Chapter 1
Measurements

Learning Objectives

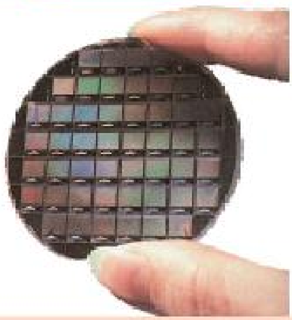
After studying this chapter, the students will be able to:

- Make reasonable estimates of value of physical quantities [of those quantities that are discussed in the topics of this grade].
- Use the conventions for indicating units, as set out in the SI units.
- Express derived units as products or quotients of the SI base units
- Analyze the homogeneity of physical equations [Through dimensional analysis]
- Derive formulae in simple cases [Through using dimensional analysis]
- Analyze and critique the accuracy and precision of data collected by measuring instruments
- Justify why all measurements contain some uncertainty.
- Assess the uncertainty in a derived quantity [By simple addition of absolute, fractional or percentage uncertainties]
- Quote answers with correct scientific notations, number of significant figures and units in all experimental and numerical results.

Physics is the most fundamental branch of physical sciences. It provides the basic principles and laws which help to understand the mysteries of other branches of sciences such as astronomy, chemistry, geology, biology and health sciences. The tools, techniques and products of Physics have transformed our dreams into realities. The comforts and pleasures added in our lives are fruitful results of science, technology and engineering in everyday life.

The Information technology has entirely changed the outlook of mankind. The fast means of communication have brought people of the entire world in so close contact that the whole world has become a global village.

Think over!



Computer chips are made from silicon, which is obtained from sand. It is up to us whether we make a sand castle or a computer out of it.

Physics is an experimental science and the scientific method emphasizes the need of accurate measurement of various measurable physical quantities. This chapter stresses in understanding the concept of measuring techniques and recording skills.

1.1 PHYSICAL QUANTITIES AND THEIR UNITS

The foundation of physics depends on physical quantities in terms of which the laws of Physics are expressed. Therefore, these quantities have to be measured accurately. These are mass, length, time, velocity, force, density, temperature, electric current, and numerous others.

Physical quantities are often divided into two categories: base quantities and derived quantities. Derived quantities are those which depend on base quantities. Examples of derived quantities are velocity, acceleration, force, etc. Base quantities are not defined in terms of other physical quantities. The base quantities are the independent physical quantities in terms of which the other physical quantities can be defined. Typical examples of base quantities are length, mass and time.

The measurement of a base quantity involves two steps: first, the choice of a standard, and second, the establishment of a procedure for comparing the quantity to be measured with the standard so that a number and a unit are determined as the measure of that quantity.

Measurements must be reliable and accurate so that they can be used, easily and effectively.

Areas of Physics

Mechanics
Heat & thermodynamics
Electromagnetism
Optics
Sound
Hydrodynamics
Special relativity
General relativity
Quantum mechanics
Atomic physics
Molecular physics
Nuclear physics
Solid state physics
Particle physics
Superconductivity
Superfluidity
Plasma physics
Magnetohydrodynamics
Space physics

Interdisciplinary areas of Physics

Astrophysics
Biophysics
Chemical physics
Engineering physics
Geophysics
Medical physics
Physical oceanography
Physics of music

1.2 INTERNATIONAL SYSTEM OF UNITS

In 1960, an international committee agreed on a set of definitions and standards to describe the physical quantities. The system that was established is called the System International (SI).

SI units are used by the world's scientific community and by almost all nations. The system International (SI) consists of two kinds of units: base units and derived units.

Base Units

There are seven base units for physical quantities namely: length, mass, time, temperature, electric current, light or luminous intensity and amount of substance (with special reference to the number of particles). Prefixes such as milli, micro, kilo, etc. may be used with them to express smaller or larger quantities.

Table 1.1

Physical Quantity	SI Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Intensity of light	candela	cd
Amount of substance	mole	mol

The names of base units for these physical quantities together with symbols are listed in Table 1.1.

Derived Units

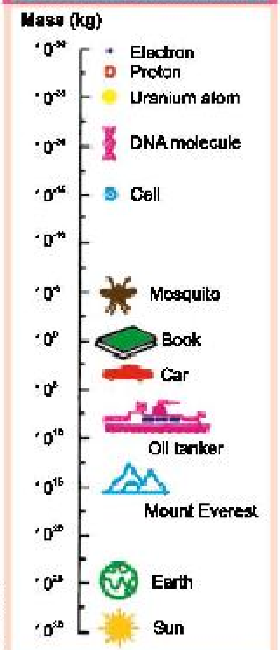
Derived units are those units which depend on the base units. Some of the derived units are given in Table. 1.2. The units of plane angle and solid angle have also been included in the list of derived units since 1995.

