Subject یاسمن سادات میرمحمد

Date ۹۴/۱۲/۰۲۲

سیگنال‌ها و سیستم‌ها

تمرین سری چهارم

۱. تبدیل فوریه

a. $x(t) = e^{-3|t|}\sin(2t)$

$$e^{-3|t|}\cdot\frac{e^{2jt}-e^{-2jt}}{2j} \longrightarrow \frac{1}{2j}\left(\underbrace{e^{2jt}}_{\text{شیفت فرکانس}}\cdot e^{-3|t|} - \underbrace{e^{2jt}}_{\text{شیفت فرکانس}}\cdot e^{-3|t|}\right)$$

$$X(w) = \frac{2(3)}{3^2+w^2} = \frac{6}{9+w^2}$$

$$= \frac{1}{2j}\left(X(w-2) - X(w+2)\right)$$

$$= \frac{1}{2j}\left(\frac{6}{9+(w-2)^2} - \frac{6}{9+(w+2)^2}\right)$$

b. $x(t) = \begin{cases} 1-t^2 & 0<t<1 \\ 0 & \text{o.w (otherwise)} \end{cases}$

$$I = \frac{e^{-jw}}{-jw} + \frac{2}{jw}\left(\frac{e^{-jw}}{-jw} + \frac{e^{-jw}}{w^2} - \frac{1}{w^2}\right)$$

$$I = \frac{e^{-jw}}{-jw} + \frac{2e^{-jw}}{w^2} + \frac{2e^{-jw}}{jw^3} - \frac{2}{jw^3}$$

$$x(t) = (1-t^2)(u(t)-u(t-1))$$

$$X(\omega) = \int_0^1 (1-t^2)\,e^{-jwt}\,dt$$

$$X(w) = \frac{1}{jw} - \frac{2\cdot e^{-jw}}{w^2} - \frac{2e^{-jw}}{jw^3} + \frac{2}{jw^3}$$

$$X(w) = \int_0^1 e^{-jwt}\,dt$$

$$I_1 = \int_0^1 t^2\cdot e^{-jwt}\,dt = \frac{t^2}{-jw}\cdot e^{-jwt}\Big|_0^1 - \int_0^1\left(\frac{-1}{jw}e^{-jwt}\right)2t\,dt$$

$$I_2 = \int_0^1 t\cdot e^{-jwt}\,dt = \underbrace{\frac{t}{-jw}e^{-jwt}\Big|_0^1}_{\frac{e^{-jw}}{-jw}} - \underbrace{\int_0^1\left(-\frac{1}{jw}e^{-jwt}\right)dt}_{\frac{-1}{jw}\left(\frac{e^{-jwt}}{-jw}\Big|_0^1\right)} = \frac{e^{-jw}}{(jw)^2} - \frac{1}{(jw)^2}$$

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c. $x(t) = \sin(2\pi t + \frac{\pi}{4})$

$\sin(2\pi t) \longrightarrow a_1 = \frac{1}{2j} , a_{-1} = \frac{-1}{2j} , w_0 = 2\pi$

$\rightarrow X_1(w) = \frac{2\pi}{2j} \delta(w-2\pi) - \frac{2\pi}{2j} \delta(w+2\pi)$

$\sin(2\pi t + \frac{\pi}{4}) \rightarrow e^{jw\frac{\pi}{4}} \cdot X_1(w)$

$\rightarrow X(w) = \frac{\pi}{j} e^{jw\frac{\pi}{4}} \delta(w-2\pi) - \frac{\pi}{j} e^{jw\frac{\pi}{4}} \delta(w+2\pi)$

$\rightarrow X(w) = \frac{\pi}{j} e^{j\frac{\pi^2}{2}} \delta(w-2\pi) - \frac{\pi}{j} e^{j\frac{\pi^2}{2}} \delta(w+2\pi)$

d. $x(t) = 1 + \cos(6\pi t + \frac{\pi}{8})$

$\cos(6\pi t) \rightarrow \frac{2\pi}{2} \delta(w-6\pi) + \frac{2\pi}{2} \delta(w+6\pi) = T(w)$

$\delta(t) \overset{F.T}{\longleftrightarrow} 1 \xrightarrow[\text{دوگانی}]{} 1 \longleftrightarrow 2\pi\delta(-w)$

$\cos(6\pi t + \frac{\pi}{6}) \longleftrightarrow e^{jw\frac{\pi}{4}} \cdot T(w) = \pi e^{j\frac{3\pi^2}{2}} \delta(w-6\pi) +$
$\pi e^{-j\frac{3\pi^2}{2}} \delta(w+6\pi)$

