برنامه ای بنویسید که
الف) ماتریس ضرایب و ماتریس ثابت نظیر به دستگاه را به عنوان ورودی بگیرد.
ب) ماتریس افزوده نظیر را نمایش دهد.
ج) با روش سطری پلکانی این دستگاه را حل ودر هر مرحله ماتریس مقدماتی نظیر و دستگاه معادل حاصل را نمایش دهد.
د) جواب دستگاه در صورت وجود به شکل بردار(های) ستونی و در غیر اینصورت به صورت پیغام 'inconsistent' نمایش داده شود.
برنامه مذکور را برای دستگاه زیر چک کنید.

$$x_1 + 3x_2 + 2x_3 - 4x_4 + 3x_5 = -3$$

$$-2x_1 - x_2 + 2x_3 + 6x_4 + 4x_5 = 19$$

$$-x_2 + 3x_3 - 5x_4 + x_5 = -2$$

$$3x_1 - 4x_2 + 2x_3 + 5x_4 - 7x_5 = -11$$

$$x_1 + 2x_2 - 8x_3 + 6x_4 + x_5 = 4$$

الف و ب)

```
def matrix_maker():
    # A basic code for matrix input from user

    R = int(input("Enter the number of rows:"))
    C = int(input("Enter the number of columns:"))

    # Initialize matrix
    matrix = []
    print("Enter the entries rowwise:")

    # For user input
    for i in range(R):  # A for loop for row entries
        a = []
        for j in range(C):  # A for loop for column entries
            a.append(int(input()))
        matrix.append(a)

        # For printing the matrix
    for i in range(R):
        for j in range(C):
            print(matrix[i][j], end=" ")
        print()
    return matrix
```

```
def solver():
    matrix = []
    """A:coefficient"""
    print("enter A:")
    print("")
    a = matrix_maker()
    print("")

    """"B:constant"""
    print("enter B:")
    print("")
    b = matrix_maker()

    augmented = np.hstack([a, b])
    print("augmented matrix:")
    print(augmented)
```

ج)

```
def add_row(A, k, i, j):
    "Add k times row j to row i in matrix A."
    n = A.shape[0]
    E = np.eye(n)
    if i == j:
        E[i, i] = k + 1
    else:
        E[i, j] = k
    return E @ A


def scale_row(A, k, i):
    "Multiply row i by k in matrix A."
    n = A.shape[0]
```

```
    E = np.eye(n)
    E[i, i] = k
    return E @ A


def switch_rows(A, i, j):
    "Switch rows i and j in matrix A."
    n = A.shape[0]
    E = np.eye(n)
    E[i, i] = 0
    E[j, j] = 0
    E[i, j] = 1
    E[j, i] = 1
    return E @ A
```

enter A:

Enter the number of rows:3

Enter the number of columns:3

Enter the entries rowwise:

6

15

1

8

7

12

2

7

8

6 15 1

8 7 12

2 7 8

enter B:

```
Enter the number of rows:3
Enter the number of columns:1
Enter the entries rowwise:
2
14
10
2
14
10

augmented matrix:
[[ 6 15  1  2]
 [ 8  7 12 14]
 [ 2  7  8 10]]
M1:
[[ 1.         2.5        0.16666667  0.33333333]
 [ 8.         7.        12.         14.        ]
 [ 2.         7.         8.        10.        ]]
M2:
[[  1.         2.5         0.16666667   0.33333333]
 [  0.       -13.         10.66666667  11.33333333]
 [  2.         7.         8.         10.        ]]
M3:
 [[  1.         2.5         0.16666667   0.33333333]
 [  0.       -13.         10.66666667  11.33333333]
 [  0.         2.         7.66666667   9.33333333]]
M4:
 [[ 1.         2.5        0.16666667  0.33333333]
 [ 0.         1.        -0.82051282 -0.87179487]
```

[0. 2. 7.66666667 9.33333333]]

M5:

[[1. 2.5 0.16666667 0.33333333]

[0. 1. -0.82051282 -0.87179487]

[0. 0. 9.30769231 11.07692308]]

M6:

[[1. 2.5 0.16666667 0.33333333]

[0. 1. -0.82051282 -0.87179487]

[0. 0. 1. 1.19008264]]

M7:

