Subject:
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یاسمن سادات میرمحمد

۹۴۳۱۰۴۲

تمرین سری سوم کنترل خطی

① درصد فراجهش $Mp = \frac{y(tp) - y(\infty)}{y(\infty)} \times 100$

Tp

Ts

$\leftarrow \omega n$

$\frac{1}{\omega n} \leftarrow \xi$

$$\boxed{\frac{\omega n^2}{s^2 + 2\xi \omega n s + \omega n^2}}$$

پاسخ به ورودی پله

$$y(t) = 1 - \frac{e^{-\xi \omega n t}}{\sqrt{1-\xi^2}} \sin(\omega d t + \theta) \quad ; \quad \theta = \cos^{-1} \xi$$

الف) $T(s) = \frac{16}{s^2 + 3s + 16}$

- $\omega n^2 = 16 \longrightarrow \boxed{\omega n = \pm 4}$
- $2\xi \omega n = 3 \longrightarrow \boxed{\xi = \frac{3}{8}}$
- $y(t) = 1 - \frac{e^{-\frac{3}{8} \times 4 t}}{3.7} \sin(3.7t + \theta) \quad ; \quad \begin{cases} \theta = \cos^{-1} \frac{1}{4} \\ \omega d = \omega n \sqrt{1-\xi^2} = 4\sqrt{1 - \frac{9}{64}} \simeq 3.7 \end{cases}$

$\omega d = 3.7$

$$\longrightarrow y(t) = 1 - \frac{e^{-\frac{3}{2} t}}{3.7} \sin(3.7t + \theta)$$

- $\omega d \, tp = \pi \longrightarrow tp = \frac{\pi}{\omega d} \longrightarrow tp = \frac{\pi}{3.7}$

- ts : زمان نشست $\begin{cases} 2\% \text{ با معیار} \longrightarrow e^{-\xi \omega n t} < 0.02 \longrightarrow \xi \omega n ts = 4 \longrightarrow ts = \frac{4}{\xi \omega n} = \frac{4}{\frac{3}{8} \times 4} = \boxed{\frac{8}{3}} \\ 5\% \text{ با معیار} \longrightarrow e^{-\xi \omega n t} < 0.05 \longrightarrow \xi \omega n ts = 3 \longrightarrow ts = \frac{3}{\xi \omega n} = \frac{3}{\frac{3}{8} \times 4} = \boxed{2} \end{cases}$

- $Mp = \frac{y(tp) - y(\infty)}{y(\infty)} \times 100 = e^{\frac{-\xi \pi}{\sqrt{1-\xi^2}}} \times 100\%$

$$Mp = e^{-\frac{3}{8} \times \pi \times \frac{1}{3.7}} \times 100$$

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$2\zeta\omega_n = \frac{2}{100}$

$\zeta \times \frac{2}{10} = \frac{1}{100}$

$\omega_n^2 = 0{,}04 \longrightarrow \boxed{\omega_n = 0{,}2}$

$T(s) = \dfrac{0{,}04}{s^2 + 0{,}02s + 0{,}04}$ (1

$2\zeta\omega_n = 0{,}02 \longrightarrow \zeta = \frac{0{,}01}{0{,}2} = 0{,}05 \longrightarrow \boxed{\zeta = 0{,}05}$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 0{,}2\sqrt{1 - \frac{25}{10000}} = 0{,}199$

$\omega_d = 0{,}2$

$y(t) = 1 - \dfrac{e^{-\frac{5}{100}\times\frac{2}{10}t}}{\sqrt{1 - 0{,}0025}} \sin(0{,}2t + \theta)$

$\sqrt{1-0{,}0025} = 1$

$\theta = \cos^{-1} 5$

$t_p = \dfrac{\pi}{\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}} = \dfrac{\pi}{0{,}2} \longrightarrow t_p = 5\pi$

$$\boxed{y(t) = 1 - e^{-\frac{t}{100}} \cdot \sin(0{,}2t + \theta)}$$

$t_s \begin{cases} 2\% \longrightarrow \dfrac{4}{\zeta\omega_n} = \dfrac{4}{\frac{5}{100}\times\frac{2}{10}} = \boxed{400}\ t_s \\ 5\% \longrightarrow \dfrac{3}{\zeta\omega_n} = \dfrac{3}{\frac{5}{100}\times\frac{2}{10}} = \boxed{300}\ t_s \end{cases}$

$M_p = e^{-\frac{0{,}05\pi}{\sqrt{1-0{,}0025}}} \times 100 \longrightarrow M_p = e^{-0{,}05\pi} \times 100$