$x(t) = x_1(t) + x_2(t) \longleftrightarrow X(w) = X_1(w) + X_2(w)$

$X(w) = 2\pi\delta(-w) + \pi e^{j\frac{3\pi^2}{2}} \delta(w-6\pi)$
$+ \pi e^{-j\frac{3\pi^2}{2}} \delta(w+6\pi)$

$$\boxed{X(w) = 2\pi\delta(-w) + \pi e^{j\frac{3\pi^2}{2}} \delta(w-6\pi) + \pi e^{-j\frac{3\pi^2}{2}} \delta(w+6\pi)}$$

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e. $x(t) = \frac{\sin 3t \cdot \cos t}{\pi t}$

$$x(t) = \frac{1}{2}\,\frac{\sin(4t)+\sin(2t)}{\pi t} \longrightarrow x(t) = \frac{2}{\pi}\left(\frac{\sin \pi \times \frac{4}{\pi} t}{\pi \times \frac{4}{\pi} t}\right) + \frac{1}{\pi}\,\frac{\sin \pi \times \frac{2}{\pi} t}{\pi \times \frac{2}{\pi} \times t}$$

$$\frac{W}{\pi}\,\mathrm{sinc}\,\frac{t}{\pi}W \xleftrightarrow{F.T} \Pi\left(\frac{w}{W}\right)$$

$$\frac{1}{2} \times \frac{t}{\pi}\ \mathrm{sinc}\ \frac{4}{\pi} t$$

$$x(t) = \frac{2}{\pi}\cdot \mathrm{sinc}\,\frac{4}{\pi}t + \frac{1}{\pi}\,\mathrm{sinc}\,\frac{2}{\pi}t$$

$$\boxed{X(w) = \frac{1}{2}\Pi\left(\frac{w}{4}\right) + \frac{1}{2}\Pi\left(\frac{w}{2}\right)}$$

f. $t e^{-2|t-1|}$

$$\longleftrightarrow \frac{4}{4+w^2}$$

$$e^{-2|t-1|} \longleftrightarrow e^{-jw} \times \frac{4}{4+w^2} = Y(w)$$

$$-jt\, e^{-2|t-1|} \longleftrightarrow \frac{d}{dw}\left(Y(w)\right) = \frac{(4+w^2)(-4je^{-jw}) - 8we^{-jw}}{(4+w^2)^2}$$

$$\longrightarrow t e^{-2|t-1|} = \frac{1}{j}\cdot \frac{e^{-jw}(16j + 4jw^2 + 8w)}{(4+w^2)^2}$$

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۲. عکس تبدیل فوریه

$a.\ X(w) = w \cdot e^{-|w|}$

$$e^{-|t|} \longleftrightarrow \frac{2}{1+w^2}$$

دوگانی

$$\frac{2}{1+t^2} \longleftrightarrow 2\pi e^{-|w|} \xrightarrow{\frac{d}{dt}} \frac{-2t(2)}{(1+t^2)} \xleftrightarrow{f.s} jw \cdot 2\pi e^{-|w|}$$

$$\longrightarrow F^{-1}\{we^{-|w|}\} = \frac{1}{2\pi j} \cdot \frac{-4t}{(1+t^2)^2} = \frac{2tj}{\pi(1+t^2)^2}$$

$$F^{-1}\{we^{-|w|}\} = \frac{2tj}{\pi(1+t^2)^2}$$

$$b.\ X(w) = \begin{cases} e^{-w}, & w>0 \\ -e^{w}, & w<0 \end{cases}$$

$$x(w) = e^{-w}u(w) - e^{w} \cdot u(-w)$$

$$e^{-t} \cdot u(t) \longleftrightarrow \frac{1}{1+jw}$$

$$\frac{1}{1+jt} \xleftrightarrow{f.s} 2\pi e^{-w} \cdot u(w) \longrightarrow F^{-1}\{e^{-w} \cdot u(w)\} = \frac{1}{2\pi} \times \frac{1}{1+jt}$$

$$\frac{1}{1-jt} \xleftrightarrow{f.s} 2\pi e^{w} \cdot u(-w) \longrightarrow F^{-1}\{e^{w} \cdot u(-w)\} = \frac{1}{2\pi} \times \frac{1}{1-jt}$$

$$\longrightarrow F^{-1}\{X(w)\} = \frac{1}{2\pi}\left\{\frac{1}{1+jt} - \frac{1}{1-jt}\right\}$$

c. $X(w) = \dfrac{۲a - jw}{۲a + jw}$

$$\underbrace{\frac{۲a}{۲a+jw}}_{\mathrm{I}} - \underbrace{\frac{jw}{۲a+jw}}_{\mathrm{II}}$$

I $\quad \dfrac{۲a}{۲a+jw} \xleftrightarrow{F.S} ۲a \left(F^{-1}\left\{\dfrac{1}{۲a+jw}\right\}\right) = ۲a \cdot e^{-۲at} \cdot u(t)$

