Department of Electrical
& Computer Engineering
Faculty of Engineering & Architectural Science

Department of Electrical, Computer, and Biomedical Engineering

Course Number	**ELE829**
Course Title	System Models & Identification
Semester/Year	Fall 2021
Instructor	Professor M.S. Zywno, Ph.D.

Final Project

Activity Title	**"Black Boxes" - Unknown Systems Identification**

Submission Date	**11/17/21**
Regular Due Date	**11/17/21**

Name	Student ID	Signature*
Yasar Zaman	*13595*	

(Note: remove the first 4 digits from your student ID: *xxxx12345*)

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www.ryerson.ca/senate/current/pol60.pdf.

Final Project Grading Sheet

Your Name: ___________Yasar Zaman____________________________	
Your Assigned OE Process Number: __48____ Your Assigned PEM Process Number: __6____	
Any deductions will be recorded here: If five in-class simulation challenges are not met (a cumulative challenge can be assigned in lieu), you will get a ZERO mark. If the two Matlab codes are not submitted, are named incorrectly, or give errors in execution, you will get a ZERO mark. If you use a wrong flag number, you will get a ZERO mark. If your codes fail to duplicate the results submitted in the report, you will receive at least a 30 point deduction.	
ONE PAGE EXECUTIVE SUMMARY Anything that is important about this report should be included on this page - it is your "bottom line". If you don't know what to include, think about a busy CEO of your company who will not want to thumb through the whole report - he/she needs to know the "why", the "what" and the end result. The rest is for "the middle management" to pore over.	**/10**
Are appropriate diagnostics for OE Model included?	**/5**
Is OE Process Identified and Validated?	**/5**
Results for OE Process and Discussion: touch on such issues as how the structure of the process was arrived at, whether the results in CT domain are consistent with the diagnostics, explain your choice of different variables that led you to a successful validation, etc. Please refer to theory learned both in ELE639 and in ELE829.	**/25**
Are appropriate diagnostics for the deterministic part of the PEM Model included?	**/5**
Are appropriate diagnostics for the stochastic part of the PEM Model included?	**/5**
Is PEM Process Identified and Validated?	**/5**
Results for PEM Process and Discussion: touch on such issues as how the structure of the process was arrived at, whether the results in CT domain are consistent with the diagnostics, explain your choice of different variables that led you to a successful validation, etc. Please refer to theory learned both in ELE639 and in ELE829. Note that you will lose marks if you do not discuss whether a BJ, a PEM or an ARMAX model is the best choice for your process.	**/30**
General: Clarity, writing style, grammar, spelling, layout of the report	**/10**
TOTAL: /100	

<u>Table of Contents:</u>

Executive Summary

The final project for ELE829 is to identify two different types of system and to extract the transfer function that was used to generate their respective data. The data will be generated through the different flags assigned. The first system is of an OE-type and is generated by *flag 48* and the second system is of a PEM-type and is generated by *flag 6*. The general approach to this project is to first run diagnostics through bode-plots, impulse response, as well as the Hankel Test. Then to generate an OE model for each system based on the diagnostics and have it validated. An OE-type system should be validated upon observing the Cross Correlation Function (CCF); however, a PEM-type system will need to check for other models that could be a suitable model to help identify the system. The PEM-type system should go through a BJ model, PEM model, as well as an ARMA model.

For the OE-type system, generated by *flag 48*, it was observed that the system had 2 zeros, 3 poles and had a sampling delay of 8. The zeros and poles were determined through the bode-plot, as well as the Hankel Test. The sampling delay was determined through the impulse response generated through MATLAB's CRA subroutine which helps in correlation analysis. Generating an OE model and comparing it with the data, shows that there was a fit-to-estimation of about 98% with FPE and MSE values of 0.001022 and 0.00102 respectively. This was then verified through the Chi-squared Test and the Autocorrelation Function (ACF) and Cross Correlation Function (CCF). The function that was produced was noted and then examined with a step response to see if the function obtained was satisfactory.

For the PEM-type system, generated by *flag 6*, it was observed that the system had 2 zeros, 3 poles and a sampling delay of 7. Similarly as before, the zeros and poles were determined through the bode-plot and Hankel Test. The delay was determined through the impulse response generated. After generating an OE model and comparing it with the data, it gave a fit-to-estimation of about 90.62% with FPE and MSE values of 0.1212 and 0.121 respectively. This was then verified through the Chi-squared test and the ACF and CCF. After using the OE Model, other models were also looked at including the BJ Model, PEM model, and the ARMA model; however, the best model was the BJ Model for my dataset. The transfer function that was created was noted and then examined with a step response as well as the bode-plot to see if the function obtained was satisfactory.

Table 1: Pole Zero and Polynomial of G(s)

Pole-Zero G(s)	$\frac{25.563(s+140.6)(s+2.001)}{(s+139.7)(s^2+1.798s+36.01)} * e^{-0.35s}$
Polynomial G(s)	$\frac{25389s^2+3646s+7195}{s^3+141.5s^2+287.2s+5032} * e^{-0.35s}$

Table 2: Pole Zero and Polynomial of G(s)

Pole-Zero G(s)	$\frac{0.24161(s-38.4)(s-0.9929)}{(s+0.819)(s^2+2.945s+2.245)} * e^{-0.3s}$
Polynomial G(s)	$\frac{0.2416s^2-9.517s+9.212}{z^3-2.817z^2+2.646z-0.8282} * e^{-0.3s}$

OE-type Model Identification

Running Diagnostics:

The first step in identifying the characteristics of the model is to create the DPRBS data with the respective flag for OE-type. The flag I received was *flag '48,'* thus I will generate the DPRBS data with that flag.

Specifications:

$$Number\ of\ Samples\ (N) = 5000$$
$$Switching\ Rate\ (K) = 10$$
$$Amplitude\ (A) = 0.1$$

I decided to initially use 5000 as the sample size to get an idea for the model through the bode-plot, impulse responses, and the hankel test. The fast switching rate makes it easier to help with diagnostics that is why I decided to use a switching rate of 10. Using these two specifications and an initial amplitude of 1, I discovered that the data that was generated was saturated and so in order to fix this saturation, I lowered my amplitude down to 0.1 to get a data set that did not have dead-zones and was not saturated. The data generated can be seen in **Figure 1**.