Table 1.2			
Physical quantity	**Unit**	**Symbol**	**In terms of base units**
Plane angle	radian	rad	dimensionless
Solid angle	steradian	sr	dimensionless
Force	newton	N	$kg\ m\ s^{-2}$
Work	joule	J	$N\ m = kg\ m^2\ s^{-2}$
Power	watt	W	$J\ s^{-1} = kg\ m^2\ s^{-3}$
Electric charge	coulomb	C	A s
Pressure	pascal	Pa	$N\ m^{-2} = kg\ m^{-1}\ s^{-2}$

In addition to base and derived units, the SI permits the use of certain additional units, including:

- The traditional mathematical units for measuring angles (degree, arcminute, and arcsecond).
- The traditional units for standard time are (minute, hour, day, and year).
- The logarithmic units bel (and its multiples, such as the decibel).
- Two metric units commonly used in ordinary life: the litre for volume and the tonne (metric ton) for large masses.
- Two non-metric scientific units are atomic mass unit (μ) and the electron volt (eV).
- The nautical mile and knot; units traditionally used at sea and in meteorology.
- The acre and hectare, common metric units of land area.
- The bar is a unit of pressure and it is commonly used as the millibar in meteorology and the kilobar in engineering.
- The angstrom and the barn, units used in physics and astronomy.

Interesting Information



Order of magnitude of some masses.

Scientific Notation

Numbers are expressed in standard form called scientific notation, which employs powers of ten. The internationally accepted practice is that there should be only one non-zero digit left of decimal. Thus, the number 134.7 should be written as 1.347×10^2 and 0.0023 be expressed as 2.3×10^{-3}.

Prefixes

Most prefixes indicate order of magnitude in steps of 1000 and provide a convenient way to express large and small numbers, to eliminate non-significant digits. SI also includes four of the other prefixes to accommodate usage already established before the introduction of SI (Table 1.3). They are centi-(10^{-2}), deci-(10^{-1}), deca-(10^{1}) and hecto-(10^{2}).

Table 1.3
Some Prefixes for Powers of Ten

Factor	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

Conventions for Using SI Units

Use of SI units requires special care, more particularly in writing prefixes. Some points to note are:

1. Each SI unit is represented by a symbol not an abbreviation. These symbols are the same in all languages. Hence, correct use of the symbol is very important.

   For example: For ampere, we should use "A" not "amp"; for seconds, "s" not "sec", SI not S.I.

2. Full name of unit does not begin with capital letter.

   For example: newton, metre, etc., except Celsius.

3. Symbols appear in lower case.

   For example: "m" for metre, "s" for second, etc., exception "L" for litre.

4. Symbols named after scientists have initial letters capital.

   For example: "N" for newton, "Pa" for pascal, "W" for watt.

5. Symbols and prefixes are printed in upright (roman) style regardless of the type style in surrounding text.

   For example: a distance of 50 m.

6. Symbols do not take plural form.

   For example: 1 mm, 100 mm, 1 kg, 60 kg.

7. No fullstop or dot is placed after the symbol except at the end of the sentence.

8. Prefix is written before and without space to base unit.

   For example: "mL" not m L or "ms" not m s.

9. Base units are written one space apart. Leave a space even between the number (value) and the symbol.

   For example: 1 kg, 10 m s^{-1}, etc.

For your information

	Interval (s)
Age of the universe	5×10^{17}
Age of the Earth	1.4×10^{17}
One year	3.2×10^{7}
One day	8.6×10^{4}
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	1×10^{-3}
Period of typical radiowaves	1×10^{-6}
Period of vibration of an atom in a solid	1×10^{-13}
Period of visible light waves	2×10^{-15}

Approximate Values of Some Time Intervals

Do you know?

Mass can be thought of as a form of energy. In effect, the mass is highly concentrated form of energy. Einstein's famous equation, $E=mc^2$ means:

Energy = mass x (speed of light)^2

According to this equation 1 kg mass is actually 9×10^{16} J of energy.

10. Compound prefixes are not allowed:

    For example: 1 μμF should be 1 pF.

11. When base unit of multiple is raised to a power, the power applies to whole multiple and not to base unit alone.

    For example: $1\ km^2 = 1\ (km)^2 = 1 \times (10^3\ m)^2 = 1 \times 10^6\ m^2$.

12. Use negative index notation ($m\ s^{-1}$) instead of solidus (m/s).

13. Use scientific notation, that is, one non-zero digit left of decimal.

    For example: $143.7 = 1.437 \times 10^2$.

14. Do not mix symbols and names in the same expression.

    For example: metre per second or $m\ s^{-1}$, not metre/sec or m/second.

15. Practical work should be recorded in most convenient units depending upon the instruments being used.

    For Example: Measurements using screw gauge should be recorded in mm but the final results must be recorded to the appropriate base units.

