Subject: یاسمن سادات میرمحمد

Date: ۹۲۳۶۰۵۳

تکلیف سری سوم سیگنال

۱.

a. $x(t) = e^{-t} \quad -1 < t < 1 \;,\; T = 2 \qquad w = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$$a_k = \frac{1}{2}\int_{-1}^{1} e^{-t} \cdot e^{-jk\pi t}\, dt = \frac{1}{2}\int_{-1}^{1} e^{-(1+jk\pi)t}\, dt$$

$$a_k = \frac{1}{2} \times \frac{-1}{1+jk\pi} \times e^{-(1+jk\pi)t}\Big|_{-1}^{1}$$

$$a_k = \frac{-1}{2(1+jk\pi)}\left(e^{-(1+jk\pi)} - e^{(1+jk\pi)}\right)$$

$$\begin{cases} a_1 = \frac{-1}{2}\left(e^{-1} - e^{1}\right) \\ a_2 = \frac{-1}{2(1+2j\pi)}\left(e^{-1(1+j\pi)} - e^{(1+j\pi)}\right) \end{cases}$$

- $x(t) = \sum_{k=-\infty}^{k=+\infty} a_k\, e^{jk\pi t}$

b. $x(t) = \begin{cases} 2 & -1 < t < 0 \\ -2 & 0 < t \leq 1 \end{cases} \;,\; T = 2 \qquad w = \frac{2\pi}{2} = \pi$

- $a_k = \frac{1}{2}\int_{-1}^{1} x(t) \cdot e^{-jk\pi t}\, dt = \overbrace{\frac{1}{2}\int_{-1}^{0} 2 \cdot e^{-jk\pi t}\, dt}^{2} + \overbrace{\frac{1}{2}\int_{0}^{1} -2 \times e^{-jk\pi t}\, dt}^{-2}$

$$a_k = \frac{-1}{jk\pi} e^{-jk\pi t}\Big|_{-1}^{0} + \frac{1}{jk\pi} e^{-jk\pi t}\Big|_{0}^{1}$$

$$\begin{cases} a_1 = \frac{-2}{j\pi} + \frac{1}{j\pi} e^{j\pi} + \frac{1}{j\pi} e^{-j\pi} \\ a_2 = \frac{-1}{j\pi} + \frac{1}{j\pi} e^{j\pi} + \frac{1}{j\pi} e^{-j\pi} \end{cases}$$

$$a_k = \frac{-1}{jk\pi} + \frac{1}{jk\pi} e^{jk\pi} + \frac{1}{jk\pi} e^{-jk\pi} - \frac{1}{jk\pi}$$

$$\begin{cases} \bullet\; a_k = \frac{-2}{jk\pi} + \frac{1}{jk\pi} e^{jk\pi} + \frac{1}{jk\pi} e^{-jk\pi} \\ \bullet\; x(t) = \sum_{k=-\infty}^{+\infty} a_k\, e^{jk\pi t} \end{cases}$$

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c. $x(t) = |\cos(w_0t)|$ $\qquad T = \frac{2\pi}{w_0}$

$$a_k = \frac{w_0}{\pi}\int_{-\frac{\pi}{2w_0}}^{\frac{\pi}{2w_0}} x(t)\cdot e^{-jkw_0t}\,dt = \frac{w_0}{\pi}\int_{-\frac{\pi}{2w_0}}^{\frac{\pi}{2w_0}} \underbrace{\cos(w_0t)\, e^{-jkw_0t}}_{\frac{e^{jw_0t}+e^{-jw_0t}}{2j}}\,dt$$

