Chapter 4 Work, Energy and Power

Learning Objectives

After studying this chapter, the students will be able to:

- Derive the formula for kinetic energy [using the equations of motion]
- Derive an expression for absolute potential energy of a body at a certain position in the gravitational field [including escape velocity]
- Deduce the work done from force-displacement graph.
- Differentiate between conservative and non-conservative forces.
- State and use the work - energy theorem in a resistive medium to solve problems.

Work is often thought in terms of physical or mental effort. In Physics, however, the termwork involves two things (i) force, and (ii) displacement. We shall begin with a simple situation in which work is done by a constant force.

4.1 WORK DONE BY A CONSTANT FORCE

Let us consider an object which is being pulled by a constant force **F**. The force displaces the object through a displacement **d** in the direction of force. In such a case, work W is defined as the product of the magnitude of the force F and magnitude of the displacement **d**. This can be written as:

$$W = Fd \qquad (4.1)$$

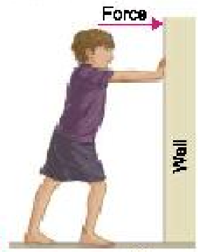


Fig. 4.1

Equation (4.1) shows that if the displacement is zero, no work is done even if a large force is applied. For example, pushing on a wall may tire your muscles, but work done is zero as shown in Fig. 4.1.

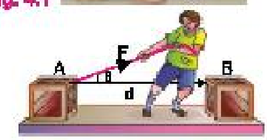


Fig. 4.2

The force applied on a body may not always be in the direction of force as shown in Fig. (4.2). If the force **F** makes an angle θ with the displacement **d** (Fig. 4.3), the work done is equal to the product of the component of force along the direction of the displacement and the magnitude of displacement. Then

$$W = (F\cos\theta)\,d = Fd\cos\theta \quad \text{.........} \quad (4.2)$$

or

$$W = \mathbf{F} \cdot \mathbf{d} \quad \text{.........} \quad (4.3)$$

Equation (4.3) shows that work is a scalar quantity.

The unit of work is joule (J). From Eq. (4.1), we have

$$1\ \text{J} = 1\ \text{N m}$$

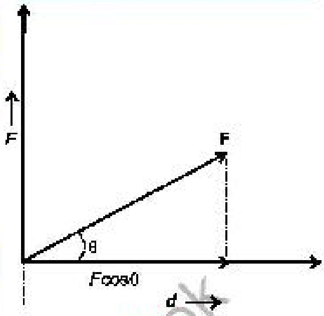


Fig. 4.3

When a constant force acts through a distance d, the event can be plotted on a simple graph (Fig. 4.4). The distance is normally plotted along x-axis and the force along y-axis. As the force does not vary, in this case, the graph will be a horizontal straight line. If the constant force F (newton) and the displacement **d** (metre) are in the same direction, then the work done is Fd (joule). Clearly shaded area in Fig. 4.4 is also Fd. Hence, the area under a force-displacement curve can be taken as to represent the work done by the constant force. In case the force F is not in the direction of displacement, the graph is plotted between $F\cos\theta$ and d.

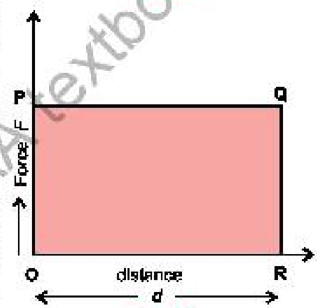


Fig. 4.4

From the definition of work, we find that:

(i) Work is a scalar quantity.

(ii) If $\theta < 90°$, work is done and it is said to be positive work.

(iii) If $\theta = 90°$, no work is done.

(iv) If $\theta > 90°$, the work done is said to be negative.

(v) SI unit of work is N m, also known as joule (J).

4.2 WORK DONE BY A VARIABLE FORCE

In many cases, the force does not remain constant during the process of doing work. For example, as a rocket moves away from the Earth, work is done against the force of gravity, which varies as the inverse square of the distance from the Earth's centre. Similarly, the force exerted by a spring increases with the amount of stretch. How can we calculate the work done in such situations?

Figure 4.5 shows the path of a particle in the x y plane as it moves from point P to point Q. The path has been divided into n short intervals of displacements $\Delta\mathbf{d}_1$, $\Delta\mathbf{d}_2$, ..., $\Delta\mathbf{d}_n$ and $\mathbf{F}_1$, $\mathbf{F}_2$, ..., $\mathbf{F}_n$ are the forces acting during these intervals, respectively.

During each small interval, the force is supposed to be approximately constant. So, the work done for the first interval can then be written as

$\Delta W_1 = \mathbf{F_1} \cdot \Delta \mathbf{d_1} = F_1 \cos\theta_1 \Delta d_1$

and in the second interval

$\Delta W_2 = \mathbf{F_2} \cdot \Delta \mathbf{d_2} = F_2 \cos\theta_2 \Delta d_2$

and so on. The total work done in moving the object can be calculated by adding all these terms.

$$W_{total} = \Delta W_1 + \Delta W_2 + \ldots\ldots + \Delta W_n$$

$$= F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + \ldots\ldots + F_n \cos\theta_n \Delta d_n$$

$$\text{or} \quad W_{total} = \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \qquad \ldots\ldots\ldots (4.4)$$

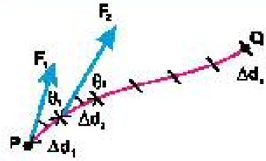


Fig. 4.5

A particle acted upon by a variable force, moves along the path shown from point P to point Q.