16. System International do not allow the use of former CGS System units such as dyne, erg, gauss, poise, torr, etc.

1.3 UNCERTAINTY IN MEASUREMENT

You can count the number of pages of a book exactly but measurement of its length needs some measuring instrument. Every instrument is calibrated to a certain smallest division mark on it and this fact puts a limit regarding its accuracy. When you take a reading with one instrument, its limit of measurement is the smallest division or graduation on its scale. Hence, every measured quantity has some uncertainty about its value. When a measurement is made, it is taken to the nearest graduation or marking on the scale. You can estimate the maximum uncertainty as being one smallest division of the instrument. This is called absolute uncertainty. It is one millimetre on a metre rule that is graduated in millimetres. For example, if one edge of the book coincides with 10.0 cm mark and the other with 33.5 cm, then the length with uncertainty is given by

$$(33.5 \pm 0.05)\ cm - (10.0 \pm 0.05)\ cm = (23{\cdot}5 \pm 0.1)\ cm$$

It means that the true length of the book is in between 23.4 cm and 23.6 cm. Hence, the maximum uncertainty is ±0.05 cm, which is equivalent to an uncertainty of 0.1 cm. Infact, it is equal to least count of the metre rule. Uncertainty may be recorded as:

$$\text{Fractional uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}}$$

or

$$\text{Percentage uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}} \times 100\%$$

Uncertainty in Digital Instruments

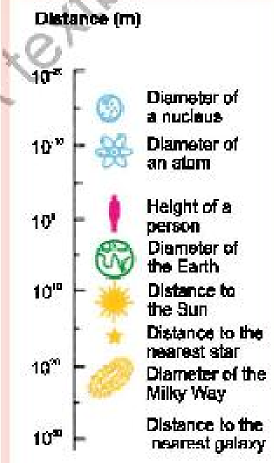
Some modern measuring instruments have a digital scale. We usually estimate one digit beyond what is certain: with a digital scale, this is reflected in some fluctuations of the last digit. If the last digit fluctuates by 1 or 2, write down that last digit. If fluctuation is more than 2 or so in the last digit, it may mean that the reading is being influenced by some factor such as air currents. Regardless of the reason, a large fluctuation may mean that the displayed digit is not really significant.

The indication of uncertainty in a recorded value has been simplified using significant figures. If a measurement is recorded using the knowledge of significant figures, then its last digit, which is an estimation, is an indication of the accuracy of the recorded value.

1.4 USE OF SIGNIFICANT FIGURES

The number of digits of a measurement about which we do feel reasonably sure are called significant figures. Infact, they reflect the use of actual instrument used for that measurement. While using a calculator, the result of any calculation contains many digits after the decimal point. The additional digits may mislead another person who uses those figures into believing them. Hence, they are to be rounded off to the correct number of significant figures. This can be done by keeping in view the uncertainty or the least count of the instrument while recording observations and also quoting results of any calculations to the correct numbers of significant figures. It is better to quote the result in scientific notation to avoid any ambiguity regarding the number of significant figures.

For your information



Order of magnitude of some distances

For example, weighing the same object with different balances:

Electronic balance : mass = 3.145 ± 0.001 g

Lever balance : mass = 3.1 ± 0.1 g

Usually, the uncertainty ± 0.001 g or ±0.1 g is dropped, and it is understood that the number quoted has an uncertainty of at least 1 unit in the last digit. All digits which are quoted are called significant figures.

> In any measurement, the accurately known digits and the first estimated or doubtful digit are called significant figures.

Proper use of significant figures ensures that we correctly represent the uncertainty of our measurements. For example, scientists immediately realize that the reported mass 3.145 g is more accurate than a reported mass of 3.1 g, reflecting the use of a better or more precise instrument. As we improve the quality of our measuring instrument and techniques, we extend the result to more and more significant figures and correspondingly improve the experimental accuracy of the result.

Working with significant figures

(I) Counting significant digits

(a) All digits 1,2,3,4,5,6,7,8,9 are significant. However, zeros may or may not be significant. In case of zeros, the following rules may be adopted:

(b) A zero between two significant figures is itself significant.

(c) Zeros to the left most significant figure are not significant. For example, none of the zeros in 0.00467 or 02.59 are significant.

(d) Zeros to the right of a significant figure may or may not be significant. In decimal fraction, zeros to the right of a significant figure are significant. For example, all the zeros in 3.570 or 7.4000 are significant. However, in integers such as 8, 000 kg, the number of significant zeros is determined by the precision of the measuring instrument. If the measuring scale has a least count of 1 kg, then there are four significant figures written in scientific notation as 8.000×10^3 kg. If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as 8.00×10^3 kg and so on.

(e) When a measurement is recorded in scientific notation or standard form, the figures other than the powers of ten are significant figures. For example, a measurement recorded as 8.70×10^4 kg has three significant figures.

(II) Multiplying or dividing numbers

Keep a number of significant figures in the product or quotient not more than that contained in the least accurate factor i.e., the factor containing the least number of significant figures. For example, the computation of the following using a calculator, gives

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^4}{1.336} = 1.45768982 \times 10^3$$

As the factor 3.64×10^4, the least accurate in the above calculation has three significant figures, the answer should be written to three significant figures only. The other figures are insignificant and should be deleted. While deleting the figures, the last significant figure to be retained is rounded off for which the following rules are followed:

> **Remember Thumb Rule**
>
> **For calculation of end result:**
> - Addition / Subtraction: same precision.
> - Multiplication / Division: same accuracy (Same number of significant figures).

(a) If the first digit dropped is less than 5, the last digit retained should remain unchanged.

(b) If the first digit dropped is more than 5, the digit to be retained is increased by one.

(c) If the digit to be dropped is 5, the previous digit which is to be retained is increased by one if it is odd and retained as such if it is even. For example, the following numbers are rounded off to three significant figures as follows. The digits are deleted one by one.

