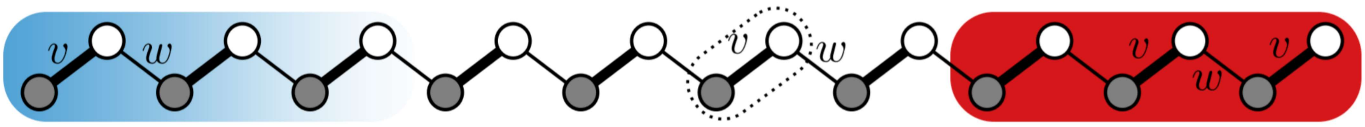
In [1]:

```
using Plots
using LinearAlgebra
```

Su-Schrieffer-Heeger (SSH) Model

$$\mathcal{H} = v \sum_i \left(a_i^\dagger b_i + \text{ h.c. }\right) + w \sum_i \left(b_i^\dagger a_{i+1} + \text{ h.c. }\right)$$



$$\tilde{\mathcal{H}} = \vec{d} \cdot \vec{\sigma}$$
$$d_x(k) = v + w\cos(k) \quad d_y(k) = w\sin(k) \quad d_z = 0$$

$$d = \pm\sqrt{d_x^2 + d_y^2} = \pm\sqrt{v^2 + +2vw\cos k + w^2}$$

In [19]:

```
sigma_x = [0 1;1 0]
sigma_y = [0 -im;im 0]
sigma_z = [1 0;0 -1]

dx(k::Float64,v=1,w=2) = v + w*cos(k)
dy(k::Float64,v=1,w=2) = w*sin(k);

d(k::Float64,v=1,w=2) = sqrt(dx(k,v,w)^2+dy(k,v,w)^2);
H(k::Float64,v=1,w=2) = dx(k,v,w)*sigma_x + dy(k,v,w)*sigma_y; # Hamiltonian

l = 2*314
ks=range(-2*pi,stop=2*pi,length=l)
dk=ks[2]-ks[1];
```

Check the Hamiltonian Properties

1) $\sigma_z h_k \sigma_z = -h_k$

In [20]:

```
k_test=rand()
sigma_z*H(k_test)*sigma_z + H(k_test) # Should equal to zero
```

Out[20]:

```
2×2 Matrix{ComplexF64}:
 0.0+0.0im  0.0+0.0im
 0.0+0.0im  0.0+0.0im
```

2) $\sigma_y h_k^* \sigma_y = -h_k$

In [21]:

```
sigma_y*conj(H(k_test))*sigma_y + H(k_test) # Should equal to zero
```

Out[21]:

```
2×2 Matrix{ComplexF64}:
 0.0+0.0im  0.0+0.0im
 0.0+0.0im  0.0+0.0im
```

3) $\sigma_x h_k \sigma_x = h_{-k}$

In [22]:

```
sigma_x*H(k_test)*sigma_x - H(-k_test) # Should equal to zero
```

Out[22]:

```
2×2 Matrix{ComplexF64}:
 0.0+0.0im  0.0+0.0im
 0.0+0.0im  0.0+0.0im
```

Dispersion relation

$$E(k) = \pm \left|v + e^{-ik} w\right| = \pm\sqrt{v^2 + w^2 + 2vw\cos k}$$

In [23]:

```julia
# Parameters chosen to look at
va = [1.0, 0.5,1.0,  0.0,0.5,  0.4,0.6]
da = [0.0, 0.5,-0.5, 0.5,-0.5, 0.2,-0.2] # delta = v - w

# w values corresponding to chosen ds
wa=round.(va+da;sigdigits=2)
logocolors = Colors.JULIA_LOGO_COLORS

# plot chosen parameters
colors=[colorant"skyblue2",
    colorant"olivedrab",colorant"slateblue4",
    colorant"violetred3",colorant"yellow2",
    colorant"maroon",colorant"sienna2",
    ]
styles=[:solid,
    :dash,:dash,
    :solid,:solid,
    :dot,:dot]
widths=[10,15,5,15,5,10,3];
```