$\cos(w_0t)$ مثبت است

$$a_k = \frac{w_0}{\pi}\int_{-\frac{\pi}{2w_0}}^{\frac{\pi}{2w_0}} \frac{e^{t(jw_0-jw_0k)}}{2j}\,dt + \frac{w_0}{\pi}\int_{-\frac{\pi}{2w_0}}^{\frac{\pi}{2w_0}} \frac{e^{t(-jw_0-jw_0k)}}{2j}\,dt$$

$$a_k = \frac{w_0}{\pi}\times\frac{1}{2j(jw_0-jw_0k)}\, e^{t(jw_0-jw_0k)}\Big|_{-\frac{\pi}{2w_0}}^{\frac{\pi}{2w_0}} + \frac{w_0}{\pi}\times\frac{1}{-2j(jw_0+jw_0k)}\, e^{t(-jw_0-jw_0k)}\Big|_{-\frac{\pi}{2w_0}}^{\frac{\pi}{2w_0}}$$

$$a_k = \frac{w_0}{\pi}\times\frac{1}{2w_0k-2w_0}\left[e^{\left(\frac{\pi j}{2}-\frac{\pi jk}{2}\right)} - e^{\left(-\frac{\pi j}{2}+\frac{\pi jk}{2}\right)}\right] + \frac{w_0}{\pi}\times\frac{1}{2w_0+2w_0k}$$

$$\times\left[e^{\left(-\frac{\pi j}{2}-\frac{\pi jk}{2}\right)} - e^{\left(\frac{\pi}{2}j+\frac{\pi jk}{2}\right)}\right]$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0t}$$

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d. $x(t) = \sin(\overbrace{۲\pi\, ۳t}^{\alpha}) \cos(\overbrace{۲\pi t}^{\beta})$

$$x(t) = \frac{1}{۲}\left[\sin(۶\pi - ۲\pi)t + \sin(۶\pi + ۲\pi)t\right] = \frac{1}{۲}\left(\sin(۸\pi t) - \sin(۴\pi t)\right)$$

$$\begin{cases} \bullet\ T_1 = \frac{۲\pi}{۸\pi} = \frac{1}{۴} \\ \bullet\ T_۲ = \frac{۲\pi}{۴\pi} = \frac{1}{۲} \end{cases}$$

$$w_0 = \frac{۲\pi}{\frac{1}{۲}} = ۴\pi$$

$$x(t) = \frac{1}{۲}\left[\sin(۸\pi t) - \sin(۴\pi t)\right] = \left[\frac{1}{۲}\left(\frac{e^{j۸\pi t} - e^{-j۸\pi t}}{۲j}\right) - \frac{1}{۲}\left(\frac{e^{j۴\pi t} - e^{-j۴\pi t}}{۲j}\right)\right]$$

$$x(t) = \frac{1}{۴j}\left(e^{j۸\pi t} - e^{-j۸\pi t} - e^{j۴\pi t} + e^{-j۴\pi t}\right) : \begin{cases} a_1 = \frac{\pm 1}{۴j} \\ a_{-1} = \frac{-1}{۴j} \\ a_۲ = \frac{1}{۴j} \\ a_{-۲} = \frac{-1}{۴j} \end{cases}$$

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a. $x(t-2) + x(t+2)$

$x(t) \rightarrow a_k$ T. شیفت زمانی

$x_1(t) = x(t-2) \rightarrow b_k = e^{-jk\omega_0 2}$

$x_2(t) = x(t-(-2)) \rightarrow c_k = e^{jk\omega_0 2}$

خطی بودن: $\begin{cases} x_1(t) \rightarrow b_k \\ x_2(t) \rightarrow c_k \end{cases} \rightarrow D = b_k + c_k \rightarrow D = e^{-jk\omega_0 2} + e^{jk\omega_0 2}$

b. $\dfrac{dx(t)}{dt}$

$x(t) \rightarrow a_k$

$\dfrac{dx}{dt} \rightarrow b_k = j\omega_0 k \cdot a_k$

$$b_k = \frac{1}{T_0}\int_{-\infty}^{+\infty} \underbrace{\frac{dx(t)}{dt}}_{dv}\, \underbrace{e^{-j\omega_0 k t}}_{u}\, dt$$

$$b_k = \frac{1}{T_0}\left[\underbrace{x(t)}_{v}\, \underbrace{e^{-jk\omega_0 t}}_{du} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{+\infty} x(t)\cdot(-jk\omega_0)\, e^{-jk\omega_0 t}\, dt \right]$$

$$b_k = jk\omega_0 \times \underbrace{\frac{1}{T_0}\int_{-\infty}^{+\infty} x(t)\cdot e^{-jk\omega_0 t}\, dt}_{a_k}$$

$$\boxed{b_k = jk\omega_0 \cdot a_k}$$

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c. $x^*(2t) + x(-\frac{t}{2})$