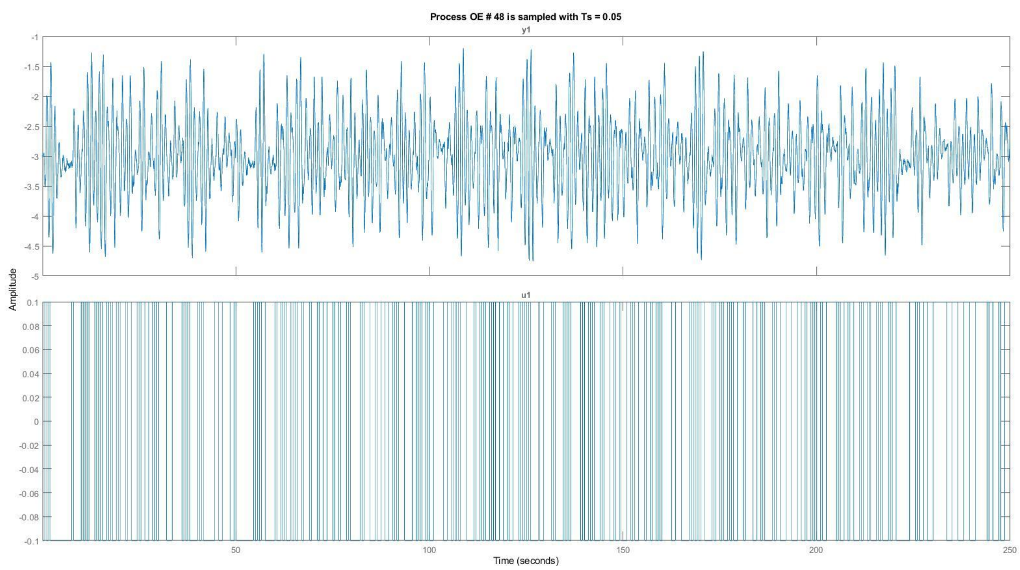


Figure 1: DPRBS Generated Data for OE-type

The next step in running the diagnostics is to first find the DC offset of the graph. We need to eliminate this DC offset in order for the *mean* of the system to be zero. This way when we create an OE model of the system, it would present a better fit-to-estimation. We can see from **Figure 2**, below, which is a zoomed in version of the first few samples of **Figure 1**, that the DC offset is approximately -3.

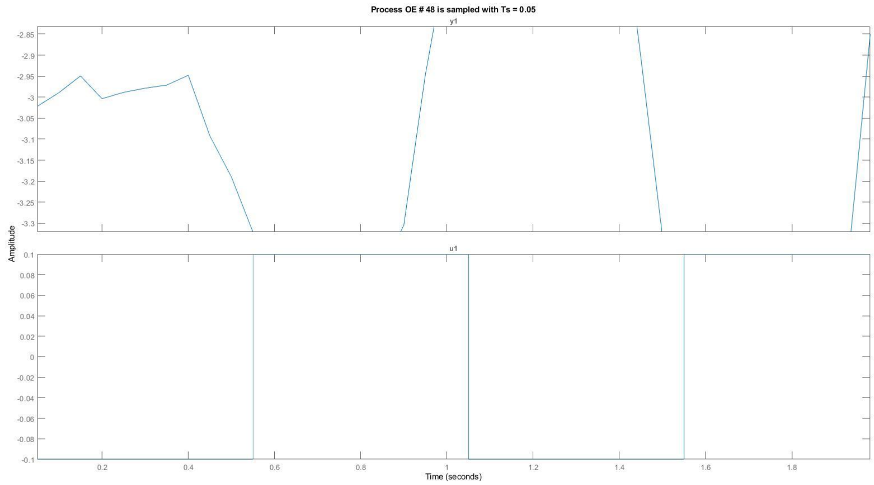


Figure 2: First Few Samples of Data Generated OE-type

After having removed the DC offset, we can now start to properly run diagnostics on our dataset. The three primary methods to use would be to initially start with the bode-plot and determine the number of zeros and poles. Upon doing that, we can generate an impulse response. MATLAB has a few subroutines that are useful for correlation analysis to determine the impulse response. There is a CRA method and Deconvolution method. Here, we will use the CRA method to determine the number of sample delays the data has. The reason we use CRA and not the Deconvolution method is because the Deconvolution method falls apart when analyzing data for noisy systems and only works best for a perfect, noise-free scenario. To double check if our analysis from the bode-plot was correct, we can use the Hankel Test to confirm our diagnostics.

As stated, the first method to determine the characteristics of the system is to display and analyse the body-plot which can be seen below in **Figure 3**. We don’t need the phase plot as we can determine the zeros and poles without it. If there was some confusion about the poles, we can refer to the phase plot; however, for now, we can omit that graph.

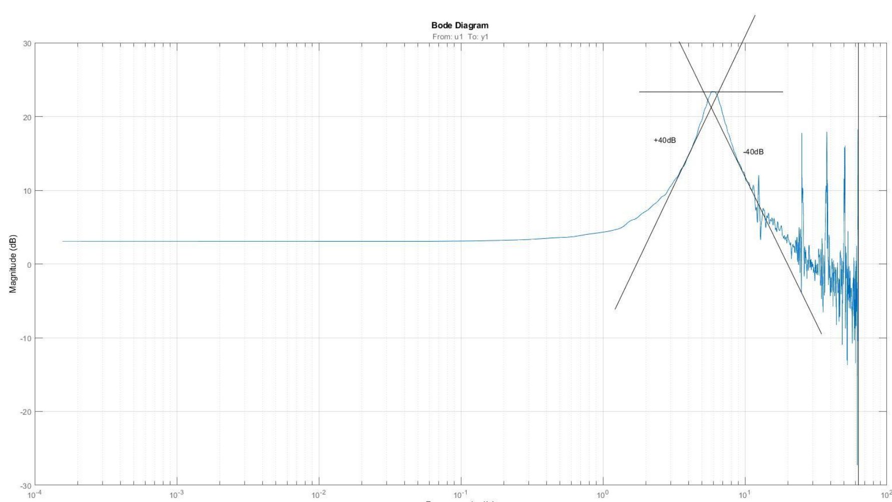


Figure 3: Bode-Plot of Data Generated OE-type

As we can clearly see, there is a positive 40dB slope indicating that there are two zeros. We also see a negative 40dB slope which would indicate two poles. Finally we see that there is a peak which indicates the presence of another pole. Going further into running the diagnostics, the primary assumption will be that the system has a total of 2 zeros and 3 poles.

To confirm if the system is of third order, we can use the Hankel Test to confirm our assumption as it should show that there is a great slope change at the third point. However, we need to determine the number of sample delays through the CRA method to generate the impulse response. This impulse response is shown in **Figure 4** and **Figure 5** below.