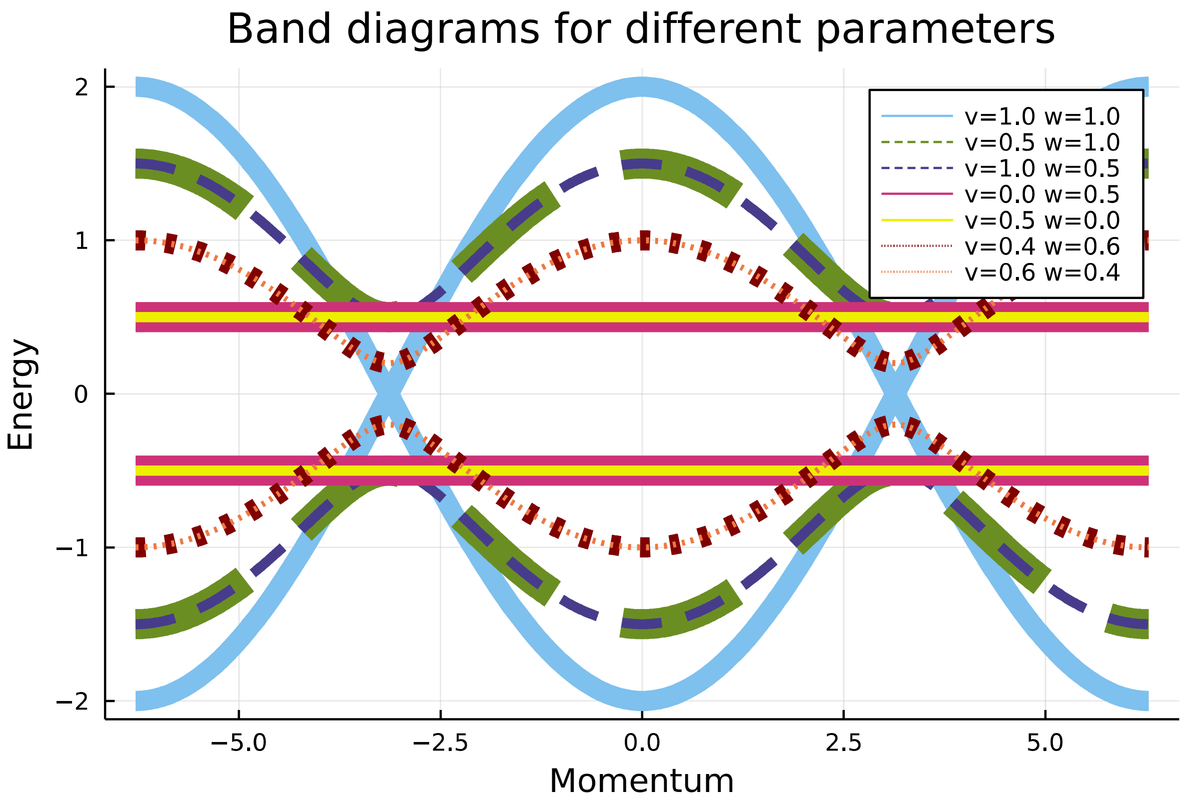
Band diagrams for different parameters

In [24]:

```
plot()
for ii in 1:length(va)
    plot!(ks,d.(ks,va[ii],wa[ii])
        ,label="v=$(va[ii]) w=$(wa[ii])"
        ,linewidth=widths[ii],color=colors[ii],linestyle=styles[ii])

    plot!(ks,-d.(ks,va[ii],wa[ii])
        ,label=""
        ,linewidth=widths[ii],color=colors[ii],linestyle=styles[ii])
end
plot!(title="Band diagrams for different parameters",
xlabel="Momentum",ylabel="Energy")
```

Out[24]:



In [8]:

