Principal Component Analysis

Principal Component Analysis

E-step

$$q(t_i) = p(t_i \mid x_i, \theta)$$

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$$q(t_i) = p(t_i \mid x_i, \theta) = \frac{p(x_i \mid t_i, \theta)p(t_i)}{Z}$$

Principal Component Analysis

E-step

$$q(t_i) = p(t_i \mid x_i, \theta) = \frac{p(x_i \mid t_i, \theta)p(t_i)}{Z}$$

$$= \mathcal{N}(\widetilde{\mu}_i, \widetilde{\Sigma}_i)$$

Principal Component Analysis

M-step

$$\max_{\theta}.\ \mathbb{E}_{q(T)} \sum_i \log p(x_i \mid t_i, \theta) p(t_i)$$

Principal Component Analysis

M-step

$$\max_{\theta}.\ \mathbb{E}_{q(T)} \sum_i \log p(x_i \mid t_i, \theta) p(t_i)$$

$$= \sum_i \mathbb{E}_{q(t_i)} \log \left(\frac{1}{Z} \exp\left(\ldots \right) \exp\left(\ldots \right) \right)$$

Principal Component Analysis

M-step

$$\max_{\theta}. \mathbb{E}_{q(T)} \sum_i \log p(x_i \mid t_i, \theta) p(t_i)$$

$$= \sum_i \log \frac{1}{Z}$$

$$+ \sum_i \mathbb{E}_{q(t_i)} \log \left(\exp \left(\ldots \right) \exp \left(\ldots \right) \right)$$

Principal Component Analysis

M-step

$$\max_{\theta}.\, \mathbb{E}_{q(T)} \sum_i \log p(x_i \mid t_i, \theta) p(t_i)$$

$$= \sum_i \log \frac{1}{Z}$$

$$+ \sum_i \mathbb{E}_{q(t_i)} \log \left(\exp\left(\ldots\right) \exp\left(-\frac{t_i^2}{2} \right) \right)$$

Principal Component Analysis

M-step

$$\max_{\theta} \mathbb{E}_{q(T)} \sum_i \log p(x_i \mid t_i, \theta) p(t_i)$$

$$= \sum_i \log \frac{1}{Z}$$

$$+ \sum_i \mathbb{E}_{q(t_i)} \log \left(\exp \left(-\frac{(x - W t_i - b)^2}{2\sigma^2} \right) \exp \left(-\frac{t_i^2}{2} \right) \right)$$

Principal Component Analysis

M-step

$$\max_{\theta}.\ \mathbb{E}_{q(T)} \sum_i \log p(x_i \mid t_i, \theta) p(t_i)$$

$$= \sum_i \log \frac{1}{Z}$$

$$+ \sum_i \mathbb{E}_{q(t_i)} \underbrace{\log \left(\exp \left(- \frac{(x - Wt_i - b)^2}{2\sigma^2} \right) \exp \left(- \frac{t_i^2}{2} \right) \right)}_{at_i^2 + ct_i + d}$$

Summary

Probabilistic formulation of PCA

- Allows for missing values
- Straightforward iterative scheme for large dimensionalities
- Can do mixture of PPCA
- Hyperparameter tuning (number of components or choose between diagonal and full covariance)