We can examine this graphically by plotting $F \cos\theta$ verses d as shown in Fig. 4.6. The displacement d has been sub-divided into the same n equal intervals. The value of $F \cos\theta$ at the beginning of each interval is indicated in the figure.

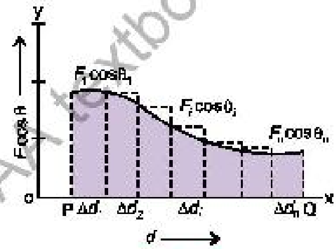


Fig. 4.6

Now the i[th] shaded rectangle has an area $F_i \cos\theta_i \Delta d$ which is the work done during the i[th] interval. Thus, the work done given by Eq. 4.4 equals the sum of the areas of all the rectangles. If we sub-divide the distance into a large number of intervals so that each Δd becomes very small, the work done given by Eq. 4.4 becomes more accurate. If we let each Δd to approach zero, then we obtain an exact result for the work done, such as:

$$W_{total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \qquad \ldots\ldots\ldots (4.5)$$

If this limit Δd approaches zero, the total area of all the rectangles (Fig. 4.6) approaches the area between the $F\cos\theta$ verses d curve and x-axis from P to Q as shown shaded in Fig. 4.7.

Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F \cos\theta$ verses d curve between the two points P and Q as shown in Fig. 4.7.

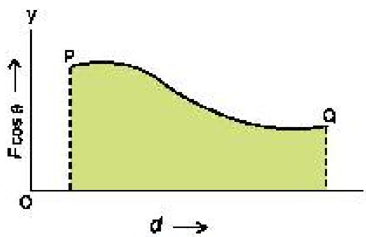


Fig. 4.7

Example 4.1 A force F acting on an object varies with distance d as shown in Fig. 4.8. Calculate the work done by the force as the object moves from $d = 0$ to $d = 6$ m.

Solution The work done by the force is equal to the total area under the curve from $d = 0$ to $d = 6$ m. This area is equal to the area of the rectangular section from $d = 0$ to

d = 4 m, plus the area of triangular section from d = 4 m to d = 6 m.

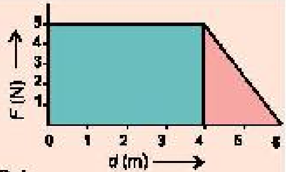


Fig. 4.8

Hence

Work done represented by the area of rectangle = 4 m x 5 N

$= 20 \text{ N m} = 20 \text{ J}$

Work done represented by the area of triangle = $\frac{1}{2}$ x 2 m x 5 N

$= 5 \text{ N m} = 5 \text{ J}$

Therefore, the total work done $= 20 \text{ J} + 5 \text{ J} = 25 \text{ J}$

4.3 CONSERVATIVE AND NON-CONSERVATIVE FORCES

Conservative Forces

The space around the Earth in which its gravitational force acts on a body is called the gravitational field. When an object is moved in the gravitational field, the work is done by the gravitational force. If displacement is in the direction of gravitational force, the work is positive. If the displacement is against the gravitational force, the work is said to be negative.

There is an interesting property of the gravitational force that when an object is moved from one place to another, the work done by the gravitational force does not depend on the choice of the path. Let us explore it.

Consider an object of mass m being displaced with constant speed from point A to B along various paths in the presence of a gravitational force (Fig. 4.9). In this case, the gravitational force is equal to the weight mg of the object.

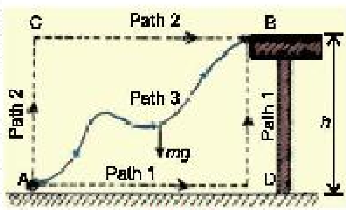


Fig. 4.9

The work done by the gravitational force along the path 1 (ADB) can be split into two parts (path AD and path DB). The work done along AD is zero, because the weight mg is perpendicular to this path, whereas the work done along DB is $(-mgh)$ because the direction of mg is opposite to that of the displacement i.e., $\theta = 180°$. Hence, the work done in displacing a body from A to B through path 1 is:

$$W_{ADB} = 0 + (-mgh) = -mgh$$

If we consider the path 2 (ACB), the work done along AC is also $(-mgh)$. Since the work done along CB is zero, therefore,

$$W_{ACB} = -mgh + 0 = -mgh$$

Now consider path 3, i.e., a curved one. Imagine the curved path to be broken down into a series of horizontal and vertical steps as shown in Fig. 4.10. There is no work done along the horizontal steps, because *mg* is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacements. During the segment CD, *mg* is not negative; it is positive. But here all Δy elements are negative, so the products of *mg* and Δy for all the elements will again be negative. Therefore, we can write:

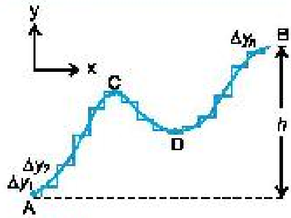


Fig. 4.10
A smooth path may be replaced by a series of infinitesimal *x* and *y* displacements. Work is done only during the *y* displacements.

$$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \Delta y_3 + \ldots\ldots + \Delta y_n)$$

As $\quad \Delta y_1 + \Delta y_2 + \Delta y_3 + \ldots\ldots + \Delta y_n = h$

Hence $\quad W_{AB} = -\,mgh$

The net amount of work done along the curved path AB is still *(-mgh)*. We conclude from the above discussion that:

Work done by gravitational force is independent of the path followed.

