Chapter 11 Special Theory of Relativity

Learning Objectives

After studying this chapter, the students will be able to:

- Distinguish between inertial and non-inertial frames of reference.
- Describe the significance of Einstein's assumption of the constancy of the speed of light.
- Describe that if c is constant then space and time become relative.
- State the postulates of Special theory of relativity
- Explain qualitatively and quantitatively the consequences of special relativity

  Specifically in the case of:
  - a. The relativity of simultaneity.
  - b. The equivalence between mass and energy.
  - c. Length contraction.
  - d. Time dilation.
  - e. Mass increase
- State that spacetime is a mathematical model in relativity that treats time as a fourth dimension of the traditional three dimensions of space

  (It can be thought of as a metaphorical sheet of paper that can bend, and when it bends it can cause effects such as stretching and compression seen when gravitational waves pass through objects.)

At the beginning of the 20th century, new experiments and theoretical calculations revealed that classical physics, based on Newton's laws, could not explain phenomena involving extremely small particles or very high velocities. This led to the development of relativistic mechanics, which offered a more comprehensive framework than classical mechanics and fundamentally changed our view of the universe. Albert Einstein's Special Theory of Relativity, introduced in 1905, addressed these issues by proposing that the laws of Physics are the same for all observers and that the speed of light is constant, regardless of the observer's motion. This theory not only resolved the conflicts between classical mechanics and electromagnetic theory but also revolutionized our understanding of time, space, and motion, forming the basis of what is now known as modern physics. This chapter will explore how Einstein's theory reshaped our view of the universe and continues to influence our understanding of the physical world.

11.1 RELATIVE MOTION

Consider throwing a ball to your right. For someone facing you, this direction appears to his left. This illustrates that direction is a relative concept. Similarly, the state of rest or motion of an object depends on the observer. For example, the walls of a moving train seem stationary to passengers inside the train but appear to be moving to someone standing on the ground. Thus, we cannot definitively say whether an object is absolutely at rest or in motion; all motions are relative to the observer or to the reference frame being used. This becomes evident with the following example: An observer in a closed train compartment uses the compartment as his frame of reference. To determine the train's motion, the observer drops a ball and measures the horizontal distance travelled by the ball, keeping the vertical distance the same in each case. It is assumed that the vertical distance is covered in "t" seconds in all scenarios.

Case (a): Suppose the train is stationary. In that case, the horizontal velocity of the ball will be zero, and the horizontal distance travelled will also be zero. In this scenario, the observations made by the observer inside the train and by someone outside the train will be identical. The ball will have fallen to a point on the floor directly below the point from where it was dropped (Fig. 11.1-a).

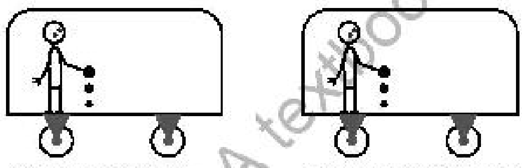


Fig. 11.1(a)

Case (b): The train is moving with a uniform velocity v_o. When the ball is released, it has an initial horizontal velocity v_o and behaves like a projectile. It travels a horizontal distance v_ot in time t it takes for the ball to reach the floor. Since both the train and the observer inside it are moving with the same velocity v_o, they both travel the same horizontal distance v_ot in the same time t. Therefore, the observer inside the train sees that the ball falls to a point on the floor directly below where it was dropped. In contrast, an observer outside the train will see the ball following a projectile path, as shown in Fig. 11.1(b). Thus, observers in different frames of reference, moving with uniform velocity relative to each other, will perceive motion differently.