[[1. 2.5 0.16666667 0.33333333]

[0. 1. 0. 0.1046832]

[0. 0. 1. 1.19008264]]

M8:

[[1. 2.5 0. 0.13498623]

[0. 1. 0. 0.1046832]

[0. 0. 1. 1.19008264]]

M9:

[[1. 0. 0. -0.12672176]

[0. 1. 0. 0.1046832]

[0. 0. 1. 1.19008264]]

X: [[-0.12672176]

[0.1046832]

[1.19008264]]

د)

```
def invertible():
    # return a.shape[0] == a.shape[1] and np.linalg.matrix_rank(a) == a.shape[0]
    try:
        print(linalg.inv(matrix_maker()))
    except:
        print("not consistent")
```

معکوس ماتریس ضرائب در مثال:

[[0.22603122 0.02982163 0.34225195 0.19732441 0.2416388]

 [0.18729097 -0.07023411 -0.48829431 -0.05351171 -0.16722408]

 [0.10953177 0.05267559 -0.13377926 0.04013378 -0.12458194]

 [0.03790412 0.06911929 -0.12263099 0.0367893 -0.01003344]

 [0.04821628 0.11733556 0.29988852 0.01003344 0.15635452]]

augmented matrix:

[[1 3 2 -4 3 -3]

 [-2 -1 2 6 4 19]

 [0 -1 3 -5 1 -2]

 [3 -4 2 5 -7 -11]

 [1 2 -8 6 1 4]]

Solution of linear equations:

[[-2.]

 [-1.]

 [-0.]

 [1.]

 [2.]]

```
Solution of linear equations: [[-2.]
 [-1.]
 [-0.]
 [ 1.]
 [ 2.]]
```

Row Operation 1:

1	3	2	-4	3	-3
-2	-1	2	6	4	19
0	-1	3	-5	1	-2
3	-4	2	5	-7	-11
1	2	-8	6	1	4

add **2** times the 1st row to the 2nd row

1	3	2	-4	3	-3
0	5	6	-2	10	13
0	-1	3	-5	1	-2
3	-4	2	5	-7	-11
1	2	-8	6	1	4

Row Operation 2:

1	3	2	-4	3	-3
0	5	6	-2	10	13
0	-1	3	-5	1	-2
3	-4	2	5	-7	-11
1	2	-8	6	1	4

add **-3** times the 1st row to the 4th row

1	3	2	-4	3	-3
0	5	6	-2	10	13
0	-1	3	-5	1	-2
0	-13	-4	17	-16	-2
1	2	-8	6	1	4

Row Operation 3:

1	3	2	-4	3	-3
0	5	6	-2	10	13
0	-1	3	-5	1	-2
0	-13	-4	17	-16	-2
1	2	-8	6	1	4

add **-1** times the 1st row to the 5th row

1	3	2	-4	3	-3
0	5	6	-2	10	13
0	-1	3	-5	1	-2
0	-13	-4	17	-16	-2
0	-1	-10	10	-2	7

Row Operation 4:

1	3	2	-4	3	-3
0	5	6	-2	10	13
0	-1	3	-5	1	-2
0	-13	-4	17	-16	-2
0	-1	-10	10	-2	7

multiply the 2nd row by **1/5**

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	-1	3	-5	1	-2
0	-13	-4	17	-16	-2
0	-1	-10	10	-2	7

Row Operation 5:

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	-1	3	-5	1	-2
0	-13	-4	17	-16	-2
0	-1	-10	10	-2	7

add **1** times the 2nd row to the 3rd row

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	21/5	-27/5	3	3/5
0	-13	-4	17	-16	-2
0	-1	-10	10	-2	7

Row Operation 6:

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	21/5	-27/5	3	3/5
0	-13	-4	17	-16	-2
0	-1	-10	10	-2	7

add **13** times the 2nd row to the 4th row

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	21/5	-27/5	3	3/5
0	0	58/5	59/5	10	159/5
0	-1	-10	10	-2	7

Row Operation 7:

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	21/5	-27/5	3	3/5
0	0	58/5	59/5	10	159/5
0	-1	-10	10	-2	7

add **1** times the 2nd row to the 5th row

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	21/5	-27/5	3	3/5
0	0	58/5	59/5	10	159/5
0	0	-44/5	48/5	0	48/5