II $\quad \dfrac{-jw}{۲a+jw} \xleftrightarrow{F.S} \dfrac{-d}{dt}\left(F^{-1}\left\{\dfrac{1}{۲a+jw}\right\}\right) = -\left(-۲a e^{-۲at} u(t) + e^{-۲at} \cdot \delta(t)\right)$

نتیجه : $F^{-1}\{X(w)\} = ۲a \cdot e^{-۲at} \cdot u(t) - \delta(t)$

d. $X(w) = \dfrac{d}{dw}\left\{\dfrac{\sin ۲w - j\cos ۲w}{1 + \frac{jw}{۳}}\right\}$

متغیرها را مرتب کنیم $\rightarrow$ I $\quad \dfrac{۳\sin ۲w}{۳+jw} = ۳\sin ۲w \times \underbrace{\dfrac{1}{۳+jw}}_{X_1(w)} = ۳ \times \dfrac{1}{۲j}\left(e^{۲jw} \cdot X_1(w) - e^{-۲jw} \cdot X_1(w)\right)$

$$\rightarrow e^{-۳t} \cdot u(t) \leftrightarrow \frac{1}{۳+jw}$$

$$\rightarrow F^{-1}\underbrace{\left\{\frac{\sin ۲w}{1+\frac{jw}{۳}}\right\}}_{\mathrm{I}} = \frac{۳j}{۳}\left(e^{-۳(t+۲)} \cdot u(t+۲) - e^{-۳(t-۲)} \cdot u(t-۲)\right) \quad \mathrm{I}$$

II $\quad ۳j\cos ۲w \times \dfrac{1}{۳+jw} = \dfrac{۳j}{۲}\left(e^{۲jw} \cdot X_1(w) + e^{-۲jw} \cdot X_1(w)\right)$

$$F^{-1}\left\{\frac{۳j\cos ۲w}{۳+jw}\right\} = \frac{۳j}{۲}\left(e^{-۳(t+۲)} \cdot u(t+۲) + e^{-۳(t-۲)} \cdot u(t-۲)\right) \quad \mathrm{II}$$

$$F^{-1}\{X(w)\} = -jt\, F^{-1}\{\mathrm{I} + \mathrm{II}\} = -jt\,(\mathrm{I} + \mathrm{II})$$

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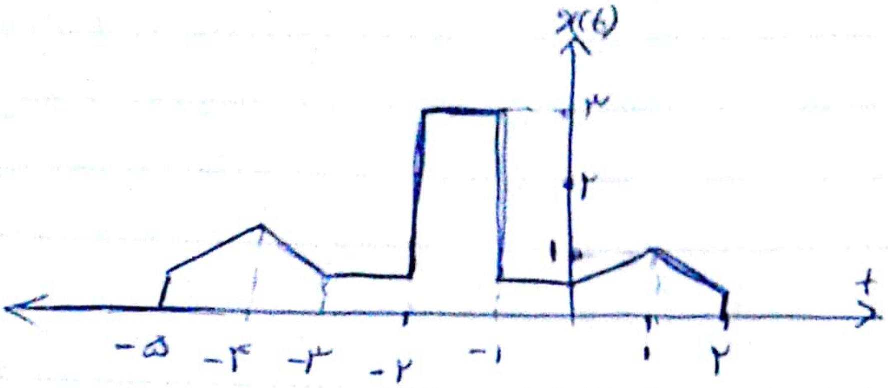
۳. $x(t) \xrightarrow{FS} X(w)$



الف. $X(0)$

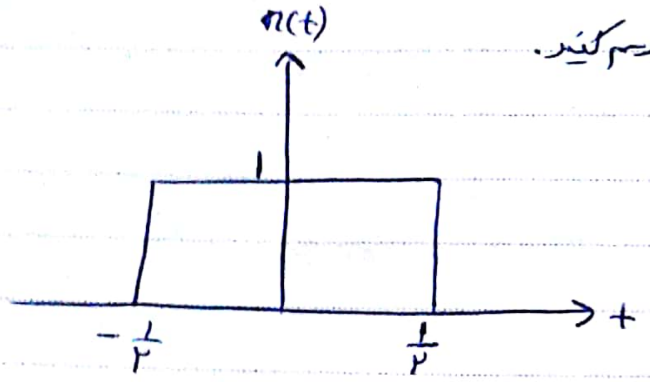
$$x(t) = \begin{cases} \frac{1}{۲}x+۳ & -۵\le t\le -۴ \xrightarrow{F.T} ۳٫۷۶\ \delta(w) + (۱,۲۵)\, i\, \delta'(w) \\ -\frac{1}{۲}x-۱ & -۴\le t\le -۳ \xrightarrow{F.T} -۷۶n\ \delta(w) + (۱,۲۷)\, i\, \delta'(w) \\ ۰٫۵ & -۳< t\le -۲ \xrightarrow{F.T} ۱,۲۵\ \delta(w) \\ ۳ & -۱\le t<۰ \xrightarrow{F.T} ۳٫۷۴\ \delta(w) \\ \frac{1}{۲}x-\frac{1}{۲} & ۰\le t\le ۱ \xrightarrow{F.T} ۱,۲۵\ \delta(w) \\ & \xrightarrow{F.T} ۱,۲۵\,\delta(w) - ۱,۲۵\, i\, \delta'(w) \\ -\frac{1}{۲}x+\frac{۳}{۲} & ۱\le t\le ۲ \xrightarrow{F.T} ۱٫۷۵\ \delta(w) + ۱,۲۵)\ \delta'(w) \end{cases}$$