$x(t) \longrightarrow a_k$ $\quad T_0 =$

$x(2t) \longrightarrow a_k$ $\quad \frac{T}{2} \longrightarrow x^*(2t) \longrightarrow a^*_{-k}$

$x(\frac{t}{2}) \longrightarrow a_k$ $\quad 2T \longrightarrow x(\frac{t}{2}) \longrightarrow a_{-k}$

$\underbrace{x^*(2t)}_{a^*_{-k}} + \underbrace{x(-\frac{t}{2})}_{a_{-k}} \longrightarrow \underbrace{a^*_{-k}}_{T} + \underbrace{a_{-k}}_{T}$ $\quad T_0 = 2T$

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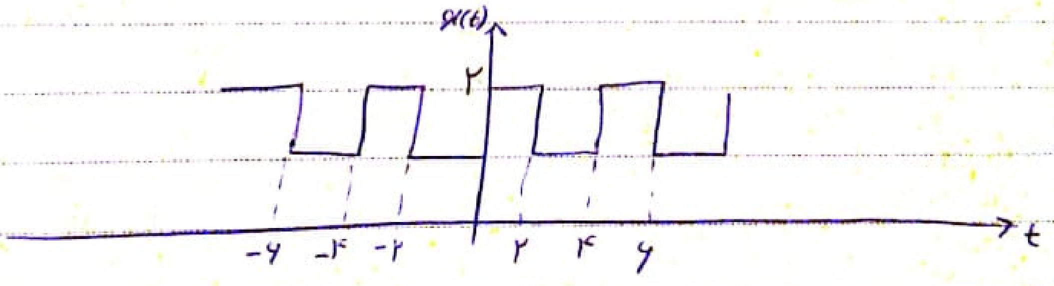
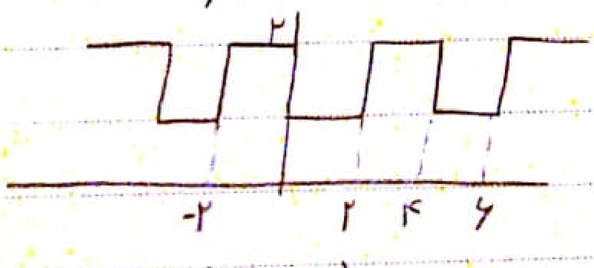
$x(t)$

$T = 4$ $\quad Y_k = (-1)^k X_k + (-1)^k X_{-k}$



$y(t) = x(t - \frac{T}{2}) + x(-t + \frac{T}{2})$



$x(t-2)$

* یادآوری:

$x(t - \frac{T}{2}) \xrightarrow{F.S} a_k e^{-jk\frac{2\pi}{T} \times \frac{T}{2}}$

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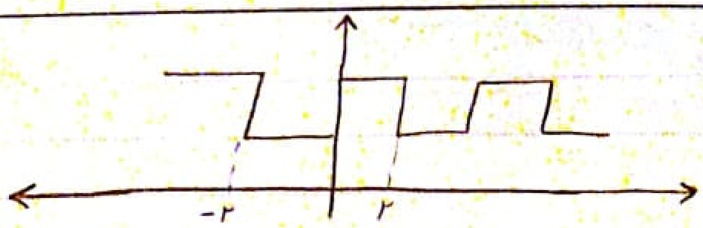
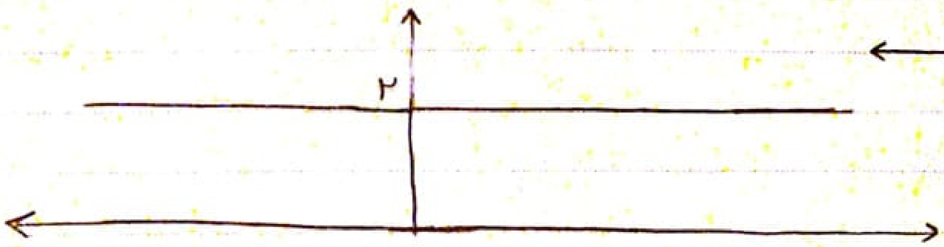