Can you prove that the work done, along a closed path, such as ACBA or ADBA (Fig. 4.9), by the gravitational force is zero?

If the work done by a force in moving an object between two points is independent of the path followed or the work done in a closed path be zero, the force is called a conservative force.

As shown above, the gravitational force is a conservative force, other examples of conservative force are electrostatic force and elastic spring force.

Non-Conservative Forces

All types of forces are not conservative forces.

A force is non-conservative if the work done by it in moving an object between two points or in a closed path depends on the path of motion.

The kinetic frictional force is a non-conservative force. When an object slides over a surface, the kinetic frictional force always acts opposite to the motion and does negative work equal in magnitude to the frictional force multiplied by the length of the path. Thus, greater amount of work is done over a longer path between any two points. Hence, the work depends on the choice of path. Moreover, the total work done by a non-conservative force in a closed path is not zero.

Other examples of non-conservative force are air resistance, tension in a string, normal force and propulsion force of a rocket.

4.4 POWER

In the definition of work, it is not clear, whether the same amount of work is done in one second or in one hour. The rate, at which work is done, is often of interest in practical applications.

Power is the measure of the rate at which work is being done.

If work ΔW is done in a time interval Δt, then the average power P_{av} during the interval Δt is defined as:

$$P_{av} = \frac{\Delta W}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.6)$$

If work is expressed as a function of time, the instantaneous power P at any instant is defined as:

$$P = \underset{\Delta t \to 0}{Limit} \frac{\Delta W}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.7)$$

where ΔW is the work done in short interval of time Δt.

Since $\Delta W = \mathbf{F} . \Delta \mathbf{d}$

Hence $P = \frac{\mathbf{F} . \Delta \mathbf{d}}{\Delta t} = \mathbf{F} . \frac{\Delta \mathbf{d}}{\Delta t}$

Since $\frac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v}$

Hence $P = \mathbf{F} . \mathbf{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.8)$

Thus, power may also be defined as the scalar product of F and **V**.

The SI unit of power is watt, defined as one joule of work done in one second.

Sometimes, for example, in the electrical measurements, the unit of work is expressed as watt second. However, a commercial unit of electrical energy is kilowatt-hour.

One kilowatt- hour is the work done in one hour by an agency whose power is one kilowatt.

Therefore $1\ kW\,h = 1000\ W \times 3600\ s$

or $1\ kW\,h = 3.6 \times 10^{6}\ J = 3.6\ MJ$

For your information

Approximate Powers

Device	Power (W)
Jumbo Jet Aircraft	1.3×10^{8}
Car at 90 km h^{-1}	1.1×10^{5}
Electric heater	2×10^{3}
Coloured TV	120
Flash light (two cells)	1.5
Pocket calculator	7.5×10^{-4}

Example 4.2 A 70 kg man runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

Solution Work done = mgh

$$\text{Power} = \frac{mgh}{t}$$

$$P = \frac{70\ kg \times 9.8\ m\,s^{-2} \times 4.5\ m}{4\ s}$$

$$P = 7.7 \times 10^{2}\ kg\,m^{2}s^{-3} = 7.7 \times 10^{2}\ W$$

Do you know?

It takes about 9×10^{9} J of energy to make a car and the car then uses about 1×10^{17} J of energy from petrol in its life time.

4.5 ENERGY

Energy of a body is its capacity to do work. There are two basic forms of energy:

(i) Kinetic energy (ii) Potential energy

Kinetic energy is the energy possessed by a body due to its motion and potential energy is the energy possessed by a body due to its changed position.

The kinetic energy and the potential energy both are the kinds of mechanical energy.

Kinetic Energy

Let us derive a formula for the kinetic energy of a moving body. Consider a car running with a constant speed on a road. If its engine is switched OFF, it will still cover some distance before stopping. As long as it is moving, it is doing work against the force of friction of the road. In other words, during this interval, it will exert a force equal in magnitude to the force of friction f. Let the distance travelled before coming to rest be d, then the work done by the car would be fd. This work is done by the car due to its motion. The ability of a body to do work due to its motion is its kinetic energy. Therefore, kinetic energy of the car is equal to fd. The acceleration can be found by using Newton's second law of motion, i.e.,

$$F = ma$$

As the car slows down and finally stops, its acceleration a is negative because it is produced by force of friction f acting apposite to the direction of motion. Thus,

$$f = -ma$$

or

$$a = -\frac{f}{m}$$

We can now determine the value of (fd) by using the third equation of motion, i.e;

$$2aS = v_f^2 - v_i^2$$

Here, Initial velocity $v_i = v$

Final velocity $v_f = 0$

Distance $S = d$

Acceleration $a = -\frac{f}{m}$

Putting values in the above equation of motion, we have

$$2 \times \left(\frac{-f}{m}\right) d = 0 - v^2$$

$$f\,d = \frac{1}{2} m v^2$$

As fd is equal to the kinetic energy of body, therefore,

$$\text{Kinetic energy} = \frac{1}{2} m v^2 \qquad (4.9)$$

For your Information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30×10^{9}
Burning 1 litre petrol	5×10^{7}
K.E. of a car at 90 km h^{-1}	1×10^{6}
Running Person at 10 km h^{-1}	3×10^{3}
Fission of one atom of uranium	1.8×10^{-11}
K.E. of a molecule of air	6×10^{-21}