$\rightarrow X(0) = ۱,۲۵\ \delta(0)$

$\boxed{X(0) = ۱,۲۵}$

ب. $\int_{-\infty}^{\infty} X(w)\, dw$

۴. تبدیل فوریه‌ی این سیگنال‌ها را بدست آورده و رسم کنید.
($\Pi(t)$ یک پالس متقارن با عرض ۱ است)

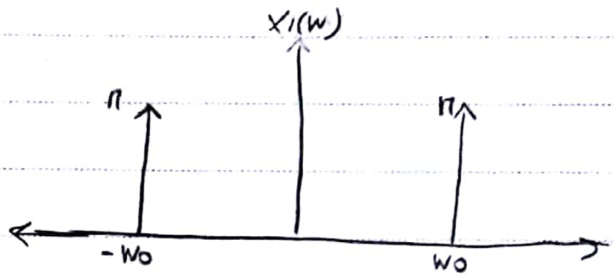


a. $x_1(t) = \cos(w_0 t)$

$$X_1(w) = \frac{\cancel{2}\pi}{\cancel{2}}\delta(w-w_0) + \frac{\cancel{2}\pi}{\cancel{2}}\delta(w+w_0)$$

$x_1(t)$ is periodic

دو تابع ضربه به این شکل می‌کشد



$x(t) = \cos(w_0 t)$

$$\boxed{X(w) = \pi\,\delta(w-w_0) + \pi\,\delta(w+w_0)}$$

b. $x_2(t) = \sin(w_0 t)$

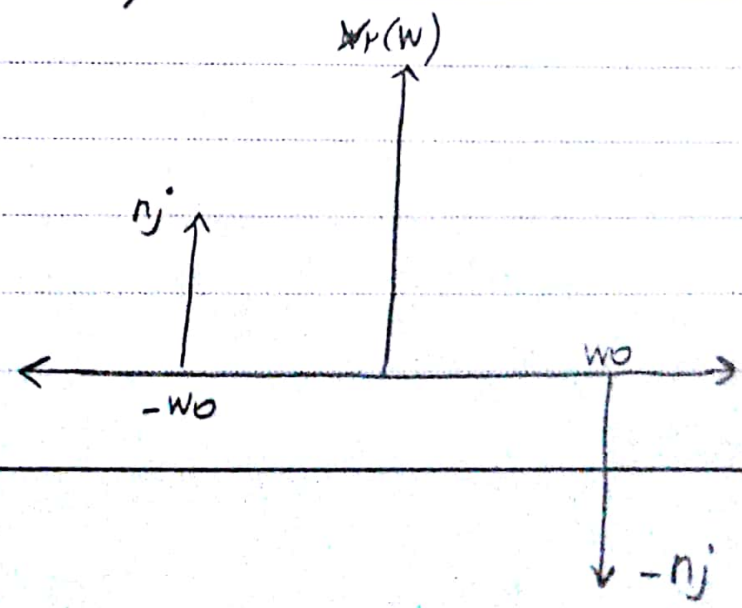
$x_2(t)$ is periodic

$x(t) = \sin(w_0 t)$

$$\boxed{X(w) = \frac{\pi}{j}\delta(w-w_0) + \left(-\frac{\pi}{j}\right)\delta(w+w_0)}$$

$$X_2(t) = \frac{\cancel{2}\pi}{\cancel{2}j}\delta(w-w_0) + \left(-\frac{\cancel{2}\pi}{\cancel{2}j}\right)\delta(w+w_0)$$

