Chapter 10 Electromagnetism

Learning Objectives

After studying this chapter, the students will be able to:

- State that a force might act on a current-carrying conductor placed in a magnetic field
- Use the equation $F=BIL\sin\theta$ [with directions as interpreted by Fleming's left-hand rule to solve problems]
- Define magnetic flux [as the product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density]
- Use $\phi = BA$ to solve problems
- Use the concept of magnetic flux linkage
- Define magnetic flux density [as the force acting per unit current per unit length on a wire placed at right angles to the magnetic field]
- Use $F=BqV\sin\theta$ to solve problems
- Describe the motion of a charged particle moving in a uniform magnetic field perpendicular to the direction of motion of the particle
- Explain how electric and magnetic fields can be used in velocity selection
- Explain experiments that demonstrate Faraday's and Lenz's laws [(a) that a changing magnetic flux can induce an emf in a circuit, (b) that the induced emf is in such a direction as to oppose the change producing it, (c) the factors affecting the magnitude of the induced emf.]
- Use Faraday's and Lenz's laws of electromagnetic induction to solve problems
- Describe how ferrofluids work [they make use of temporary soft magnetic materials suspended in liquids to develop fluids that react to the poles of a magnet and have many applications in fields such as electronics]
- Explain how seismometers make use of electromagnetic induction to the earthquake detection [specifically in terms of:
  (i) any movement or vibration of the rock on which the seismometer rests (buried in a protective case) results in relative motion between the magnet and the coil (Suspended by a spring from the frame.)
  (ii) The emf induced in the coil is directly proportional to the displacement associated

We have already studied that a magnetic field is produced around a current-carrying conductor. Also, a changing magnetic field gives rise to a current in a conductor placed in it. Electromagnetism is a key area of physics that studies how electric charges and magnetic fields interact.

In 1820, Hans Christian Oersted found that electricity and magnetism are correlated.

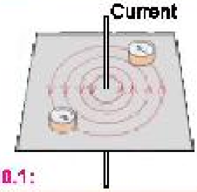


Fig. 10.1: The direction of magnetic field produced by a current

For your Information

Magnetism started with lodestone, a natural mineral discovered in ancient Türkiye. Lodestone, or magnetite (Fe_3O_4), can attract metals like iron and steel and aligns with the Earth's magnetic poles, leading to the invention of the compass.

Electromagnetism is crucial for modern technology, including phones, computers, and medical devices. In this chapter, we will explore basic concepts like electric fields, magnetic forces, and electromagnetic induction, and see how these principles affect both natural phenomena and technology. Understanding these concepts help us appreciate how electromagnetism influences our world and drives innovation.

10.1 FORCE ON A CURRENT-CARRYING CONDUCTOR IN A UNIFORM MAGNETIC FIELD

It has been observed experimentally that a current-carrying conductor placed in a magnetic field experiences a force. Consider a straight conductor carrying a steady current placed perpendicular to uniform magnetic field. Assume the direction of the current is out of the paper as shown by ⊙ in Fig. 10.2. The direction of magnetic field produced by the current is also shown.

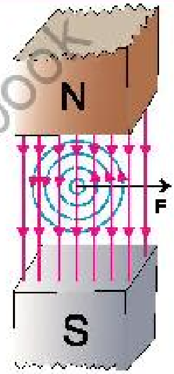


Fig. 10.2

Just as two magnets exert forces on each other through their magnetic fields, a current-carrying conductor experiences a force due to the interaction between its own magnetic field and the external magnetic field. To determine the direction of this force, consider the interaction between the two fields.

The magnetic field produced by the current and the external uniform magnetic field reinforce each other on the left side of the conductor and oppose each other on the right side. Consequently, the conductor moves towards the side where the field is weaker. That is, the force on the conductor is directed to the right as viewed from the front. Thus, the force *F* is perpendicular to both the conductor and the magnetic field. Fleming's left-hand rule is used to predict the direction of the force experienced by a current-carrying conductor in a magnetic field. To apply the rule:

> Position your left hand such that the first finger points in the direction of magnetic field, the second finger points in the direction of current, the thumb will then point in the direction of force.

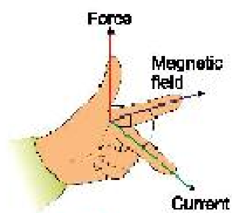


Fig. 10.3

The Fleming's left hand rule is illustrated in Fig. 10.3.

However, direction of force can also be found by using right hand rule that can be stated as:

> Curl fingers of your right hand from current to magnetic field through smaller angle, the stretched thumb will indicate the direction of force.

Let us now determine the magnitude of the force on a current-carrying conductor placed inside a magnetic field. Experimentally, it has been observed that the magnitude of the force acting on the conductor is directly proportional to the current I in the conductor, the length L of the conductor, and the strength of the external magnetic field B. The strength of the magnetic field is also known as the magnetic induction B, which has the same direction as the field. Thus, the force F on a conductor of length L, carrying a current I and placed perpendicular to a magnetic field of strength B, is given by

$$F \propto BIL$$

$$F = kBIL$$

In SI units, the value of k = 1. Therefore,

$$F = BIL \quad \text{.....................} (10.1)$$

From Eq. (10.1) we can see that $B = \frac{F}{IL}$, so we can define B as:

The magnetic strength is numerically equal to the force exerted on a conductor of length one metre carrying one ampere current, placed perpendicular to the magnetic field.

Equation $B = \frac{F}{IL}$ also gives us the unit of B. The SI unit of B is tesla (T).

$$1\text{T} = 1\text{ NA}^{-1}\text{ m}^{-1}$$

It may be noted that magnetic induction is a vector quantity. Its direction is the same as that of magnetic field.

We can also consider a vector **L** which has a magnitude equal to the length of the conductor and its direction is along the flow of current.

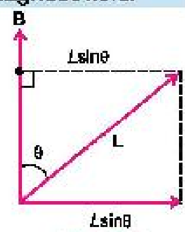


Fig. 10.4

Now consider a conductor **L** placed at an angle 'θ' w.r.t the magnetic field, then we will use the component of **L** perpendicular to **B** i.e., (L sinθ), as shown in Fig. 10.4. Then the Eq. (10.1) will become,

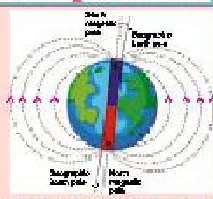
$$F = BIL\sin\theta \quad \text{.......................} (10.2)$$

In the vector form the Eq. 10.2 can be written as

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B} \quad \text{.........................} (10.3)$$

Equation (10.2) shows that the force will be maximum (BIL) when the conductor is perpendicular to the field, i.e., θ = 90°, and it will be zero when the conductor is along the field i.e., θ = 0.

Interesting Information



The Earth's magnetic field is approximately that of a dipole, like that of the fictitious bar magnet, where the south magnetic pole is towards the geographic north pole and the north magnetic pole is towards the geographic south pole.

