CSC 370

Database Systems

Assignment 1

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Problem 3

As the problem states, the functional dependencies are:

(i) $AB \rightarrow C$

(ii) $BC \rightarrow D$

(iii) $CD \rightarrow A$

(iv) $AD \rightarrow B$

1. To find all the non-trivial FDs that follow from the above functional dependencies, first we need to find the closures using the closure algorithm. We use the algorithm based on the different possible sets of attributes:

   (a) $\{A\}$. As there is no FD with A on its left-hand side, so $\{A\}^+ = \{A\}$.

   (b) $\{B\}$. As there is no FD with B on its left-hand side, so $\{B\}^+ = \{B\}$.

   (c) $\{C\}$. As there is no FD with C on its left-hand side, so $\{C\}^+ = \{C\}$.

   (d) $\{D\}$. As there is no FD with D on its left-hand side, so $\{D\}^+ = \{D\}$.

   (e) $\{A,B\}$. Based on FD (i), $C \in \{A,B\}^+$. So far A,B,C are in $\{A,B\}^+$. FD (ii) suggests that $D \in \{A,B\}^+ = \{A,B,C\}^+$. So $\{A,B\}^+ = \{A,B,C,D\}$.

   (f) $\{B,C\}$. Based on FD (ii), $D \in \{B,C\}^+$. So far B,C,D are in $\{B,C\}^+$. FD (iii) suggests that $A \in \{B,C\}^+ = \{B,C,D\}^+$. So $\{B,C\}^+ = \{A,B,C,D\}$.

   (g) $\{C,D\}$. Based on FD (iii), $A \in \{C,D\}^+$. So far A,C,D are in $\{C,D\}^+$. FD (iv) suggests that $B \in \{C,D\}^+ = \{A,C,D\}^+$. So $\{C,D\}^+ = \{A,B,C,D\}$.

   (h) $\{A,D\}$. Based on FD (iv), $B \in \{A,D\}^+$. So far A,B,D are in $\{A,D\}^+$. FD (i) suggests that $C \in \{A,D\}^+ = \{A,B,D\}^+$. So $\{A,D\}^+ = \{A,B,C,D\}$

   (i) $\{A,C\}$. As there is no FD with AC, A or C on its left-hand side, so $\{A,C\}^+ = \{A,C\}$.

   (j) $\{B,D\}$. As there is no FD with BD, B or D on its left-hand side, so $\{B,D\}^+ = \{B,D\}$.

   (k) $\{A,B,C\}$: As it is showed in (e), $\{A,B,C\}^+ = \{A,B,C,D\}$.

   (l) $\{A,B,D\}$: As it is showed in (h), $\{A,B,D\}^+ = \{A,B,C,D\}$.

   (m) $\{A,C,D\}$: As it is showed in (g), $\{A,C,D\}^+ = \{A,B,C,D\}$.

   (n) $\{B,C,D\}$: As it is showed in (f), $\{B,C,D\}^+ = \{A,B,C,D\}$.

   (o) $\{A,B,C,D\}$: Obviously, $\{A,B,C,D\}^+ = \{A,B,C,D\}$.

LHS Set X	X^+	$X^+ \setminus X$	Resulting FDs
A	A	$\emptyset$	$----$
B	B	$\emptyset$	$----$
C	C	$\emptyset$	$----$
D	D	$\emptyset$	$----$
AB	$ABCD$	CD	$AB \rightarrow CD$
AC	AC	$\emptyset$	$----$
AD	$ABCD$	BC	$AD \rightarrow BC$
BC	$ABCD$	AD	$BC \rightarrow AD$
BD	BD	$\emptyset$	$----$
CD	$ABCD$	AB	$CD \rightarrow AB$
ABC	$ABCD$	D	$ABC \rightarrow D$
ABD	$ABCD$	C	$ABD \rightarrow C$
ACD	$ABCD$	B	$ACD \rightarrow B$
BCD	$ABCD$	A	$BCD \rightarrow A$
$ABCD$	$ABCD$	$\emptyset$	$----$

Table 1: Illustration of finding all implied FDs for T

Table 1 shows the non trivial FDs which are followed from FDs (i), (ii), (iii) and (iv).

2. As it mentioned and computed in the previous part, the keys are $\{A,B\},\{B,C\},\{C,D\}$ and $\{A,D\}$. That is because they determine all other attributes and they are *minimal*, i.e. their proper subsets (subsets of size one) cannot functionally determine all other attributes.

3. As every superset of a key is a superkey, so the non-key superkeys are:

$$\{A,B,C\}$$

$$\{A,B,D\}$$

$$\{A,C,D\}$$

$$\{B,C,D\}$$

$$\{A,B,C,D\}$$