Row Operation 8:

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	21/5	-27/5	3	3/5
0	0	58/5	59/5	10	159/5
0	0	-44/5	48/5	0	48/5

multiply the 3rd row by **5/21**

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	1	-9/7	5/7	1/7
0	0	58/5	59/5	10	159/5
0	0	-44/5	48/5	0	48/5

Row Operation 9:

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	1	-9/7	5/7	1/7
0	0	58/5	59/5	10	159/5
0	0	-44/5	48/5	0	48/5

add **-58/5** times the 3rd row to the 4th row

1	3	2	-4	3	-3
0	1	6/5	-2/5	2	13/5
0	0	1	-9/7	5/7	1/7
0	0	0	48/7	0	48/7

$$\begin{bmatrix} 0 & 0 & \frac{58}{5} & \frac{59}{5} & 10 & \frac{159}{5} \\ 0 & 0 & \frac{-44}{5} & \frac{48}{5} & 0 & \frac{48}{5} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & \frac{187}{7} & \frac{12}{7} & \frac{211}{7} \\ 0 & 0 & \frac{-44}{5} & \frac{48}{5} & 0 & \frac{48}{5} \end{bmatrix}$$

Row Operation 10:

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 2 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{-9}{7} & \frac{5}{7} & \frac{1}{7} \\ 0 & 0 & 0 & \frac{187}{7} & \frac{12}{7} & \frac{211}{7} \\ 0 & 0 & \frac{-44}{5} & \frac{48}{5} & 0 & \frac{48}{5} \end{bmatrix}$$

add **44/5** times the 3rd row to the 5th row

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 2 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{-9}{7} & \frac{5}{7} & \frac{1}{7} \\ 0 & 0 & 0 & \frac{187}{7} & \frac{12}{7} & \frac{211}{7} \\ 0 & 0 & 0 & \frac{-12}{7} & \frac{44}{7} & \frac{76}{7} \end{bmatrix}$$

Row Operation 11:

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 2 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{-9}{7} & \frac{5}{7} & \frac{1}{7} \\ 0 & 0 & 0 & \frac{187}{7} & \frac{12}{7} & \frac{211}{7} \\ 0 & 0 & 0 & -12 & 44 & 76 \end{bmatrix}$$

multiply the 4th row by **7/187**

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 2 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{-9}{7} & \frac{5}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & \frac{12}{187} & \frac{211}{187} \\ 0 & 0 & 0 & -12 & 44 & 76 \end{bmatrix}$$

			7	7	7

			$\frac{}{7}$	7	7

Row Operation 12:

1	3	2	$-\frac{4}{}$	3	-3
0	1	$\frac{6}{5}$	$-\frac{2}{5}$	2	$\frac{13}{5}$
0	0	1	$-\frac{9}{7}$	$\frac{5}{7}$	$\frac{1}{7}$
0	0	0	1	$\frac{12}{187}$	$\frac{211}{187}$
0	0	0	$-\frac{12}{7}$	$\frac{44}{7}$	$\frac{76}{7}$

add **12/7** times the 4th row to the 5th row

1	3	2	$-\frac{4}{}$	3	-3
0	1	$\frac{6}{5}$	$-\frac{2}{5}$	2	$\frac{13}{5}$
0	0	1	$-\frac{9}{7}$	$\frac{5}{7}$	$\frac{1}{7}$
0	0	0	1	$\frac{12}{187}$	$\frac{211}{187}$
0	0	0	0	$\frac{1196}{187}$	$\frac{2392}{187}$

Row Operation 13:

1	3	2	$-\frac{4}{}$	3	-3
0	1	$\frac{6}{5}$	$-\frac{2}{5}$	2	$\frac{13}{5}$
0	0	1	$-\frac{9}{7}$	$\frac{5}{7}$	$\frac{1}{7}$
0	0	0	1	$\frac{12}{187}$	$\frac{211}{187}$

multiply the 5th row by **187/1196**

1	3	2	$-\frac{4}{}$	3	-3
0	1	$\frac{6}{5}$	$-\frac{2}{5}$	2	$\frac{13}{5}$
0	0	1	$-\frac{9}{7}$	$\frac{5}{7}$	$\frac{1}{7}$
0	0	0	1	$\frac{12}{}$	$\frac{211}{}$

$$\begin{array}{cccc|cc} 0 & 0 & 0 & 0 & \frac{1196}{187} & \frac{2392}{187} \end{array}$$

$$\begin{array}{cccc|cc} & & & & 187 & 187 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}$$

