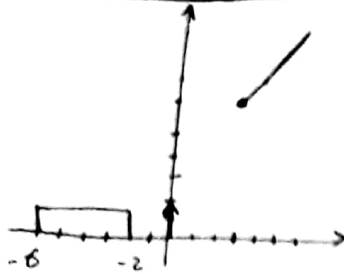
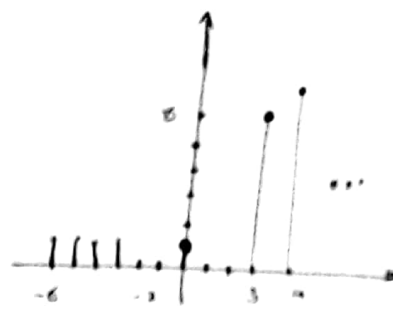


# HW1

1) a.



b.



$$2) a. E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-6}^{-2} |\sin^2(t)| dt = \int_{-6}^{-2} (1 - \cos(2t)) dt = \left[ t - \frac{1}{2} \sin(2t) \right]_{-6}^{-2}$$

$$= 6 - \sin(6)$$

$$b. E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \sum_{k=3}^3 \sin^2[k] = 0 + 2\sin^2(1) + 2\sin^2(2) + 2\sin^2(3)$$

$$3) a. x(t) = \sin\left(\frac{5\pi t}{3}\right) \xrightarrow{\text{دوره}} T = \frac{2\pi}{5\pi/3} = \frac{6}{5}$$

$$b. x[n] = \sin\left(\frac{5\pi n}{3}\right) \rightarrow N = \left(\frac{2\pi}{5/3}\right) \times K = 6$$

$$c. x(t) = e^{j\frac{4\pi}{7}t} + e^{j\frac{\pi}{5}t} \rightarrow T_1 = \frac{2\pi}{4\pi/7} = \frac{7}{2}, T_2 = \frac{2\pi}{\pi/5} = 10 \rightarrow T = [T_1, T_2] = 70$$

$$d. x[n] = e^{j\frac{4\pi}{7}n} + e^{j\frac{\pi}{5}n} \rightarrow N_1 = 7, N_2 = 10 \Rightarrow N = 70$$

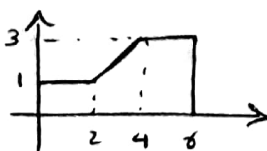
$$f. x[n] = \cos\left(\frac{3}{5}\pi n^2\right) \rightarrow x[n] = x[n+N] \Rightarrow \cos\left(\frac{3}{5}\pi n^2\right) = \cos\left(\frac{3}{5}\pi (n+N)^2\right)$$

$$\Rightarrow \cos\left(\frac{3}{5}\pi n^2\right) = \cos\left(\frac{3\pi}{5}(n^2 + 2nN + N^2)\right) \Rightarrow \frac{3}{5}\pi n^2 + 2\pi nN \pm \frac{3}{5}\pi (n^2 + 2nN + N^2)$$

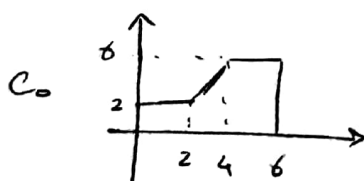
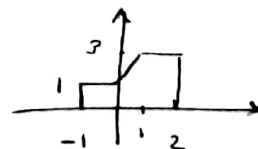
$$\Rightarrow \frac{3}{5}(2nN + N^2) = 2K \Rightarrow \text{دوره } N \Rightarrow N = 10 \Rightarrow N_0 = 10$$

$$4) \text{ حل) } x(t) = u(t+2) + tu(t) - (t-2)u(t-2) - 3u(t+4)$$

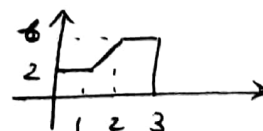
ج) a.



b.



d.