Since, kinetic energy is equal to work which the body is capable of doing, so the unit of kinetic energy must be that of work, i.e. joule (J).

Example 4.3 A car weighing 18620 N is running with a speed of 16 m s^{-1}. Brakes are applied and it is brought to rest in a distance of 60 m. Determine the average force of friction acting on it.

Solution

Given that $v = 16\ \text{m s}^{-1}$, $d = 80\ \text{m}$, $w = 18620\ \text{N}$ and $f = ?$

The kinetic energy of the car is equal to the work done by it before stopping, i.e.,

$$\frac{1}{2}mv^2 = fd$$

Here $$m = \frac{w}{g} = \frac{18620\ \text{N}}{9.8\ \text{m s}^{-2}} = 1900\ \text{kg}$$

Putting the value in the above equation, we have

$$\frac{1}{2} \times 1900\ \text{kg} \times (16\ \text{m s}^{-1})^2 = f \times 80$$

or $$f = 3040\ \text{N}$$

Potential Energy

The potential energy is possessed by a body because of its position in a force field, e.g. gravitational field or because of its constrained state.

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state. This form of energy is called the elastic potential energy.

Absolute Potential Energy

> The absolute gravitational potential energy of an object at a certain position is the work done by the gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero.

The relation for the calculation of the work done by the gravitational force or potential energy is mgh, which is true only near the surface of the Earth where the gravitational force is nearly constant. But if the body is displaced through a large distance in space, let it be from point 1 to N (Fig. 4.11) in the gravitational field, then the gravitational force will not remain constant, since it varies inversely to the square of the distance.

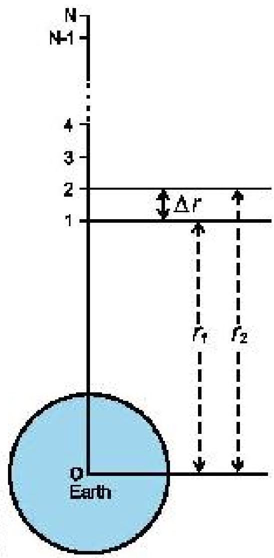


Fig. 4.11

In order to overcome this difficulty, we divide the distance between points 1 and N into small steps each of length Δr so that the value of the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps. If r_1 and r_2 are the distances of points 1 and 2 respectively, from the centre O of the Earth (Fig. 4.11.), the work done during the first step i.e., displacing a body from point 1 to point 2 can be calculated as below. The distance between the centre of this step and centre of the Earth will be:

$$r = \frac{r_1 + r_2}{2}$$

As $\quad r_2 - r_1 = \Delta r \quad$ then $\quad r_2 = r_1 + \Delta r$

Hence $\quad r = \dfrac{r_1 + r_1 + \Delta r}{2} = r_1 + \dfrac{\Delta r}{2}$ .... (4.10)

The gravitational force F at the centre of this step is:

$$F = G\frac{Mm}{r^2} \quad \text{..........} \quad (4.11)$$

where m = mass of an object , M = mass of the Earth
and G = Gravitational constant

Squaring Eq. 4.10

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$r^2 = r_1^2 + 2r_1\frac{\Delta r}{2} + \left(\frac{\Delta r}{2}\right)^2$$

As $(\Delta r)^2 << r_1^2$, so $(\Delta r)^2$ can be neglected as compared to r_1^2.

Hence $\quad r^2 = r_1^2 + r_1 \Delta r$

Putting the value of $\Delta r = r_2 - r_1$

$$r^2 = r_1^2 + r_1 (r_2 - r_1) = r_1 r_2$$

Hence, Eq. 4.11 becomes

$$F = G\frac{Mm}{r_1 r_2} \quad \text{.....................} \quad (4.12)$$

As this force is assumed to be constant during the interval Δr, so the work done is:

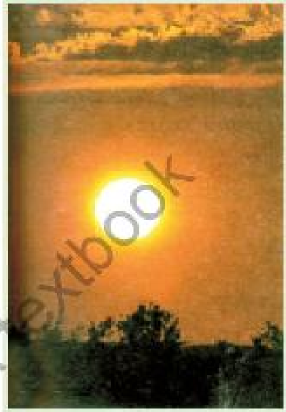
$$W_{1\to 2} = \mathbf{F}.\Delta\mathbf{r} = F\Delta r \cos 180^0 = - GMm\frac{\Delta r}{r_1 r_2}$$

The negative sign indicates that the work has to be done on the body from point 1 to 2 because displacement is opposite to gravitational force. Putting the value of Δr, we have

$$W_{1\to 2} = -GMm\frac{r_2 - r_1}{r_1 r_2}$$

or $$W_{1\to 2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Do you know?