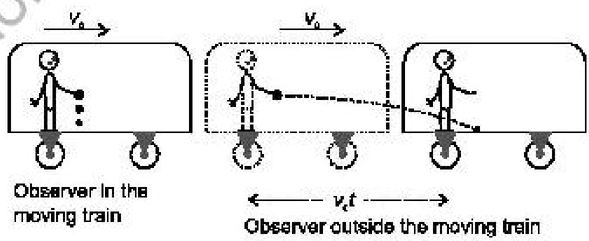


Fig. 11.1(b)

11.2 FRAMES OF REFERENCE

We have discussed the most commonly used Cartesian Coordinate System. In effect, a frame of reference is any coordinate system relative to which measurements are taken. For example, the position of a table in a room can be described relative to the walls of the room, making the room the frame of reference. Similarly, the laboratory is the reference frame for measurements taken there. If the same experiment is performed in a moving train, the train becomes the frame of reference. The position of a spaceship can be described relative to the positions of distant stars. A coordinate system based on these stars is then the frame of reference.

Inertial and Non-inertial Frame of Reference

An inertial frame of reference is defined as a coordinate system in which the law of inertia is valid. This means a body at rest remains at rest unless acted upon by an unbalanced force that produces acceleration. Other laws of nature also apply in such a system. For instance, a body placed on the Earth remains at rest unless an unbalanced force acts upon it, indicating that the Earth can be considered an inertial frame of reference. A body in a car moving with uniform velocity relative to the Earth also remains at rest, so the car is also an inertial frame of reference. Thus, any frame of reference moving with uniform velocity relative to an inertial frame is also an inertial frame.

However, if the moving car is suddenly stopped or accelerated, the body inside no longer remains at rest. In such cases, the car is not an inertial frame of reference. Therefore, an accelerated frame of reference is a non-inertial frame. While Earth is rotating and revolving, making it strictly speaking a non-inertial frame, it is often treated as an inertial frame due to its relatively small acceleration.

For your information

Relativity is the study of the way in which observers from moving frame of reference affect your perception of the world.

11.3 SPECIAL THEORY OF RELATIVITY

The theory of relativity deals with how observers in different states of relative motion describe physical phenomena. The special theory of relativity addresses problems involving inertial (non-accelerating) frames of reference. There is another theory, called the general theory of relativity, that deals with problems involving frames of reference that are accelerating relative to one another. The special theory of relativity is based on two postulates, which can be stated as follows:

Do you know?

The speed of light c emitted by the flashlight is measured same by two observers, one moving in the car with speed v and other standing on the road.

1. The laws of physics are the same in all inertial frames (Principle of Relativity).
2. The speed of light in free space has the same value for all observers, regardless of the state of motion of the source or the observer (Principle of Constancy of Light).

The first postulate generalizes the fact that all physical laws are the same in frames of reference moving with uniform velocity relative to one another. If the laws of Physics differed for observers in relative motion, those observers could determine which was stationary and which was moving. However, such a distinction does not exist, implying that there is no way to detect absolute uniform motion.

The second postulate states the experimental fact that the speed of light in free space is a universal constant, denoted as c ($c = 3 \times 10^8$ m s^{-1}). Since c is constant, space and time become relative. For example, if you are sitting in a train moving at the speed of light and you hold up a mirror in front of you at arm's length, you will still see your reflection in the mirror. This is because, according to the principle of relativity, no experiment can detect the constant motion of the train relative to the person inside it.

These simple postulates have far-reaching consequences. They include phenomena such as the slowing down of clocks and the contraction of lengths in moving reference frames as observed by a stationary observer. Some interesting results of the special theory of relativity can be summarized as follows, without going into their mathematical details.

The Relativity of Simultaneity

If two events in different locations are observed by one observer to be simultaneous, they will generally not be observed as simultaneous by another observer in a different frame of reference moving relative to the first observer. In other words, whether two events are seen as simultaneous depends on the observer's frame of reference.

Consider a train equipped with light-operated doors. The light switch is located in the centre of the roof and is operated by a traveler standing in the middle of the compartment. When the train is travelling at half the speed of light, the traveler turns on the light. The light travels forward and backward at equal speed and reaches both doors at the same time. Consequently, the traveler sees both doors opening simultaneously. However, an observer outside the train will see the back door open before the front door. This is because the back door is moving towards the light waves, while the front door is moving away from the light waves.