43.75	is rounded off as	43.8
56.8546	is rounded off as	56.9
73.650	is rounded off as	73.6
64.350	is rounded off as	64.4

Following this rule, the correct answer of the computation given in section (ii) is 1.46×10^3.

Thumb Rule for Uncertainty

For average value of many readings:

- **Mean deviation from an average value.**
- **Periodic Uncertainty:**
  Divide least count of timing device by the number of oscillations.

(iii) In adding or subtracting numbers

The number of decimal places retained in the answer should be equal to the smallest number of decimal places in any of the quantities being added or subtracted. In this case, the number of significant figures is not important. It is the position of decimal that matters. For example, suppose we wish to add the following quantities expressed in metres.

	(a)	(b)
	72.1	2.7543
	3.42	4.10
	0.003	1.273
	75.523	8.1273
Correct answer:	75.5 m	8.13 m

Quick Quiz

1. Give the correct number of significant figures for 0.0054 m, 0.03030 m, 40.0 m, 0.6 m, 8.20×10^7 m.
2. Give the answer to the appropriate number of significant figures.
   2602 kg + 36.02 kg + 54.1 kg = ?
3. Give the answer to the appropriate number of significant figures.
   3.54 kg - 2.4 kg = ?
4. Give the answer to the appropriate number of significant figure.
   2.45×10^3 m x 2.46 m / 3.6 m = ?

In case (a), the number 72.1 has the smallest decimal places, thus the answer is rounded off to the same position which is then 75.5 m. In case (b), the number 4.10 has the smallest number of decimal places and hence, the answer is rounded off to the same decimal position which is then 8.13 m.

Limitations of Significant Figures

Significant figures deal with only one source of uncertainties that inherent in reading the scale. Real experimental uncertainties have many contributions, including personal errors and sometimes hidden systematic errors. One cannot do better than that what the scale reading allows, but the total uncertainty may well be more than what the significant figure of the measurements would suggest.

1.5 PRECISION AND ACCURACY

Remember Thumb Rule

- Precision: Less absolute uncertainty.
- Accuracy: Less % age uncertainty.

The terms precision and accuracy are frequently used in physics measurements. They should be distinguished clearly. The precision of a measurement is determined by the instrument or device being used. The smaller the least count the more precise is the measurement. Accuracy is defined as the closeness of a measurement to the exact or accepted value of a physical quantity. It is expressed by the fractional or percentage uncertainty. The smaller the fractional or percentage uncertainty, the more accurate is the measurement.

For example, the length of an object is recorded as 25.5 cm by using a metre rule having smallest division in millimetre. Its precision or absolute uncertainty (least count) = ± 0.1 cm.

$$\text{Fractional uncertainty} = \frac{0.1\ \text{cm}}{25.5\ \text{cm}} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1\ \text{cm}}{25.5\ \text{cm}} \times 100\% = 0.4\%$$

Another measurement taken by Vernier Callipers with least count 0.01 cm is recorded as 0.45 cm. It has precision or absolute uncertainty (least count)= ± 0.01 cm.

$$\text{Fractional uncertainty} = \frac{0.01\ \text{cm}}{0.45\ \text{cm}} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.01\ \text{cm}}{0.45\ \text{cm}} \times 100\% = 2\%$$

Thus, the reading 25.5 cm taken by metre rule is although less precise but is more accurate having less percentage uncertainty or error.

Whereas the reading 0.45 cm taken by Vernier Callipers is more precise but is less accurate. In fact, it is the relative measurement which is important. The smaller a physical quantity, the more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such as micrometer screw gauge, with least count 0.001 cm, should have been used. Hence, we can conclude that:

A precise measurement is the one which has less precision or absolute uncertainty and an accurate measurement is the one which has less fractional or percentage uncertainty.

We can never make an exact measurement. The best we can do is to come as close as possible with in the limitation of the measuring instrument.

1.6 ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT

Knowing the uncertainties in all the factors involved in a calculation, the maximum possible uncertainty or error in the final result can be found as follows:

For your information

We use many devices to measure physical quantities, such as length, time, and temperature. They all have some limit of precision.

For your information

These are not decoration pieces of glass but are the earliest known exquisite and sensitive thermometers, built by the Accademia del Cimento (1657-1667), in Florence. They contained alcohol, some times coloured red for easier reading.

1. For addition and Subtraction

Absolute uncertainties are added. For example, the distance between two positions $x_1 = 15.4 \pm 0.1$ cm and $x_2 = 25.6$ cm ± 0.1 cm is recorded as:

$x = x_2 - x_1 = 10.2 \pm 0.2$ cm

and addition of two lengths is:

$\ell_1 = 8.5 \pm 0.1$ cm and $\ell_2 = 12.6 \pm 0.1$ cm recorded as:

$\ell = \ell_1 + \ell_2 = 21.1 \pm 0.2$ cm

For your information

Colour printing uses just four colours cyan, magenta, yellow and black to produce the entire range of colours. All the colours in this book have been made from just these four colours.