Example 10.1 A 20.0 cm wire carrying a current of 10.0 A is placed in a uniform magnetic field of 0.30 T. If the wire makes an angle of 40° with the direction of magnetic field, find the magnitude of the force acting on the wire.

Solution Length of the wire $= L = 20.0 \text{ cm} = 0.20 \text{ m}$

Current	$= I$	$= 10.0\text{A}$
Strength of magnetic field	$= B$	$= 0.30\text{T}$
Angle	$= \theta$	$= 40°$

Substituting the values in Eq. (10.2): $F = 10.0\text{A} \times 0.30\text{T} \times 0.20\text{m} \times \sin 40° = 0.39\text{N}$

10.2 MAGNETIC FLUX AND FLUX DENSITY

We can represent the strength of a magnetic field B by the lines of force in the same way as for electric field. Then, the population of these lines in the field per unit area passing through a surface perpendicular to the field will represent the magnetic flux. Thus,

The magnetic flux through a patch of area A is the number of magnetic lines passing through this area.

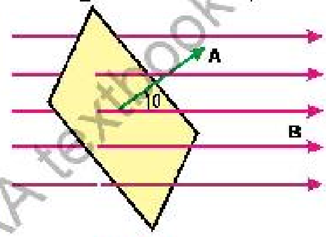


Fig. 10.5

If B represents the number of lines passing through unit area placed perpendicular to the field, then the total flux through area A perpendicular to the field will be:

$$B = \frac{\phi_B}{A} \quad \text{.........................(10.4)}$$

The surface A may not be perpendicular to the field, that is the normal to the surface makes an angle θ with **B** as shown in Fig. 10.5. Then, we will have to use component of **B** ($B\cos\theta$) along the vector area **A** (Fig. 10.6). Therefore, the flux passing through the surface will be:

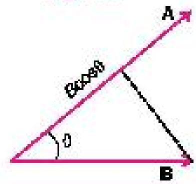


Fig. 10.6

$$\phi_B = BA\cos\theta \quad \text{..................(10.5)}$$

As **B** and **A** both are vectors, so we can write

$$\phi_B = \mathbf{B}.\mathbf{A} \quad \text{........................(10.6)}$$

Equation (10.6) shows that ϕ_B is a scaler quantity. Therefore, we can define magnetic flux as:

The magnetic flux ϕ_B, through a plane element of area **A** in a uniform magnetic field **B** is given by dot product of **B** and **A**.

Note that **A** is a vector whose magnitude is the area of the element and whose direction is along the normal to the surface of the element and θ is the angle between the directions of the vectors **B** and **A** (Fig. 10.5). In Fig. 10.7(a), the field is directed along the normal to the area, so θ is zero ($\cos 0°=1$) and the flux is maximum, equal to BA. When the field is parallel to the plane of the area (Fig. 10.7-b), the angle between the field and normal to area is 90° i.e., $\theta = 90°$ ($\cos 90°=0$), so the flux through the area in this orientation is zero.

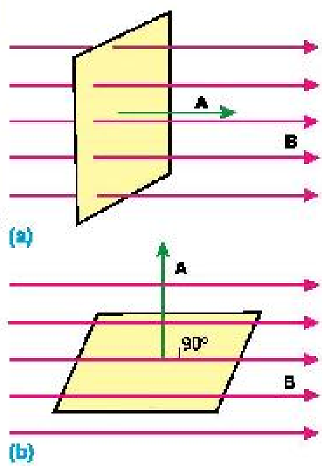


Fig. 10.7

In case of a curved surface placed in a uniform magnetic field, the curved surface is divided into a number of small surface elements, each element being assumed plane and the flux through the whole curved surface is calculated by the sum of the contributions from all the elements of the surface using Eq.(10.5).

From the definition of tesla, the unit of magnetic flux is N m A^{-1} which is called weber (Wb). According to Eq. 10.5, the magnetic induction **B** is the flux per unit area of a surface perpendicular to **B**, hence, it is also called as magnetic flux density. Its unit is Wb m^{-2}. Therefore, magnetic induction, i.e. the magnetic field strength is measured in terms of Wb m^{-2} or N A^{-1} m^{-1} (tesla).

Example 10.2 A rectangular loop of wire is placed in a uniform magnetic field of magnitude 1.2 T. If the loop is 25 cm long and 20 cm wide, determine the magnetic flux through the loop for the three orientations as shown in Fig. 10.8.

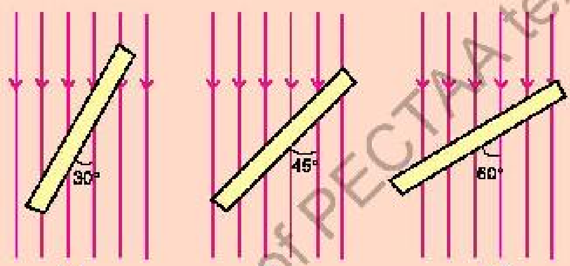


Fig. 10.8

Solution For orientation (a), angle between **B** and area vector **A** is $\theta = 60°$

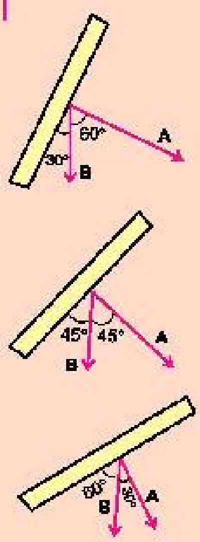


Fig. 10.9

Using $\qquad \phi_B = BA\cos\theta$

$$= 1.2\,T \times 20\,cm \times 25\,cm \times \cos 60°$$

$$= 1.2\,T \times 5 \times 10^{-2}\,m^2 \times 0.5$$

$$= 3 \times 10^{-2}\,Wb$$

For orientation (b), angle $\theta = 45°$

$$\phi_B = 1.2T \times 5 \times 10^{-2}\,m^2 \times 0.707$$

$$= 4.2 \times 10^{-2}\,Wb$$

For orientation (c), angle $\theta = 30°$

$$\phi_B = 1.2\,T \times 5 \times 10^{-2}\,m^2 \times 0.866$$

$$= 5.2 \times 10^{-2}\,Wb$$

10.3 MAGNETIC FLUX LINKAGE

Magnetic flux linkage is a key concept in electromagnetism, particularly in the study of inductance and electromagnetic induction. Magnetic flux linkage refers to the product of the magnetic flux through a coil and the number of turns in the coil. It essentially measures how much magnetic flux is linked with the coil due to its multiple turns, and is an important factor in understanding how coils and inductors operate in electrical circuits.

Magnetic flux linkage $= \phi = N\,\phi_B$ ....(10.7)

where ϕ_B is the magnetic flux through a single loop of area A and N is the total number of turns of the coil. Magnetic flux linkage plays a crucial role in the design and operation of transformers, electric motors, generators, and inductors. This concept is particularly important in Faraday's law of electromagnetic induction, which we will explore later in this chapter.

