

Obtaining POS Expression

- Simplified POS expression can be obtained by collecting maxterm (I.e. 0) for the given function.
- Example

Given $F = \sum m(0, 1, 2, 3, 5, 7, 8, 9, 10, 11)$, we start with draw K-map, then cluster the maxterm

		A			
		AB		11	10
C	CD	00	01	11	10
	00	1	0	0	1
	01	1	1	0	1
	11	1	1	0	1
	10	1	0	0	1

Obtaining POS Expression

		A			
		AB		01	11
C	CD	00	01	11	10
	00	1	0	0	1
	01	1	1	0	1
	11	1	1	0	1
	10	1	0	0	1

Groupings: B (columns 01, 11), D (rows 01, 11), C (rows 11, 10)

		A			
		AB		01	11
C	CD	00	01	11	10
	00	0	1	1	0
	01	0	0	1	0
	11	0	0	1	0
	10	0	1	1	0

Groupings: B (columns 01, 11), D (rows 01, 11), C (rows 11, 10)

- Given SOP for F' is:

$$F' = BD' + AB$$

- To obtain POS for F , we do:

$$\begin{aligned}
 F &= (BD' + AB)' \\
 &= (BD')'(AB)' && \text{DeMorgan} \\
 &= (B' + D)(A' + B') && \text{DeMorgan}
 \end{aligned}$$

Don't Care Condition

- In certain problems, some of the output is not determined
- The output can be '1' or '0'
- This is known as don't care which is mark by X
- Example: In odd parity executor for BCD code, 6 is not used

No.	A	B	C	D	P
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	X
11	1	0	1	1	X
12	1	1	0	0	X
13	1	1	0	1	X
14	1	1	1	0	X
15	1	1	1	1	X

Don't Care Condition

- Don't care condition can be used to help us in simplifying Boolean expression in K-map
- It can be '1' or '0' depends on which expression is simpler

Don't Care Condition

- As a comparison
 - Without don't cares

$$P = A'B'C'D' + A'B'CD + A'BC'D + A'BCD' + AB'C'D$$

- With don't cares

$$P = A'B'C'D' + B'CD + BC'D + BCD' + AD$$

		C			
		D			
A	AB	00	01	11	10
	00	1		1	
	01		1		1
	11				
	10		1		

		C			
		D			
A	AB	00	01	11	10
	00	1		1	
	01		1		1
	11	X	X	X	X
	10		1	X	X

Simplification of SOP Expression

- Example 1

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

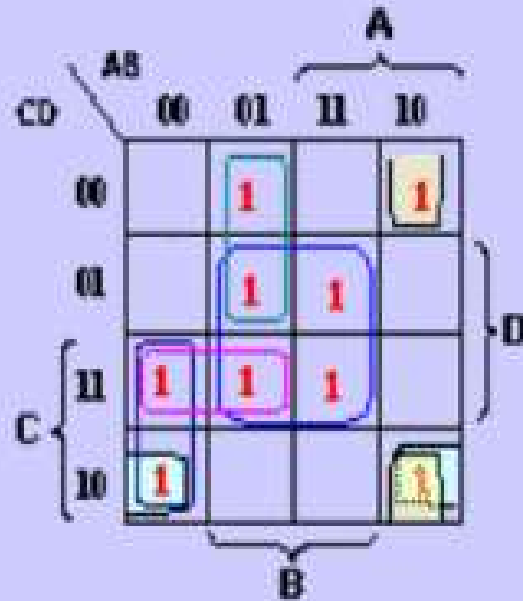
		A			
		00	01	11	10
C	00		1		1
	01		1	1	
	11	1	1	1	
	10	1			1
		B			

← Fill in the 1's.

