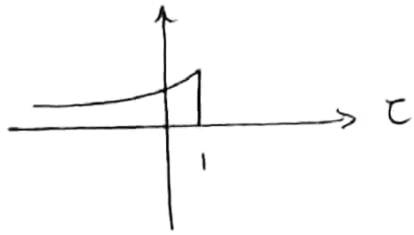
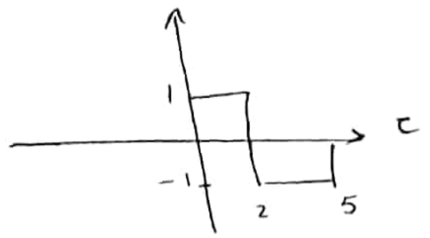
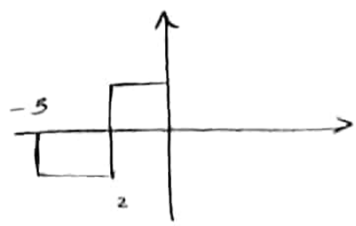
1) **a.** $y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$

$x(\tau) = e^{2\tau} u(1-\tau)$ $\quad h(\tau) \quad h(-\tau)$



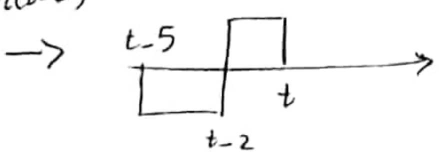


$\rightarrow$



$h(t-\tau)$

$\rightarrow$



حالت ۱ : $t-5 > 1 \Rightarrow t > 6 \Rightarrow y = 0$

حالت ۲ : $\begin{cases} t-5 < 1 \\ t-2 > 1 \end{cases} \Rightarrow 3 < t < 6 \Rightarrow h(t-\tau) = -1$

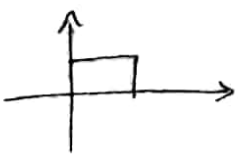
$\Rightarrow y(t) = \int_{t-5}^{1} e^{2\tau}(-1)\, d\tau = -\left[\frac{e^{2\tau}}{2}\right]_{t-5}^{1} = \frac{1}{2}\left(e^{2t-10} - e^{2}\right)$

حالت ۳ : $\begin{cases} t-2 < 1 \\ t > 1 \end{cases} \Rightarrow 1 < t < 3 \Rightarrow \begin{cases} h(t-\tau) = -1, & t-5<\tau<t-2 \\ h(1-\tau) = -1, & t-2 < \tau < 1 \end{cases} \Rightarrow y(t) = \int_{t-5}^{t-2} -e^{2\tau} d\tau$

$+ \int_{t-2}^{1} e^{2\tau} d\tau = \left[\frac{-e^{2\tau}}{2}\right]_{t-5}^{t-2} + \left[\frac{e^{2\tau}}{2}\right]_{t-2}^{1} = -e^{2t-4} + \frac{e^{2t-10}}{2} + \frac{e^{2}}{2}$

حالت ۴ : $t < 1 \Rightarrow y(t) = \int_{t-5}^{t-2} -e^{2\tau} d\tau + \int_{t-2}^{t} e^{2\tau} d\tau = -e^{2t-4} + \frac{1}{2}e^{2t-10} + \frac{e^{2t}}{2}$

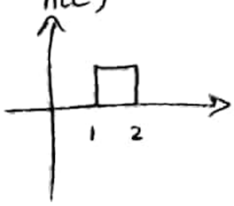
b. $x(\tau)$ $\quad x(-\tau)$ $\quad x(t-\tau)$