For your information

A magnetic strip on the ATM card contains millions of tiny magnetic domains held together by a resin binder. The machine reads the information encoded on the magnetic strip and it makes your access to your account.

10.4 MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

We have observed that a current-carrying conductor placed perpendicularly in a uniform magnetic field experiences a force **F**. Since a current is the flow of electric charges, it raises the question: do individual charges moving through a magnetic field also experience a force? The answer is yes. Experiments show that a charged particle does experience a force when it moves across a magnetic field. We can calculate this force by examining the behaviour of a current-carrying conductor in a magnetic field.

Consider a conductor of length *L* through which *N* charged particles, each with charge *q*, are passing in time *t*. The motion of these charged particles produces a current *I* in the conductor, which is given by

Do you know?

Like electric field lines, magnetic field lines also never cross each other. However, they can attract or repel each other.

$$I = \frac{Q}{t} = \frac{Nq}{t}$$

where *Q* is the total charge flowing in time *t*. If **v** is the velocity of charged particles, then the velocity of the particles along the conductor is

$$\mathbf{v} = v\hat{\mathbf{L}}$$

where $\hat{\mathbf{L}}$ is the unit vector in the direction of the current. The sign of the force depends on whether the charge *q* is positive or negative. However, the unit vector $\hat{\mathbf{L}}$ is directed along the direction of the current, which is the direction of motion of positive charges.

Since the particles take time *t* to move across the conductor of length *L*, therefore,

$$t = \frac{L}{v}$$

Then $$I = \frac{Nq}{t} = \frac{Nqv}{L} \quad \text{.......................(10.8)}$$

Point to ponder!

Why does a picture become distorted when a magnetic bar is brought near to the screen of TV, computer monitor or oscilloscope?

If this conductor is placed in a uniform magnetic field **B**, it will experience a force **F**, as given by Eq.10.3.

$$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B} \quad \text{....................... (10.9)}$$

As $$\mathbf{L} = L\hat{\mathbf{L}} \quad \text{....................... (10.10)}$$

Substituting the values of I and L in Eq. (10.3), we have

$$\mathbf{F} = \frac{Nqv}{L}(L\hat{\mathbf{L}} \times \mathbf{B})$$

or $$\mathbf{F} = Nqv\hat{\mathbf{L}} \times \mathbf{B} \quad \text{.......................(10.11)}$$

As velocity **v** is directed along the conductor **L**, so we can write: $v\hat{\mathbf{L}} = v\hat{\mathbf{v}} = \mathbf{v}$ (as $\hat{\mathbf{L}} = \hat{\mathbf{v}}$)

$$\mathbf{F} = Nq\,(\mathbf{v} \times \mathbf{B}) \quad \text{.......................(10.12)}$$

Therefore the force on a single particle will be

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \quad \text{.......................(10.13)}$$

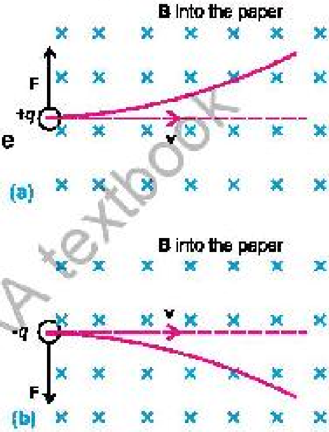


Fig. 10.10

If θ is the angle between **B** and **v**, then magnitude of force **F** is given by

$$F = qvB\sin\theta \quad \text{....................... (10.14)}$$

Therefore, the force is maximum when **B** is perpendicular to **v**, i.e., $\theta = 90^\circ$, and force is zero when **B** is in the direction of **v**, i.e., $\theta = 0$. The direction of force can be known by applying Fleming's left hand rule or right hand rule for vector product.

(a) The positively charged particle enters into the magnetic field along the dotted line on plane of paper. It experiences a force in the upward direction due to which it is deflected along a curved path (Fig. 10.10-a).

(b) The negatively charged particle is deflected downward by the force acting on it downwards (Fig. 10.10-b).

Example 10.3 An electron enters into a uniform magnetic field perpendicularly with a speed of 10^4 m s^{-1}. What path the electron will move along inside the field?

($B = 2.5$ Wb m^{-2}, $m = 9.11 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)

Solution The Force F acting on the electron will be:

$$F = qvB\sin\theta$$

As the velocity **v** is perpendicular to **B**, i.e., $\theta = 90^\circ$, the charge $q = e$, so

$$F = evB \times 1 = evB$$

Since F acts perpendicular to **v**, so this force provides centripetal force to the electron to keep it in a circle of radius *r*. Then

$$evB = \frac{mv^2}{r}$$

$$r = \frac{mv}{eB}$$

Putting the values in above equation, we have

$$r = \frac{9.11 \times 10^{-31}\ \text{kg} \times 10^{4}\ \text{m s}^{-1}}{1.6 \times 10^{-19}\ \text{C} \times 2.5\ \text{Wb m}^{-2}}$$

$$r = 2.3 \times 10^{-8}\ \text{m}$$

The path of electron will be a circle of radius 2.3×10^{-8} m.

10.5 VELOCITY SELECTOR

A velocity selector is a device used to determine the velocity of a charged particle. In this device, electric and magnetic forces are applied to the moving particle in such a way that they balance each other only for one value of velocity, allowing the particle to continue moving with a constant velocity.

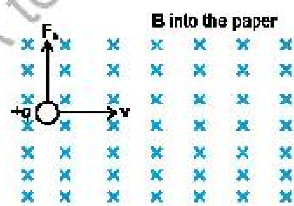


Fig. 10.11

Consider a particle with a positive charge +q that enters a uniform magnetic field **B** at a right angles to it, with a velocity **v**. The magnetic force acts on the particle in the upward direction, as shown in Fig.10.11. To balance this magnetic force, an electric force must act downward on the particle.

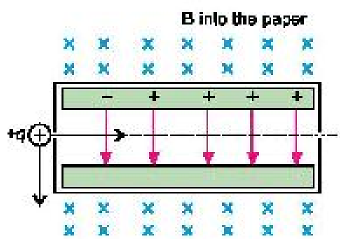


Fig. 10.12

A velocity selector consists of a cylindrical tube located within a magnetic field **B**. Inside the tube is a parallel plate capacitor that creates a uniform electric field **E**. The electric field **E** is oriented perpendicular to the magnetic field **B**, as shown in Fig. 10.12.

When the charged particle enters the left end of the tube, the magnetic force acts upward, while the electric force acts downward in the direction of the electric field **E** on the positively charged particle. If the strengths of the electric and magnetic fields are adjusted appropriately, these forces will cancel each other out. With no net force acting on the particle, its velocity **v** remains constant in accordance with Newton's first law. As a result, the particle moves in a straight line at a constant velocity and exits the right end of the tube.

