

linear control systems HW3 solutions

1. الف) $\omega_n^2 = 16 \text{ r/s}^2$, $2\zeta\omega_n = 3 \Rightarrow \zeta = 0.375$
 $\omega_n = 4$

$$T_s = \frac{4}{\zeta\omega_n} = 2.667$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 0.847$$

$$\%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \cdot 100 = 28.06\%$$

ب) $\omega_n^2 = 0.04$, $2\zeta\omega_n = 0.02 \Rightarrow \zeta = 0.05$

$$T_p = 15.73$$

$$\omega_n = 0.2$$

$$\%OS = 85.45\%$$

$$T_s = 400s$$

ج) $\omega_n^2 = 1.05 \times 10^7$, $2\zeta\omega_n = 1.6 \times 10^3 \Rightarrow \zeta = 0.247$

$$T_p = 0.001$$

$$\omega_n = 3240$$

$$\%OS = 44.92\%$$

$$T_s = 0.005$$

2. $\frac{V_c(s)}{V_i(s)} = \frac{1/s}{1/s + R} = \frac{0.703}{s + 0.703}$, $V_i(s) = 5/s$

$$V_c(s) = \frac{5}{s} \left(\frac{0.703}{s + 0.703} \right) = \frac{5}{s} - \frac{5}{s + 0.703} \Rightarrow V_c(t) = 5 - 5e^{-0.703t}$$

$$T = \frac{1}{0.703} = 1.422, T_r = \frac{2.2}{0.703} = 3.129, T_s = \frac{4}{0.703} = 5.69$$

مسألة ٣

$$O.S. = a = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \times 100 \Rightarrow \ln\left(\frac{a}{100}\right) = \frac{-\pi \zeta}{\sqrt{1-\zeta^2}} \Rightarrow \ln^2\left(\frac{a}{100}\right) = \frac{\pi^2 \zeta^2}{1-\zeta^2}$$

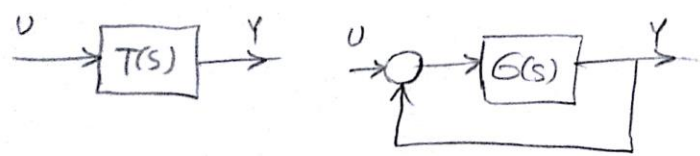
$$\Rightarrow (1-\zeta^2) \frac{\ln^2\left(\frac{a}{100}\right)}{\pi^2} = \zeta^2 \Rightarrow \left[1 + \frac{\ln^2\left(\frac{a}{100}\right)}{\pi^2}\right] \zeta^2 = \frac{\ln^2\left(\frac{a}{100}\right)}{\pi^2}$$

$$\Rightarrow \zeta = \sqrt{\frac{\ln^2\left(\frac{a}{100}\right)}{\pi^2 + \ln^2\left(\frac{a}{100}\right)}} = \frac{|\ln^2\left(\frac{a}{100}\right)|}{\sqrt{\pi^2 + \ln^2\left(\frac{a}{100}\right)}} = \frac{-\ln\left(\frac{a}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{a}{100}\right)}}$$

$$\Rightarrow \zeta = \frac{-\ln(12.3/100)}{\sqrt{\pi^2 + \ln^2(12.3/100)}} = 0.5549$$

$$T_s = \frac{4}{\zeta \omega_n} \Rightarrow 1 = \frac{4}{0.5549 \omega_n} \Rightarrow \omega_n = 7.208$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} = \frac{51.96}{s^2 + 8s + 51.96}$$



$$T(s) = \frac{5000}{s^2 + 75s + 5000}$$

تابع انتقال

$$G(s) = \frac{5000}{s(s + 75)}$$

تابع انتقال باز

a) $\omega_n = \sqrt{5000} = 70.71 \text{ rad/s}$

$$2\zeta\omega_n = 75 \Rightarrow \zeta = \frac{75}{2 \times 70.71} = 0.53$$

$$\%OS = e^{\frac{-\zeta}{\sqrt{1-\zeta^2}}} \times 100 = 14.01 \%$$

b) $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{75/2} = 0.107 \text{ s}$

$$G(s) = \frac{5000}{s(s + 75)} \Rightarrow \text{Type} = 1$$

c) $k_p = \lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{5000}{75s} = \infty$ $e_{ss} = \frac{5}{1+k_p} = 0$

d) $k_v = \lim_{s \rightarrow 0} sG(s) = \lim_{s \rightarrow 0} \frac{5000}{75} = 66.67$ $e_{ss} = \frac{5}{k_v} = 0.075$

e) $k_a = \lim_{s \rightarrow 0} s^2 G(s) = \lim_{s \rightarrow 0} \frac{5000s}{75} = 0$ $e_{ss} = \frac{5}{k_a} = \infty$