There is more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.

Similarly, the work done during the second step in which the body is displaced from point 2 to 3 is:

$$W_{2\rightarrow 3} = -GMm\left(\frac{1}{r_2}-\frac{1}{r_3}\right)$$

and the work done in the last step is:

$$W_{N-1\rightarrow N} = -GMm\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)$$

Tidbits

More coal has been used since 1945 than was used in the whole of history before that.

Hence, the total work done in displacing a body from point 1 to N is calculated by adding up the work done during all these steps.

$$W_{total} = W_{1\rightarrow 2} + W_{2\rightarrow 3} + \ldots\ldots\ldots + W_{N-1\rightarrow N}$$

$$= -G\,Mm\left[\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\left(\frac{1}{r_2}-\frac{1}{r_3}\right)+\ldots\ldots+\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)\right]$$

On simplification, we have

$$W_{total} = -GMm\left(\frac{1}{r_1}-\frac{1}{r_N}\right)$$

If the point N is situated at an infinite distance from the Earth, then

$r_N = \infty$; so $\frac{1}{r_N} = \frac{1}{\infty} = 0$

Hence $W_{total} = -\frac{GMm}{r_1}$

This total work by definition is the absolute potential energy (P.E) as stated earlier represented by U.

$$U = -\frac{GMm}{r}$$

This is also known as the absolute value of gravitational potential energy of a body at a distance r from the centre of the Earth.

Note that when r increases, U becomes less negative i.e., U increases. It means when we raise a body above the surface of the Earth, its P.E. increases. Therefore, if we want to raise the body up to infinite distance, we will have to do work on it equal to $\frac{GMm}{R}$, so that its P.E. becomes zero.

Now the absolute potential energy on the surface of the Earth is found by putting $r = R$ (Radius of the Earth), so

$$\text{Absolute potential energy} = U_g = -\frac{GMm}{R} \quad \ldots\ldots \quad (4.13)$$

The negative sign shows that the Earth's gravitational field for mass m is attractive. The above expression gives the work or the energy required to take the body out of the

Earth's gravitational field, where its potential energy with respect to Earth is zero.

4.6 ESCAPE VELOCITY

It is our daily life experience that an object projected upward comes back to the ground after rising to a certain height. This is due to the force of gravity acting downward. With increased initial velocity, the object rises to the greater height before coming back. If we go on increasing the initial velocity of the object, a stage comes when it will not return to the ground. It will escape from the influence of gravity.

For your information

Some Escape speeds ($km\ s^{-1}$)

Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

> The initial velocity of an object with which it goes out of the Earth's gravitational field, is known as escape velocity.

The escape velocity corresponds to the initial kinetic energy gained by the body, which carries it to an infinite distance from the surface of the Earth.

$$\text{Initial } K.E. = \frac{1}{2} m v_{esc}^2$$

We know that the work done in lifting a body from Earth's surface to an infinite distance is equal to the increase in potential energy.

$$\text{Incerase in } P.E. = 0-(-G\frac{Mm}{R}) = G\frac{Mm}{R}$$

where M and R are the mass and radius of the Earth respectively. The body will escape out of the gravitational field if the initial $K.E.$ of the body is equal to increase in $P.E.$ Then

$$\frac{1}{2} m v_{esc}^2 = G\frac{Mm}{R}$$

or $$v_{esc} = \sqrt{\frac{2GM}{R}} \quad \text{..........} \quad (4.14)$$

As $$g = \frac{GM}{R^2} \quad \text{or} \quad gR = \frac{GM}{R}$$

Hence $$v_{esc} = \sqrt{2gR} \quad \text{..........} \quad (4.15)$$

The value of v_{esc} comes out to be approximately 11 $km\ s^{-1}$.

4.7 WORK-ENERGY THEOREM

Whenever work is done on a body, it increases its energy. For example, if a force F acts on a body of mass m, initially moving with velocity v_i, through a distance d and increases its velocity to v_f, then the acceleration produced will be:

$$2ad = v_f^2 - v_i^2$$

or $$a = \frac{1}{2d}(v_f^2 - v_i^2) \quad .... \quad (4.16)$$

From the second law of motion:

$$F = ma$$

or $$a = \frac{F}{m} \quad .... \quad (4.17)$$

Comparing Eqs. 4.16 and 4.17, we have

$$\frac{F}{m} = \frac{1}{2d}(v_f^2 - v_i^2)$$

or $$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad .... \quad (4.18)$$

Tidbits

All the food you eat in one day has about the same energy as 1/3 litre of petrol.

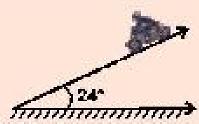
This expression is the work-energy theorem. It states that:

> The change in kinetic energy of an object is equal to the work done on it by a net force.
>
> W = Change in kinetic energy = $(K.E)_f - (K.E)_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

This is also known as work-energy principle.

The work-energy theorem is applicable for any direction of the force relative to the displacement. For instance, an object with kinetic energy can perform work if it is allowed to push or pull on another object. In this case, the work will be taken as negative and the kinetic energy of the object will decrease. The theorem remains valid even if the force may vary from point to point.