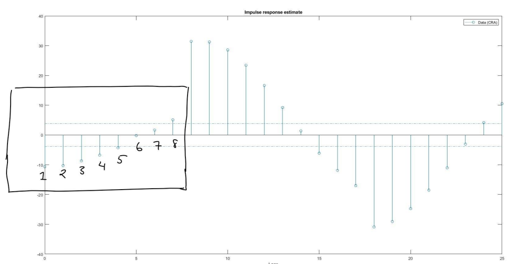


Figure 4: CRA with 50 Lags

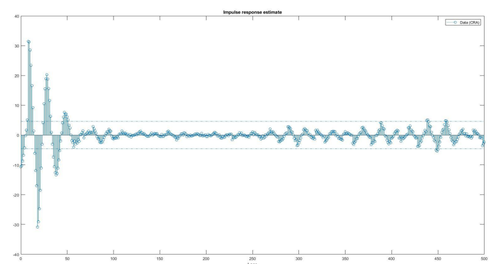


Figure 5: CRA with 500 Lags

Clearly, from **Figure 4** we see that there is a sample delay of 8 for sure. We can observe the CRA with 500 Lags as shown in **Figure 5** to understand and validate our assumption that the order is at least a second order since there is a large overshoot. Since it is at least a second order, we know that the system can have 3 poles.

In order to validate our assumption for the order of the system, we can use the Hankel Test as shown in **Figure 6** and **Figure 7**.

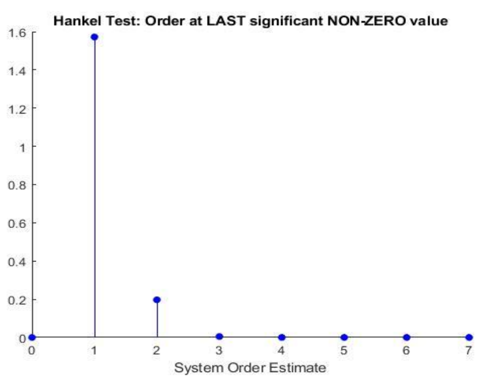


Figure 6: Hankel Test (Linear)

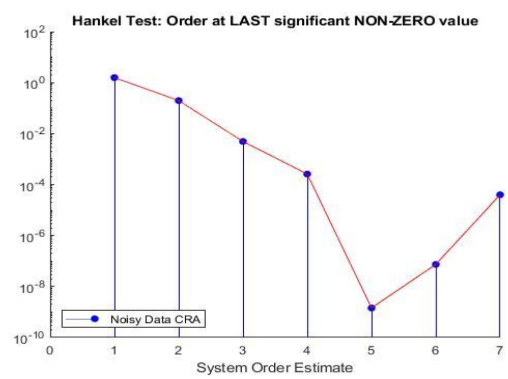


Figure 7: Hankel Test (Logarithmic)

We see that the Hankel Test does not give us a clear indication that the system is order 3, and instead tells us that it is at least order 2. This is satisfactory as we can change the order of the system to 2, should 3 not give a good fit-to-estimation when comparing with the OE model.

Generating OE Model:

Going into creating the OE model, our assumption is that the system has a structure vector of [2 3 8]. Where 2 is the number of zeros, 3 is the number of poles, and 8 is the sampling delay. Using these characteristics to generate an OE model, we will then compare it with newly generated data that has a slower switching rate as shown in **Figure 8**.

Specifications:

$$\begin{aligned} &Number\ of\ Samples\ (N) &&= 5000 \\ &Switching\ Rate\ (K) &&= 100 \\ &Amplitude\ (A) &&= 0.1 \end{aligned}$$

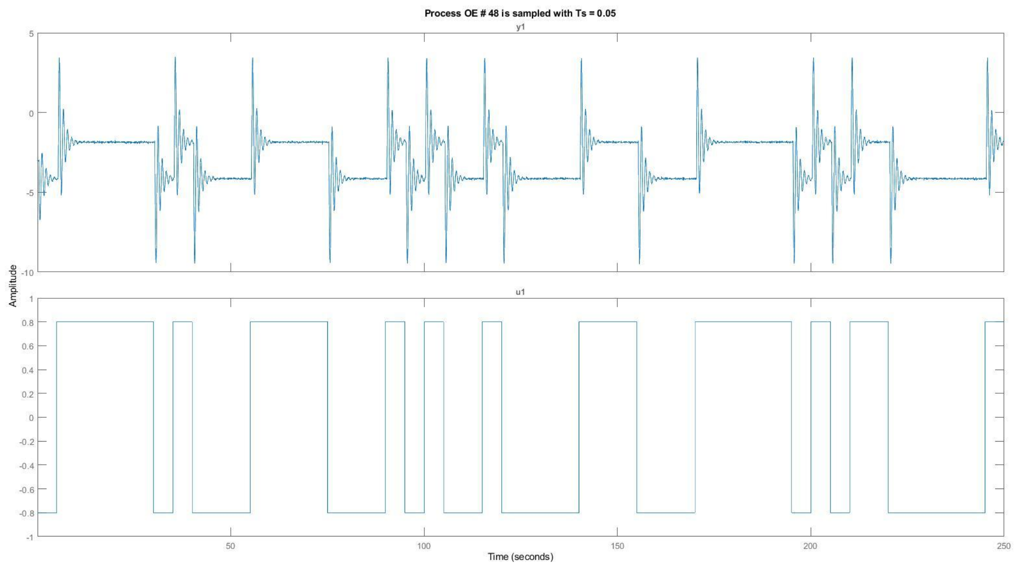


Figure 8: New DPRBS Generated Data with Slower Switching Rate OE-type

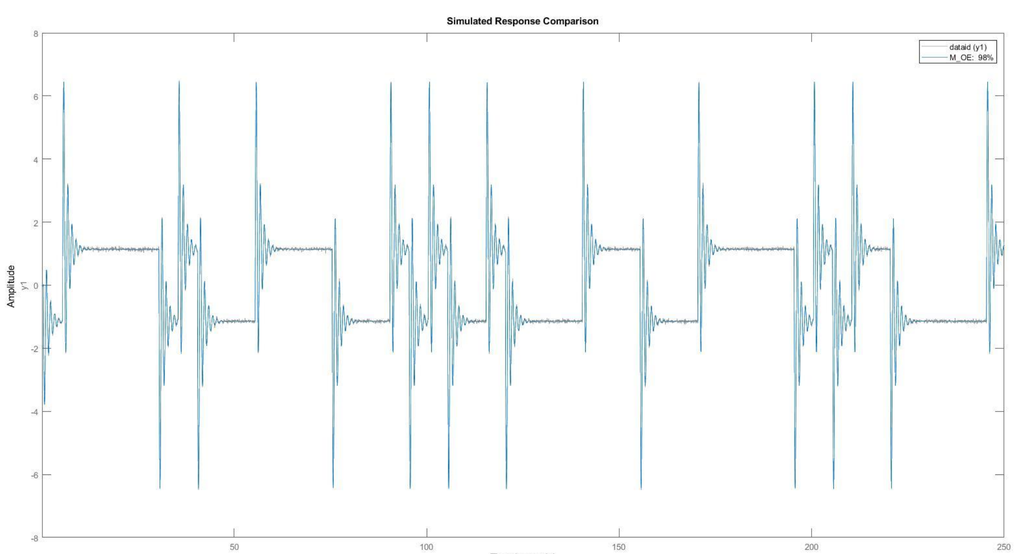


Figure 9: OE Model Comparison with New DPRBS Generated Data OE-type

Validation

We see from **Figure 9** that the fit-to-estimation is an amazing 98%. We know that this is a good fit by looking at the FPE and MSE values. According to MATLAB, the FPE and MSE values were 0.001022 and 0.00102 respectively. Since these numbers are very small, we know that the room for error when generating and comparing the OE model with the data is small.