Row Operation 14:

$$\begin{array}{cccc|cc} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 2 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{-9}{7} & \frac{5}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & \frac{12}{187} & \frac{211}{187} \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}$$

add -**12/187** times the 5th row to the 4th row

$$\begin{array}{cccc|cc} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 2 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{-9}{7} & \frac{5}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}$$

Row Operation 15:

$$\begin{array}{cccc|cc} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 2 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{-9}{7} & \frac{5}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}$$

add **-5/7** times the 5th row to the 3rd row

$$\begin{array}{cccc|cc} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 2 & \frac{13}{5} \\ 0 & 0 & 1 & \frac{-9}{7} & 0 & \frac{-9}{7} \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}$$

Row Operation 16:

$$\begin{array}{cccc|cc} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & -2 & 2 & \frac{13}{5} \end{array}$$

add **-2** times the 5th row to the 2nd row

$$\begin{array}{cccc|cc} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & -2 & 0 & -7 \end{array}$$

$$\begin{bmatrix} & & & 5 & & \\ 0 & 0 & 1 & -\frac{9}{7} & 0 & -\frac{9}{7} \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} & & & 5 & & 5 \\ 0 & 0 & 1 & -\frac{9}{7} & 0 & -\frac{9}{7} \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Row Operation 17:

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 3 & -3 \\ 0 & 1 & \frac{6}{5} & -\frac{2}{5} & 0 & -\frac{7}{5} \\ 0 & 0 & 1 & -\frac{9}{7} & 0 & -\frac{9}{7} \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

add **-3** times the 5th row to the 1st row

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 0 & -9 \\ 0 & 1 & \frac{6}{5} & -\frac{2}{5} & 0 & -\frac{7}{5} \\ 0 & 0 & 1 & -\frac{9}{7} & 0 & -\frac{9}{7} \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Row Operation 18:

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 0 & -9 \\ 0 & 1 & \frac{6}{5} & -\frac{2}{5} & 0 & -\frac{7}{5} \\ 0 & 0 & 1 & -\frac{9}{7} & 0 & -\frac{9}{7} \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

add **9/7** times the 4th row to the 3rd row

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 0 & -9 \\ 0 & 1 & \frac{6}{5} & -\frac{2}{5} & 0 & -\frac{7}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Row Operation 19:

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 0 & -9 \end{bmatrix}$$

add **2/5** times the 4th row to the 2nd row

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 0 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & \frac{6}{5} & \frac{-2}{5} & 0 & \frac{-7}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & \frac{6}{5} & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Row Operation 20:

$$\begin{bmatrix} 1 & 3 & 2 & -4 & 0 & -9 \\ 0 & 1 & \frac{6}{5} & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

add **4** times the 4th row to the 1st row

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & -5 \\ 0 & 1 & \frac{6}{5} & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Row Operation 21:

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & -5 \\ 0 & 1 & \frac{6}{5} & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

add **-6/5** times the 3rd row to the 2nd row

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Row Operation 22:

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

add **-2** times the 3rd row to the 1st row

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Row Operation 23:

1	3	0	0	0	-5
0	1	0	0	0	-1
0	0	1	0	0	0
0	0	0	1	0	1
0	0	0	0	1	2

add **-3** times the 2nd row to the 1st row

1	0	0	0	0	-2
0	1	0	0	0	-1
0	0	1	0	0	0
0	0	0	1	0	1
0	0	0	0	1	2

augmented matrix:

[[1 3 2 -4 3 -3]

[-2 -1 2 6 4 19]

[0 -1 3 -5 1 -2]

[3 -4 2 5 -7 -11]

[1 2 -8 6 1 4]]

M1: [[1. 3. 2. -4. 3. -3.]

[0. 5. 6. -2. 10. 13.]

[0. -1. 3. -5. 1. -2.]

[3. -4. 2. 5. -7. -11.]

[1. 2. -8. 6. 1. 4.]]

M2: [[1. 3. 2. -4. 3. -3.]