$x(-t+۲)$

$y(t) = x(t-۲) + x(-t+۲)$



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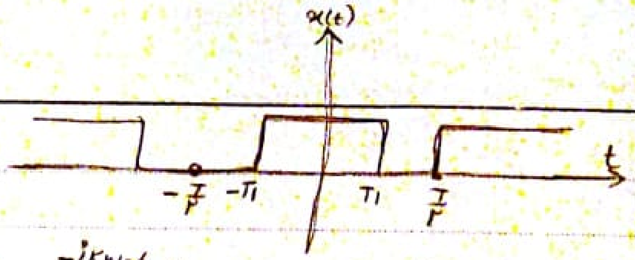

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$$a_k = \frac{1}{T}\int_{-\frac{T}{۲}}^{\frac{T}{۲}} x(t) \cdot e^{-jk\omega_0 t}\, dt \qquad \omega_0 = \frac{۲\pi}{T}$$

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} 1 \times e^{-jk\omega_0 t}\, dt = \frac{1}{T} \times \frac{1}{-jk\left(\frac{۲\pi}{T}\right)} \times e^{-jk\cdot\frac{۲\pi}{T}\cdot t}\Big|_{-T_1}^{T_1}$$

$$a_k = -\frac{1}{jk۲\pi}\left[e^{-jk\left(\frac{۲\pi}{T}\right)T_1} - e^{jk\left(\frac{۲\pi}{T}\right)T_1}\right]$$

$$a_k = \frac{1}{k\pi}\left[\frac{e^{jk\frac{۲\pi}{T}T_1} - e^{-jk\frac{۲\pi}{T}T_1}}{۲j}\right]$$

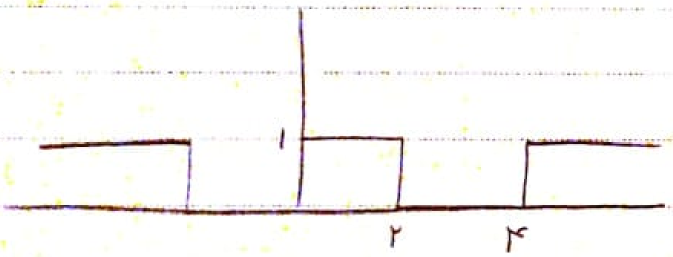
$\sin \frac{k۲\pi}{T} \cdot T_1$

$$* \; a_k = \frac{1}{k\pi}\sin\left(k\frac{۲\pi}{T}\cdot T_1\right) \xrightarrow[T = ۴T_1]{} \begin{cases} a_0 = \frac{۲T_1}{T} = \frac{۲T_1}{۴T_1} = \frac{1}{۲} & (k=0) \\ \frac{\sin\left(\frac{k\pi}{۲}\right)}{k\pi} & k \neq 0 \end{cases}$$

$(k \neq 0)$

ب. شیفت یافته‌ی شکل الف است.

$x_1(t) = x(t-1)$



$$\boxed{b_k = e^{-jk\frac{\pi}{۲}\times 1} \cdot a_k}$$

$$b_k \begin{cases} e^{-jk\frac{\pi}{۲}} \times \frac{\sin\left(\frac{k\pi}{۲}\right)}{k\pi} & (k \neq 0) \\ \frac{1}{۲} & (k=0) \end{cases}$$