For your information

If you are in a frame of reference moving at constant velocity from which you cannot see any other frame of reference, there is no way to know if you are moving or at rest.

Time Dilation

According to the special theory of relativity, time is not an absolute quantity; it depends on the motion of the frame of reference.

Suppose an observer is stationary in an inertial frame and measures the time interval between two events in this frame. Let this time interval be t_o. This is known as proper time. If the observer is moving with respect to the frame of events with relativistic velocity v, or if the frame of events is moving with respect to the observer with a uniform relativistic velocity v, the time measured by the observer will not be t_o, but rather t, given by

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{.................................} \quad (11.1)$$

As the quantity $\sqrt{1-\frac{v^2}{c^2}}$ is always less than one, so t is greater than t_o i.e., time has dilated or stretched due to the relative motion of the observer and the frame of reference of the events. This astonishing result applies to all timing processes—physical, chemical, and biological. Even the aging process of the human body is slowed by motion at very high speeds or relativistic speeds.

For example, if a traveler on a plane moving at 0.8 c picks up and opens a book, the event takes one second as measured by the traveler. However, to a person standing outside the plane, the same event takes 1.7 seconds.

Length Contraction

The distance from Earth to a star measured by an observer in a moving spaceship would appear smaller than the distance measured by an observer on the Earth. In other words, if you are in motion relative to two points that are a fixed distance apart, the distance between the two points appears shorter than if you were at rest relative to them. This effect is known as length contraction. Length contraction occurs only along the direction of motion; no such contraction is observed perpendicular to the direction of motion. The length of an object or the distance between two points measured by an observer who is at rest relative to them is called the proper length ℓ_o. If an object and an observer are in relative motion with speed v, then the contracted length 'ℓ' is given by

$$\ell = \ell_o\sqrt{1-\frac{v^2}{c^2}} \quad \text{..................................} \quad (11.2)$$

Let a train that is measured to be 100 metres long when at rest travel at 80% of the speed of light (0.8 c). A person inside the train will measure its length as 100 metres. However, a person standing by the side of the track will observe the train to be only 60 metres long. This effect of relativity, which is the shortening of length in the direction of motion, is due to length contraction.

Mass Variation

According to the special theory of relativity, the mass of an object is a variable quantity that depends on the object's speed. An object whose mass is measured at rest is called its rest mass m_o, will have an increased mass m when observed to be moving at speed v. They are related by

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{..................................} \quad (11.3)$$

The increase in mass indicates the increase in inertia that an object has at high speeds. As v approaches c, it requires a greater force to change the object's speed.

As $\quad v \to c,\ \frac{v}{c} \to 1, \quad$ therefore, $\quad \sqrt{1-\frac{v^2}{c^2}} \to 0$

Thus $\quad m \to \infty$

An infinite mass would require an infinite force to accelerate it. Since infinite forces are not available, an object cannot be accelerated to the speed of light c in free space.

In our everyday life, we deal with speeds that are extremely small compared to the speed of light. Even Earth's orbital speed is only 30 km s^{-1}, while the speed of light in free space is 300,000 km s^{-1}. This is why Newton's laws are valid in everyday situations. However, when dealing with subatomic particles moving at velocities approaching the speed of light, relativistic effects become very prominent, and experimental results cannot be explained without considering Einstein's equations.

11.4 THE EQUILANCE BETWEEN MASS AND ENERGY

According to the special theory of relativity, mass and energy are distinct entities but are interconvertible. The total energy E and mass m of an object are related by the expression:

$$E = mc^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11.4)$$

where m depends on the speed of the object. At rest, the energy equivalent of an object's mass m_o is called its rest mass energy E_o. Thus,

$$E_o = m_o c^2$$

As mc^2 is greater than $m_o c^2$, the difference of energy $(mc^2 - m_o c^2)$ is due to the motion, and it represents the kinetic energy of the mass. Hence,

$$K.E. = (m - m_o)c^2$$

From Eq. 11.4, the change in mass m due to change in energy ΔE is given by

$$\Delta m = \frac{\Delta E}{c^2}$$

Because c^2 is a very large quantity, this implies that small changes in mass require very large changes in energy. In our everyday world, energy changes are too small to provide measurable mass changes. However, energy and mass changes in nuclear reactions are found to be exactly in accordance with the aforementioned equations.