Based on the OE model comparisons presented in **Figure 9**, we can infer that our initial assumption was correct, and can proceed to validate it through CCFs and ACFs.

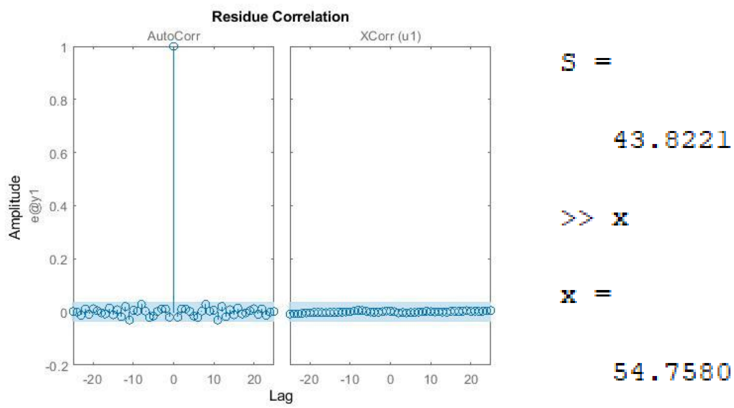


Figure 10: Whiteness Visual and Numerical Test

We see from **Figure 10** that the system passes the Whiteness Visual and Numerical Tests thus verifying the system specifications. To validate it, we need to check the Autocorrelation Function and the Cross Correlation Functions.

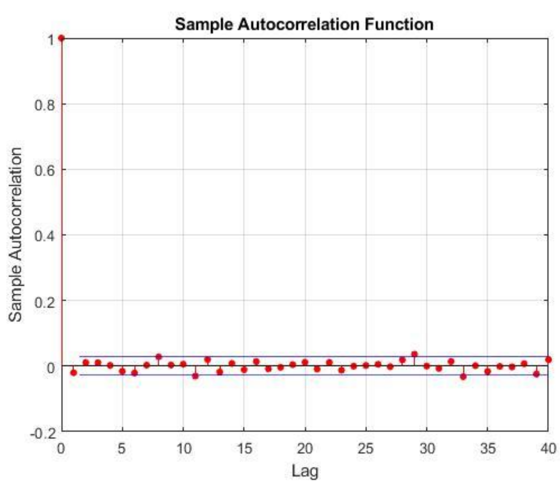


Figure 11: Autocorrelation Function OE-type

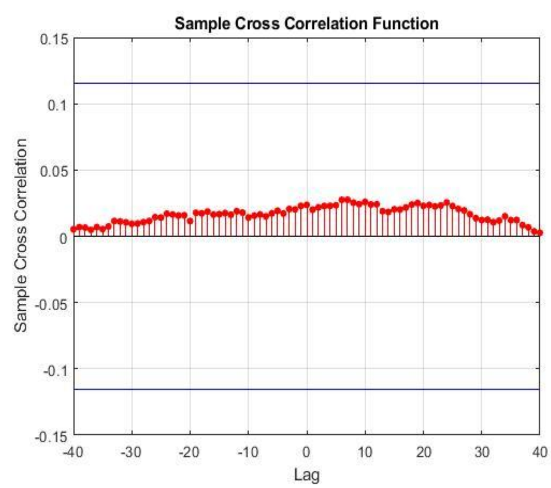


Figure 12: Cross Correlation Function OE-type

As we can see, the system passes both the autocorrelation function and the cross correlation function for validation. Which means that our system is validated.

The final part is to determine the G(s) for the system. We identify the black box OE-type system as having the equation of

Table 1: Pole Zero and Polynomial of G(s)

Pole-Zero G(s)	$\frac{25.563(s+140.6)(s+2.001)}{(s+139.7)(s^2+1.798s+36.01)} * e^{-0.35s}$
Polynomial G(s)	$\frac{25389s^2+3646s+7195}{s^3+141.5s^2+287.2s+5032} * e^{-0.35s}$

Comparison:

If we generate the step response as well as the bode-plot of the given system with the original data, we can determine if the system is correct. **Figure 13** and **Figure 14** show that both are very similar and is a good comparison. This means the G(s) we calculated was correct.

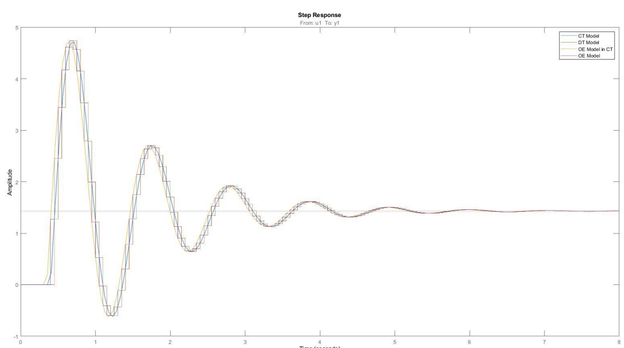


Figure 13: Step Response Comparison

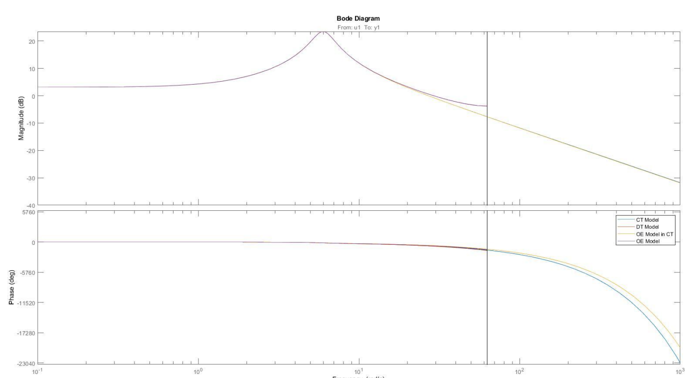


Figure 14: Bode Plot Comparison

PEM-type Model Identification

Running Diagnostics

For the PEM-type model, the initial first steps are the same; running diagnostics using bode-plots, impulse responses and hankel tests, then creating an OE model. If the model passes in CCF (Cross Correlation Function), then we can proceed with verifying the actual model of the system.

Similar to last time, the first step in identifying the characteristics of the system, we need to generate the data using the correct flag for a PEM-type model. The flag that I received was *flag '06,'* thus I would generate the data with that flag.