Simplification of SOP Expression

- Example 1

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

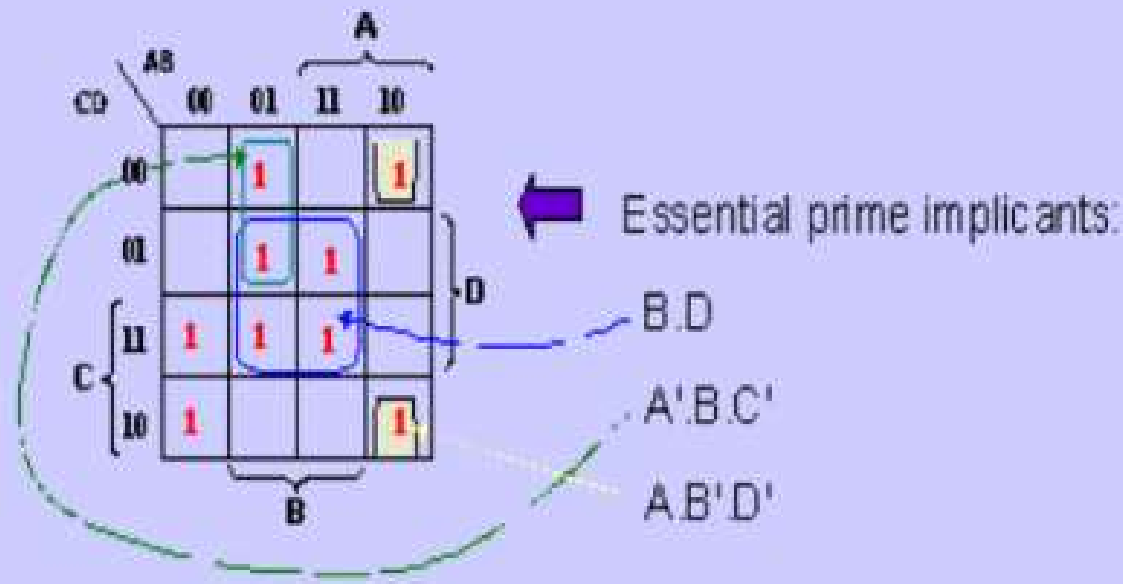


These are all the prime implicants; but do we need them all?

Simplification of SOP Expression

- Example 1

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$



Simplification of SOP Expression

- Example 1

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

		A			
		00	01	11	10
CD	00		1		1
	01		1	1	
C	11	1	1	1	
	10	1			1
		B			



Minimum cover.