11.5 SPACE-TIME IN RELATIVITY

Space is said to be a three-dimensional extent in which all objects and events occur. It provides a framework to define the position and motion of various objects under the influence of some force.

Time measures the sequence and duration of events. In the theory of relativity, time is not absolute; it is considered the fourth dimension. For example, oscillatory motion, such as that of a swinging pendulum, relies on time to determine the frequency of oscillations. Another example is time dilation, a phenomenon discussed earlier in this chapter, where time passes more slowly for an observer moving at extremely high speeds compared to one at rest. The special theory of relativity explains that space and time are related to each other. It describes how space and time are influenced by gravity and speed, such as the bending of light around massive objects like stars.

Space-time is, in fact, a mathematical model that unifies space-time into a single continuum. It is a concept used to describe all points of space and time and their relation to each other. According to Einstein's theory, space-time is curved especially near massive bodies and for speeds approaching the speed of light. We can hypothetically visualize this as a fabric sheet. If a heavy ball is placed over this sheet, it curves as shown in Fig. 11.2.

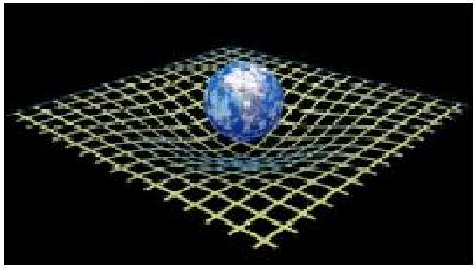


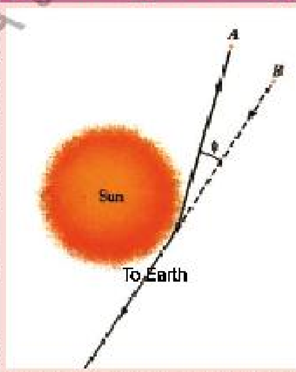
Fig. 11.2

Objects such as stars and planets cause space-time to curve around themselves, much like an elastic fabric deforms when holding a ball. The more massive the object, the deeper the curve. Consequently, we do not speak of a force of gravity acting on bodies; instead, we say that bodies and light rays move along geodesics (analogous to straight lines in plane geometry) in curved space-time. Thus, a body at rest or moving slowly near a massive object would follow a geodesic toward that object.

Einstein's theory provides a physical picture of how gravity works. Newton discovered the inverse square law of gravity but explicitly stated that he offered no explanation for why gravity should follow this law. Einstein's theory also describes gravity as following an inverse square law (except in strong gravitational fields), but it explains why this is so. This is why Einstein's theory is considered an advancement over Newton's, even though it encompasses Newton's theory and yields the same results as Newton's theory in all but very strong gravitational fields.

The bending of starlight caused by the Sun's gravity was measured during a solar eclipse in 1919. The results matched Einstein's theory rather than Newton's, leading to Einstein's theory being hailed as a scientific triumph. Another success of Einstein's theory was the detection of gravitational waves, produced by some celestial events causing disturbances (squeezes and stretches) in the curvature of space-time. These waves were detected in 2015 and announced in 2016.

Interesting Information



Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ. Einstein predicted that ϕ = 1.745 seconds of angle which was found to be the same during the solar eclipse of 1919.

Example 11.1 The period of a pendulum is measured to be 3.0 s in the inertial reference frame of the pendulum. What is its period measured by an observer moving at a speed of 0.95 c with respect to the pendulum?