Specifications:

$$Number\ of\ Samples\ (N) \ = \ 20000$$
$$Switching\ Rate\ (K) \ = \ 1$$
$$Amplitude\ (A) \ = \ 2$$

I decided to initially use 20000 as the sample size to get an idea for the model through the bode-plot, impulse responses, and the hankel test. The fast switching rate makes it easier to help with diagnostics that is why I decided to use a switching rate of 1. Through trial and error, I discovered that an amplitude of 2 gives a good read for the data; data that isn't saturated or has a dead-zone effect. The DPRBS generated data can be seen in **Figure 15**.

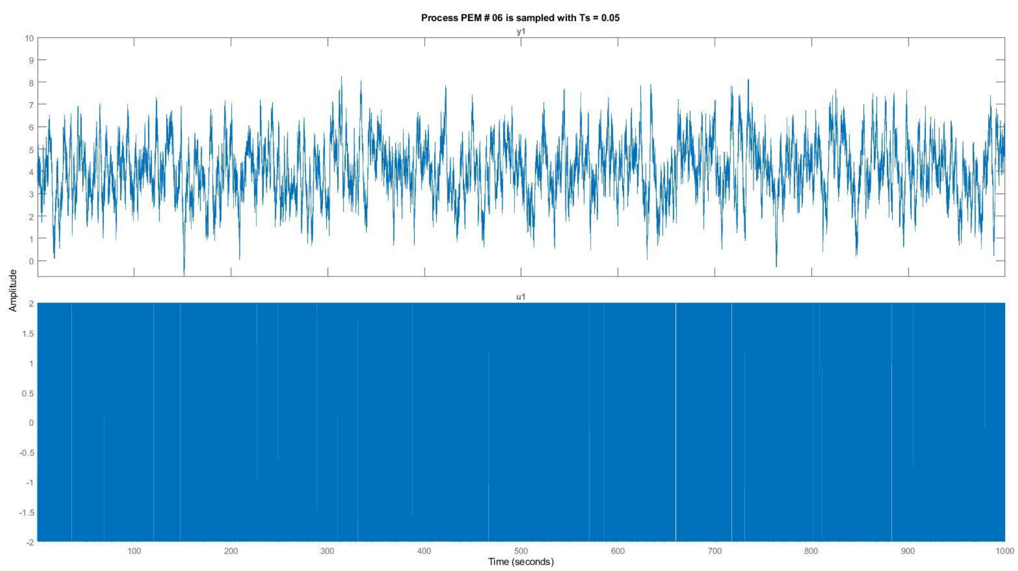


Figure 15: DPRBS Generated Data for PEM-Type

Once again, we will need to remove the DC offset so that the *mean* of the system can be zero. This is to omit any correlation issues later one. Zooming into **Figure 15** we see that the DC offset will be approximately 4. This was determined after running the data multiple times and seeing that the DC offset was always in between 3.5 and 4.5. **Figure 16**, below, shows an example of this.

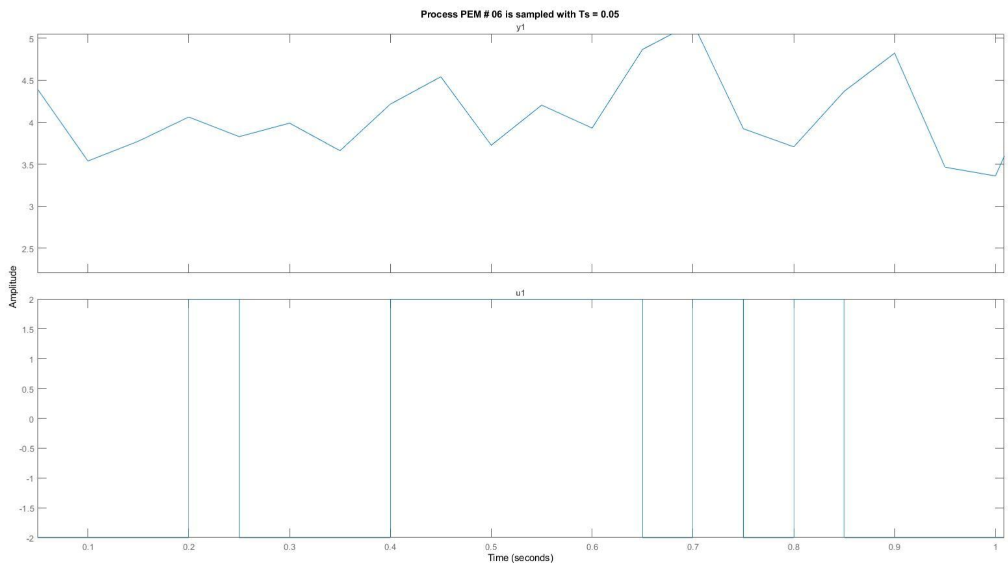


Figure 16: First Few Samples of Data Generated PEM-type

For this particular I tried to observe what would happen if there was a slow switching rate (50). This graph can be observed in **Figure 17**. We can clearly see that there is a negative dip in the graph indicating that there is a zero in the system, something that wouldn't have been seen with **Figure 15**.

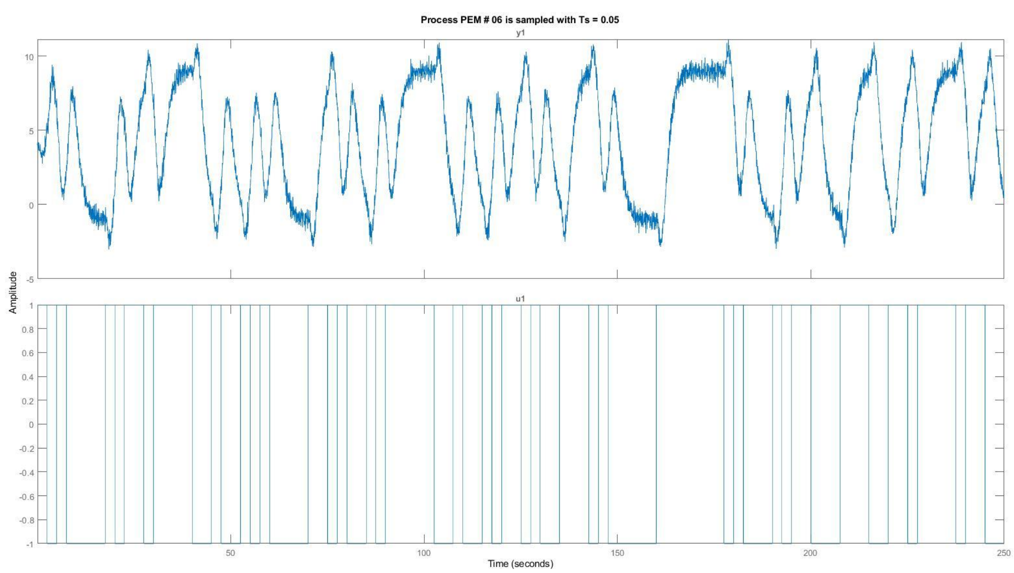


Figure 17: DPRBS Generated Data with Slow Switch Rate for PEM-type

Removing the DC offset lets us run the diagnostics properly. The first step being to analyze the bode-plot, determine the number of zeros and poles. Then creating an impulse response using the CRA correlation method to determine the number of sample delays. Finally, to check the Hankel Test to see if our initial analysis was either correct or if our assumptions were on the right track.