[0. 5. 6. -2. 10. 13.]

[0. -1. 3. -5. 1. -2.]

[0. -13. -4. 17. -16. -2.]

[1. 2. -8. 6. 1. 4.]]

Temp: [[1. 3. 2. -4. 3. -3.]

[0. 5. 6. -2. 10. 13.]

[0. -1. 3. -5. 1. -2.]

```
 [ 0. -13.  -4.  17. -16.  -2.]
 [ 0.  -1. -10.  10.  -2.   7.]]
M3: [[ 1.   3.   2.  -4.   3.  -3. ]
 [ 0.   1.   1.2  -0.4  2.   2.6]
 [ 0.  -1.   3.  -5.   1.  -2. ]
 [ 0. -13.  -4.  17. -16.  -2. ]
 [ 0.  -1. -10.  10.  -2.   7. ]]
M4: [[ 1.   3.   2.  -4.   3.  -3. ]
 [ 0.   1.   1.2  -0.4  2.   2.6]
 [ 0.   0.   4.2  -5.4  3.   0.6]
 [ 0. -13.  -4.  17. -16.  -2. ]
 [ 0.  -1. -10.  10.  -2.   7. ]]
M5: [[ 1.   3.   2.  -4.   3.  -3. ]
 [ 0.   1.   1.2  -0.4  2.   2.6]
 [ 0.   0.   4.2  -5.4  3.   0.6]
 [ 0.   0.  11.6  11.8  10.  31.8]
 [ 0.  -1. -10.  10.  -2.   7. ]]
M6: [[ 1.  3.  2. -4.  3. -3. ]
 [ 0.  1.  1.2 -0.4  2.  2.6]
 [ 0.  0.  4.2 -5.4  3.  0.6]
 [ 0.  0. 11.6 11.8 10. 31.8]
 [ 0.  0. -8.8  9.6  0.  9.6]]
M7: [[ 1.         3.         2.        -4.         3.        -3.       ]
 [ 0.         1.         1.2       -0.4        2.         2.6      ]
 [ 0.         0.         1.        -1.28571429  0.71428571  0.14285714]
 [ 0.         0.        11.6       11.8       10.        31.8      ]
 [ 0.         0.        -8.8        9.6        0.         9.6      ]]
M8: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00 -4.00000000e+00
   3.00000000e+00 -3.00000000e+00]
```

```
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00 -4.00000000e-01
   2.00000000e+00  2.60000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00 -1.28571429e+00
   7.14285714e-01  1.42857143e-01]
 [ 0.00000000e+00  0.00000000e+00  1.77635684e-15  2.67142857e+01
   1.71428571e+00  3.01428571e+01]
 [ 0.00000000e+00  0.00000000e+00 -8.80000000e+00  9.60000000e+00
   0.00000000e+00  9.60000000e+00]]
M9: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00 -4.00000000e+00
   3.00000000e+00 -3.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00 -4.00000000e-01
   2.00000000e+00  2.60000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00 -1.28571429e+00
   7.14285714e-01  1.42857143e-01]
 [ 0.00000000e+00  0.00000000e+00  1.77635684e-15  2.67142857e+01
   1.71428571e+00  3.01428571e+01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.71428571e+00
   6.28571429e+00  1.08571429e+01]]
M10: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00 -4.00000000e+00
   3.00000000e+00 -3.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00 -4.00000000e-01
   2.00000000e+00  2.60000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00 -1.28571429e+00
   7.14285714e-01  1.42857143e-01]
 [ 0.00000000e+00  0.00000000e+00  6.64946410e-17  1.00000000e+00
   6.41711230e-02  1.12834225e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.71428571e+00
   6.28571429e+00  1.08571429e+01]]
M11: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00 -4.00000000e+00
```

```
  3.00000000e+00 -3.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00 -4.00000000e-01
   2.00000000e+00  2.60000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00 -1.28571429e+00
   7.14285714e-01  1.42857143e-01]
 [ 0.00000000e+00  0.00000000e+00  6.64946410e-17  1.00000000e+00
   6.41711230e-02  1.12834225e+00]
 [ 0.00000000e+00  0.00000000e+00  1.13990813e-16 -2.88657986e-15
   6.39572193e+00  1.27914439e+01]]
M12: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00 -4.00000000e+00
   3.00000000e+00 -3.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00 -4.00000000e-01
   2.00000000e+00  2.60000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00 -1.28571429e+00
   7.14285714e-01  1.42857143e-01]
 [ 0.00000000e+00  0.00000000e+00  6.64946410e-17  1.00000000e+00
   6.41711230e-02  1.12834225e+00]
 [ 0.00000000e+00  0.00000000e+00  1.78229783e-17 -4.51329795e-16
   1.00000000e+00  2.00000000e+00]]
M13: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00 -4.00000000e+00
   3.00000000e+00 -3.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00 -4.00000000e-01
   2.00000000e+00  2.60000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00 -1.28571429e+00
   7.14285714e-01  1.42857143e-01]
 [ 0.00000000e+00  0.00000000e+00  6.53509205e-17  1.00000000e+00
   2.77555756e-17  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.78229783e-17 -4.51329795e-16
   1.00000000e+00  2.00000000e+00]]
```