2. For multiplication and division

Percentage uncertainties are added. For example, the maximum possible uncertainty in the value of resistance R of a conductor determined by the potential difference V applied across the conductor resulting in current flowing through it is estimated as under:

Let $V = 3.4 \pm 0.1$ V

$I = 0.68 \pm 0.05$ A

using $R = \frac{V}{I}$

Percentage uncertainty in $V = \frac{0.1 \text{ V}}{3.4 \text{ V}} \times 100\% = 3\%$

Percentage uncertainty in $I = \frac{0.05 \text{ A}}{0.68 \text{ A}} \times 100\% = 7\%$

Hence, total percentage uncertainty in the value of R is 3 + 7 = 10%

The value of R will be written as:

$$R = \frac{3.4 \text{ V}}{0.68 \text{ A}} = 5.0 \text{ ohm}$$

Hence, $R = 5.0 \pm 0.5$ ohms, uncertainty being an estimate only, is recorded by one significant figure.

Thumb Rule for Total Uncertainty

- **For addition and subtraction:** Absolute uncertainties are added.
- **For multiplication and division:** Percentage uncertainties are added.
- **For power factor:** Power factor x Percentage uncertainty

For your information

Travel time of light

Moon to Earth	1 min 20 s
Sun to Earth	8 min 20 s
Pluto to Earth	5 h 20 s

3. For Power Factor

The percentage uncertainty is multiplied by the power factor in the formula. For example, the calculation of cross-sectional area of a cylinder of radius $r = 1.25$ cm using formula for Area $A = \pi r^2$ is given by the %age uncertainty which is $A = 2 \times$ %age uncertainty in radius r. As uncertainty is multiplied by power factor, it increases the precision demand of measurement. When the radius of a small sphere is measured as 1.25 cm by Vernier Callipers with least count 0.01 cm, then

The radius r is recorded as $r = 1.25 \pm 0.01$ cm

%age uncertainty in radius r is $r = \frac{0.01}{1.25} \times 100\% = 0.8\%$

Total percentage uncertainty in area $A = 2 \times 0.8 = 1.6\%$

Thus $A = \pi r^2$

$= 3.14(1.25)^2 = 4.906$ cm^2 with 1.6% uncertainty

Thus, the result should be recorded as $A = 4.91 \pm 0.08\ cm^2$

Example 1.1 The length, breadth and thickness of a metal sheet are 2.03 m, 1.22 m and 0.95 cm respectively. Calculate the volume of the sheet correct up to the appropriate significant digits.

Solution Given Length $\ell = 2.03\ m$

Breadth $b = 1.22\ m$

Thickness $h = 0.95\ cm = 0.95 \times 10^{-2}\ m$

Volume $V = \ell \times b \times h = 2.03\ m \times 1.22\ m \times 0.95 \times 10^{-2}\ m$

$= 2.35277 \times 10^{-2}\ m^3$

As the factor 0.95 cm has minimum number of significant figures equal to two, therefore, volume is recorded up to 2 significant figures, hence, $V = 2.4 \times 10^{-2}\ m^3$

Example 1.2 The mass of a metal box measured by a lever balance is 3.25 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct up to the appropriate precision?

Solution

Total mass when silver coins are added to box $= 3.25\ kg + 0.01001\ kg + 0.01002\ kg$

$= 3.27003\ kg$

Since least precise mass is 3.25 kg, having two decimal places, hence, total mass should be reported to 2 decimal places which is the appropriate precision.

Thus Total mass = 3.27 kg

Example 1.3 The diameter and length of a metal cylinder measured with the help of Vernier Callipers of least count 0.01 cm are 1.25 cm and 3.35 cm, respectively. Calculate the volume V of the cylinder and uncertainty in it.

Solution Given

Diameter $d = 1.25\ cm$ with least count 0.01 cm

Length $\ell = 3.35\ cm$ with least count 0.01 cm

Absolute uncertainty in length = 0.01 cm

%age uncertainty in length $= (0.01\ cm / 3.35\ cm) \times 100\% = 0.3\%$

Absolute uncertainty in diameter = 0.01 cm

%age uncertainty in diameter $= (0.01\ cm / 1.25\ cm) \times 100\% = 0.8\%$

As Volume $= \pi r^2 \ell = \pi \frac{d^2}{4} \ell$

Total uncertainty in V = 2 (%age uncertainty in diameter) + (%age uncertainty in length)

$= 2 \times 0.8\% + 0.3\% = 1.9\%$

Then $V = 3.14 \times (1.25\ cm)^2 \times 3.35\ cm/4 = 4.1089842\ cm^3$ with 1.9 % uncertainty

Thus $V = (4.11 \pm 0.08)\ cm^3$

where $4.11\ cm^3$ is calculated volume and $0.08\ cm^3$ is the uncertainty in it.

1.7 DIMENSIONS OF PHYSICAL QUANTITIES

Any physical quantity can be described by certain familiar properties such as length, mass, time, temperature, electric current, etc. These measurable properties are called dimensions. Dimensions deal with the qualitative nature of a physical quantity in terms of fundamental quantities. The quantities such as length, depth, height, diameter, light year are all measured in metre and denoted by the same dimension, basically known as length given by symbol *L* written within square brackets [L]. Similarly, the other fundamental quantities, mass, time, electric current and temperature are denoted by specific symbols [M], [T], [A] and [θ], respectively. These five dimensions have been chosen as being basic because they are easy to measure in experiments.