To determine the number of zeros and poles, we need to examine the bode-plot of the system which is displayed in **Figure 18**, below. Once again, the phase plot is not needed so it can be omitted from the display.

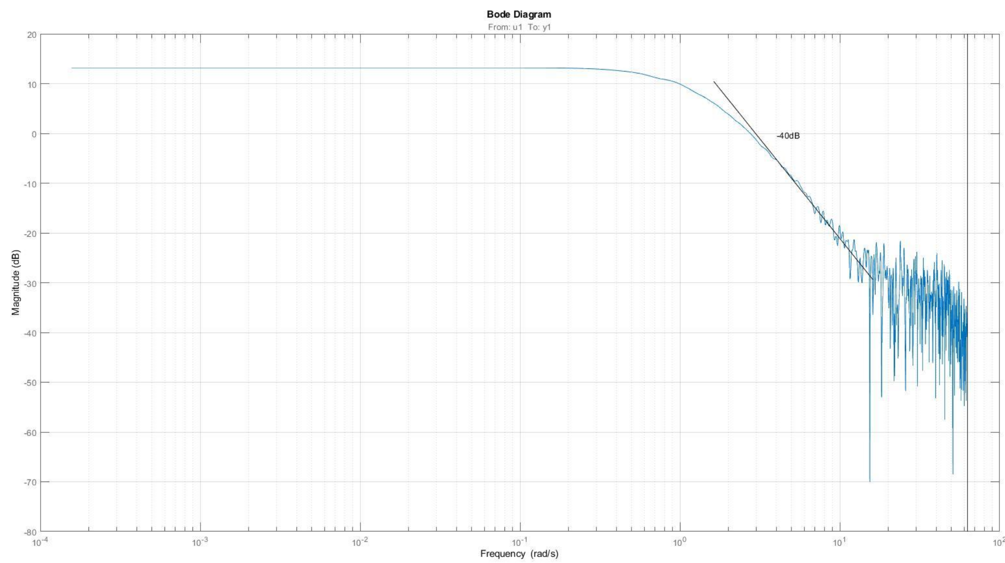


Figure 18: Bode-Plot of Data Generated PEM-type

As we see in **Figure 18**, we can see that there are no zeros, and two poles; however, we noticed in **Figure 17** that there is actually a zero. To make sense of this, we first need to continue with our diagnostics and determine the delay so that we can conduct the Hankel Test.

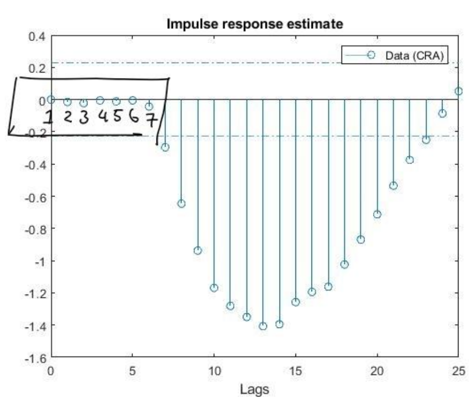


Figure 19: CRA with 50 Lags PEM-type

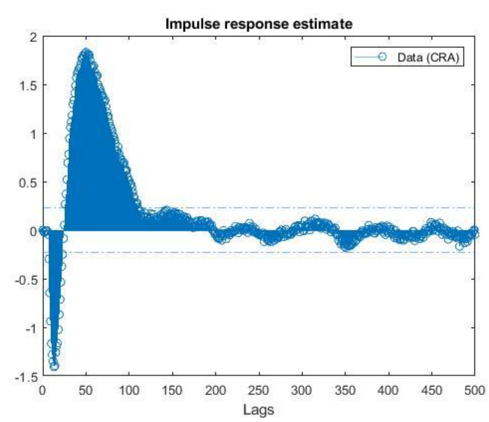


Figure 20: CRA with 1000 Lags PEM-Type

We can clearly see that from **Figure 19**, we have a sampling delay of 7. From **Figure 20**, we can observe that it is at least a second order system. **Figure 20** also shows an undershoot which implies that there is a zero. To understand why, we have to think back to planes and understand that in order for a zero to not show on the bode plot, if there is a RHP zero at +a, then there would need to be a LHP pole at -a for it to cancel out.

To get a better understanding of the order of the system, we can use the Hankel Test as shown below in **Figure 21** and **Figure 22**.

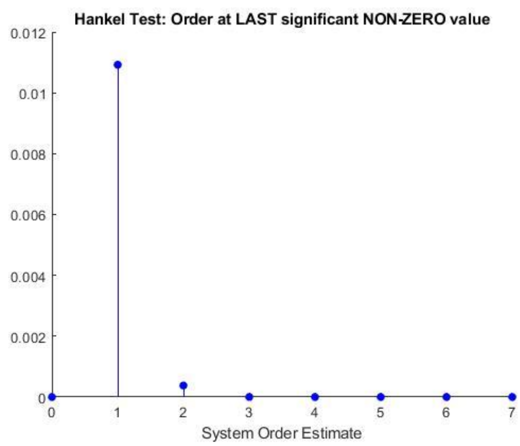


Figure 21: Hankel Test (Linear)

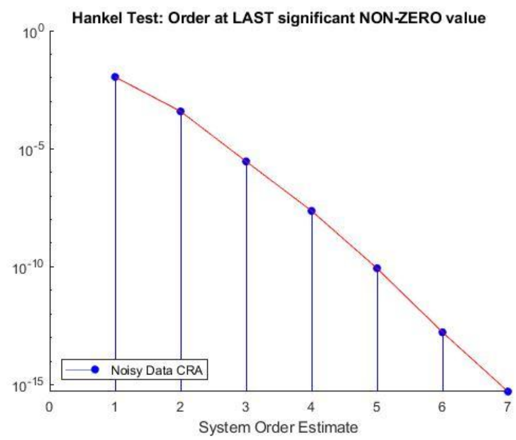


Figure 22: Hankel Test (Logarithmic)

Based on the Hankel Tests, we can only really confirm that the order is at least 2; however it does show very slightly that there is a change in slope at 3, thus suggesting an order of 3.

Based on the information gathered so far, we need to update our initial structure from no zeros, two poles to having one zero and three poles. When creating the OE model, check for two DT structures; [2 3 7] and [3 3 7].