$\sqrt{1-0{,}0025} \simeq 1$

$\omega_n^2 = 1.05\times10^7 \Rightarrow \sqrt{1050\times10^6} = 32.4\times10^3$

$\omega_n = 3240$

$T(s) = \dfrac{1.05\times10^7}{s^2 + 1.6\times10^3 s + 1.05\times10^7}$ (2

$2\zeta\omega_n = 1.6\times10^3 \longrightarrow 2\times\zeta\times3240 = 1600 \longrightarrow \zeta = 0{,}246$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\omega_d = 3240\sqrt{1-(\frac{24}{100})^2}$

$\omega_d = 3240$

$y(t) = 1 - \dfrac{e^{-\frac{24}{100}\times3240t}}{\sqrt{1-(\frac{24}{100})^2}} \cdot \sin(3240t + \theta)$

$\sqrt{1-(\frac{24}{100})^2} = 0{,}97 \simeq 1$

$$\longrightarrow \boxed{y(t) = 1 - e^{-777.6t} \cdot \sin(3240t + \theta)}$$

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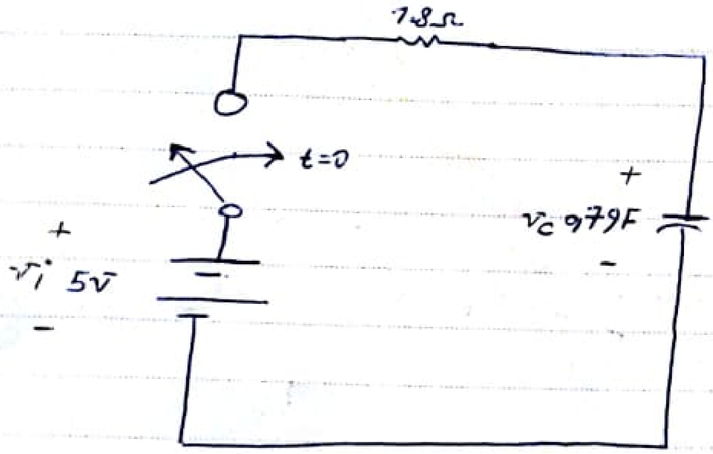

(2)

- $\tau = RC \rightarrow$ ثابت زمانی
- t_r
- t_s

$v_c(t) = e^{-t} \cdot u(t)$

$V_c(s) = \frac{1}{c}$

$v_i(t) = 5 \cdot e^{-t} \cdot u(t)$

$V_i(s) = \frac{5}{s}$

$v_i \rightarrow$ input

$v_c \rightarrow$ output

$v_i(t) + v_R(t) + v_c(t) = 0$

$$\boxed{\frac{V_c(s)}{V_i(s)} = \frac{\frac{1}{c}}{\frac{1}{c} + R}}$$

$$\rightarrow \frac{V_c(s)}{\frac{5}{s}} = \frac{0.79}{s + 0.79} \rightarrow V_c(s) = \frac{5}{s}\left(\frac{0.79}{s+0.79}\right)$$

$$V_c(s) = \frac{5}{s} - \frac{5}{s+0.79} \rightarrow V_c(s) = \frac{5}{s} - \frac{5}{s+0.79}$$

$$\xrightarrow{L^{-1}} \boxed{V_c(t) = 5 - 5 \cdot e^{-0.79}}$$

- $\tau = \frac{1}{0.79} \rightarrow \tau = 1.2658$
- $T_r = \frac{\pi - \cos^{-1}\delta}{\omega_n\sqrt{1-\delta^2}} = \frac{0.79 + 1.8}{0.79} = \frac{2.59}{0.79} \rightarrow T_r = 3.278$
- $T_s = \frac{4}{0.79}$, $\frac{3}{0.79} \rightarrow \boxed{T_s = 5.06}$, $\boxed{T_s = 3.797}$

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③ شکل کلی تابع تبدیل سیستم مرتبه‌ی دوم:

$$\boxed{\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}}$$

- $M_p = e^{\frac{-\zeta}{\sqrt{1-\zeta^2}}\pi} = \frac{12.3}{100} \rightarrow \ln\frac{12.3}{100} = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \rightarrow \frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{\ln\frac{123}{1000}}{-\pi}$