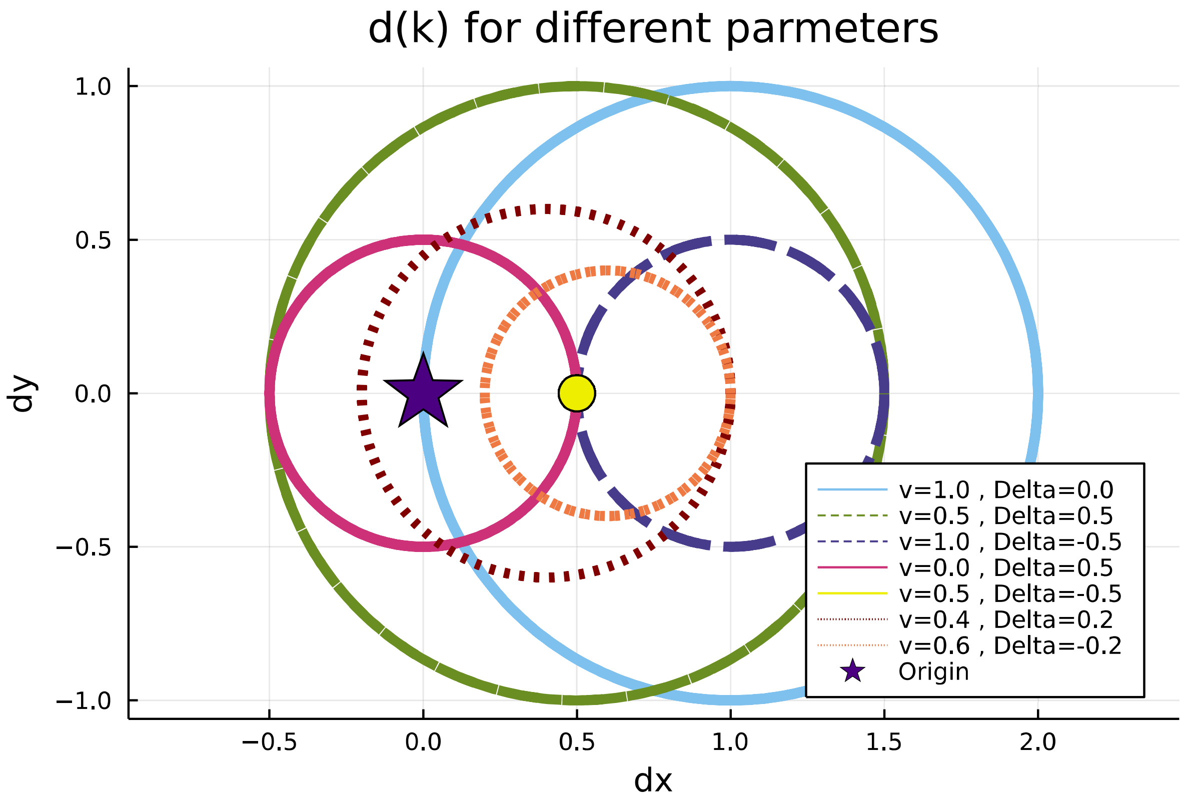
```julia
# Delta = v - w
plot()
for ii in 1:length(va)
    plot!(dx.(ks,va[ii],wa[ii]),
        dy.(ks,va[ii],wa[ii])
    ,label="v=$(va[ii]) , Delta=$(da[ii])"
    ,linewidth=5,color=colors[ii],linestyle=styles[ii])
end

statval=5
scatter!(dx.(ks,va[statval],wa[statval]),dy.(ks,va[statval],wa[statval])
    ,label="",markersize=10,color=colors[statval])

scatter!([0],[0],label="Origin",
        markersize=20,markershape=:star5,color=colorant"indigo")

plot!(title="d(k) for different parmeters",
xlabel="dx", ylabel="dy",legend=:bottomright,aspect_ratio=1)
```

Out[8]:



Plotting the Phase

In [25]:

```
function Winding_phi(k,v,w)
    dum2=(um2.(k[2:end],v,w).-um2.(k[1:(end-1)],v,w))
    return 1/(2π*im)*sum(dum2./um2.(k[2:end],v,w) )
end
```

Out[25]:

```
Winding_phi (generic function with 1 method)
```

In [26]:

```
um1=-1/sqrt(2)

function um2(k::Float64,v=1,w=2)
    return 1/(sqrt(2)*d(k,v,w))*(dx(k,v,w)+im*dy(k,v,w))
end
```