In order to create an OE model, we need to generate two data sets with a slower switching rate (50). We create the OE model with one dataset and then compare with the seconds dataset. The new generated data sets are shown below in **Figure 23** and **Figure 24**.

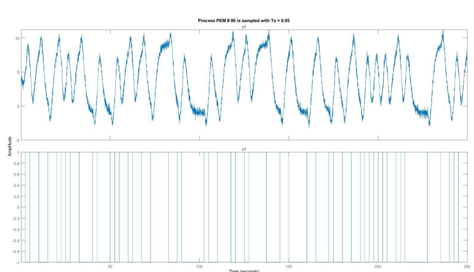


Figure 23: First New DPRBS Generated Data

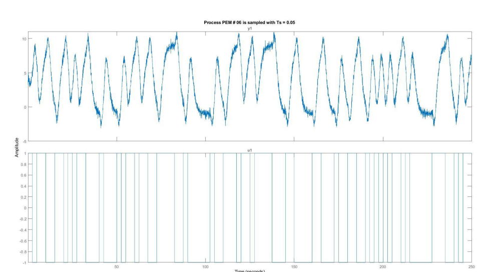


Figure 24: Second New DPRBS Generated Data

After trying both [2 3 7] and [3 3 7], I determined that [2 3 7] was a better fit than the other as the other's fit-to-estimation was often 30% less than [2 3 7]. The FPE and MSE values were also a lot larger than [2 3 7].

Generating OE Model:

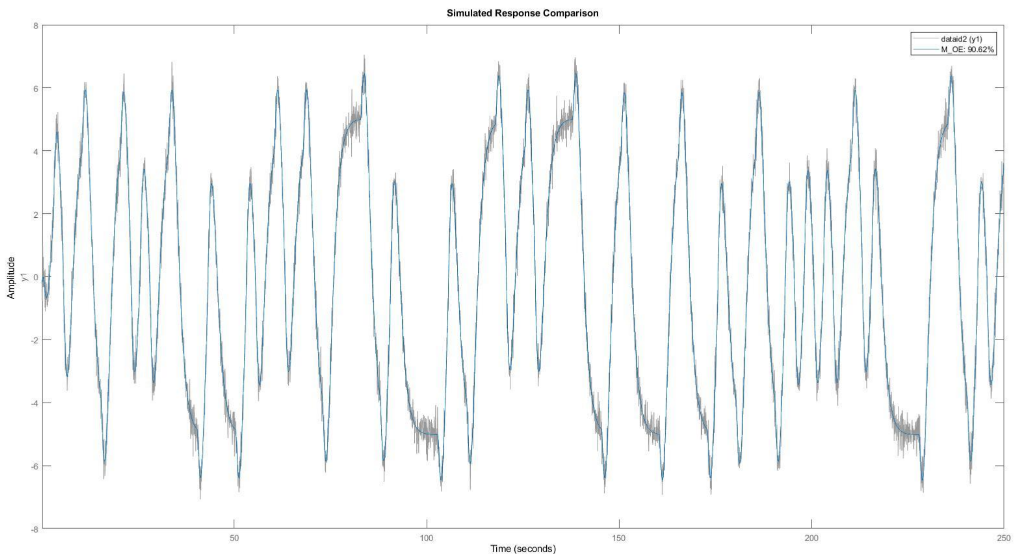
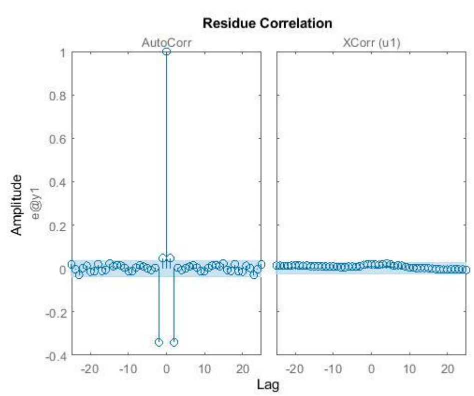


Figure 25: OE Model Comparison with New DPRBS Generated Data PEM-type

We can see from **Figure 25** that the fit-to-estimation is 90.62% which is a good fit. We can also look at the FPE and MSE values, which are 0.1212 and 0.121 respectively. These numerical values are small , meaning that room for error when creating the OE model is relatively small.

Validation:

Now that we have our OE model, we can verify and validate with the whiteness visual test as well as checking the autocorrelation and cross correlation functions.



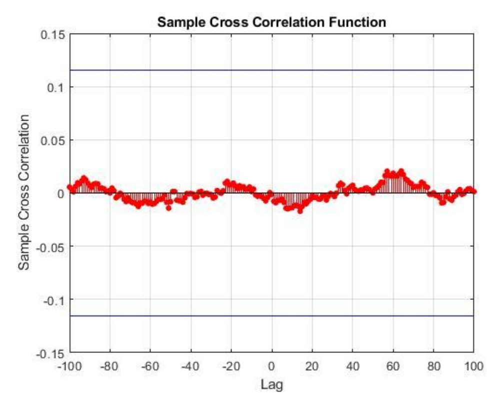
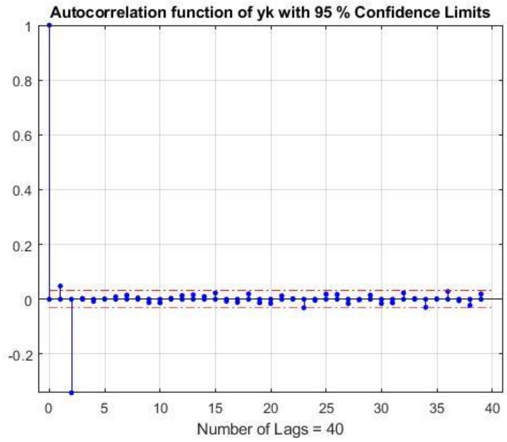


Figure 26: Whiteness Test for PEM-type **Figure 27:** Cross Correlation Function PEM-type

We see from **Figure 26** that it passes the whiteness test, and from **Figure 27** that the CCF is correct. This means we can now check the ACF (Autocorrelation Function) and PAC (Partial Autocorrelation Function) to figure out what ARMA structure it has.