$\rightarrow$

$\rightarrow$

$h(\tau)$

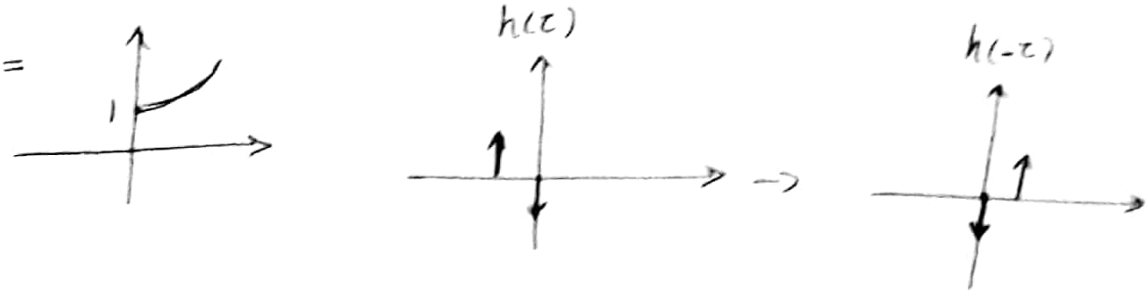


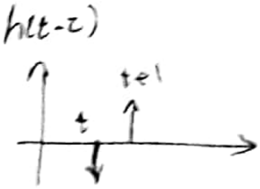
1) $t < 1 \Rightarrow y(t) = 0$

2) $1 < t < 2 \Rightarrow y(t) = \int_{1}^{t} d\tau = t - 1$

3) $2 < t < 3 \Rightarrow y(t) = \int_{t-1}^{2} d\tau = 3 - t$ $\quad$ 4) $t > 3 \Rightarrow y(t) = 0$

c. $x(\tau) =$



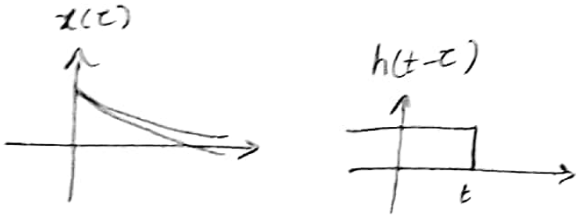


1) $t+1<0 \Rightarrow t<-1 \longrightarrow y(t)=0$

2) $-1<t<0 \Rightarrow y(t) = \int e^{\tau}\cdot\delta(\tau-(t+1))\,d\tau = e^{t+1}$

3) $0<t \Rightarrow y(t) = e^{t+1} - e^{t}$

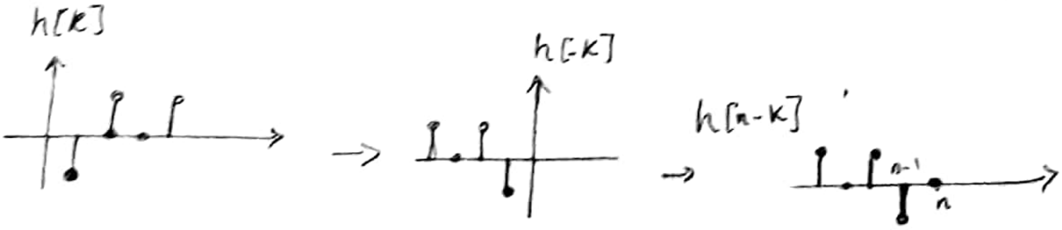
d.

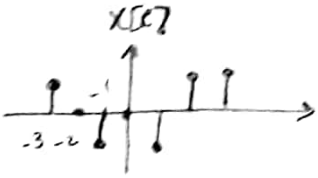


1) $t<0 \Rightarrow y(t)=0$

2) $t>0 \Rightarrow y(t) = \int_0^t e^{-a\tau}\,d\tau = \left[-\frac{1}{a}e^{\tau}\right]_0^t = \frac{1}{a}\left(1-e^{-at}\right)$

2) $h[n] = -\delta[n-1] + \delta[n-2] + \delta[n-4]$, $y[n] = x[n] * h[n]$





$n<-2 \rightarrow y[n]=0$

$y[-2] = -1$, $y[-1] = 0+0+1 = 1$

$y[0] = 0+1 + 0+0 = 1$, $y[1] = 0+0-1+1 = 0$

$y[2]=1$, $y[3] = -3$, $y[4]=0$, $y[5]=0$

$y[6]=1$, $y[7]=1$

3)

$$A = \underbrace{\int_{-1.5}^{0.5} (t+3t^2)\,\delta(3t^2-3)\,dt}_{A_1} + \underbrace{\int_{-1.5}^{0.5} e^{-\cos t + 2\sin t}\,\delta(t-\tfrac{\pi}{6})\,dt}_{A_2}$$

A_1 : $\delta(f(t))$, $f(t) = K(t-a_1)(t-a_2)\ldots$ درصورتی که آرگومان δ چندجمله ای باشد به صورت زیر می توانیم تجزیه کنیم:

$$\Rightarrow \delta(f(t)) = \sum_{m=1}^{n} \frac{1}{|f'(a_m)|}\,\delta(t-a_m)$$

$$f(t) = 3t^2-3 \Rightarrow \delta(3t^2-3) = \frac{1}{6}\delta(t-1) + \frac{1}{6}\delta(t+1)$$

$$\Rightarrow A_1 = \int_{-1.5}^{0.5} \frac{1}{6}(t+3t^2)\,\delta(t+1)\,dt + \cancel{\int_{-1.5}^{0.5} \frac{1}{6}(t+3t^2)\,\delta(t-1)\,dt} \to 0$$

$$= \frac{1}{6}(-1+3) = 1/3$$

$$A_2 : \int_{-1.5}^{0.5} e^{-\cos t + 2\sin t}\,\delta(t-\pi/6)\,dt = 0 \quad (\pi/6 > 0.5)$$