Out[26]:

```
um2 (generic function with 3 methods)
```

In [27]:

```
vaa=repeat(range(0,1,length=100),1,100)
waa=transpose(vaa)

φaa=zeros(Complex{Float64},100,100)
for ii in 1:100
    for jj in 1:100
        φaa[ii,jj]=Winding_phi(ks,vaa[ii,jj],waa[ii,jj])
    end
end
```

In [28]:

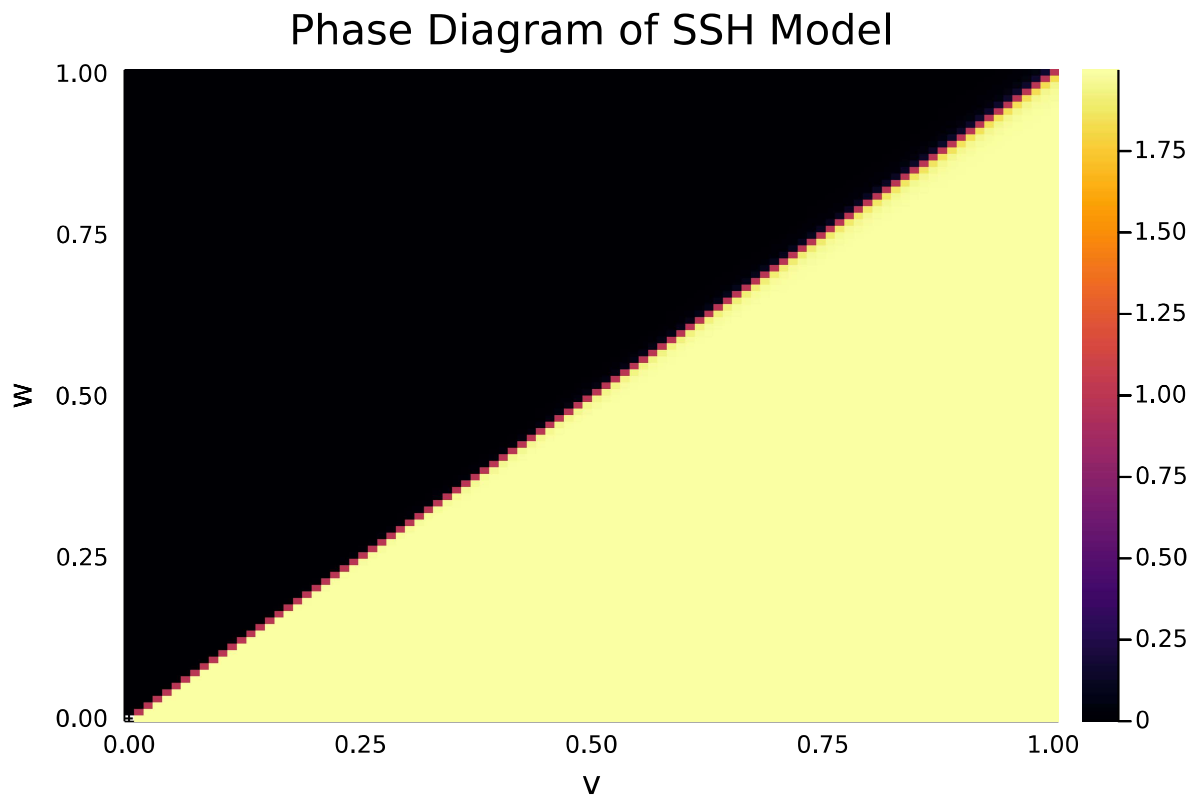
```
vaa=repeat(range(0,1,length=100),1,100)
waa=transpose(vaa)

φaa=zeros(Complex{Float64},100,100)
for ii in 1:100
    for jj in 1:100
        φaa[ii,jj]=Winding_phi(ks,vaa[ii,jj],waa[ii,jj])
    end
end
```

In [29]:

```
heatmap(vaa[:,1],waa[1,:],real.(φaa))
plot!(xlabel="v",ylabel="w", title="Phase Diagram of SSH Model")
```

Out[29]:



Topological phase: v < w , Trivial phase: v>w

In [ ]:

```

```

In [ ]:

```

```