دو تابع ضربه‌ی مختلط به شکل زیر می‌شود



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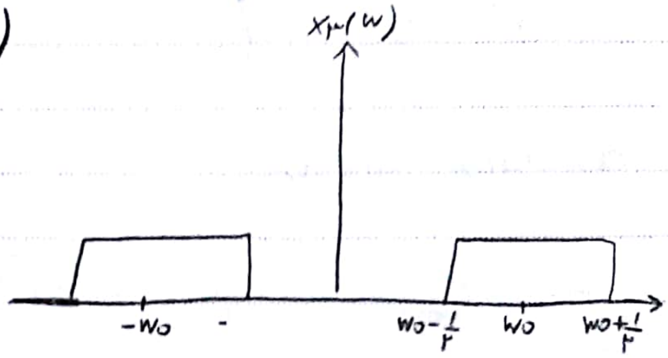
c. $x_3(t) = x_1(t) \cdot F^{-1}\{\Pi(w)\}$

$$X_3(w) = \frac{1}{2\pi}\left(X_1(w) * \Pi(w)\right) =$$

$$X_3(w) = \frac{1}{2\pi}\left(\pi\delta(w-w_0) * \Pi(w) + \pi\,\delta(w+w_0) \cdot \Pi(w)\right)$$

$$X_3(w) = \frac{1}{2}\Pi(w-w_0) + \frac{1}{2}\Pi(w+w_0)$$

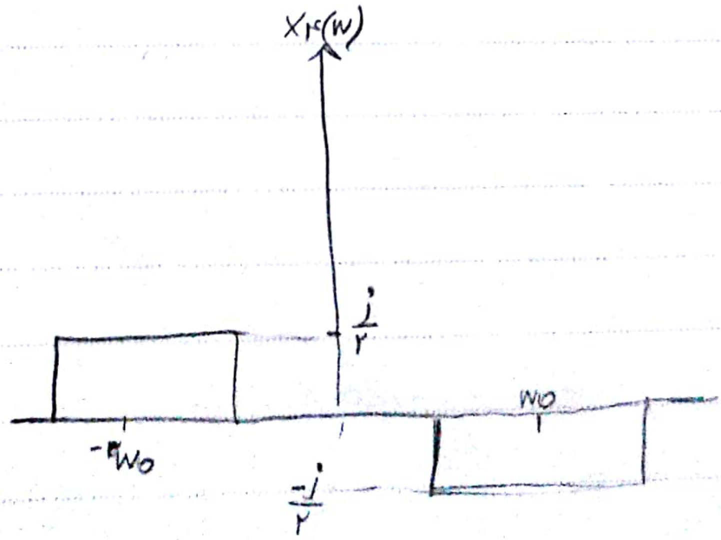


d. $x_4(t) = x_3(t) \cdot F_1\{\Pi(w)\}$

$$X_4(w) = \frac{1}{2\pi}\left(X_2(w) * \Pi(w)\right)$$

$$X_4(w) = \frac{1}{2\pi}\left(\pi j\,\delta(w+w_0) * \Pi(w) - \pi j\,\delta(w-w_0) * \Pi(w)\right)$$

$$X_4(w) = \frac{j}{2}\Pi(w+w_0) - \frac{j}{2}\Pi(w-w_0)$$



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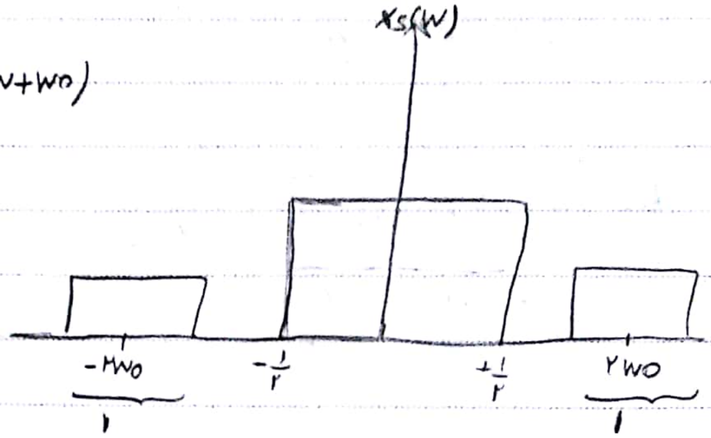
e. $x_s(t) = x_3(t) \cdot x_1(t)$

$$X_s(w) = \frac{1}{2\pi}\left(X_3(w) * X_1(w)\right)$$

$$X_s(w) = \frac{1}{2\pi}\left(\pi\delta(w-w_0) * X_3(w) + \pi\delta(w+w_0) * X_3(w)\right)$$

$$X_s(w) = \frac{1}{2} X_3(w-w_0) + \frac{1}{2} X_3(w+w_0)$$



۵. (سیستم زمان پیوسته‌ی علّی)

$$\left.\begin{array}{l} \text{input}: x(t) \\ \text{output}: y(t) \end{array}\right\} \longrightarrow \frac{d}{dt}y(t) + y(t) = x(t)$$

