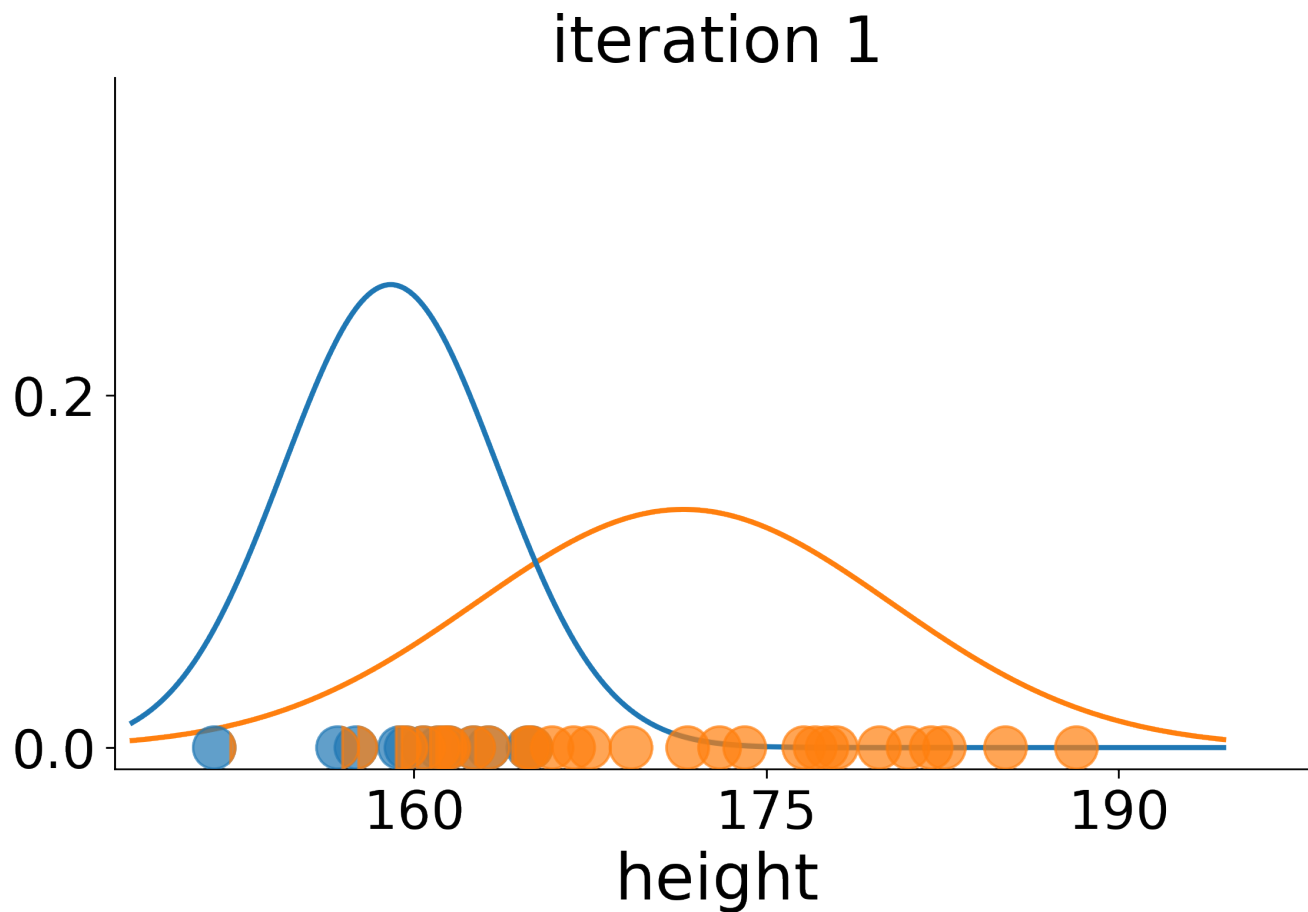
GMM EM local maximum example



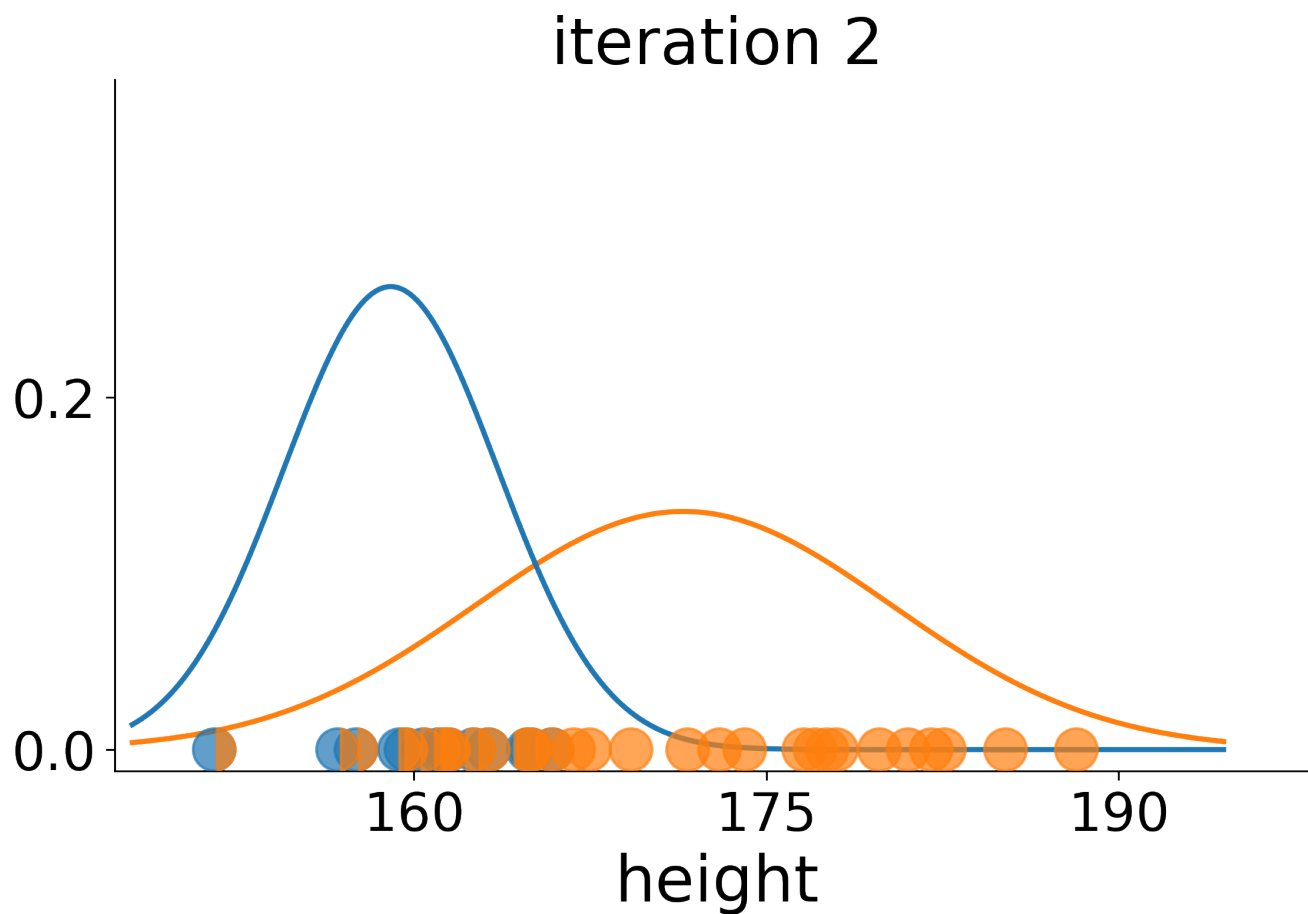
GMM EM local maximum example



