Metropolis-Hastings

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Apply rejection sampling to Markov Chains

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For k = 1, 2, …

- Sample x' from a wrong $Q(x^k \to x')$

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How to choose A: $\pi(x') = \sum_x \pi(x)T(x \to x')$

Detailed Balance

$$\pi(x') = \sum_{x} \pi(x) T(x \rightarrow x')$$

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If $$\pi(x)T(x \to x') = \pi(x')T(x' \to x)$$

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Proof $$\sum_x \pi(x)T(x \to x') = \sum_x \pi(x')T(x' \to x)$$

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Proof $$\begin{aligned} \sum_x \pi(x)T(x \to x') &= \sum_x \pi(x')T(x' \to x) \\ &= \pi(x') \sum_x T(x' \to x) \end{aligned}$$

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$$= \pi(x') \underbrace{\sum_x T(x' \to x)}_{1}$$

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&= \pi(x') \sum_x T(x' \to x) \\
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