The dimensions of other quantities indicate how they are related to the basic quantities and are combination of fundamental dimensions. For example, speed *v* is measured in metres per second, so it has the dimensions of length [L] divided by time [T].

$$[v] = [L]/[T] = [L][T^{-1}] = [LT^{-1}]$$

As the acceleration $a = \Delta v / \Delta t$

Dimensions of acceleration are

$$[a] = [v]/[T] = [LT^{-1}]/[T] = [LT^{-2}]$$

Also, dimensions of force can be written as

$$[F] = [m][a] = [M][LT^{-2}] = [MLT^{-2}]$$

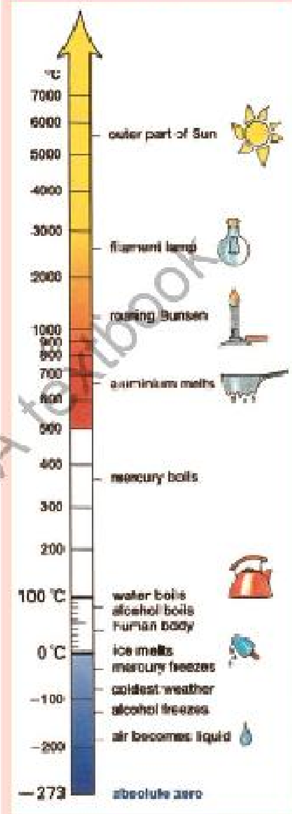
By the use of dimensionality, we can check the homogeneity (correctness) of a physical equation, and also, we can derive formula for a physical quantity.

Homogeneity of Physical Equations

The correctness of an equation can be checked by showing that the dimensions of quantities on both sides of the equation are the same. This is known as principle of homogeneity.

Suppose a car starts from rest ($v_i = 0$) and covers a distance *S* in time *t* moving with an acceleration a. The

Interesting Information



Some Specific Temperatures

For your information

Atomic Clock

The cesium atomic frequency standard at the National Institute of Standards and Technology in Colorado (USA). It is the primary standard for the unit of time.

equation of motion is given by,

$$S = v_i t + \frac{1}{2}at^2$$

or

$$S = \frac{1}{2}at^2$$

Numerical factors like 1/2 have no dimensions, so they can be ignored. By putting the dimensions of both sides of the equation:

$$[S] = [a][t^2]$$

Writing the symbols of dimensions $[L] = [LT^{-2}][T^2]$

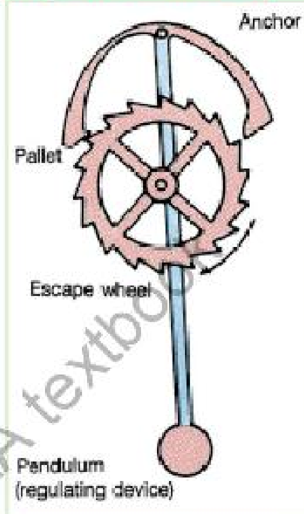
$$[L] = [LT^{-2}T^2]$$

or

$$[L] = [L]$$

This shows that dimensions on both sides of equation are the same, therefore, the equation is dimensionally correct.

Do you know?



The device which made the pendulum clock practical.

Derivation of a Formula

Dimensionality can be used to derive a possible formula for a physical quantity by correct estimation of various factors on which the quantity depends.

Example 1.4 Derive a formula for the centripetal force required to keep an object moving along a circle with uniform speed. Assuming that centripetal force depends on mass of the object, radius of the circle and uniform speed.

Solution As force depends on mass m of the object, radius r of the circle and uniform speed v, we can write:

$$F \propto m^a v^b r^c$$

$$F = (\text{constant})\, m^a v^b r^c \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

where the exponents (powers) a, b and c are to be determined. By the principle of homogeneity, the dimensions on both sides of the equation should be the same. Since, constant has no dimension so by ignoring it, we write the above equation in terms of dimensions as,

$$[F] = [m^a][v^b][r^c]$$

$$[MLT^{-2}] = [M^a][L/T]^b[L]^c$$

$$[MLT^{-2}] = [M^a][LT^{-1}]^b[L]^c$$

$$[MLT^{-2}] = [M^a L^{b+c} T^{-b}] \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Comparing the powers of dimensions on both sides of the above equation, we have

$$a = 1$$

Beware!

Calculators are designed to yield as many digits as the memory of the calculator chip permits. Hence, be sure to round off the final answers of calculations down to correct number of significant figures.

$$b + c = 1$$
$$-b = -2$$

Solving the above equations, we have $a = 1$, $b = 2$, $c = -1$

Putting the values of a, b and c in equation (i), we have

$$F = (\text{constant})\, mv^2 r^{-1}$$

or
$$F = (\text{constant})\, mv^2/r$$

The numerical value of the constant cannot be determined by dimensional analysis. However, it can be found by experiments. In the above equation, numerical value of the constant happens to be "1", so the equation reduces to:

$$F = mv^2/r$$

Limitations in Dimensional Analysis

The dimensional method cannot identify where an equation is wrong. Even if an equation is proved correct, we can only say the equation might be correct, for the reason that the method does not provide a check on any numerical factor or constant. That can only be determined by experiments or plotting some suitable graph between the variables.