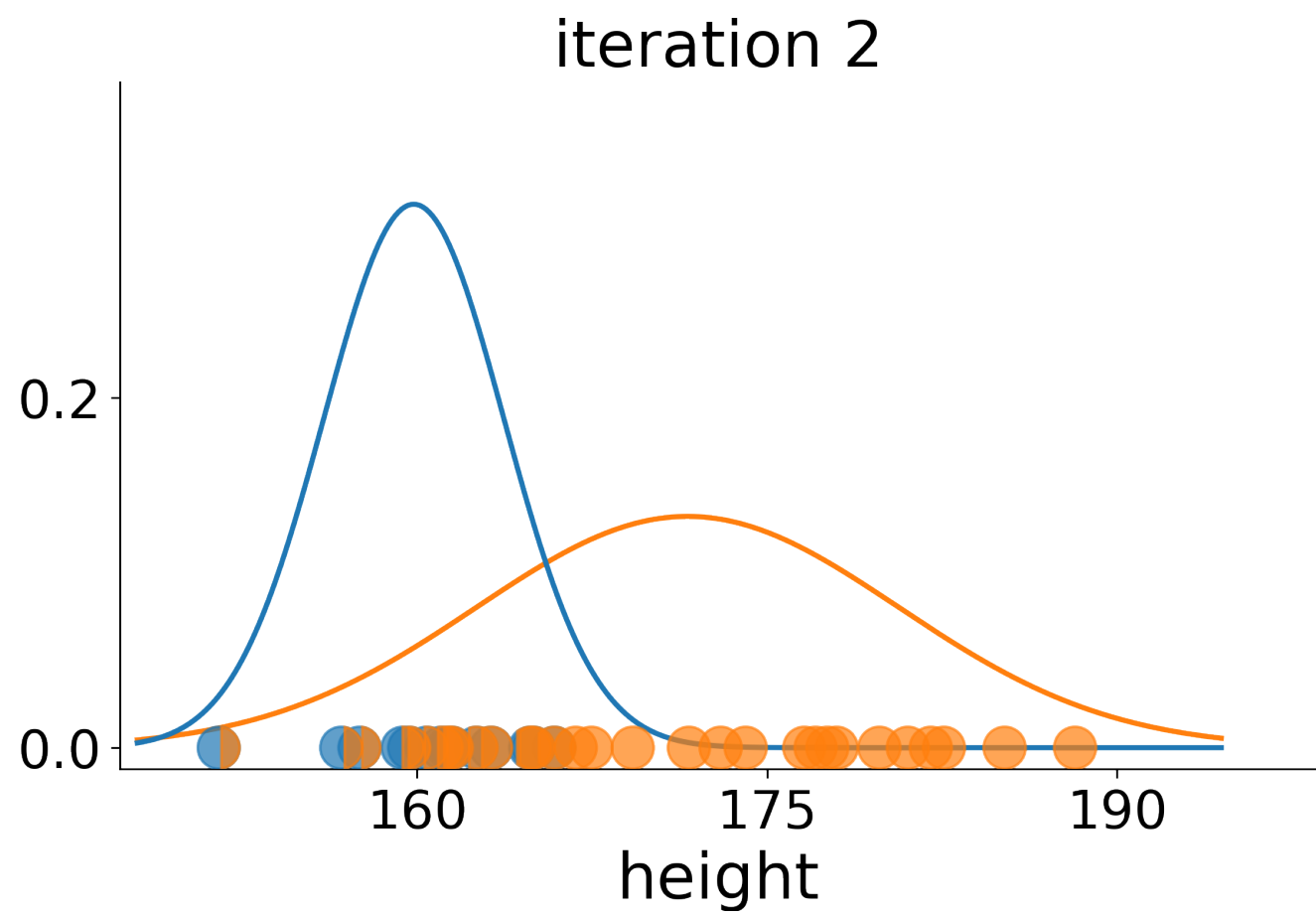
GMM EM local maximum example



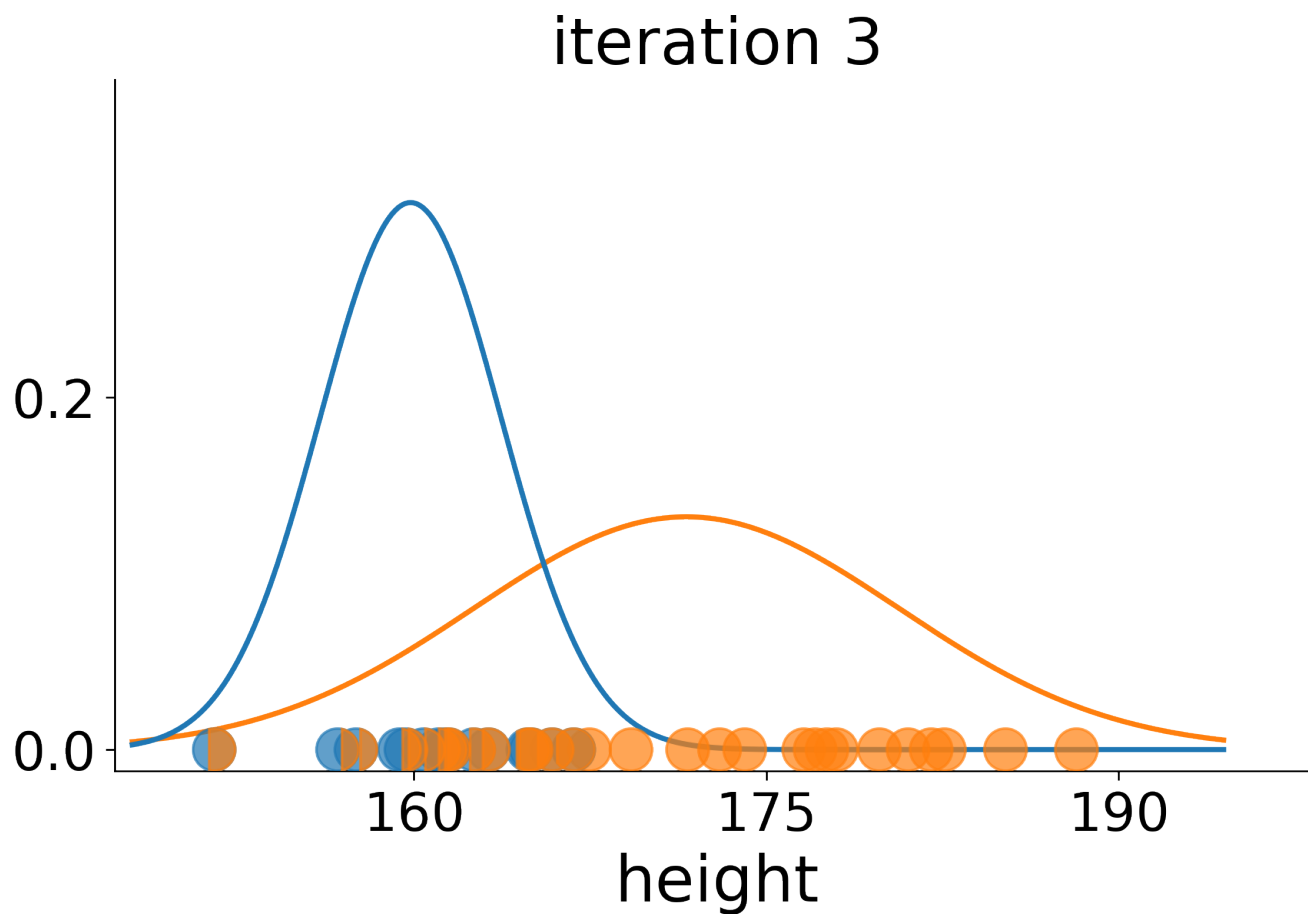
GMM EM local maximum example