ضرایب سری فوریه‌ی خروجی $y(t)$ را برای هر یک از ورودی‌های زیر بیابید:

a. $x(t) = \cos(3\pi t + \frac{\pi}{4})$

$$\frac{d}{dt}y(t) + y(t) = x(t)$$

$$jkw_0 b_k + b_k = a_k \longrightarrow b_k = \frac{a_k}{1+jw_0k}$$

$$\left.\begin{array}{l} x(t) \to a_k \\ y(t) \to b_k \end{array}\right\} \to \boxed{\frac{d}{dt}y(t) \to jkw_0b_k} \;\text{و}\; \begin{cases} x(t) = \sum a_k e^{jkw_0t} \\ y(t) = \sum b_k e^{jkw_0t} \end{cases}$$

$$b_k = a_k \cdot \underline{H(kw_0)} \qquad \frac{1}{1+jkw_0}$$

حل a

$\cos(3\pi t) \to w_0 = 3\pi$ و $a_1 = a_{-1} = \frac{1}{2}$

$$\cos(3\pi t + \frac{\pi}{4}) \to \begin{cases} a_1 = e^{-j3\pi(-\frac{\pi}{4})} \\ a_{-1} = e^{j3\pi(-\frac{\pi}{4})} \end{cases} \;\text{و}\; \begin{cases} a'_1 = \frac{1}{2}e^{\frac{3\pi^2}{4}j} \\ a'_{-1} = \frac{1}{2}e^{-\frac{3\pi^2}{4}j} \end{cases}$$

$$b_k \begin{cases} b_1 = \dfrac{\frac{1}{2}e^{\frac{3\pi^2}{4}j}}{1+3\pi j} \\ b_{-1} = \dfrac{\frac{1}{2}e^{-\frac{3\pi^2}{4}j}}{1-3\pi j} \end{cases} \longrightarrow y(t) = \sum b_k e^{jk3\pi t}$$

$$\boxed{y(t) = b_1 e^{j3\pi t} + (b_{-1}) \cdot e^{-j3\pi t}}$$

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b. $\sin(\pi t) + \cos(3\pi t + \frac{\pi}{6})$

$\sin(\pi t) \to \begin{cases} c_1 = \frac{1}{2j} \\ c_{-1} = \frac{-1}{2j} \end{cases}$, $w_0 = \pi$

$\cos(3\pi t) \to \begin{cases} d'_1 = \frac{1}{2} \\ d'_{-1} = \frac{1}{2} \end{cases}$, $w_0 = 3\pi$

$\begin{cases} d_1 = e^{-j(3\pi)(-\frac{\pi}{6})} , \; d'_1 = \frac{e^{\frac{j\pi^2}{2}}}{2} \\ d_{-1} = e^{+j(3\pi)(-\frac{\pi}{6})} , \; d'_{-1} = \frac{e^{\frac{-j\pi^2}{2}}}{2} \end{cases}$

$\to a_K = c_K + d_K$, $\underbrace{w_0 = 3\pi}_{\text{ک.م.م}}$

دوره تناوب

$\begin{cases} a_1 = \frac{1}{2j} + \frac{e^{\frac{j\pi^2}{2}}}{2} \\ a_{-1} = \frac{-1}{2j} + \frac{e^{\frac{-j\pi^2}{2}}}{2} \end{cases}$

$b_K = \frac{a_K}{1 + jKw_0}$

میدانیم

$\to \begin{cases} b_1 = \frac{a_1}{1 + 3\pi j} \\ b_{-1} = \frac{a_{-1}}{1 - 3\pi j} \end{cases}$

$\xrightarrow{\text{و}} y(t) = \sum_{-\infty}^{+\infty} b_K \cdot e^{jK3\pi t}$

$y(t) = b_1 \cdot e^{j3\pi t} + b_{-1} \cdot e^{-j3\pi t}$

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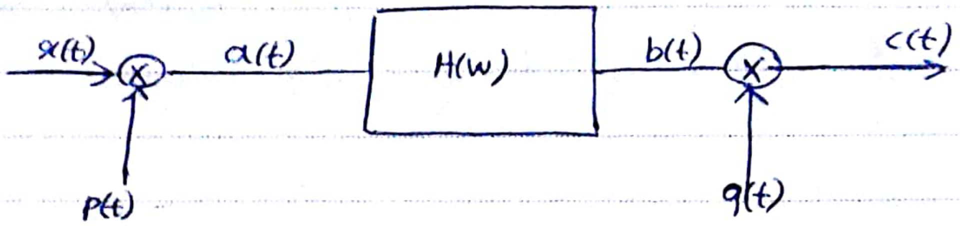


$x(t) = \frac{\sin \pi t}{\pi t}$ ، $p(t) = \cos 4\pi t$ ، $q(t) = \frac{\sin 5\pi t}{\pi t}$