QUESTIONS

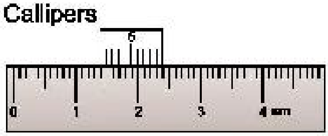
Multiple Choice Questions

Tick (✓) the correct answer.

1.1 The purpose of study and discoveries in Physics is:
- (a) the probing of interstellar spaces
- (b) the betterment of mankind
- (c) the development of destructive technology in warfare
- (d) development in aesthetics for the world

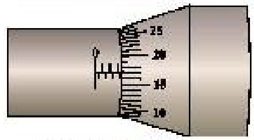
1.2 The length of a steel pipe is in between 0.7 m to 0.8 m. Identify from the following, the appropriate instrument to be used for an accuracy of 0.001 m.
- (a) A micrometer screw gauge (b) A metre rule
- (c) A ten metres measuring tape (d) A Vernier Callipers

1.3 The diameter of a steel ball is measured using a Vernier Callipers and its reading is shown in the figure. What is the diameter of the steel ball?



- (a) 1.30 cm (b) 1.39 cm
- (c) 1.40 cm (d) 1.31 cm

1.4 The figure shows the reading on a micrometer screw gauge used to measure diameter of a thin rod. One complete turn of the thimble is 0.50 mm and there are 50 lines on the circular scale. The diameter of the rod is:



(a) 3.67 mm (b) 3.17 mm (c) 4.17 mm (d) 4.20 mm

1.5 The number of significant figures of a measurement are defined as:

(a) they reflect the accuracy of the observation in a measurement

(b) they are the figures which are reasonably reliable

(c) they are the accurately known digits and the first doubtful digit of a measurement

(d) all of the above

1.6 The number of significant figures in the measured mass 2500.0 kg is:

(a) two (b) three (c) four (d) five

1.7 The sum 12 kg + 2.02 kg + 5.1 kg according to appropriate precision is:

(a) 19 kg (b) 19.0 kg (c) 19.1 kg (d) 19.12 kg

1.8 The answer to appropriate precision for the subtraction (1.126-0.97268) is:

(a) 0.15 (b) 0.153 (c) 0.1533 (d) 0.15332

1.9 The answer of the product (2.8723x1.6) to the appropriate number of significant figures is:

(a) 4.59568 (b) 4.595 (c) 4.59 (d) 4.6

1.10 The answer to the mathematical division ($45.2 \div 6.0$) in appropriate number of significant figures is:

(a) 7.5 (b) 7.53 (c) 7.533 (d) 7.5333

1.11 The answer to the following mathematical operation 24.4 m x 100 m / 5.0 m to the appropriate number of significant figures is:

(a) 4880 m (b) 4900 m (c) 4.88×10^3 m (d) 4.9×10^2 m

1.12 The ratio of the dimensions of force and energy is:

(a) T (b) T^{-1} (c) L (d) L^{-1}

1.13 Identify which pair from the following does not have identical dimensions.

(a) Work and torque

(b) Angular momentum and Planck's constant

(c) Moment of Inertia and moment of force

(d) Impulse and momentum

1.14 The following figures are of the same Vernier Callipers. Figure (1) shows the reading when the jaws are closed while

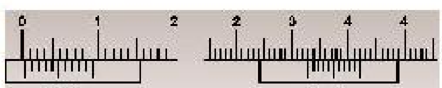


Fig. 1 Fig. 2

Fig. (2) shows the reading when a solid cylinder is placed between the jaws. The length of the cylinder is:

(a) 3.26 cm (b) 3.30 cm (c) 3.34 cm (d) 4.20 cm

1.15 The least count of an instrument determines:

(a) precision of a measurement
(b) accuracy of a measurement
(c) fractional uncertainty of a measurement
(d) percentage uncertainty of a measurement

1.16 A measuring tape has been graduated with a minimum scale division of 0.2 cm. The allowed reading using this tape may be:

(a) 80.5 cm (b) 80.6 cm (c) 80.65 cm (d) 80.7 cm

Short Answer Questions

1.1 What are base units and derived units? Give some examples of both these units.

1.2 How many significant figures should be retained in the following?
(i) Multiplying or dividing several numbers (ii) Adding or subtracting numbers

1.3 How is the Vernier scale related to the main scale of a Vernier Callipers? What is meant by L.C. of the Vernier Callipers?

1.4 Write the following numbers in scientific notation:
a) 143.7 (b) 206.4×10^{2}

1.5 Write the following numbers using correct prefixes:
(a) 580×10^{2} g (b) 0.45×10^{-5} s

1.6 Kinetic energy of a body of mass m moving with speed v is given by $1/2\ mv^2$. What are the dimensions of kinetic energy?