Example 4.4 A motorcycle rider weighing 60 kg is coasting down a 24° slope. The weight of motorcycle is 30 kg. At the top of the slope, the speed of motorcycle is 3.2 m s^{-1}. If the kinetic frictional force is 100 N, what will be the speed of the motorcycle 72 m downhill?



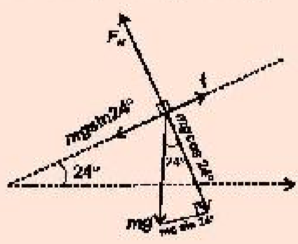
Solution The normal force F_N is balanced by the component of weight ($mg\cos 24°$) perpendicular to the slope, Let the kinetic frictional force is f, then the net force F is:

$$F = mg\sin 24° - f \qquad \text{where } m = \text{total mass} = 60\text{ kg} + 30\text{ kg} = 90\text{ kg}$$

or $$F = (90\text{ kg} \times 9.8\text{ m s}^{-2} \times 0.4) - 100\text{ N}$$

$$F = 252.8\text{ N}$$

Wok done $W = Fd = 252.8\text{ N} \times 72\text{ m} = 18201.6\text{ J}$



As work is positive, so applying work – energy theorem,

$$W = (K.E.)_f - (K.E.)_i$$

Form here,

$$(K.E.)_f = W + (K.E.)_i$$

Putting the values, we have

$$\frac{1}{2}mv_f^2 = W + \frac{1}{2}mv_i^2$$

$$\frac{1}{2}\times 90\ \text{kg}\times v_f^2 = 18201\ \text{J} + \frac{1}{2}\times 90\ \text{kg}\times(3.2\ \text{m s}^{-1})^2$$

This gives, $v_f^2 = 414.7\ \text{m}^2\ \text{s}^{-2}$

or $v_f = \sqrt{414.7\ \text{m}^2\ \text{s}^{-2}} = 20.4\ \text{m s}^{-1}$

4.8 INTERCONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

Consider a body of mass m at rest, at a height h above the surface of the Earth as shown in Fig. 4.12. At position A, the body has $P.E. = mgh$ and $K.E. = 0$. We release the body and as it falls, we can examine how kinetic and potential energies associated with it interchange.

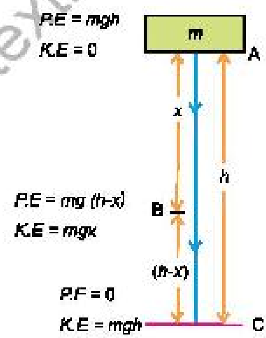


Fig. 4.12

Let us calculate $P.E.$ and $K.E.$ at the position B when the body has fallen a distance x, ignoring air friction.

Now, height from the ground is $(h-x)$, so that

$$P.E. = mg\,(h-x)$$

and $$K.E. = \frac{1}{2}mv_B^2$$

Velocity v_B, at position B, can be calculated from the relation,

$$v_f^2 = v_i^2 + 2gS$$

as $v_f = v_B$, $v_i = 0$, $S = x$

$$v_B^2 = 0 + 2gx$$

$$v_B^2 = 2gx$$

Therefore $$K.E. = \frac{1}{2}m\,(2gx)$$

$$= mgx$$

Total energy at position B = $P.E. + K.E.$

Total energy $= mg\,(h-x) + mgx = mgh$ .... (4.19)

At position C, just before the body strikes the Earth, $P.E. = 0$ and $K.E. = \frac{1}{2}mv_C^2$, where v_C can be found out by the following expression.

$$v_c^2 = v_i^2 + 2gh = 2gh \quad \text{as} \quad v_i = 0$$

i.e., $$K.E = \frac{1}{2}mv_c^2 = \frac{1}{2}m \times 2gh = mgh$$

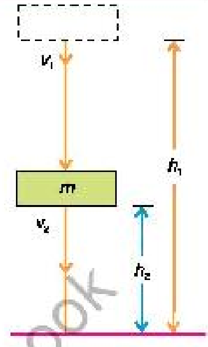


Fig. 4.13

Thus, at point C, kinetic energy is equal to the original value of the potential energy of the body. Actually, when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its kinetic energy. On the other hand, as the body falls, its height decreases and hence, its potential energy also decreases. Thus, we see (Fig. 4.13) that:

Loss in *P.E.* = Gain in *K.E*

$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2) \quad .... \quad (4.20)$$

where v_1 and v_2 are he velocities of the body at the heights h_1 and h_2 respectively. This result is true only when frictional force is not considered.

If we assume that a frictional force f is present during the downward motion, then a part of P.E. is used in doing work against friction equal to fh. The remaining *P.E.* = $mgh - fh$ is converted into K.E.

Hence $$mgh - fh = \frac{1}{2}mv^2$$

or $$mgh = \frac{1}{2}mv^2 + fh \quad .... \quad (4.21)$$

Thus Loss in *P.E.* = Gain in *K.E.* + Work done against friction

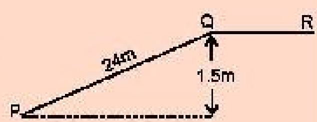
Conversely,

Loss of *K.E.* = Gain in *P.E.* + Work done against friction

Example 4.5

A car weighing 1100 kg is moving with a velocity of 12 m s^{-1}. When it is at point P, its engine stops. If the frictional force is 120 N, what will be its velocity at point Q? How far beyond Q will it go before coming to rest?