The particles with velocities different from **v** will be deflected and will not exit at the right end of the tube.

The magnitude of the velocity selected can be determined as below.

As the velocity **v** is perpendicular to both **B** and **E**, therefore,

Magnetic force (upward) $= Bqv$

Electric force (downward) $= qE$

For no deflection of particle, $Bqv = qE$

or $$v = \frac{E}{B} \quad \text{.............................. (10.15)}$$

Example 10.4 Alpha particles ranging in speed from 1000 m s^{-1} to 2000 m s^{-1} enter into a velocity selector where the electric intensity is 300 V m^{-1} and the magnetic induction 0.20 T. Which particle will move undeviated through the field?

Solution $E = 300\ \text{V m}^{-1} = 300\ \text{N C}^{-1}$, $B = 0.20\ \text{T}$

Only those particles will be able to pass through the plate for which the electric force qE acting on the particles balances the magnetic force Bqv on the particles as shown in the Fig. 10.12.

Therefore $qE = Bqv$

Thus, the selected speed is:

$$v = \frac{E}{B} = \frac{300\ \text{N C}^{-1}}{0.20\ \text{N A}^{-1}\text{m}^{-1}} = 1500\ \text{m s}^{-1}$$

Point to ponder!

A force is exerted on a moving charged particle in a magnetic field. In what direction it should move that the force is not exerted on it?

The alpha particles having a speed of 1500 m s^{-1} will move undeviated through the field.

Example 10.5 A charged particle moves through a velocity selector at a constant velocity in a straight line. The electric field of the velocity selector is 4.8×10^{3} N C^{-1}, while the magnetic field is 0.2 T. When the electric filed is turned OFF, the charged particle travels on a circular path of radius 3.0 cm. Find the charge to mass ratio of the particle.

Solution Since the particle is moving in a direction perpendicular to both **E** and **B**, so the magnitude of velocity **v** will be given by

$$qE = Bqv \quad \text{or} \quad v = \frac{E}{B}$$

When the electric filed is turned off, the particle will move along a circular path of radius r. Then the magnetic force provides necessary centripetal force. Then

$$Bqv = \frac{mv^2}{r}$$

So, charge to mass ratio becomes:

$$\frac{q}{m} = \frac{v}{Br}$$

Putting the value of $v = \frac{E}{B}$, we have

$$\frac{q}{m} = \frac{E}{B^2 r}$$

Putting the values of *E*, *B* and *r*, we have

$$\frac{q}{m} = \frac{4.8\times10^{3}\,\mathrm{N\,C^{-1}}}{(0.20\,\mathrm{T})^{2}(3\times10^{-2}\,\mathrm{m})}$$

$$= \frac{4.8\times10^{3}\,\mathrm{N\,C^{-1}}}{1.2\times10^{-3}\,\mathrm{m}} = 4\times10^{6}\,\mathrm{C\,kg^{-1}}$$

10.6 INDUCED EMF AND FARADAY'S LAW

It has been observed experimentally that when a conductor moves across a magnetic field, an electromotive force (emf) is induced between its ends. The induced emf in the moving conductor is similar to that of a battery. That is, if the ends of the conductor are connected by a wire to form a closed circuit, a current will flow through it.

The emf induced by the motion of a conductor across a magnetic field is called motional emf.

Consider an experiment as shown in Fig. 10.13. A conducting rod of length *L* is placed on two parallel metal rails separated by a distance *L*. A galvanometer is connected between the ends c and d of the rails. This forms a complete conducting loop abcda. A uniform magnetic field **B** is applied directed into the page. Initially, when the rod is stationary, galvanometer indicates no current in the loop. If the rod is pulled to the right with constant velocity **v**, the galvanometer indicates a current flowing through the loop. Obviously, the current is induced due to the motion of the conducting rod across the magnetic field. The moving rod is acting as a source of emf $\varepsilon = V_b - V_a = \Delta V$.

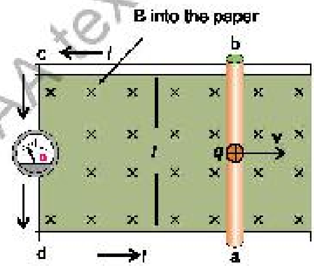


Fig. 10.13

When the rod moves, a charge *q* within the rod also moves with the same velocity **v** in the magnetic field **B** and experiences a force given by, Eq. 10.13.

$$\mathbf{F} = q\,(\mathbf{v}\times\mathbf{B})$$

The magnitude of the force is:

$$F = q\,v\,B\sin\theta$$

Since angle θ between *v* and *B* is 90°, so

$$F = qvB \quad \text{............} \quad (10.16)$$

Applying right hand rule, we see that the force **F** acting on the charge *q* is directed from point a to point b along the rod. As a result, charges migrate to the top end of the conductor. As more charges move, a concentration of charge builds up at the top end b, while there is a deficiency of charges at the bottom end a. This redistribution of charge creates an electrostatic field *E* directed from b to a. The electrostatic force on the charge is $F_e = qE$ directed from b to a. The system quickly reaches an equilibrium state where these two forces on the charge are balanced. If E is the electric field intensity in this state, then

$$qE = qvB$$

or $$E = vB \text{ .... } (10.17)$$

The motional emf ε will be equal to the potential difference $\Delta V = V_b - V_a$ between the two ends of the moving conductor in this equilibrium state. The gradient of potential is given by $\Delta V/L$. As the electric intensity is given by the negative of the gradient, therefore,

$$E = -\frac{\Delta V}{L} \text{ ....} (10.18)$$

or $$\Delta V = -LE = -(LvB)$$

The motional emf is:

$$\varepsilon = \Delta V = LvB \text{ ....} (10.19)$$

This is the magnitude of motional emf. However, if the angle between **v** and **B** is θ, then

$$\varepsilon = -vBL\sin\theta \text{ ....} (10.20)$$

Due to the induced emf, positive charges flow along the path abcda; therefore, the induced current is anticlockwise in the diagram. As the current flows, the quantity of charge at the top decreases, which reduces the electric field intensity, while the magnetic force remains unchanged. This imbalance disturbs the equilibrium in favour of the magnetic force. Consequently, as the charges reach the end 'a' of the conductor due to the current flow, they are carried back to the top end 'b' by the unbalanced magnetic field, and the current continues to flow.

Do you know?

Wireless charging works under the principle of electromagnetic induction.

Point to ponder!

This heater operates on the principle of electromagnetic induction. The water in the metal pot is boiling whereas that in the glass pot is not. Even the glass top of the heater is cool to touch. The coil just beneath the top carries AC that produces changing magnetic flux. Flux linking with pots induces emf in them. Current is generated in the metal pot that heats up the water, but no current flows through the glass pan, why?