EPs: $B.D$, $A'.B.C'$, $A.B'.D'$

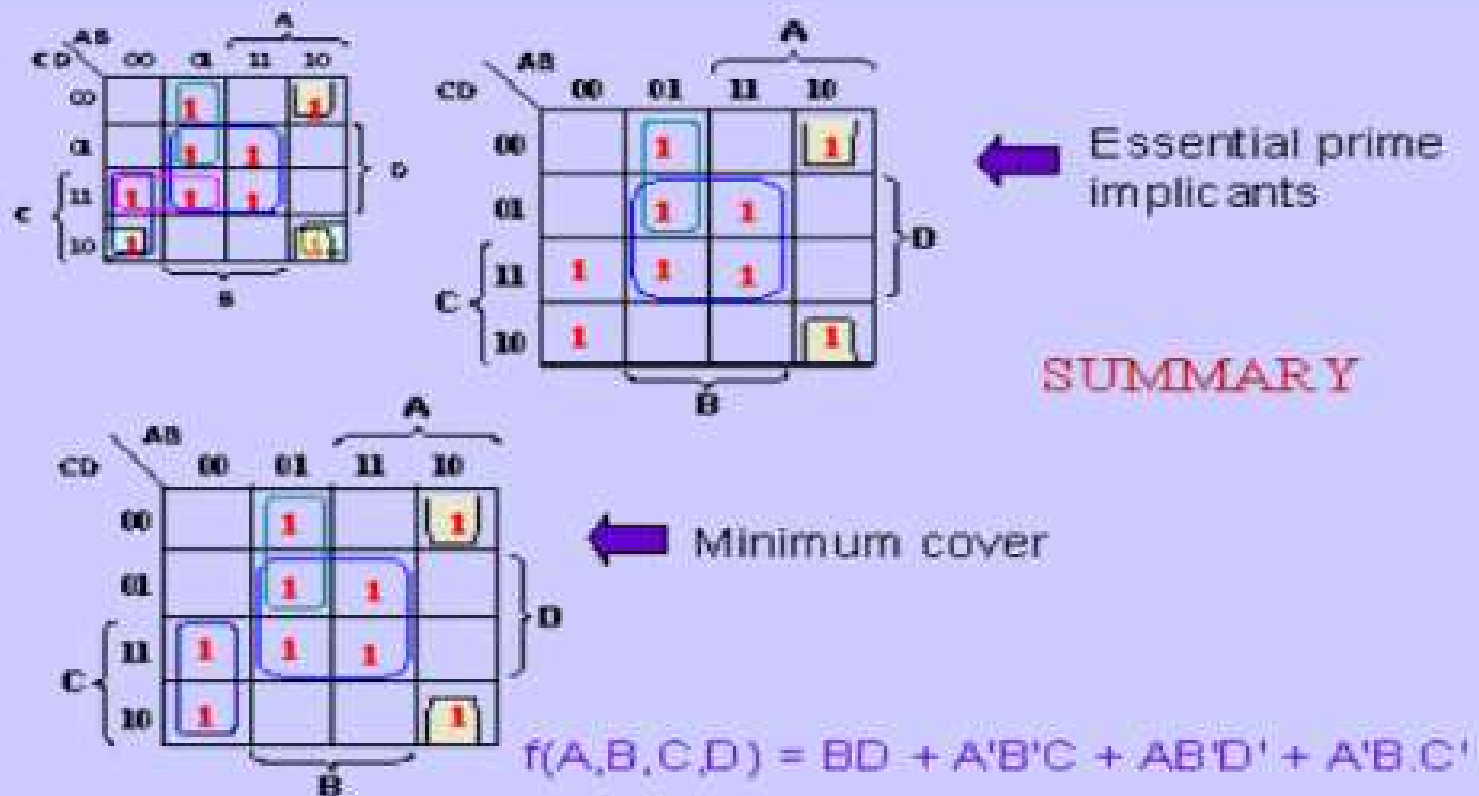
+

$A'.B'.C$

$$f(A,B,C,D) = B.D + A'.B.C' + A.B'.D' + A'.B'.C$$

Simplification of SOP Expression

- Example 1

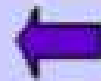


Simplification of SOP Expression

- Example 2

$$f(A,B,C,D) = A.B.C + B'.C.D' + A.D + B'.C'.D'$$

		A			
		00	01	11	10
C	00	1			1
	01			1	1
	11			1	1
	10	1		1	1
		B			

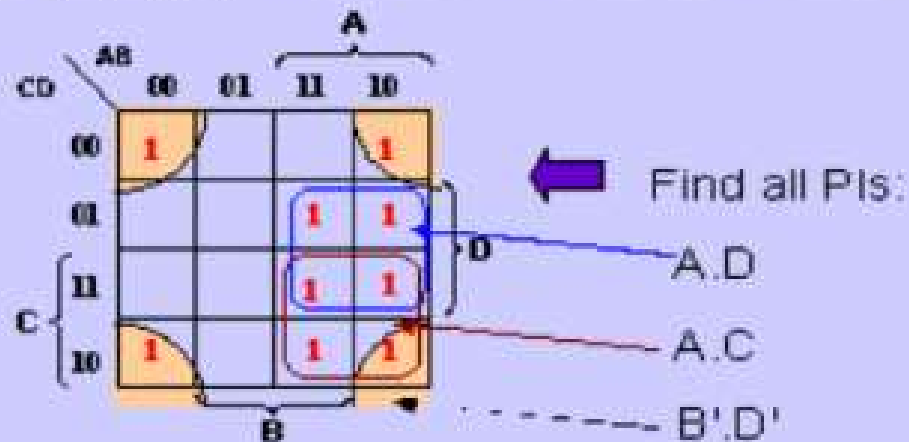


Fill in the 1's.