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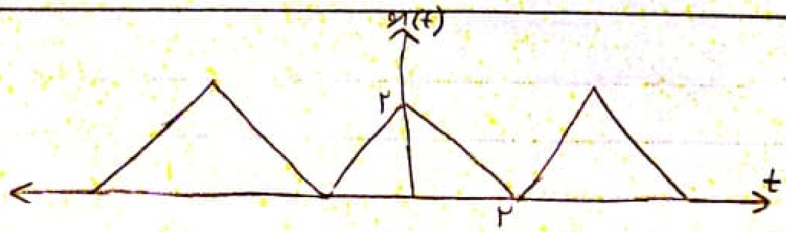
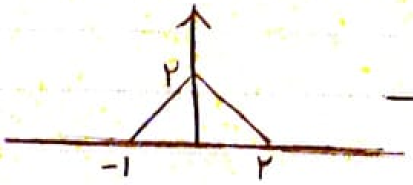
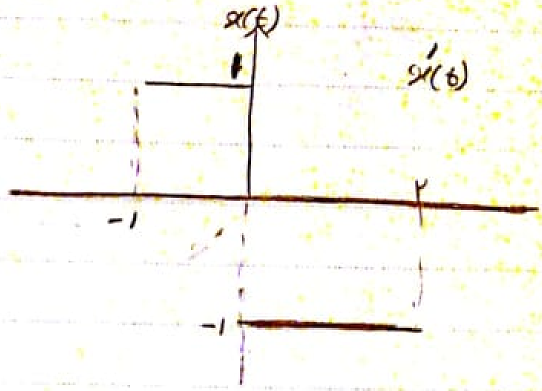
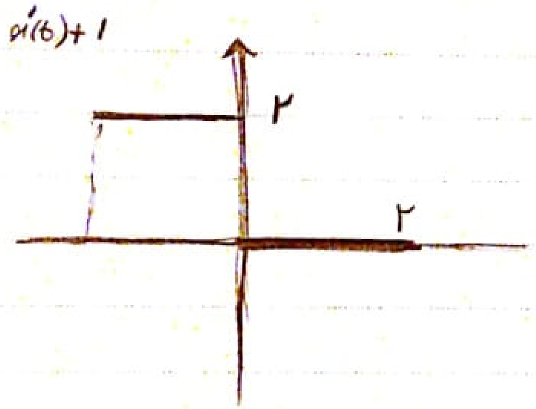
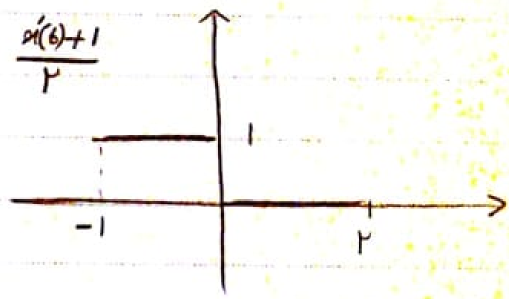




$x(t)$







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$$y(t) = \left(x'\left(\tfrac{t}{2} - \tfrac{1}{4}\right) + 1\right) \times \tfrac{1}{2}$$

- $x(t) \xrightarrow{FS} b_k$
- $x'(t) \longrightarrow jk\omega_0 \cdot b_k$
- $x'(t - \tfrac{1}{2}) \longrightarrow jk\omega_0 \cdot b_k \cdot e^{-jk\omega_0 \frac{1}{2}}$
- $x'(\tfrac{t}{2} - \tfrac{1}{4}) \longrightarrow jk\omega_0 \cdot b_k \cdot e^{-jk\omega_0 \cdot \frac{1}{2}}$

$$\longrightarrow \quad x'\left(\tfrac{t}{2} - \tfrac{1}{4}\right) = 2y(t) - 1 \longrightarrow 2a_{k-1} = Fs\left(y'\left(\tfrac{t}{2} - \tfrac{1}{4}\right)\right)$$

$$2a_{k-1} = jk\omega_0 \cdot e^{-jk\omega_0 \cdot \frac{1}{2}} \cdot b_k$$

$$\boxed{b_k = \frac{2a_{k-1} \cdot e^{jk\omega_0 \cdot \frac{1}{2}}}{jk\omega_0}}$$

۵.