- $t_s \begin{cases} 2\% \rightarrow \frac{4}{\zeta\omega_n} = 1 \\ 4\% \rightarrow \frac{3}{\zeta\omega_n} = 1 \end{cases}$

$$\rightarrow \frac{\zeta^2}{1-\zeta^2} = \frac{\ln^2(\frac{12.3}{100})}{\pi^2}$$

$$\rightarrow \zeta^2 \cdot \pi^2 = \ln^2\left(\frac{12.3}{100}\right) - \zeta^2\ln^2\left(\frac{12.3}{100}\right)$$

$$\rightarrow \zeta^2\pi^2 + \zeta^2\ln^2\left(\frac{12.3}{100}\right) = \ln^2\left(\frac{12.3}{100}\right)$$

$$\rightarrow \zeta^2\left(\pi^2 + \ln^2\left(\frac{12.3}{100}\right)\right) = \ln^2\left(\frac{12.3}{100}\right)$$

$$\rightarrow \zeta = \frac{\ln\left(\frac{123}{1000}\right)}{\sqrt{\pi^2+\ln^2\left(\frac{123}{1000}\right)}}$$

$$\rightarrow \zeta = 0.55$$

$2\% \rightarrow \frac{4}{\frac{55}{100}\times\omega_n} = 1 \rightarrow \omega_n \times \frac{55}{100} = 4$

$\omega_n = 7.27$

$4\% \rightarrow \frac{3}{\frac{55}{100}\times\omega_n} = 1 \rightarrow \omega_n \times \frac{55}{100} = 3$

$\omega_n = 5.45$

* چون سوال نگفته از چه دقتی و معیاری استفاده می‌شود، دو تا مقدار برای ω_n به دست میاید.

حالت ① $\omega_n = 7.27 \rightarrow$

$$G(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{52.85}{s^2+2\times 7.27\times 0.55\, s+52.85} = \frac{52.85}{s^2+7.9s+52.85}$$

حالت ② $\omega_n = 5.45 \rightarrow$

$$G(s) = \frac{29.70}{s^2+2\times 0.55\times 5.45\, s+29.7} = \frac{29.7}{s^2+5.9s+29.7}$$

PAPCO

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$$T(s) = \frac{5000}{s^2 + 75s + 5000}$$ (5)

a. $2\xi\omega_n = 75 \rightarrow 2 \times \xi \times 70.71 = 75 \rightarrow \xi = 0{,}53$

$\omega_n^2 = 5000 \Rightarrow \omega_n \cong 70.71$

$\sqrt{1-\left(\frac{53}{100}\right)^2} = 0{,}847$

$$\rightarrow M_p = \left(e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right) \times 100$$

$$M_p = e^{\frac{-0{,}53\pi}{0{,}847}} \times 100$$

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b. $t_s \begin{cases} 2\% \rightarrow \dfrac{4}{\xi\omega_n} = \dfrac{4}{0{,}53 \times 70{,}71} \rightarrow t_s = 0{,}106 \\ 4\% \rightarrow \dfrac{3}{\xi\omega_n} = \dfrac{3}{0{,}53 \times 70.71} \rightarrow t_s = 0{,}080 \end{cases}$

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c. $\begin{cases} x(t) = 5 \cdot u(t) \\ K_p = \lim\limits_{s\to 0} G(s) = \lim\limits_{s\to 0} \dfrac{5000}{s^2+75s+5000} = \infty \end{cases}$

خطای حالت دائم ـ ورودی پله : $e_{ss} = \frac{1}{1+K_p}$

مرتبه‌ی قطب $s=0$ ← نوع سیستم

$$\rightarrow e_{ss} = \frac{1}{1+K_p} = 0$$

$e_{ss} = 0$ ← سیستم از نوع I است

* ~~~~~~~~ *

d. $\begin{cases} e_{ss} = \dfrac{1}{K_V} \\ x(t) = 5t\, u(t) \end{cases}$, $K_V = \lim\limits_{s\to 0} s \cdot G(s) = \lim\limits_{s\to 0} s \cdot \dfrac{5000}{s^2+75s+5000} =$

$$\rightarrow \lim_{s\to 0} \frac{5000}{s+75+\frac{5000}{s}} \rightarrow K_V = \frac{5000}{75} = 66.6 \rightarrow e_{ss} = \frac{5}{K_V} = 0{,}075$$

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e. $x(t) = 5 \cdot t^2 \cdot u(t) \longrightarrow k_a = \lim_{s \to 0} s^2 \cdot G(s) = \lim_{s \to 0} s^2 \cdot \frac{5000}{s^2 + 75s + 5000}$

$k_a = 0$

$\longrightarrow e_{ss} = \frac{1}{k_a} = \infty \longleftarrow$ سیستم نوع I است

پایان