linear control systems HW3 solutions

1. a) $\omega_n^2 = 16$ r/s, $2\zeta\omega_n = 3 \Rightarrow \boxed{\zeta = 0.375}$

$\boxed{\omega_n = 4}$

$\boxed{T_s = \frac{4}{\zeta\omega_n} = 2.667}$

$\boxed{T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.847}$

$\boxed{\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 28.06\%}$

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b) $\omega_n^2 = 0.04$, $2\zeta\omega_n = 0.02 \Rightarrow \boxed{\zeta = 0.05}$

$\boxed{\omega_n = 0.2}$

$\boxed{T_s = 400s}$

$\boxed{T_p = 15.73}$

$\boxed{\%OS = 85.45\%}$

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c) $\omega_n^2 = 1.05 \times 10^7$, $2\zeta\omega_n = 1.6 \times 10^3 \Rightarrow \boxed{\zeta = 0.247}$

$\boxed{\omega_n = 3240}$

$\boxed{T_s = 0.005}$

$\boxed{T_p = 0.001}$

$\boxed{\%OS = 44.92\%}$

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2. $$\frac{V_c(s)}{V_i(s)} = \frac{1/Cs}{1/Cs + R} = \frac{0.703}{s + 0.703}, \quad V_i(s) = 5/s$$

$$V_c(s) = \frac{5}{s}\left(\frac{0.703}{s+0.703}\right) = \frac{5}{s} - \frac{5}{s+0.703} \Rightarrow v_c(t) = 5 - 5e^{-0.703}$$

$$T = 1/0.703 = 1.422, \quad T_r = \frac{2.2}{0.703} = 3.129, \quad T_s = \frac{4}{0.703} = 5.69$$

سوال ۳

$$O.S. = a = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 \Rightarrow \ln\left(\frac{a}{100}\right) = \frac{-\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \ln^2\left(\frac{a}{100}\right) = \frac{\pi^2\zeta^2}{1-\zeta^2}$$

$$\Rightarrow (1-\zeta^2)\frac{\ln^2\left(\frac{a}{100}\right)}{\pi^2} = \zeta^2 \Rightarrow \left[1+\frac{\ln^2\left(\frac{a}{100}\right)}{\pi^2}\right]\zeta^2 = \frac{\ln^2\left(\frac{a}{100}\right)}{\pi^2}$$

$$\Rightarrow \zeta = \sqrt{\frac{\ln^2\left(\frac{a}{100}\right)}{\pi^2+\ln^2\left(\frac{a}{100}\right)}} = \frac{\left|\ln^2\left(\frac{a}{100}\right)\right|}{\sqrt{\pi^2+\ln^2\left(\frac{a}{100}\right)}} = \frac{-\ln\left(\frac{a}{100}\right)}{\sqrt{\pi^2+\ln^2\left(\frac{a}{100}\right)}}$$

$$\Rightarrow \zeta = \frac{-\ln(12.3/100)}{\sqrt{\pi^2+\ln(12.3/100)}} = 0.5549$$

$$T_s = \frac{4}{\zeta\omega_n} \Rightarrow 1 = \frac{4}{0.5549\,\omega_n} \Rightarrow \omega_n = 7.208$$

$$G(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{51.96}{s^2+8s+51.96}$$

سوال ۴

$$T(s)=\frac{5000}{s^2+75s+5000}$$ تابع تبدیل حلقه بسته

$$G(s)=\frac{5000}{s(s+75)}$$ تابع تبدیل حلقه باز

a) $\omega_n=\sqrt{5000}=70.71\ rad/s$

$$2\zeta\omega_n=75 \Rightarrow \zeta=\frac{75}{2\times 70.71}=0.53$$

$$\%OS=e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}\times 100=14.01\ \%$$

b) $$T_s=\frac{4}{\zeta\omega_n}=\frac{4}{75/2}=0.107\ s$$

$$G(s)=\frac{5000}{s(s+75)} \Rightarrow Type=1$$

c) $$k_p=\lim_{s\to 0}G(s)=\lim_{s\to 0}\frac{5000}{75s}=\infty \qquad e_{ss}=\frac{5}{1+k_p}=0$$

d) $$k_v=\lim_{s\to 0}sG(s)=\lim_{s\to 0}\frac{5000}{75}=66.67 \qquad e_{ss}=\frac{5}{k_v}=0.075$$

e) $$k_a=\lim_{s\to 0}s^2G(s)=\lim_{s\to 0}\frac{5000s}{75}=0 \qquad e_{ss}=\frac{5}{k_a}=\infty$$