Faraday's Law

The motional emf induced in a rod moving perpendicular to a magnetic field is $\varepsilon = -vBL$. The motional emf as well as other induced emfs can be described in terms of magnetic flux. Consider the experiment shown in Fig. 10.14 again. Let the conducting rod be moving from position 1 to position 2 in a

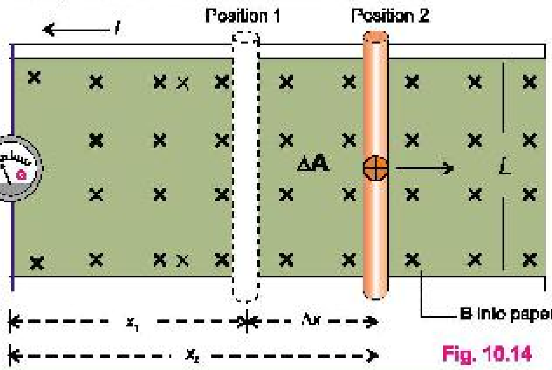


Fig. 10.14

small interval of time Δt. The distance travelled by the rod in time Δt is $x_2 - x_1 = \Delta x$.

Since the rod is moving with constant velocity **v**, therefore,

$$v = \frac{\Delta x}{\Delta t} \quad \text{..................(10.21)}$$

Putting this value of v in Eq. 10.19, we have

$$\varepsilon = -vBL = -\frac{\Delta x}{\Delta t}BL \quad \text{..................(10.22)}$$

As the rod moves through the distance Δx, the increase in the area of loop is given by $\Delta A = \Delta x L$. This increases the flux through the loop by $\Delta\phi_B = (\Delta A)B$. Putting $(\Delta x L)B = \Delta\phi_B$ in Eq. 10.22, we have

$$\varepsilon = \frac{\Delta\phi_B}{\Delta t} \quad \text{..................(10.23)}$$

Eqation(10.20) shows that if the magnetic flux is changing through the single loop of a conducting coil, then the negative of the rate of change of magnetic flux is equal to the emf induced in the loop.

If there is a coil of N loops instead of a single loop, then the induced emf will become N times.

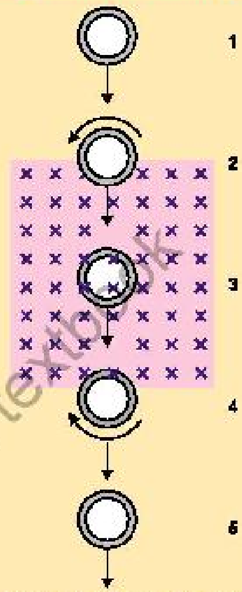
$$\text{i.e.,} \quad \varepsilon = -N\frac{\Delta\phi_B}{\Delta t} \quad \text{..................(10.24)}$$

Although the above expression is derived on the basis of motional emf, but it is true in general. This conclusion was first arrived at by Faraday, so this is known as Faraday's law of electromagnetic induction which states that:

> The average emf induced in a conducting coil of N loops is equal to the negative of the rate at which the magnetic flux through the coil is changing with time.

The minus sign indicates that the direction of the induced emf is such that it opposes the change in flux.

Point to ponder!



A copper ring passes through a rectangle region where a constant magnetic field is directed into the page. What do you guess about the current in the ring at the positions 2, 3 and 4 ?

For your information

Faraday's homopolar generator with which he was able to produce a continuous induced current.

10.7 LENZ'S LAW AND DIRECTION OF INDUCED EMF

In the previous section, a mathematical expression for Faraday's law of electromagnetic induction was derived as:

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

The minus sign in the expression is very important; it relates to the direction of the induced emf. To determine the direction, we use a principle based on the discovery made by the Russian physicist Heinrich Lenz in 1834. He found that the polarity of an induced emf always produces an induced current that opposes the change in the magnetic field that causes the emf. This principle is known as Lenz's Law, which states that;

> The direction of the induced current is always such that it opposes the change that causes the current.

Lenz's Law specifically applies to induced currents and not directly to induced emf. This means we can apply Lenz's Law to closed conducting loops or coils. If the loop is not closed, we can imagine it as if it were closed to determine the direction of the induced current, and from this, we can infer the direction of the induced emf.

Let us apply Lenz's law to a coil in which a current is induced by the movement of a bar magnet. A current-carrying coil generates a magnetic field similar to that of a bar magnet, with one face of the coil acting as the north pole and the other as the south pole. To oppose the motion of the bar magnet, the face of the coil towards the magnet must become a north pole (Fig. 10.15). This arrangement causes the two north poles to repel each other. According to the right-hand rule, the induced current in the coil must flow anticlockwise when viewed from the side of the bar magnet.

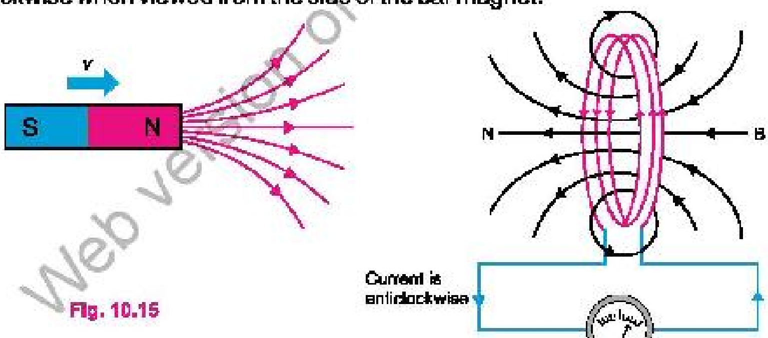


Fig. 10.15

According to Lenz's law, the "push" of the magnet is the "change" that produces the induced current, and the current acts to oppose the push. On the other hand, if we pull the magnet away from the coil, the induced current will oppose the "pull" by creating a south pole on the face of coil towards the bar magnet.

Lenz's law is also a manifestation of the law of conservation of energy and can be conveniently applied to circuits involving induced currents. To understand this, let us revisit the experiment depicted in Fig. 10.16. When the rod moves to the right, an emf is induced in it, causing an induced current to flow through the loop in anticlockwise direction. Because the current-carrying rod is moving within the magnetic field, it experiences a magnetic force F_m with the magnitude of $F_m = ILB\sin 90°$.

According to the right-hand rule, the direction of the magnetic force F_m is opposite to that of the velocity **v**, so it tends to stop the rod (Fig. 10.16-a). To keep the rod moving with a constant velocity, an external force equal in magnitude to F_m but opposite in direction must be applied. This external force provides the energy necessary for the induced current to flow. Thus, electromagnetic induction adheres to the law of conservation of energy.