Solution



The kinetic energy possessed by the car at point P will partly be converted into *P.E.* and partly used up in doing work against friction as it reaches point Q. Therefore,

Loss of *K.E.* = Gain in *P.E.* + Work against friction

$$\frac{1}{2}m(v_i^2 - v_f^2) = wh + fd$$

$$\frac{1}{2}\times 1100\text{ kg }(144\text{ m}^2\text{ s}^{-2} - v_f^2) = (1100\text{ kg} \times 9.8\text{ m s}^{-2} \times 1.5\text{ m}) + 120\text{ N} \times 24\text{ m}$$

$$550\text{ kg }(144\text{ m}^2\text{ s}^{-2} - v_f^2) = 16170\text{ kg m}^2\text{ s}^{-2} + 2880\text{ kg m}^2\text{ s}^{-2}$$

$$(144\text{ m}^2\text{ s}^{-2} - v_f^2) = \frac{16170\text{ kg m}^2\text{ s}^{-2} + 2880\text{ kg m}^2\text{ s}^{-2}}{550\text{ kg}} = 34.6\text{ m}^2\text{ s}^{-2}$$

$$v_f^2 = 144\text{ m}^2\text{ s}^{-2} - 34.6\text{ m}^2\text{ s}^{-2} = 109.4\text{ m}^2\text{ s}^{-2}$$

Velocity at point Q, $v_f = \sqrt{109.4\text{ m}^2\text{ s}^{-2}} = 10.5\text{ m s}^{-1}$

Now if the car stops at point R, then using the formula:

$$\frac{1}{2}mv^2 = fS$$

$$\frac{1}{2}\times 1100\text{ kg} \times 109.4\text{ m}^2\text{ s}^{-2} = 120\text{ kg m s}^{-2} \times S$$

$$S = 501\text{ m approximately}$$

Example 4.6 An object of mass 3 kg falls from a height of 15 m. If it strikes the ground with a velocity of 16 m s^{-1}, calculate the average frictional force of the air.

Solution Loss of $P.E.$ = Gain in $K.E.$ + Work done against friction

$$\because v_i = 0, \quad mgh = \frac{1}{2}mv^2 + fh$$

$$3\text{ kg} \times 9.8\text{ m s}^{-2} \times 15\text{ m} = \frac{1}{2}\times 3\text{ kg }(16\text{ m s}^{-1})^2 + f \times 15\text{ m}$$

$$441\text{ kg m}^2\text{ s}^{-2} = 384\text{ kg m}^2\text{ s}^{-2} + 15\text{ m} \times f$$

or

$$f = \frac{441\text{ kg m}^2\text{ s}^{-2} - 384\text{ kg m}^2\text{ s}^{-2}}{15\text{ m}} = 3.8\text{ kg m s}^{-2} = 3.8\text{ N}$$

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

4.1 A 1 kg mass has potential energy of 1 joule relative to the ground when it is at a height of:

(a) 0.102 m (b) 1 m (c) 9.8 m (d) 32 m

4.2 An iron sphere whose mass is 30 kg has the same diameter as an aluminium sphere whose mass is 10.5 kg. The spheres are simultaneously dropped from a cliff. When they are 10 m from the ground, they have identical:

(a) accelerations (b) momentums (c) potential energies (d) kinetic energies

4.3 A body at rest may have:

(a) speed (b) velocity (c) momentum (d) energy

4.4 The height above the ground of a child on a swing varies from 0.5 m of his lowest point to 1.5 m at his highest point. The maximum speed of the child is approximately:

(a) $1.5\ m\,s^{-1}$ (b) $4.4\ m\,s^{-1}$
(c) $9.8\ m\,s^{-1}$ (d) Depends upon child's mass

4.5 When a ball is thrown vertically upward and then falls back to the ground, which force can be considered conservative in this scenario?

(a) Air resistance (b) Gravity
(c) Friction between ball and air (d) Contact force with hand

4.6 According to work-energy principle in linear motion, the work done on body is equal to:

(a) change in *K.E.* (b) change in *P.E.*
(c) zero (d) sum of *K.E.* and *P.E.*

4.7 Power of a lamp is 6 W. How much energy does a lamp give out in 2 min?

(a) 12 J (b) 20 J (c) 3 J (d) 720 J

4.8 A dry battery can deliver 3000 J of energy to a 2 W small electric motor before the battery is exhausted. For how many minutes does the battery run?

(a) 1500 min (b) 100 min (c) 50 min (d) 25 min

4.9 The kinetic energy acquired by a mass m after travelling a fixed distance from rest under the action of a constant force is directly proportional to:

(a) $\sqrt{m}$ (b) $1/\sqrt{m}$ (c) m (d) independent of m

4.10 A body moves a distance of 10 m along a straight line under the action of 5 N force. If the work done is 25 J, the angle which the force makes with the direction of motion of the body is:

(a) 0° (b) 30° (c) 60° (d) 90°

Short Answer Questions

4.1 Why is electrical power required at all when the elevator is descending? Why should be there a limit on the number of passengers in this case?

