Linear Knapsack Problem

September 16, 2022

Q1 The Knapsack Problem (KP) is considered to be a combinatorial optimization problem. A Knapsack model serves as an abstract model with broad spectrum applications such as: Resource allocation problems, Portfolio optimization, Cargo-loading problems and Cutting stock problems. In linear KP the objective function and constraint(s) are linear. Formulate the linear KP mathematically using the following data. [2 Marks]

Linear Knapsack Problem: Consider the following pairs
$(v_i, w_i) = \{(2, 7), (6, 3), (8, 3), (7, 5), (3, 4), (4, 7), (6, 5),$
$(5, 4), (10, 15), (9, 10), (8, 17), (11, 3), (12, 6), (15, 11), (6, 6),$
$(8, 14), (13, 4), (14, 8), (15, 9), (16, 10), (26, 24)\}$ with profit v_i and weight w_i for the i-th item; total capacity $W = 30$.

Q2 Use the following greedy algorithm to solve the above problem in Q1: [5 Marks]

Algorithm 1: Greedy Algorithm

1. Identify the available items with their weights and values and take note of the maximum capacity of the bag.
2. Use a score or efficiency function, i.e. the profit to weight ratio: $\frac{v_i}{w_i} \left(\frac{v_i}{w_i} \geq \frac{v_j}{w_j} \cdots\right)$
3. Sort the items non-increasingly according to the efficiency function.
4. Add into knapsack the items with the highest score, taking note of their accumulative weights until no item can be added.
5. Return the set of items that satisfies the weight limit and yields maximum profit.

Q3 Construct a penalty function of the maximization problem in Q1 with penalty parameter $R = 25$. Maximize the linear KP problem in Q1 via maximizing the penalty function using the iterative improvement local search (IILS). IILS uses passes and epochs. Each Pass executes a number of Epochs and each Epoch lock a variable. Epoch 1 always begins with x^0. IILS operates as follows: [8 Marks]

- Epochs within a Pass continue locking variables until an overall best solution (better than x^0) is found when a new pass begins (with Epoch 1).
- When all the Epochs in a Pass is unable to find an overall best solution (better than x^0) then IILS stops with x^0 as the minimum value. Note that execution of all Epochs in a Pass means all variables are locked.
- You must start your initial solution $x^0 = (x_1, x_2, \cdots, x_{21})^T$ such that $x_1 = x_2 = x_3 = x_4 = x_5 = 1$, and $x_i = 0$ for all $i = 6, 7, \cdots, 21$.

Implementation of Algorithm 1 and results: 5 Marks;
Implementation of Algorithm 2 and results: 10 Marks;

Submit your single pdf file consisting of your computer program and results via email attachment by 5pm 14th October 2022.