5) a. Linearity:  $\begin{cases} y_1(t) = \sin(x_1(t)) \\ y_2(t) = \sin(x_2(t)) \end{cases} \Rightarrow \begin{cases} y_3 = \sin(x_3(t)) \\ x_3(t) = ax_1(t) + bx_2(t) \end{cases}$

$\Rightarrow ay_1(t) + by_2(t) \neq y_3(t) \rightarrow \underline{\text{Non-Linear}}$

T.I.:  $y(t) = \sin(x(t))$

$x_2 = x(t-t_0) \Rightarrow y_2(t) = \sin(x(t-t_0)) \Rightarrow y(t-t_0) = y_2(t) \rightarrow \underline{\text{T.I.}}$

b. L:  $\begin{cases} y_1(t) = 3x_1(t/4) \\ y_2(t) = 3x_2(t/4) \end{cases}, y_3(t) = 3(ax_1(t/4) + bx_2(t/4))$

$\Rightarrow y_3(t) = ay_1(t) + by_2(t) \rightarrow \underline{\text{Linear}}$

T.I.:  $y(t) = 3x(t/4)$

$x_2 = x(t-t_0) \Rightarrow y_2(t) = 3x(t/4 - t_0) \Rightarrow y(t-t_0) \neq y_2(t) \rightarrow \underline{\text{T.V.}}$

c.  $\begin{cases} y_1(t) = 3x_1(t/4) - 1 \\ y_2(t) = 3x_2(t/4) - 1 \end{cases}, y_3 = 3(ax_1(t/4) + bx_2(t/4)) - 1$

$y_3(t) \neq ay_1(t) + by_2(t) \rightarrow \underline{\text{Non-Linear}}$

T.I.:  $y(t) = 3x(t/4) - 1$

$x_2 = x(t-t_0) \Rightarrow y_2(t) = 3x(t/4 - t_0) - 1 \Rightarrow y(t-t_0) \neq y_2(t) \rightarrow \underline{\text{T.V.}}$

d. Linearity:  $\begin{cases} y_1(t) = \frac{d}{dt} x_1(t) \\ y_2(t) = \frac{d}{dt} x_2(t) \end{cases}, y_3(t) = \frac{d}{dt} (ax_1(t) + bx_2(t))$

$\Rightarrow ay_1 + by_2 = y_3 \Rightarrow \underline{\text{Linear}}$

T.I.:  $y = \frac{d}{dt} x(t)$

$x_2 = x(t-t_0) \Rightarrow y_2(t) = \frac{d}{dt} x(t-t_0)$

$\Rightarrow y(t-t_0) = y_2(t) \rightarrow \underline{\text{T.I.}}$

e.  $y_1(t) = \begin{cases} 3x_1(t)e^2, & t > 0 \\ -4x_1(t) + 3, & t < 0 \end{cases}$

$y_3(t) = \begin{cases} 3(ax_1 + bx_2) + 2, & t > 0 \\ -4(ax_1 + bx_2) + 3, & t < 0 \end{cases}$

$y_2(t) = \begin{cases} 3x_2(t)e^2, & t > 0 \\ -4x_2(t) + 3, & t < 0 \end{cases}$

$y_3(t) \neq ay_1(t) + by_2(t) \rightarrow \underline{\text{Non-Linear}}$

$$T.I. 8 \quad y(t) = \begin{cases} 3x(t) + 2, & t > 0 \\ -4x(t) + 3, & t < 0 \end{cases}$$

$$x_2 = x(t-t_0) \Rightarrow y_2(t) = \begin{cases} 3x(t-t_0) + 2, & t > 0 \\ -4x(t-t_0) + 3, & t < 0 \end{cases} \Rightarrow y(t-t_0) \neq y_2(t) \quad \underline{T.V}$$

$$f. \quad \begin{cases} y_1[n] = x_1[n]x_1[n-1] \\ y_2[n] = x_2[n]x_2[n-1] \end{cases}, \quad y_3[n] = (ax_1[n] + bx_2[n])(ax_1[n-1] + bx_2[n-1])$$

$$\underline{y_3[n] \neq ay_1[n] + by_2[n] \rightarrow \text{Non-Linear.}}$$

$$y[n] = x[n]x[n-1]$$

$$x_2[n] = x[n-n_0] \rightarrow y_2[n] = x[n-n_0]x[n-n_0-1] \Rightarrow y_2[n] = y[n-n_0] \rightarrow \underline{T.I.}$$

$$g. \quad \begin{cases} y_1[n] = \sum_{k=-\infty}^n x_1[k] \\ y_2[n] = \sum_{k=-\infty}^n x_2[k] \end{cases}, \quad y_3[n] = \sum_{k=-\infty}^n (ax_1[k] + bx_2[k])$$