4.) $x(t) = \delta(2t-2\tau) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \delta(t-3\tau)$

خاصیت δ

$$x(t) = \tfrac{1}{2}\,\delta(t-\tau) \to \boxed{} \to \delta(t-3\tau)$$

همگنی $\delta(t-\tau) \longrightarrow \boxed{} \to 2\,\delta(t-3\tau)$

همگنی $x(\tau)\,\delta(t-\tau) \to \boxed{} \to 2x(\tau)\,\delta(t-3\tau)$

خطی بودن $\int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau \to \boxed{} \to \int_{-\infty}^{+\infty} 2x(\tau)\,\delta(t-3\tau)\,d\tau$

$$x(t) \longrightarrow \boxed{} \to y(t) = 2\int x(\tau)\,\delta(t-3\tau)\,d\tau$$

$$= 2\int x(\tau)\,\tfrac{1}{3}\,\delta(\tfrac{t}{3}-\tau)\,d\tau = \tfrac{2}{3}\,x(t/3) \Rightarrow y(t) = \tfrac{2}{3}\,x(t/3)$$

5) a. علّی نیست زیرا به ازای $n<0$ مقدار غیر صفر دارد — پایدار است. زیرا مقدار $\sum_{n=-\infty}^{+\infty} |h[n]|$ محدود باقی می‌ماند ($5^{-3}+5^{-2}+5^{-1}+1+\dots$) — حافظه‌دار است. زیرا شرط حافظه‌دار نبودن این است که $h[n]=k\delta[n]$ باشد.

b. علّی نیست. (شرط $h[n]=0 \Rightarrow n<0$ برقرار نیست.) — پایدار نیست ($\sum_{k=-\infty}^{+\infty} |h[n]| = \frac{1}{2}+4+8+\dots$)
حافظه‌دار است. ($h[n] \neq k\delta[n]$)

c. علّی است. ($h[n]=0 \Leftarrow n<0$) ناپایدار است. (وقتی $n\to\infty$ مقدار $\infty \Leftarrow h[n]$) .
حافظه‌دار است. ($h[n] \neq k\delta[n]$)

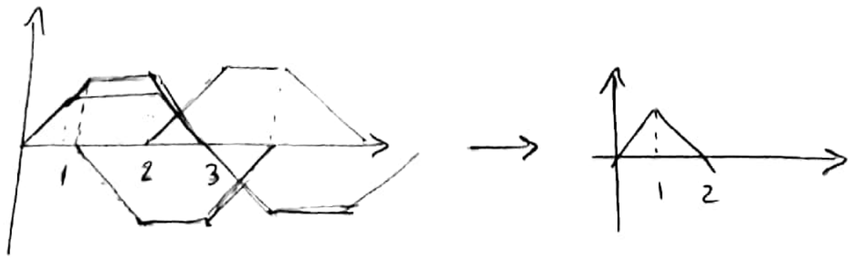
d. علّی نیست — پایدار است — حافظه‌دار است.

e. علّی نیست — پایدار است — حافظه‌دار است.

6)

$$x_2(t) = x_1(t) - x_1(t-1) + x_1(t-2) - x_1(t-3) + \dots$$

$$\Rightarrow y_2(t) = y_1(t) - y_1(t-1) + y_1(t-2) - y_1(t-3) + \dots$$



7)

$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau \qquad x(kt) * y(kt) = \int_{-\infty}^{+\infty} x(k\tau)\, y(kt-k\tau)\, d\tau$$

$$\Rightarrow \lambda = k\tau \Rightarrow d\lambda = |k|\, d\tau \Rightarrow \int_{-\infty}^{+\infty} x(\lambda)\, y(kt-\lambda)\, \frac{1}{|k|}\, d\lambda = \frac{1}{|k|} z(kt)$$

8)

$$x_3(t) = x_1(t) + x_1(t-1) - x_2(t)$$ با استفاده از خاصیت خطی بودن.

$$\Rightarrow y_3(t) = y_1(t) + y_1(t-1) - y_2(t)$$