1.7 How many significant figures are there in the following measurements?
(i) 37 km (ii) 0.002953 m (iii) 7.50034 cm (iv) 200.0 m

1.8 Write the dimensions of: (i) Planck's constant (ii) angular velocity

Constructed Response Questions

1.1 Why do we find it useful to have two units for the amount of a substance, the kilogram and the mole?

1.2 Three students measured the length of a rod with a scale on which minimum division is 1 mm and recorded as: (i) 0.4235 m (ii) 0.42 m (iii) 0.424 m. Which record is correct and why?

1.3 Why is the kilogram (not the gram), the base unit of mass.

1.4 Consider the equation; $P = Q + R$
If Q and R both have the dimensions of [MLT], what are the dimensions of P? What

are the units of P in SI? If the dimensions of Q were different from those of R, could we determine dimensions of P?

1.5 What is the least count of a clock if it has:

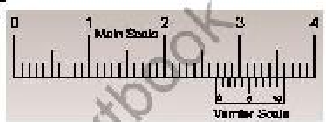
(a) Hour's hand, minute's hand and second's hand

(b) Hour's hand and minute's hand

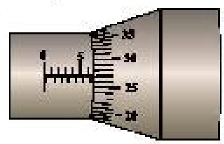
1.6 How can the diameter of a round pencil be measured using metre rule with the same accuracy as that of Vernier Callipers? Describe.

1.7 How would be the readings differ if the screw gauge is used instead of a Vernier Callipers to measure the thickness of a glass plate?

1.8 Write the correct reading of the length of a solid cylinder as shown in the figure if there is an error of +0.02 cm in the Vernier Callipers.

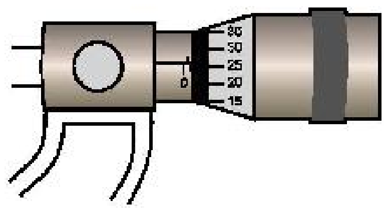


1.9 There are 50 divisions on the circular scale of a screw gauge. If the head (thimble) of the screw is given 10 revolutions, then the spindle advances by 5 mm. There is also zero error as the 2nd division of the circular scale coincides with the datum line and zero of circular scale is below the datum line. What is the thickness of a glass slab as measured by the described screw gauge shown in the figure?



1.10 What is meant by a dimensionless quantity? Give one example.

1.11 A student uses a screw gauge to determine the thickness of a sheet of paper. The student folds the paper three times and measures the total thickness of the folded sheet. Assume that there is no zero error in the screw gauge. The reading of screw gauge is shown in the figure. Find the thickness of the sheet.



1.12 Round off each of the following numbers to 3 significant figures and write your answer in scientific notation.

(a) 0.02055 (b) 4656.5

Comprehensive Questions

1.1 What is meant by uncertainty in a measurement? How the uncertainty in a digital instrument is indicated?

1.2 Differentiate between the terms precision and accuracy with reference to measurement of physical quantities.

1.3 (a) What is meant by significant figures? Write two reasons for using them in measurements. How to find the uncertainty in a timing experiment such as the time period of a simple pendulum?

(b) The mass of a solid cylinder is 12.85 g. Its length is 3.35 cm and diameter is 1.25 cm. Find the density of its material expressing the uncertainty in the density.

1.4 Explain with examples the writing of physical quantities into their dimensions. Write its two benefits.

1.5 Check the homogeneity of the relation:

$$v = \sqrt{\frac{T \times \ell}{m}}$$

where v is the speed of transverse wave on a stretched string of tension T, length ℓ and mass m.

Numerical Problems

1.1 Astronomers usually measure astronomical distances in light years. One light year is the distance that light travels in one year. If speed of light is 3×10^8 m s^{-1}, what is one light year in metres? (Ans: 9.5×10^{15} m)

1.2 Write the estimated answer of the following in standard form.

(a) How many seconds are there in 1 year?

(b) How many years are in 1 second?

(Ans: (a) 3.2×10^7 s (b) 3.1×10^{-8} years)

1.3 The length and width of a rectangular plate are measured to be 18.3 cm and 14.60 cm, respectively. Find the area of the plate and state the answer to correct number of significant figures. (Ans: 267 cm^2)

1.4 Find the sum of the masses given in kg up to appropriate precision:

(i) 3.197 (ii) 0.068 (iii) 13.9 (iv) 3.28 (Ans: 20.4 kg)

1.5 The diameter and length of a metal cylinder measured with the help of a Vernier Callipers of least count 0.01 cm are 1.22 cm and 5.35 cm respectively. Calculate its volume and uncertainty in it. (Ans: 6.2 ± 0.1 cm^3)

1.6 Show that the expression: $v_f^2 - v_i^2 = 2aS$ is dimensionally correct, where v_i is the initial velocity, a is the acceleration and v_f is the velocity after covering a distance S.

1.7 Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

1.8 Derive a formula for the time period of a simple pendulum using dimensional analysis. The various possible factors on which the time period T may depend are:

(i) length of the pendulum ℓ

(ii) mass of the bob m

(iii) angle θ which the thread makes with the vertical

(iv) acceleration due to gravity g. (Ans: $T = \text{constant}\sqrt{\frac{\ell}{g}}$)