M14: [[1.00000000e+00 3.00000000e+00 2.00000000e+00 -4.00000000e+00
  3.00000000e+00 -3.00000000e+00]
 [0.00000000e+00 1.00000000e+00 1.20000000e+00 -4.00000000e-01
  2.00000000e+00 2.60000000e+00]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00 -1.28571429e+00
 -1.11022302e-16 -1.28571429e+00]
 [0.00000000e+00 0.00000000e+00 6.53509205e-17 1.00000000e+00
  2.77555756e-17 1.00000000e+00]
 [0.00000000e+00 0.00000000e+00 1.78229783e-17 -4.51329795e-16
  1.00000000e+00 2.00000000e+00]]
M15: [[1.00000000e+00 3.00000000e+00 2.00000000e+00 -4.00000000e+00
  3.00000000e+00 -3.00000000e+00]
 [0.00000000e+00 1.00000000e+00 1.20000000e+00 -4.00000000e-01
  0.00000000e+00 -1.40000000e+00]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00 -1.28571429e+00
 -1.11022302e-16 -1.28571429e+00]
 [0.00000000e+00 0.00000000e+00 6.53509205e-17 1.00000000e+00
  2.77555756e-17 1.00000000e+00]
 [0.00000000e+00 0.00000000e+00 1.78229783e-17 -4.51329795e-16
  1.00000000e+00 2.00000000e+00]]
M16: [[1.00000000e+00 3.00000000e+00 2.00000000e+00 -4.00000000e+00
  0.00000000e+00 -9.00000000e+00]
 [0.00000000e+00 1.00000000e+00 1.20000000e+00 -4.00000000e-01
  0.00000000e+00 -1.40000000e+00]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00 -1.28571429e+00
 -1.11022302e-16 -1.28571429e+00]
 [0.00000000e+00 0.00000000e+00 6.53509205e-17 1.00000000e+00
  2.77555756e-17 1.00000000e+00]
 [0.00000000e+00 0.00000000e+00 1.78229783e-17 -4.51329795e-16

```
  1.00000000e+00  2.00000000e+00]]
M17: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00 -4.00000000e+00
  0.00000000e+00 -9.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00 -4.00000000e-01
  0.00000000e+00 -1.40000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  2.22044605e-16
 -7.53365624e-17 -2.22044605e-16]
 [ 0.00000000e+00  0.00000000e+00  6.53509205e-17  1.00000000e+00
  2.77555756e-17  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.78229783e-17 -4.51329795e-16
  1.00000000e+00  2.00000000e+00]]
M18: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00 -4.00000000e+00
  0.00000000e+00 -9.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00  8.88178420e-16
  1.11022302e-17 -1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  2.22044605e-16
 -7.53365624e-17 -2.22044605e-16]
 [ 0.00000000e+00  0.00000000e+00  6.53509205e-17  1.00000000e+00
  2.77555756e-17  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.78229783e-17 -4.51329795e-16
  1.00000000e+00  2.00000000e+00]]
M19: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00  1.33226763e-15
  1.11022302e-16 -5.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  1.20000000e+00  8.88178420e-16
  1.11022302e-17 -1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  2.22044605e-16
 -7.53365624e-17 -2.22044605e-16]
 [ 0.00000000e+00  0.00000000e+00  6.53509205e-17  1.00000000e+00
  2.77555756e-17  1.00000000e+00]
```

```
 [ 0.00000000e+00  0.00000000e+00  1.78229783e-17 -4.51329795e-16
   1.00000000e+00  2.00000000e+00]]
M20: [[ 1.00000000e+00  3.00000000e+00  2.00000000e+00  1.33226763e-15
   1.11022302e-16 -5.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  2.22044605e-16  6.21724894e-16
   1.01506105e-16 -1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  2.22044605e-16
  -7.53365624e-17 -2.22044605e-16]
 [ 0.00000000e+00  0.00000000e+00  6.53509205e-17  1.00000000e+00
   2.77555756e-17  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.78229783e-17 -4.51329795e-16
   1.00000000e+00  2.00000000e+00]]
M21: [[ 1.00000000e+00  3.00000000e+00  4.44089210e-16  8.88178420e-16
   2.61695427e-16 -5.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  2.22044605e-16  6.21724894e-16
   1.01506105e-16 -1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  2.22044605e-16
  -7.53365624e-17 -2.22044605e-16]
 [ 0.00000000e+00  0.00000000e+00  6.53509205e-17  1.00000000e+00
   2.77555756e-17  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.78229783e-17 -4.51329795e-16
   1.00000000e+00  2.00000000e+00]]
M22: [[ 1.00000000e+00  0.00000000e+00 -2.22044605e-16 -9.76996262e-16
  -4.28228881e-17 -2.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  2.22044605e-16  6.21724894e-16
   1.01506105e-16 -1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  2.22044605e-16
  -7.53365624e-17 -2.22044605e-16]
 [ 0.00000000e+00  0.00000000e+00  6.53509205e-17  1.00000000e+00
```

