Subject: Homework 3 Solutions (Q1 - Q3)
Year. Month Date

1-a) $x(t) = e^{-t} \quad -1<t<1$

$$a_k = 1/T \int x(t) e^{-j w k t} dt = 1/2 \int_{-1}^{1} e^{-t} e^{-t j k w_0} dt = 1/2 \int_{-1}^{1} e^{-(1+jkw_0)} dt =$$

$$= \frac{1}{2} \times \frac{e^{-(1+jkw_0)}}{-(1+jkw_0)} \Big|_{-1}^{1} = 1/2 \times \frac{e^{-(1+jkw_0)} - e^{(1+jkw_0)}}{-(1+jkw_0)}$$

$$\Rightarrow a_k = \frac{j}{1+jkw_0} \times \sin(1+jkw_0) = j\pi \operatorname{sinc}(1+jkw_0)$$

1-b) $x(t) = \begin{cases} 2 & -1<t<0 \\ -2 & 0<t\leq 1 \end{cases}$, $T=2$, $w_0 = 2\pi/T = \pi$

$$a_k = 1/2 \left[\int_{-1}^{0} 2e^{-jkw_0 t} dt - 2\int_{0}^{1} e^{-jkw_0 t} dt \right] =$$

$$= \int_{-1}^{0} e^{-jkw_0 t} dt - \int_{0}^{1} e^{-jkw_0 t} dt = \frac{e^{-jkw_0 t}}{-jkw_0} \Big|_{-1}^{0} - \frac{e^{-jkw_0 t}}{-jkw_0} \Big|_{0}^{1} =$$

$$= \frac{2 + e^{jkw_0} - e^{-jkw_0}}{-jkw_0} = \frac{2 + e^{jk\pi} - e^{-jk\pi}}{-jk\pi}$$

1-c) $x(t) = |\cos(w_0 t)|$, $T = \pi/w_0$, $w_0' = 2\pi / (\pi/w_0) = 2w_0$

$$a_k = 1/T \int x(t) e^{-jw_0 t} dt = 1/k/w_0 \int_{-\pi/2w_0}^{\pi/2w_0} \cos(w_0 t) e^{-jk2w_0 t} dt =$$

$$= \frac{w_0}{\pi} \int_{-\pi/2w_0}^{\pi/2w_0} \left(\frac{e^{jw_0 t} + e^{-jw_0 t}}{2} \right) e^{-jk2w_0 t} dt = \frac{j}{\pi j} \left[\frac{(-1)^k + (-1)^k}{1-2k} + \frac{(-1)^k + (-1)^k}{1+2k} \right] =$$

$$= \frac{(-1)^4}{\pi} \left(\frac{2}{1-4k^2} \right) = 1/\pi \left(2/1-4k^2 \right)$$

1-d) $x(t)=\sin(2\pi 3t)\cos(2\pi t) = 1/2\left(\sin(6\pi-2\pi)t+\sin(6\pi+2\pi)t\right)$

$= \frac{1}{2}\left(\sin(8\pi t)-\sin(4\pi t)\right)$ $\quad T_1=1/4,\ T_2=1/2 \quad \omega_0=\frac{2\pi}{1/2}=4\pi$

$$x(t)=1/2\left[\sin(8\pi t)-\sin(4\pi t)\right]=1/2\left[\frac{e^{j8\pi t}-e^{-j8\pi t}}{2j}-\frac{e^{j4\pi t}-e^{-j4\pi t}}{2j}\right]$$

$$=1/4j\left(e^{j8\pi t}-e^{-j8\pi t}-e^{j4\pi t}+e^{j4\pi t}\right)$$

$a_1=-1/4j \quad a_{-1}=-1/4j \quad a_2=1/4j \quad a_{-2}=1/4j$

2 a) $x(t-2)+x(t+2) \Rightarrow b_k=a_k e^{-jk\omega_0(2)}+a_k e^{jk\omega_0}$

$b_k=a_k\left(e^{-j2k\omega_0}+e^{jk2\omega_0}\right),\ \omega_0=2\pi/t$

b) $dx(t)/dt \Rightarrow b_k=(jk\omega_0)a_k$

c) $y(t)=x^*(t)+x(-t/2) \Rightarrow \begin{cases} y_1(t)=x^*(2t) \Rightarrow b_{1k}=a^*_k,\ T_1=T/2 \\ y_2(t)=x(-t/2) \Rightarrow b_{2k}=a_{-k},\ T_2=2t \end{cases}$

$b_k=b_{1k}+b_{2k} \Rightarrow T=$ ک.م.م $(T/2, 2T)=2T$

3 $y_k=(-1)^k x_k+(-1)^k x_{-k}=e^{-jk\pi}x_k+e^{-j\pi}x_{-k}$

$y(t)=x(t-T/2)+x(-t+T/2)$

* We know that: $x(t-T/2)\longrightarrow a_k e^{-jk\frac{2\pi}{T}\times T/2}$

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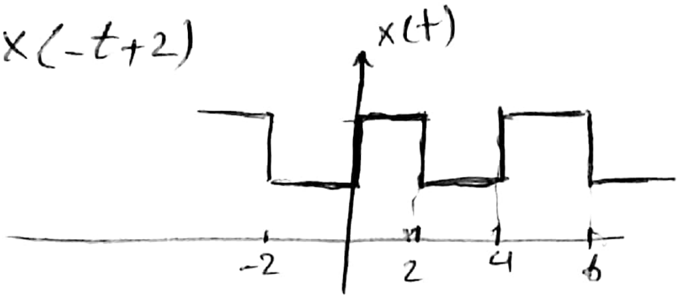
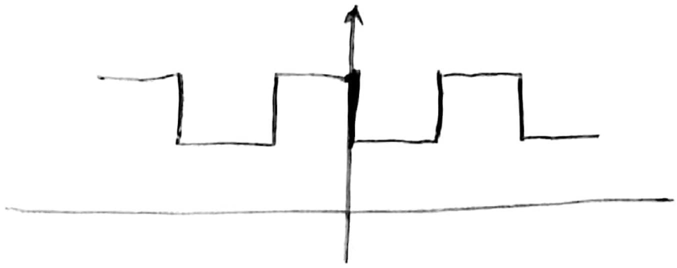
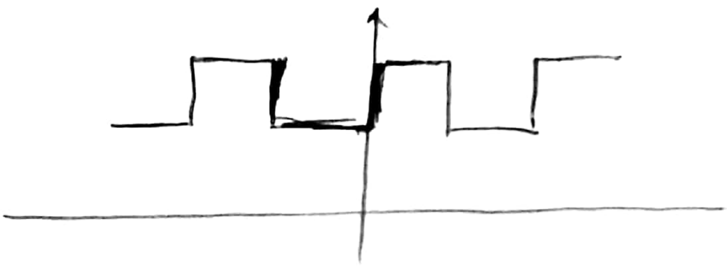
$$y(t) = x(t-2) + x(-t+2)$$



$x(t-2) \rightarrow$



$x(-t+2)$



$y(t) \Rightarrow$