Week summary

Monte Carlo methods to approximate expected values

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How to sample from distribution known up to normalization constant?

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How to sample from distribution known up to normalization constant?

Two MCMC approaches:

- Gibbs sampling – reducing multidimensional sampling to a sequence of 1d
- Metropolis Hastings – rejection sampling for Markov Chains (gives more freedom)

Monte Carlo vs Variational Inference

Monte Carlo

$$\mathbb{E}_{p(x)} f(x) \approx \frac{1}{M} \sum_{s=1}^{M} f(x_s)$$

$$x_s \sim p(x)$$

Unbiased estimate (larger M => better accuracy)

Monte Carlo vs Variational Inference

Monte Carlo

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$$x_s \sim p(x)$$

Unbiased estimate (larger M => better accuracy)

$$\mathbb{E}_{p(x)} \frac{1}{M} \sum_{s=1}^{M} f(x_s) = \mathbb{E}_{p(x)} f(x)$$

Monte Carlo vs Variational Inference

Monte Carlo

$$\mathbb{E}_{p(x)} f(x) \approx \frac{1}{M} \sum_{s=1}^{M} f(x_s)$$

$$x_s \sim p(x)$$

Unbiased estimate (larger M => better accuracy)

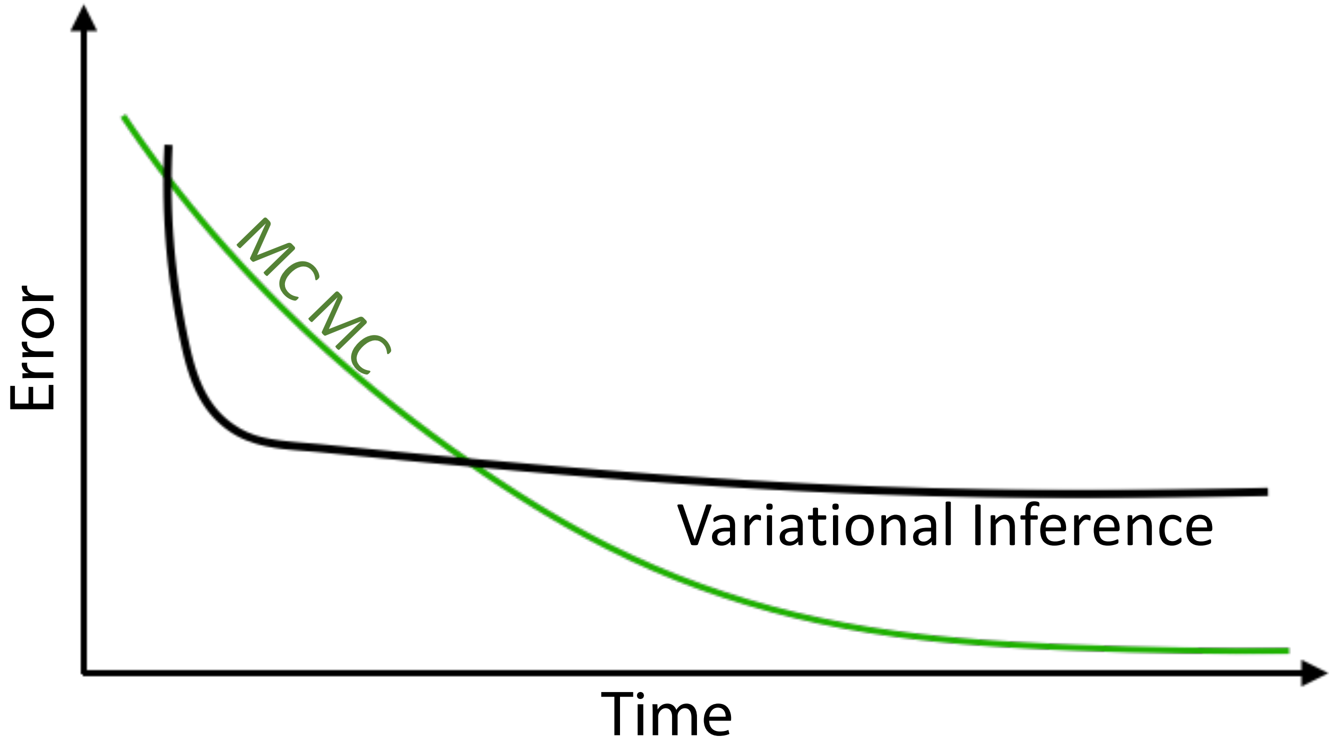
Variational Inference (week 3)

$$p(x) \approx q(x)$$

$$\mathbb{E}_{p(x)} f(x) \approx \mathbb{E}_{q(x)} f(x)$$

Monte Carlo vs Variational Inference

Schematic illustration



Methods

Best

- Full inference $p(T, \theta | X)$

Worst

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Best

- Full inference $p(T, \theta | X)$
    - Mean field $q(T)q(\theta) \approx p(T, \theta | X)$

Worst

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  - Mean field $q(T)q(\theta) \approx p(T, \theta | X)$
  - MCMC $T_s, \Theta_s \sim p(T, \Theta \mid X)$

Worst

Methods

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- Full inference $p(T, \theta | X)$
    - Mean field $q(T)q(\theta) \approx p(T, \theta | X)$
    - MCMC $T_s, \Theta_s \sim p(T, \Theta \mid X)$
- EM algorithm $q(T),\ \theta = \theta_{\mathrm{MP}}$

Worst

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  - Mean field $q(T)q(\theta) \approx p(T, \theta | X)$
  - MCMC $T_s, \Theta_s \sim p(T, \Theta \mid X)$
- EM algorithm $q(T),\ \theta = \theta_{\mathrm{MP}}$
  - Variational EM $q_1(T_1) \ldots q_d(T_d),\ \theta = \theta_{\mathrm{MP}}$

Worst

Methods

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- Full inference $p(T, \theta | X)$
    - Mean field $q(T)q(\theta) \approx p(T, \theta | X)$
    - MCMC $T_s, \Theta_s \sim p(T, \Theta \mid X)$
- EM algorithm $q(T),\ \theta = \theta_{\mathrm{MP}}$
    - Variational EM $q_1(T_1) \ldots q_d(T_d),\ \theta = \theta_{\mathrm{MP}}$
    - MCMC EM $T_s \sim p(T \mid \Theta, X), \Theta = \Theta_{\mathrm{MP}}$

Worst

Summary of Markov Chain Monte Carlo

Pros

- Easy to implement
- Easy to parallelize
- Unbiased estimates (wait more => more accuracy)

Cons

- Usually slower than alternatives