- $x(t) = |\sin(3t)| \qquad \omega_0 = 3$
- $L = \sum_{k=-\infty}^{+\infty} a_k^2 = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt \longleftarrow a_k = \frac{1}{T_0} \int_{T_0} x(t)\, dt$

$$T_0 = \frac{2\pi}{3} \xrightarrow{L=} \frac{3}{2\pi} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^2(3t)\, dt \qquad (\sin^2 3t = 1 - \cos^2 3t) \quad (\cos 6t = 2\cos^2 3t - 1)$$

$$\longrightarrow \quad L = \frac{3}{2\pi} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1 - \frac{(\cos 6t + 1)}{2}\, dt$$

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$$\angle = \frac{3}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\,dt + \left(-\frac{3}{2\pi}\right)\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\cos(6t)}{2}\,dt \quad \frac{3}{2\pi}\times\frac{1}{3}\sin(3t)\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$\angle = \frac{3}{2\pi}\left(\frac{\pi}{6}+\frac{\pi}{6}\right) - \frac{1}{2\pi}\Big(\underbrace{\sin\pi}_{0} + \underbrace{\sin\pi}_{0}\Big) =$$

$$\angle = \frac{3}{\pi}\cdot\left(\frac{\pi}{3}\right)$$

$$\boxed{\angle = \frac{1}{2}} \longrightarrow \boxed{\angle = \frac{1}{2}}$$

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- $T = 6$
- a_k
- $k = 0,\ |k| > 2 \longrightarrow a_k = 0$
- $x(t) = -x(t-3)$
- $\int_{-3}^{+3} x^2(t)\,dt = 3$
- $a_1 \rightarrow$ حقیقی و مثبت

سیگنال حقیقی $\longrightarrow$ $a_k = a_{-k}^*$

$$\begin{cases} a_1 = a_{-1} \neq 0 \quad , \quad a_1 = a_{-1}^* \\ a_2 = a_{-2} \neq 0 \quad , \quad a_2 = a_{-2}^* \end{cases}$$

$x(t) = -x(t-3) = 0 \rightarrow$ ها a_k ضرایب a_k های زوج صفر هستن و فقط مرد می‌مونن *

$$a_k = -a_k(-1)^k : \begin{cases} k \text{ زوج} \longrightarrow a_k = -a_k \ : \ a_k = 0 \\ k \text{ فرد} \longrightarrow a_k = a_k \ \checkmark \end{cases} \longrightarrow a_0 = a_2 = 0$$

(ـ از اول a_k های زوج صفره!)

$$\int_{-3}^{3} |x(t)|^2 dt = 3 \longrightarrow \frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = \frac{3}{6}$$

$$\xrightarrow[\text{پارسوال}]{\text{تساوی}} \sum |a_k|^2 = \frac{1}{2} \longrightarrow a_1^2 + a_{-1}^2 = \frac{1}{2}$$

a_1 حقیقی و مثبت است $\longrightarrow a_1 = a_1^*$ ، از طرفی $a_{-k} = a_k^* \longrightarrow a_1 = a_{-1}$

$$\Longrightarrow a_1^2 + a_{+1}^2 = \frac{1}{2} \longrightarrow 2a_1^2 = \frac{1}{2} \longrightarrow a_1^2 = \frac{1}{4} \longrightarrow \boxed{a_1 = \pm\frac{1}{2}} \longrightarrow a_1 = \frac{1}{2}$$

$$\longrightarrow \boxed{x(t) = \sum a_k e^{jk\omega_0 t} = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}}$$

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$\longrightarrow a_k = -a_k\, e^{-jk\omega_0 \times ۳} \longrightarrow -1 = e^{-jk\omega_0 \times ۳}$

$\longrightarrow k = \frac{1}{۳} e^{-j\omega_0}$

۷.

$x(t) \longrightarrow \boxed{} \longrightarrow y(t)$

$(*)\ \cdot\ \frac{dy(t)}{dt} + y(t) = x(t) \longrightarrow t+1=0 \longrightarrow t=-1$

$(**)\ \cdot\ x(t) = \cos(۲\pi t)$

$\longrightarrow y_h = Ae^{-t}$ ، جواب خصوصی : $K\cos(۲\pi t)$

جواب کلی $\longrightarrow y(t) = Ae^{-t} + K\cos(۲\pi t)$ جایگذاری در * و **

$\underbrace{-\cancel{Ae^{-t}} - ۲\pi K\sin(۲\pi t)}_{\dot{y}(t)} + \underbrace{\cancel{Ae^{-t}} + K\cos(۲\pi t)}_{y(t)} = \cos(۲\pi t)$

$-۲\pi \sin(۲\pi t) = (1-K)\cos(۲\pi t)$