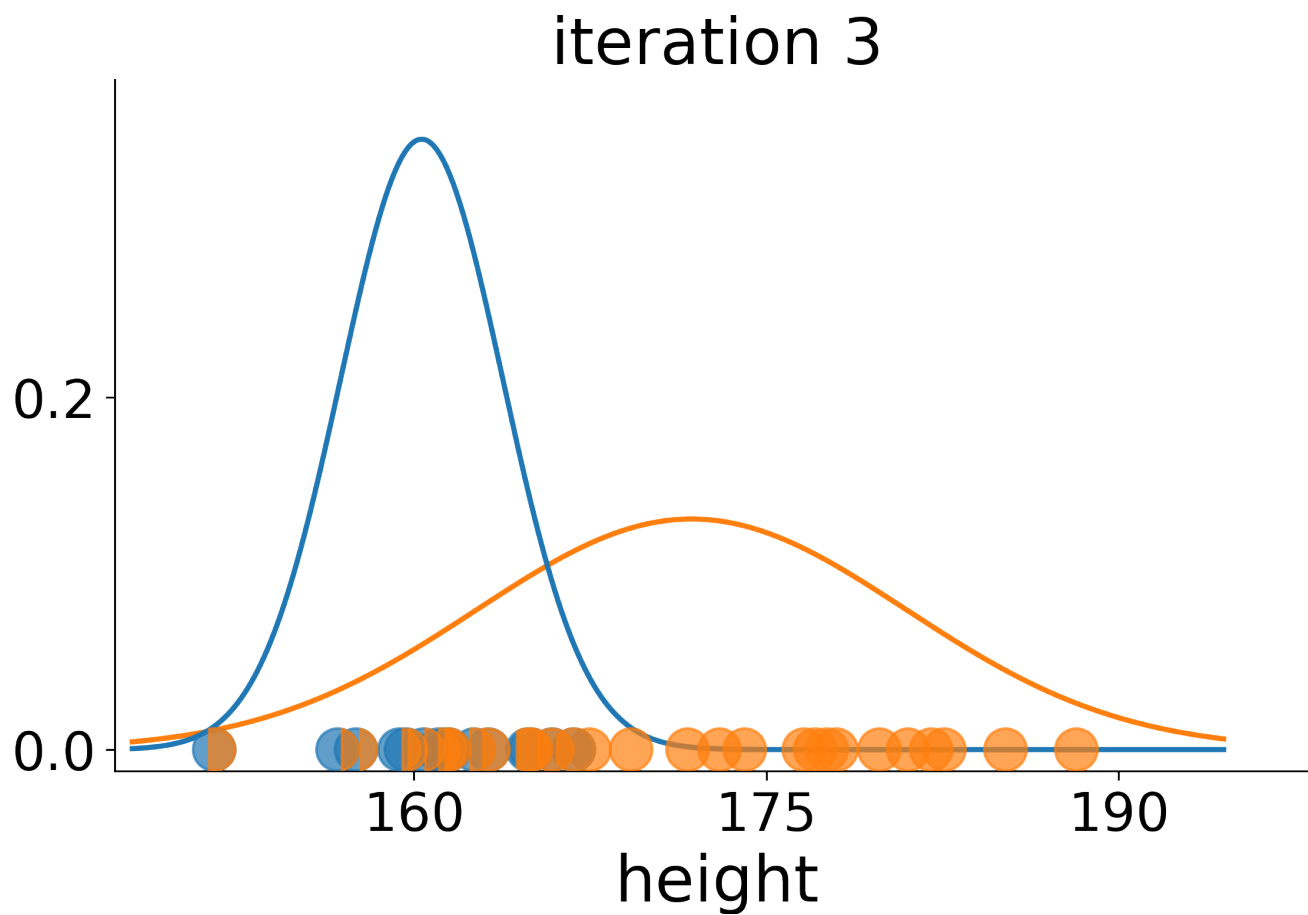
GMM EM local maximum example



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GMM EM local maximum example



Summary

- Gaussian Mixture Model is a flexible probabilistic approach to clustering problem
- Expectation Maximization algorithm can train GMM faster than Stochastic Gradient Descent and also handles complicated constraint
- Expectation Maximization suffers from local maxima (the exact solution is NP-hard)