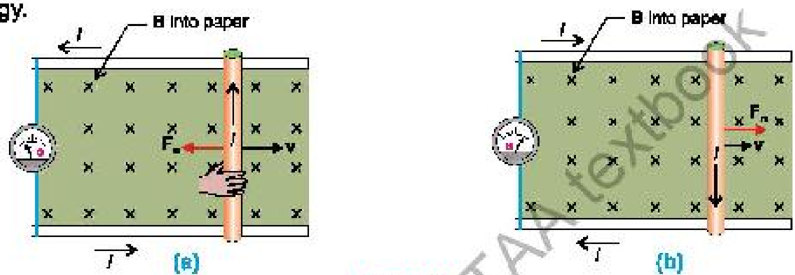


Fig.10.16

The Lenz's law forbids the induced current directed clockwise in this case, because the force F_m would be, then, in the direction of **v** that would accelerate the rod towards right (Fig.10.16-b). This in turn would induce a stronger current, the magnetic field due to it also increases and the magnetic force increases further. Thus, the motion of the wire is accelerated more and more. Starting with a minute quantity of energy, we obtain an ever-increasing kinetic energy of motion apparently from nowhere. Consequently, the process becomes self-perpetuating which is against the law of conservation of energy.

Example 10.6 A metal rod of length 25 cm is moving at a speed of 0.5 m s^{-1} in a direction perpendicular to a 0.25 T magnetic field. Find the emf produced in the rod.

Solution

Speed of rod $= v = 0.5\ \text{m s}^{-1}$

Length of rod $= L = 25\ \text{cm} = 0.25\ \text{m}$

Magnetic flux density $= B = 0.25\ \text{T} = 0.25\ \text{N A}^{-1}\ \text{m}^{-1}$

Induced emf $= \varepsilon = ?$

Point to ponder!

By neglecting the resistance, can a constant current in a coil set up a potential difference across the coil?

Using the relation,

$$\varepsilon = vBL$$

$$\varepsilon = 0.5\ \text{m s}^{-1} \times 0.25\ \text{NA}^{-1}\ \text{m}^{-1} \times 0.25\ \text{m}$$

$$\varepsilon = 3.13 \times 10^{-2}\ \text{J C}^{-1} = 3.13 \times 10^{-2}\ \text{V}$$

Example 10.7 A loop of wire is placed in a uniform magnetic field that is perpendicular to the plane of the loop. The strength of the magnetic field is 0.6 T. The area of the loop begins to shrink at a constant rate of $\frac{\Delta A}{\Delta t}$ = 0.8 $m^2 s^{-1}$. What is the magnitude of emf induced in the loop while it is shrinking?

Solution Rate of change of area = $\frac{\Delta A}{\Delta t}$ = 0.8 $m^2 s^{-1}$

Magnetic flux density = B = 0.6 T = 0.6 N $A^{-1} m^{-1}$

Number of turns = N = 1

Induced emf = ε = ?

Rate of change of flux = $\frac{\Delta\phi}{\Delta t} = B\frac{\Delta A}{\Delta t}\cos 0° = B\frac{\Delta A}{\Delta t}$

Applying Faraday's law, magnitude of induced emf is:

$$\varepsilon = N\frac{\Delta \Phi}{\Delta t} = NB\frac{\Delta A}{\Delta t}$$

$$= 1 \times 0.6\ \text{N A m}^{-1} \times 0.8\ \text{m}^2\ \text{s}^{-1}$$

$$\varepsilon = 0.48\ \text{J C}^{-1} = 0.48\ \text{V}$$

10.8 FACTORS AFFECTING EMF

1. Rate of Change of Magnetic Flux

Faraday's law suggests that faster changes in magnetic flux result in greater induced emf.

2. Number of Turns of the Coil

According to Faraday's law, induced emf is also proportional to the number of turns of the coil. More turns result in a greater induced emf.

3. Relative Speed

The speed of the coil (or conductor) through a magnetic field also affects the magnitude of the induced emf. Faster speed increases the rate of change of magnetic flux that results into an increase in the induced emf.

10.9 FERROFLUIDS

Ferrofluid is a unique material that exhibits both liquid and magnetic properties. It operates through a combination of magnetic and fluid dynamics principles.

Essentially, the ferrofluid is a colloidal suspension of magnetic particles in a carrier fluid (such as oil or water). Typically, the magnetic particles are iron oxide, ground to the nano-scale and approximately 10 nanometres in size. These particles are coated with a surfactant, a substance that reduces surface tension. This coating prevents the particles from clumping together, ensuring they remain evenly dispersed in the fluid. The viscosity of the fluid, the nanometre size of the particles, and their constant movement prevent the particles from settling down.

When there is no magnet around, a ferrofluid acts like a liquid, but when there is a magnet nearby, the particles are temporarily magnetized and the fluid becomes a magnet. They form structures within the fluid causing the ferrofluid to act more like a solid. When the magnet is removed, the particles are demagnetized and the ferrofluid acts like a liquid again.

For your information

The first ferrofluid developed by NASA in 1960, was ground from natural magnetite. Ferrofluid was invented to move liquids through space.

This phenomenon is due to the competition between magnetic forces, surface tension and gravity. In the presence of strong magnetic field, the formation of chain-like structures is a result of magnetic forces pulling the fluid upwards while gravity and surface tension work to pull it back down. These chains align along the magnetic field lines and increase the viscosity, making it behave like a solid bulging in certain directions. These are commonly known as spikes. However, the spikes are formed where the magnetic forces overcome the other forces (Fig.10.17).

Fig.10.17

The following experiment will exhibit this phenomenon.

Experiment

You need some laser printer toner, some cooking oil, a test tube, a glass bottle, a small stick and a magnet.

Procedure

Pour some toner in the test tube. Remember that laser printer toner contains 40 % iron oxide in nanometre particle size. Add some cooking oil in it and mix it well with the stick to form ferrofluid. Put this fluid in the bottle. The fluid will act like a liquid on shaking the bottle. Now bring the magnet near to fluid outside of the bottle.

You will observe that the fluid jumps towards the magnet, because it has itself become a magnet. If we hold the magnet on the side of the bottle, you will see a structure with spikes formed by the fluid as shown in Fig. 10.18.

Fig.10.18

Applications of Ferrofluids

There are many applications of ferrofluids in the fields of electronics, medicine, engineering, and active research in Physics and Material science. In **electronics,** ferrofluids are used in rotary seals for computer hard drives and other rotating shaft motors. In loudspeakers, ferrofluids are used to cool the voice coil which can heat up during operation. The magnetic field holds the fluid in place around the coil, allowing it to absorb and dissipate heat more effectively. Ferrofluids are also used in speakers to dampen vibrations and improve sound quality.

In **medical** applications, ferrofluids can be directed to specific areas in the body using external magnets, allowing for targeted drug delivery. The magnetic particles can carry drugs directly to a tumor or other targeted site, reducing side effects and improving treatment efficiency. Ferrofluids can also be used as contrast agents in magnetic resonance imaging (MRI).

Other applications of ferrofluids include damping or precisely controlling the flow of liquids by manipulating the magnetic field.