Simplification of SOP Expression

- Example 2

$$f(A,B,C,D) = A.B.C + B'.C.D' + A.D + B'.C'.D'$$



Are all '1's covered by the PIs? Yes, so the answer is: $f(A,B,C,D) = A.D + A.C + B'.D'$

Simplification of SOP Expression

- Example 3

$$f(A,B,C,D) = \sum m(2,8,10,15) + \sum d(0,1,3,7)$$

		A			
		00	01	11	10
CD	00	X			1
	01	X			
C	11	X	X	1	
	10	1			1



Fill in the 1's and X's.

Simplification of SOP Expression

- Example 3

$$f(A,B,C,D) = \sum m(2,8,10,15) + \sum d(0,1,3,7)$$

		A			
		00	01	11	10
CD	00	X			1
	01	X			
C	11	X	X	1	
	10	1			1
		B			

Do we need to have an additional term $A'B'$ to cover the 2 remaining x's?

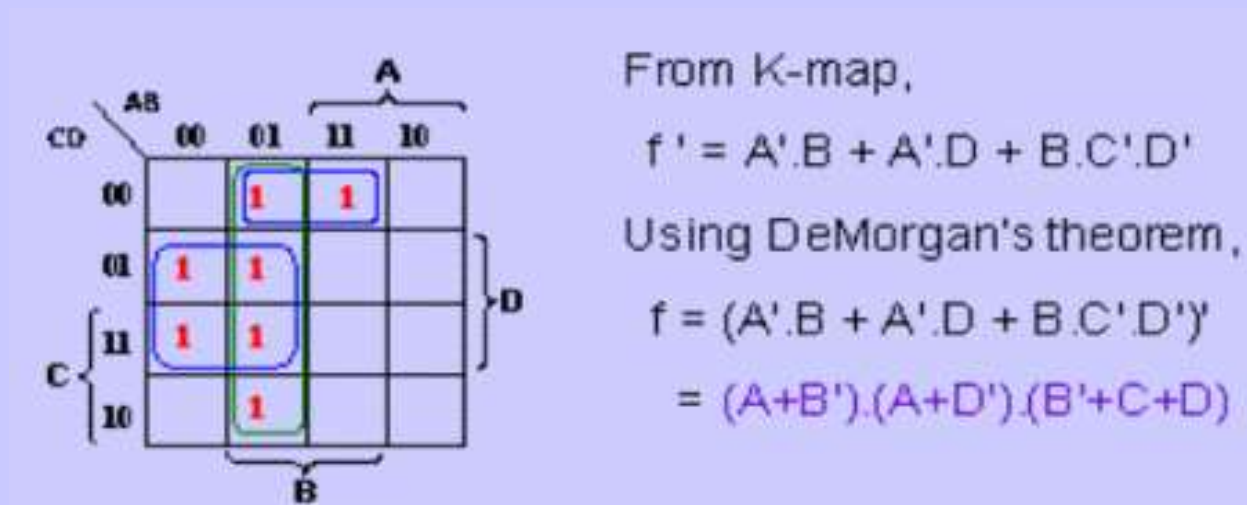
No, because all the 1's (minterms) have been covered.

$$f(A,B,C,D) = B'D' + B.C.D$$

Simplification of POS Expression

- To obtain POS expression for example 2

$$f(A,B,C,D) = A.B.C + B'.C.D' + A.D + B'.C'.D'$$
- Draw K-map for f complement which is f'



Simplification of POS Expression

- To obtain POS expression for example 3

$$f(A,B,C,D) = \sum m(2,8,10,15) + \sum d(0,1,3,7)$$
- Draw K-map for f complement which is f'

$$f'(A,B,C,D) = \sum m(4,5,6,9,11,12,13,14) + \sum d(0,1,3,7)$$

		A			
		AB			
CD	00	00	01	11	10
	00	x	1	1	
	01	x	1	1	1
	11	x	x		1
C		10		1	1
		B			

From K-map,

$$f' = B.C' + B.D' + B'.D$$

Using DeMorgan's theorem,

$$\begin{aligned} f &= (B.C' + B.D' + B'.D)' \\ &= (B'+C).(B'+D).(B+D') \end{aligned}$$