$$\Rightarrow y_3[n] = ay_1[n] + by_2[n] \rightarrow \underline{\text{Linear}}$$

$$y[n] = \sum_{k=-\infty}^n x_0[k]$$

$$x_2[n] = x[n-n_0] \rightarrow y_2[n] = \sum_{k=-\infty}^n x[k-n_0] = \sum_{k=-\infty}^{n-n_0} x[k] \Rightarrow y_2[n] = y[n-n_0] \rightarrow \underline{T.I.}$$

$$h. \quad \begin{cases} y_1[n] = x_1[n]x_1[1] \\ y_2[n] = x_2[n]x_2[1] \end{cases}, \quad y_3[n] = (ax_1[n] + bx_2[n])(ax_1[1] + bx_2[1])$$

$$\Rightarrow y_3[n] \neq ay_1[n] + by_2[n] \rightarrow \text{Non-Linear.}$$

$$y[n] = x[n]x[1]$$

$$x_2[n] = x[n-n_0] \rightarrow y_2[n] = x[n-n_0]x[1] \Rightarrow y_2[n] = y[n-n_0] \rightarrow \underline{T.I.}$$

$$6) a. \quad t < t+5 \rightarrow \text{غير محدد} \xrightarrow{\hat{t}=-1} y(-1) = 4x(4)$$

$$b. \quad t=0 \Rightarrow y(0) = x(0)x(3) \rightarrow \text{غير محدد}$$

$$c. \quad n \geq -|n| \rightarrow \text{غير محدد}$$

$$\underline{7. a.} |x(t)| < A \Rightarrow -4A+5 < 4x+5 < 4A+5 \Rightarrow |y(t)| < B \rightarrow \text{میدار}$$

b. در صورتی که  $x(t)$  ورودی باشد ممکن است  $\frac{dx(t)}{dt}$  محدود باشد. (در صورتی که ناپوشانی باشد) سی پایداری نیست.

$$c. |x(t)| < A \Rightarrow \left| \frac{\sin(x(t))}{x(t)} \right| < B$$

$$\star \text{ توهم داریم باشد } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$d. n \rightarrow \infty \Rightarrow (n-1) \rightarrow \infty \Rightarrow (n-1)x[n] \rightarrow \infty \Rightarrow \text{ناپایداری}$$

$$\underline{8. a.} y(t) = 2x\left(\frac{t}{2}\right) \quad \forall y_1 = y_2 \stackrel{?}{\Rightarrow} x_1 = x_2$$

$$y_1(t) = y_2(t) \Rightarrow 2x_1(t/2) = 2x_2(t/2) \Rightarrow x_1(t) = x_2(t) \rightarrow \text{برگشت پذیر}$$

$$b. \quad \forall y_1 = y_2 \stackrel{?}{\Rightarrow} x_1 = x_2$$

$$y_1[n] = y_2[n] \Rightarrow |x_1[n]| = |x_2[n]| \Rightarrow x_1[n] = \pm x_2[n] \rightarrow \text{معکوس پذیر نیست}$$

$$\underline{c.} \quad n > 10 \text{ و } n < 1 \text{ و } 1 \leq n \leq 10 \text{ در خروجی نقش دارند. یعنی تمام } n \text{ ها نقش دارند. حال بابر}$$

بررسی کنیم که آیا از روی خروجی به ورودی می‌توانی رسم یا نه.

$$x[n] = 0 \rightarrow x[1] \dots x[10] \quad \star \rightarrow \text{معکوس پذیر نیست.}$$

$$\underline{9. a.} \omega_1 = \frac{5\pi}{4} \Rightarrow \omega_2 < \omega_1$$

$$\omega_2 = \frac{3\pi}{4}$$

$$b. \quad N_1 = \frac{2\pi}{\frac{5\pi}{4}} \times k = 8 \rightarrow \text{فرکانس‌ها برابر}$$

$$N_2 = \frac{2\pi}{\frac{3\pi}{4}} \times k = 8$$

$\star$  توهم داریم باشد در سبک (های) گسسته، حوزه فرکانس متناوب با دوره متناوب  $2\pi$  است.