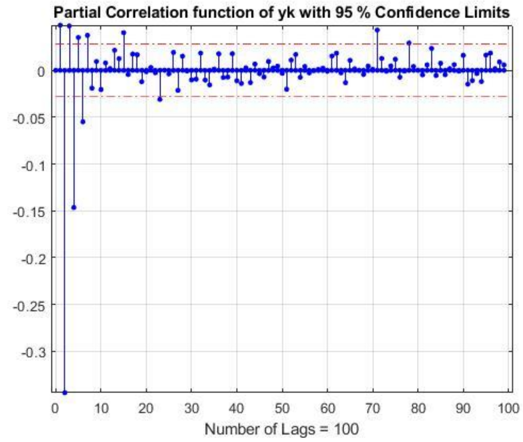
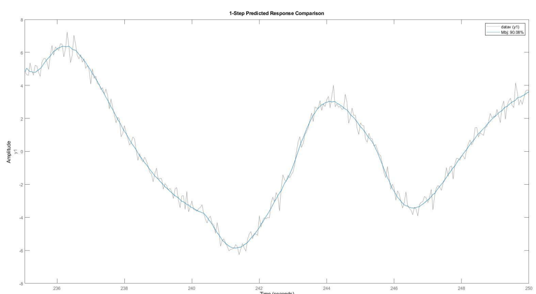


Figure 28: Autocorrelation Function PEM-type **Figure 29:** Partial Autocorrelation Function PEM-type

Generating BJ/Other Models + Validation:

As we see, it's difficult to analyze this; however, an assumption can be made that it is an MA(1) based on the ACF from **Figure 28** and the PACF from **Figure 29**. We can now adjust our model and try different models to get a good fit. The default first attempt should be the BJ model. **Figure 30** shows the BJ model comparison.



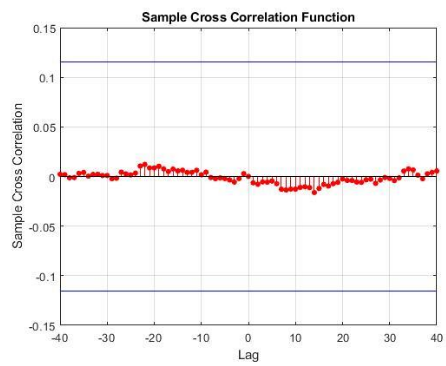


Figure 30: BJ Model Comparison **Figure 31:** CCF with BJ Model PEM-type

The fit-to-estimation with the BJ model gives us a 90.08% fit. The numerical values for the FPE and MSE are 0.123 and 0.121 respectively, thus indicating that there is little error in the model. Looking at the CCF of the model in **Figure 31**, we can see that it works and is a good fit. Based on the BJ model, it does not look like the PEM or ARMA structures are applicable due to the data matrix it makes in MATLAB with the identification data.

Based on the above data analysis, I determined that the BJ was the most appropriate model to use. Using this, the G(s) model that was used to generate our data would be:

Table 2: Pole Zero and Polynomial of G(s)

Pole-Zero G(s)	$\frac{0.24161(s-38.4)(s-0.9929)}{(s+0.819)(s^2+2.945s+2.245)} * e^{-0.3s}$
Polynomial G(s)	$\frac{0.2416s^2-9.517s+9.212}{z^3-2.817z^2+2.646z-0.8282} * e^{-0.3s}$

Comparison:

This G(s) when compared to the data, gives us a step response shown below in **Figure 32** and the bode-plot shown below in **Figure 33**. **Figure 32** shows that there may be a delay error in creating the step response; however, for the most part, it is very similar to G(s). This is also the case for the bode-plot shown below in **Figure 33**. This means that the G(s) that was calculated was correct.

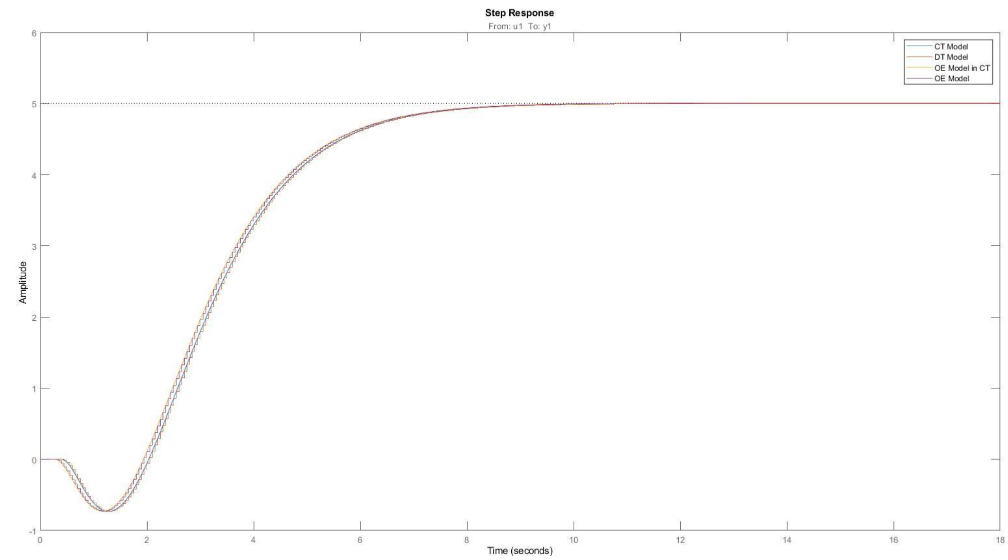


Figure 32: Step Response Comparison PEM-type

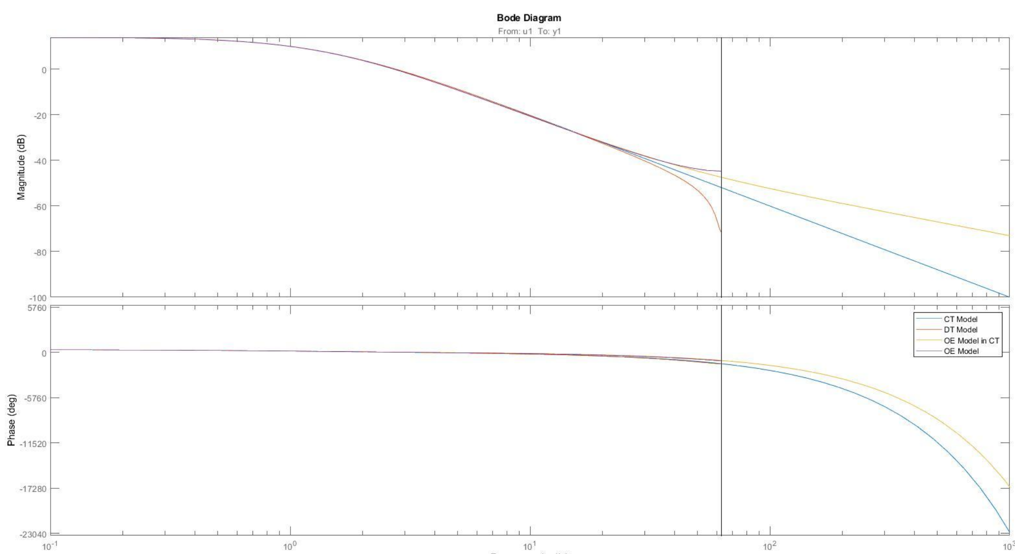


Figure 33: Bode-Plot Comparison PEM-type