2.77555756e-17 1.00000000e+00]

[0.00000000e+00 0.00000000e+00 1.78229783e-17 -4.51329795e-16

1.00000000e+00 2.00000000e+00]]

X: [[-2.00000000e+00]

[-1.00000000e+00]

[-2.22044605e-16]

[1.00000000e+00]

[2.00000000e+00]]

Solution of linear equations:

[[-2.]

[-1.]

[-0.]

[1.]

[2.]]

```
M22: [[ 1.00000000e+00  0.00000000e+00 -2.22044605e-16 -9.76996262e-16
  -4.28228881e-17 -2.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  2.22044605e-16  6.21724894e-16
   1.01506105e-16 -1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  2.22044605e-16
  -7.53365624e-17 -2.22044605e-16]
 [ 0.00000000e+00  0.00000000e+00  6.53509205e-17  1.00000000e+00
   2.77555756e-17  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.78229783e-17 -4.51329795e-16
   1.00000000e+00  2.00000000e+00]]
X: [[-2.00000000e+00]
 [-1.00000000e+00]
 [-2.22044605e-16]
 [ 1.00000000e+00]
 [ 2.00000000e+00]]
Solution of linear equations: [[-2.]
 [-1.]
 [-0.]
 [ 1.]
 [ 2.]]
```

ب)

برنامه ای بنویسید که
الف) ماتریس A را به عنوان ورودی دریافت کند
ج) بعد فضای صفر نظیر به ماتریس A را نمایش دهد.
ب) رتبه ماتریس A را در خروجی نمایش دهد.

برنامه را برای ماتریس ضرایب تمرین قبل چک کنید.

```
A = matrix_maker()
ns = null_space(A)
print("null space:", ns)
dim_ns = ns.shape
print("dimension of null space:", dim_ns)

print("rank A:")
print(matrix_rank(A))
```

Enter the number of rows:2

Enter the number of columns:2

Enter the entries rowwise:

1 1

1 1

null space:

[[-0.70710678]

 [0.70710678]]

dimension of null space: (2, 1)

input:

1 3 2 -4 3

-2 -1 2 6 4

0 -1 3 -5 1

3 -4 2 5 -7

1 2 -8 6 1

null space: []

dimension of null space: (5, 0)

rank A: 5

```
1 3 2 -4 3
-2 -1 2 6 4
0 -1 3 -5 1
3 -4 2 5 -7
1 2 -8 6 1
null space: []
dimension of null space: (5, 0)
rank A:
5
```