4.2. A body is being raised to a height H from surface of the Earth. What is the sign of work done by both (body and the Earth)? Justify.

4.3 A body falls towards the Earth in air. Will its total mechanical energy be conserved during fall? Justify.

4.4 Calculate power of a crane in kilowatt which lifts a mass of 1000 kg to a height of 100 m in 20 second.

4.5 A trolley of mass 1500 kg carrying sand bags of 500 kg is moving uniformly with a speed of 40 km h^{-1} on a frictionless track. After some time, sand starts leaking out of whole sand bags on the road at a rate of 0.05 kg s^{-1}. What is the speed of the trolley after entire sand bags are empty?

4.6 Give absolute and gravitational units of work in M.K.S and C.G.S systems.

4.7 A body dropped from a height of H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate work done by air friction.

4.8 A bicycle has a *K.E.* of 150 J. What *K.E.* would the bicycle have if it had:

(i) same mass but has speed double?

(ii) three times mass and was moving with one half of the speed?

4.9 What will be the effect on *K.E.* of the body having mass *m*, moving with velocity *v* when its momentum becomes double? Justify.

4.10 Does the international space station have gravitational *P.E.* or kinetic energy or both? Explain.

Constructed Response Questions

4.1 When will you say that a force is conservative? Give two conditions.

4.2 A light and a heavy body have same linear momentum, which one has greater *K.E.*?

4.3 A motorcycle is running with constant speed on a horizontal track. Is any work being done on the motorcycle, if no net force is acting on it?

4.4 A force acts on a ball moving with 14 m s^{-1} speed and brings its speed to 6 m s^{-1}. Has the force done positive or negative work? Explain your answer.

4.5 A slow moving truck can have more kinetic energy than a fast moving car. How is this possible?

4.6 Why work done against friction is non-conservative in nature? Explain briefly.

4.7 Does wind contain kinetic energy? Explain.

Comprehensive Questions

4.1 Define *K.E.* and derive an expression for the same.

4.2 How is work done by a:

(i) constant force (ii) variable force?

4.3 Define conservative field. Show that gravitational field is conservative in nature.

4.4 What is meant by absolute P.E.? Derive an expression for absolute P.E.

4.5 State and explain work-energy theorem in a resistive medium.

4.6 Define escape velocity. Show that an expression for escape velocity can be expressed as $\sqrt{2Rg}$, where *R* and *g* denote radius of the Earth and acceleration due to gravity, respectively. Also find its numerical value near the surface of the Earth.

Numerical Problems

4.1 A machine gun fires 6 bullets per minute with a velocity of 700 m s^{-1}. If each bullet has a mass of 40 g, then find power developed by the gun? (Ans: 980 W)

4.2 A family uses 10 kW of power. Direct solar energy is incident on horizontal surface at an average rate of 300 W per square metre. If 75% of this energy can be converted into useful electrical energy, how large area is needed to supply 10 kW?

(Ans: 44.44 m^2)

4.3 The mass of the Earth is 6.0×10^{24} kg and mass of the Sun is 1.99×10^{30} kg. The Sun is 160 million km away from the Earth. Find the value of gravitational *P.E.* of the Earth. (Ans: $\approx -4.97 \times 10^{33}$ J)

4.4 An object weighing 98 N is dropped from a height of 10 m. Its speed just before hitting the ground is 12 m s^{-1}. What is the frictional force acting on it? (Ans: 26 N)

4.5 A 75 watt fan is used for 8 hours daily for 30 days. Find:

(i) energy consumed in electrical units

(ii) electricity bill if one unit costs Rs. 22.5? [Ans: (i) 18 units (ii) Rs. 405]

4.6 If an object of mass 2 kg thrown up from ground reaches a height of 5 m and falls back to the Earth (neglecting air resistance), calculate:

(i) work done by gravity when the object reaches at 5 m height.

(ii) work done by gravity when the object comes back to the Earth.

(iii) total work done by gravity in upward and downward motion. Also mention physical significance of the result.

[Ans: (i) -98 J (ii) +98 J (iii) 0 J work done in a closed path in a conservative field is zero]

4.7 An electrical motor of one horse power is used to run a water pump. Water pump takes 15 minutes to fill a tank of 400 litres at a height of 10 m (1 hp 746 watts). Find:

(a) actual input work done by electric motor to fill the tank

(b) actual output work done [Ans: (a) 671.4 kJ, (b) 39.2 kJ]

4.8 A passenger just arrives at the airport and dragging his suitcase to luggage checks in at the desk. He pulls strap with a force of 200 N at an angle of 45° to the floor to displace it 50 m to the desk. Determine the value of work done by him on the suitcase. (Ans: 7 kJ)

4.9 A 1200 kg car is running at a speed of 40 km h^{-1}. How much power will be expended by it to accelerate at 2 m s^{-2}? (Ans: 26.67 kW)

4.10 A 200 g apple is lifted to 10 m and then dropped. What is its velocity when it hits the ground? Assume that 75% of work done in lifting the apple is transferred to *K.E.* by the time it hits the ground. (Ans: 12.1 m s^{-1})