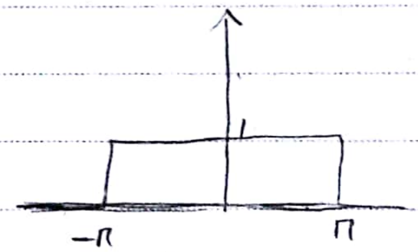
$$H(w) = \begin{cases} 1 & |w| \geq 2\pi \\ 0 & |w| < 2\pi \end{cases}$$

الف. $A(w) = ?$ $\quad$ plot +
$B(w) = ?$
$C(w) = ?$

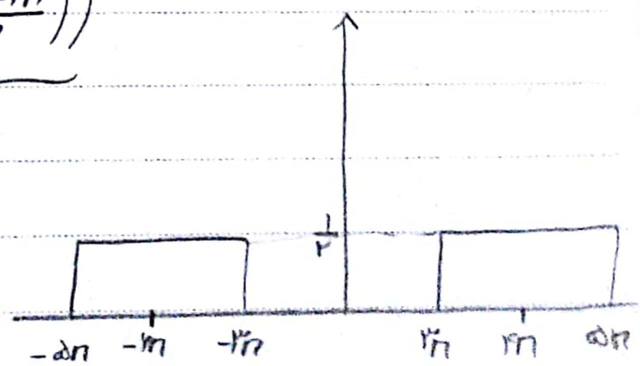
$a(t) = x(t) \times p(t)$

$a(t) = \text{sinc}\, t \times \cos 4\pi t$

- $\Pi\left(\frac{t}{T}\right) \xleftrightarrow{F.S} T\, \text{sinc}\left(\frac{w}{\pi} \times \frac{T}{2}\right)$
- $\text{sinc}\left(\frac{t}{\pi} \times \frac{2\pi}{2}\right) \xleftrightarrow{F.S} \underbrace{\frac{1}{\cancel{2\pi}}\left(\cancel{2\pi} \cdot \Pi\left(\frac{-w}{2\pi}\right)\right)}_{\Pi\left(\frac{w}{2\pi}\right)}$

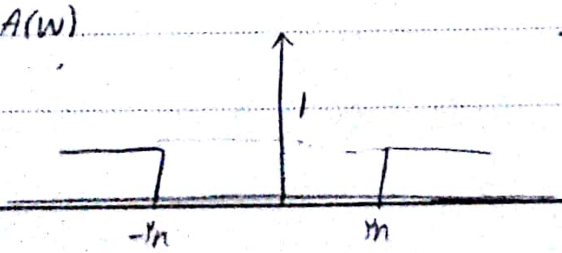


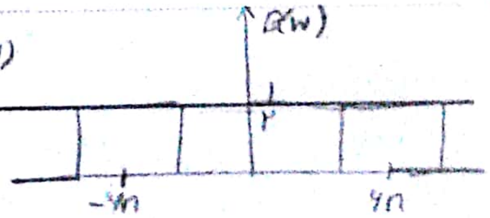
$$\cos 4\pi t \cdot \text{sinc}\, t \longleftrightarrow \frac{1}{2}\left(X(w+4\pi) + X(w-4\pi)\right)$$
$$\underbrace{\frac{1}{2}\left(\Pi\left(\frac{w+4\pi}{2\pi}\right) + \Pi\left(\frac{w-4\pi}{2\pi}\right)\right)}_{A(w)}$$



$b(t) = a(t) * h(t)$

$B(w) = H(w) \cdot A(w)$





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b. $c(t) = ?$

(به جای استفاده از کانولوشن در حوزه زمان، میتوان در حوزه فرکانس، سیگنالها را در هم ضرب کرد)

$$c(t) = b(t) \times q(t)$$

$$A(w) = B(w) \longrightarrow F^{-1}\{A(w)\} = a(t) = F^{-1}\{B(w)\} = b(t)$$

$$\longrightarrow c(t) = a(t) \times q(t) = \underbrace{x(t) \times h(t)}_{a(t)} \times q(t)$$

$$\longrightarrow c(t) = \frac{\sin \pi t}{\pi t} \cdot \cos 4\pi t \cdot \frac{\sin 2\pi t}{\pi t}$$