10.10 A SEISMOMETER

A seismometer is an instrument that responds to any movement of the rocks under the ground or vibration caused by earthquakes, volcano eruption and explosion. A seismometer detects earthquakes by using electromagnetic induction to convert ground motion into electrical signals. Typically, a seismometer includes a weight suspended by a spring. When an earthquake occurs, the ground moves but the weight tends to stay stationary due to inertia. This results in relative motion between the weight and the frame of the seismometer which is attached to the ground. The weight is often attached to a magnet which moves inside a coil of wire (Fig.10.19). This setup works according to Faraday's law of electromagnetic induction, that is, the changing magnetic flux through the coil induces an emf in the coil. This gives rise to an induced electric current.

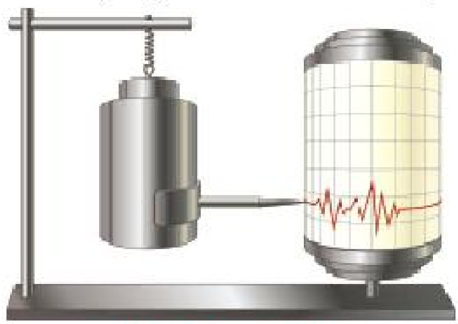


Fig. 10.19 Seismometer

The induced current is proportional to the velocity of the ground motion. The electrical signals generated are then amplified and recorded. Thus, data is provided on the amplitude, frequency and the duration of the earthquake waves.

For your information

Most earthquakes are caused by plate tectonics (displacement) and occur at a depth of 60 km. These earthquakes are categorized as shallow. Intermediate and deep intermediate can be as deep, as 280 km beneath the crust, while deep earthquakes can reach depths past 280 km.

The data is analyzed to determine various characteristics of the earthquake, such as its location, magnitude and depth.

For your information

There are two main types of seismic waves that generate earthquakes i.e., P-waves are primary waves which are longitudinal and S-waves are secondary waves which are transverse in nature.

Usually, a seismometer is buried under the ground at a depth of 50-1000 metres. It is placed in a protective case called a vault. This is a cylindrical steel tank that is approximately 1 metre wide and 2 metres deep with a concrete pad at the bottom (Fig.10.20).

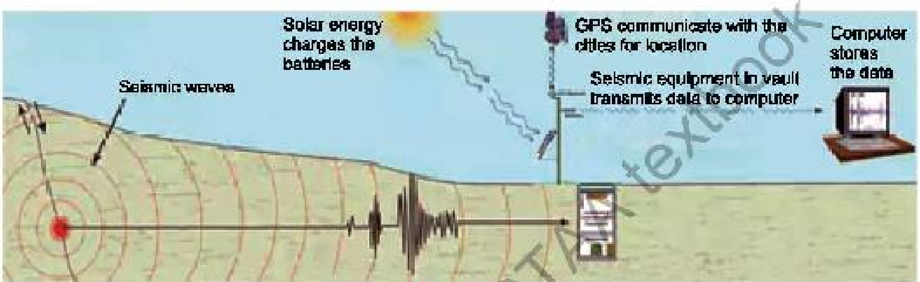


Fig. 10.20

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

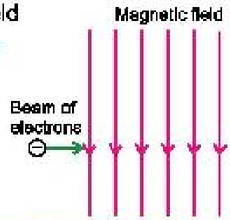
10.1 A current is flowing towards north along a power line. The direction of the magnetic field over the wire is directed towards:

(a) north (b) south (c) east (d) west

10.2 The radius of curvature of the path of a charged particle in a uniform magnetic field is directly proportional to:

(a) the particle's charge (b) the particle's momentum
(c) the particle's energy (d) the flux density of the field

10.3 The diagram shows a beam of electrons entering a magnetic field. What is the effect of magnetic field on the electrons?



(a) They are deflected into the plane of the diagram.
(b) They are deflected out of the plane of the diagram.
(c) They are deflected towards the bottom of the diagram.
(d) They are deflected towards the top of the diagram.

10.4 The force exerted on a wire of 1 metre length carrying 1 ampere current placed at right angle to the magnetic field is called:

(a) magnetic field intensity (b) magnetic flux
(c) magnetic induction (d) none of these

10.5 The unit of flux density is:

(a) $NA^{-1}m^{-1}$ (b) NAm^{-1} (c) NmA^{-2} (d) NmA

10.6 A moving charged particle is surrounded by:

(a) electric field only (b) magnetic field only

(c) both electric and magnetic field (d) no field

10.7 Magnetic force on the charge q moving parallel to magnetic field with velocity v is:

(a) $qvB\sin\theta$ (b) qvB (c) zero (d) ILB

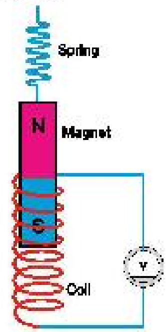
10.8 The unit $NA^{-1}m^{-1}$ is called:

(a) weber (b) tesla (c) coulomb (d) none of these

10.9 Electrons while moving perpendicularly through a uniform magnetic field are:

(a) deflected towards north pole (b) deflected towards south pole

(c) deflected along circular path (d) not deflected at all



10.10 A magnet is suspended from a spring. The magnet oscillates and moves in and out of the coil connected to a galvanometer. When the magnet oscillates, the galvanometer shows:

(a) deflection to the left and to the right with constant amplitude

(b) deflected on one side

(c) no deflection

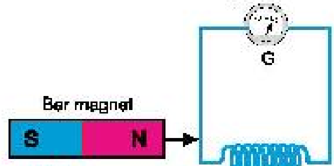
(d) deflection to the left and right, but the amplitude steadily decreases

Short Answer Questions

10.1 It is said that Lenz's law specifically applies to induced currents and not directly to induce emf. Explain briefly.

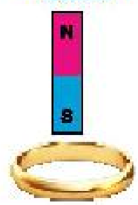
10.2 A square loop of wire is moving through a uniform magnetic field. The normal to the loop is oriented parallel to the magnetic field. Is an emf induced in the loop? Give a reason for your answer.

10.3 Does the induced emf always act to decrease the magnetic flux through a circuit?



(a)

10.4 When a magnet is pushed into the solenoid, as shown in the figure (a), the galvanometer indicates a small current. Why is the current produced? What will be the magnetic pole produced at the left end of the solenoid?



(b)

10.5 A bar magnet falls through a fixed metal ring (Fig-b). Will the magnet fall with an acceleration of a freely falling body? Give reason.

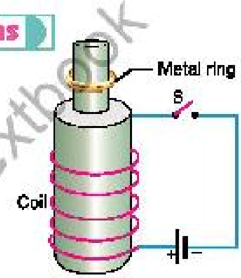
10.6 Which of the two charged particles of the same mass will be deflected most in the magnetic field (a) fast moving (b) slow moving?

10.7 An electron and a proton are projected into a magnetic field at right angles to it with a certain velocity. Which of the particles will suffer greater deflection? Why?