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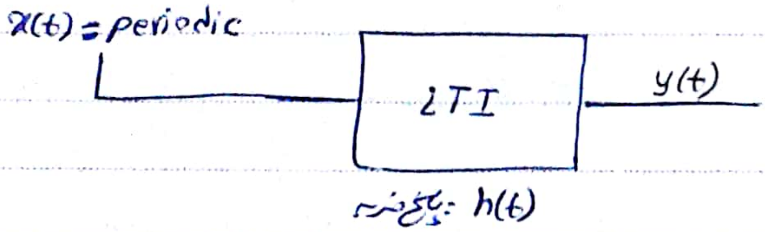
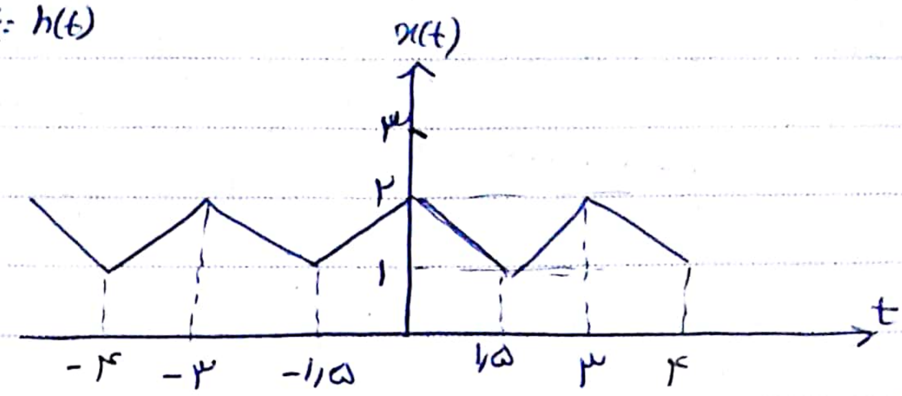
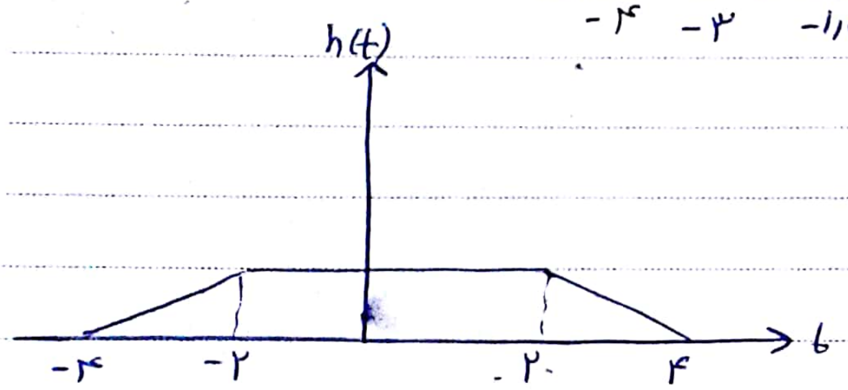
۷.

توان خروجی سیستم چقدر است؟







$$P_T = \frac{1}{T}\int_T |y(t)|^2 dt$$

$$Y(w) = X(w)\cdot H(w)$$

$$y(t) = \frac{1}{2\pi}\{X(w) * H(w)\}$$

$$x(t) * y(t) \xleftrightarrow{F.T} X(w)\cdot H(w)$$

$$x(t)*h(t) = \int x(\tau)\, h(t-\tau)\, d\tau$$

$T = 8$

$$P_8 = \frac{1}{T}\int_{-4}^{4} \overbrace{|y(t)|^2}^{x(t)} dt$$

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۸. تبدیل فوریه

$$x(t) = \frac{A}{2}\left(1 + \cos\left(\frac{\pi t}{T}\right)\right) \Pi\left(\frac{t}{2T}\right)$$

$$x(t) = \frac{A}{2} \cdot \Pi\left(\frac{t}{2T}\right) + \frac{A}{2} \cdot \cos\left(\frac{\pi t}{T}\right) \cdot \Pi\left(\frac{t}{2T}\right)$$

$$\Pi\left(\frac{t}{2T}\right) \xleftrightarrow{F.T} 2T \operatorname{sinc}\left(\frac{wT}{\pi}\right) = Z(w)$$

$$\cos\left(\frac{\pi}{T} t\right) \Pi\left(\frac{t}{2T}\right) \xleftrightarrow{F.T} \frac{1}{2}\left(Z\left(w+\frac{\pi}{T}\right) + Z\left(w-\frac{\pi}{T}\right)\right)$$

$$\longrightarrow X(w) = \frac{A}{2} \cdot Z(w) + \frac{A}{4}\left(Z\left(w+\frac{\pi}{T}\right)\right) + \frac{A}{4} \cdot Z\left(w-\frac{\pi}{T}\right)$$

$$\boxed{X(w) = AT \operatorname{sinc}\left(\frac{w}{\pi} T\right) + \frac{AT}{2} \operatorname{sinc}\left(\frac{\left(w+\frac{\pi}{T}\right)}{\pi} \cdot T\right) + \frac{AT}{2} \operatorname{sinc}\left(\frac{\left(w-\frac{\pi}{T}\right)}{\pi} T\right)}$$