10.8 Can a single moving proton produce magnetic field?

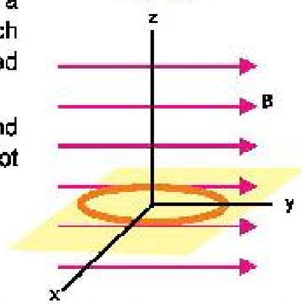
10.9 A magnetic field is necessary if there is to be a magnetic flux passing through a coil of wire. Yet, just because there is a magnetic field does not mean that a magnetic flux will pass through a coil. Account for this situation.

Constructed Response Questions



10.1 A charge is lying stationary between the opposite poles of two magnets. Is a magnetic force exerted on it? Why?

10.2 When the switch in the circuit is closed, a current is established in the coil and the metal ring jumps upward. Why? Describe what would happen to the ring if the battery polarity were reversed?



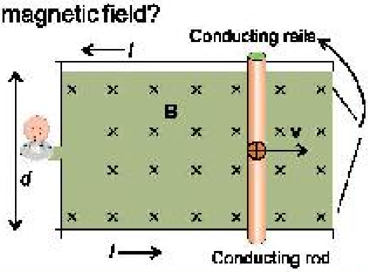
10.3 The figure shows a coil of wire in the x – y plane with a magnetic field directed along the y-axis. Around which of the three-coordinate axis should the coil be rotated in order to generate an emf and a current in the coil?

10.4 Is it possible to change both the area of the loop and the magnetic field passing through the loop and still not have an induced emf in the loop? Give reason.

10.5 Does the application of uniform magnetic field to a moving charged particle result in a change in kinetic energy of the particle? Explain.

10.6 A uniform electric field and a magnetic field act in the same direction. A proton is projected, into the space, with a uniform velocity in opposite direction. What will happen to the proton?

10.7 A conductor moves in a magnetic field when a current is passed through the conductor. Would you expect the reverse effect to occur? That is, would a current be produced if a conductor is moved across the magnetic field?



10.8 Consider a conducting rod of length L moving with velocity v to the right as shown in the figure. Left ends of the conducting rails are connected to a bulb. Due to motion of the rod through the magnetic field, an emf is produced across the ends of the rod. This emf gives rise to a current I. As a result, the bulb lights up. Explain where does the electrical energy consumed by the bulb come from?

10.9 What will you do if you want to save a sensitive instrument from stray magnetic fields?

Comprehensive Questions

10.1 Distinguish between magnetic flux and flux density. How are they related?

10.2 Find an expression for the force exerted on a current-carrying conductor placed in a uniform magnetic field.

10.3 State and explain Faraday's law and Lenz's law. Also describe factors affecting the induced emf.

10.4 Determine the force acting on a charged particle moving through a uniform magnetic field.

10.5 What is a velocity selector? Explain its working.

10.6 Explain how ferrofluids work?

Numerical Problems

10.1 A positively charged particle is projected perpendicularly into a magnetic field at a speed of 1500 m s^{-1}. It experiences a force of magnitude F. At what angle θ with the field, the particle should be projected at a speed of 2000 m s^{-1}, so that it experiences the same magnitude of force? (Ans: $\theta = 49°$)

10.2 Electrons are accelerated from rest through a potential difference of 15 kV in an oscilloscope. The electrons then pass through a 0.35 T magnetic field that deflects them to the desired position on the screen. Find the magnitude of the maximum force that an electron can experience. (Ans: 4.1×10^{-12} N)

10.3 A square coil of side 15 cm each consists of 60 turns. Initially, it is located in a uniform magnetic field of magnitude 0.8 T such that plane of the coil is perpendicular to the field. The coil is then turned through an angle of $\theta = 30°$ in a time of 2 s. Determine the average induced emf. (Ans: 0.07 V)

10.4 A metallic rod is moving through a uniform magnetic field of 0.2 T. The emf induced across its ends is found to be 0.8 V. It is required to induce an emf of 2.4 V across its ends. How much field strength is needed for this? (0.6 T)

10.5 A copper ring has a radius of 4.0 cm and resistance of 1.0 mΩ. A magnetic field is applied over the ring, perpendicular to its plane. If the magnetic field increases from 0.2 T to 0.4 T in a time interval of 5×10^{-3} s, what is the current in the ring during this interval? (Ans: 201 A)

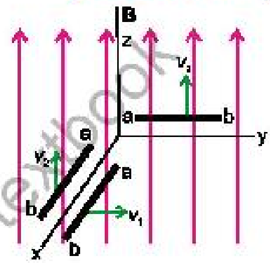
10.6 A coil of 10 turns and 35 cm^2 area is in a perpendicular magnetic field of 0.5 T. The coil is pulled out of the field in 1.0 s. Find the induced emf in the coil as it is pulled out of the field. (Ans: 1.75×10^{-2} V)

10.7 A proton is accelerated by a potential difference of 6×10^{5} volts. It then enters perpendicularly in a uniform magnetic field B =1.0 weber m^{-2}. Find the radius of curvature of the path of the proton. $m=1.67\times10^{-27}$ kg, $e = 1.6\times10^{-19}$ C.

(Ans: 11.2 cm)

10.8 A proton enters a uniform magnetic field B = 0.3 weber m^{-2} in a direction making an angle 45° with the magnetic field. What will be the radius of the circular path if the velocity of proton is 10^{4} m s^{-1}. (Ans: 2.46×10^{-4} m)

10.9 Three identical conducting rods L_1, L_2 and L_3 are moving in different planes with the same speeds $v_1 = v_2 = v_3$ = 2.5 m s^{-1} as shown in the figure. The length of each rod is 60 cm. A constant magnetic field of magnitude B = 0.5 T is directed along z-axis. Find the magnitude of emf induced in each rod and indicate which end of the rod is positive.



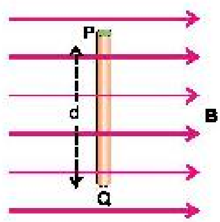
[Ans: (Rod L_1) emf = 0.75 V, and end a is positive, (Rod L_2) emf = 0 (Rod L_3) emf = 0]

10.10 An emf of 0.5 V is induced across the ends of a metal rod moving through a magnetic field of 0.4 T. If an emf of 1.5 V has to be induced, what field strength would be needed for that? Assume that all other factors remain the same.

(Ans: 1.2 T)

10.11 A charged particle moves through a velocity selector at a constant velocity of 4.96×10^{4} m s^{-1} in a direction perpendicular to both **E** and **B**. If the magnetic field strength is 0.114 T, what should be the magnitude of electric field intensity so that the particle moves undeflected? (Ans: 5.65×10^{3} N C^{-1})

10.12 A current-carrying conductor PQ of length 2 m is placed perpendicularly to a magnetic field of flux density 0.5 T as shown in the figure. The resulting force on the conductor is 1 N acting into the plane of the paper. What is the magnitude and direction of the current?



(Ans: 1 A, Q to P)