Solution

$t_o = 3.0\,\text{s}$, $\quad v = 0.95\,c$, $\quad t = ?$

Using $$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{3.0\ s}{\sqrt{1-\frac{(0.95\ c)^2}{c^2}}} = \frac{3.0\ s}{\sqrt{1-(0.95)^2}} = 9.6\ s$$

Example 11.2 A bar 1.0 m in length and located along x-axis moves with a speed of 0.75 c with respect to a stationary observer. What is the length of the bar as measured by the stationary observer?

Solution

$\ell_o = 1.0$ m , $v = 0.75\ c$, $\ell = ?$

Using $$\ell = \ell_o\sqrt{1-\frac{v^2}{c^2}}$$

$$\ell = 1.0\ m \times \sqrt{1-\frac{(0.75\ c)^2}{c^2}} = 1.0\ m \times \sqrt{1-(0.75)^2} = 0.66\ m$$

For your information

The faster you are moving or close to a strong source of gravity, the slower the time goes for you.

Example 11.3 Find the mass m of a moving object with speed 0.8 c.

Solution

Using $$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

or $$m = \frac{m_o}{\sqrt{1-\frac{(0.8\ c)^2}{c^2}}} = \frac{m_o}{\sqrt{1-(0.8)^2}} = 1.67\ m_o$$

or $$m = 1.67\ m_o$$

Interesting Information

If you are on some spaceship moving extremely fast through space near a black hole like in movie, "Interstellar" then you could miss 7 years on the Earth in every hour.

Hypothetical Example of Space Time

Let a spaceship be travelling to a star with half of the speed of light. Let it takes eight years to reach to the star, from the point of view of the observer on the Earth. From the Earth's point of view, the clocks on the spaceship are moving slowly, so that less time passes on the spaceship compared to the Earth.

For the spaceship occupants, the length of the journey has contracted which they cover in less time. The occupants of the spaceship record 7 years to reach their destination, rather than 8 years.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

11.1 Relativistic mechanic yields results different from classical mechanics for objects moving with:

(a) low velocity (b) velocity equal to that of sound waves
(c) velocity greater than sound waves (d) velocity approaching that of light

11.2 If an observer is moving in the same direction as a sound wave, the velocity of the wave seems to be:

(a) more (b) less
(c) constant (d) sum of the two velocities

11.3 If the rest mass of a particle m_o increases to m due to its high speed, then its kinetic energy is:

(a) $1/2\ mc^2$ (b) $1/2\ mv^2$ (c) $(m - m_o)\ c^2$ (d) $1/2\ (m - m_o)\ c^2$

11.4 The speed of beam light of a car while moving with high speed as compared to its rest position is:

(a) greater (b) less (c) same (d) zero

11.5 A photon is a particle of light. What is its mass when it moves with 0.9 c?

(a) 9.1×10^{-31} kg (b) 1.67×10^{-19} kg (c) 1.67×10^{-27} kg (d) Zero

Short Answer Questions

11.1 What is meant by inertial frame of reference and a non-inertial frame of reference?
11.2 What are the two postulates of special theory of relativity?
11.3 Describe why it is impossible for a material particle to move with speed of light.
11.4 Does theory of relativity contradicts Newton's laws of motion? Explain briefly.
11.5 What is meant by proper time, and proper length?
11.6 What is meant by relativistic mass, length and time?
11.7 Why mass of a moving object increases?
11.8 All motion are relatives. Does space-time is absolute? Explain briefly.
11.9 Explain that speed of light is an ultimate limit for any object.
11.10 Give examples where the results of special theory of relativity have been verified.

Constructed Response Questions

11.1 Speed of sound is affected by relative motion between the observer and the source. Does this apply to speed of light as well? Describe briefly.

11.2 Is it ever possible to see a star moving away from us at a uniform velocity equal to the velocity of light?

11.3 If the speed of light is just 50 m s^{-1}, how would every day events appear to?

11.4 If the speed of light were infinite, what would the equations of special theory of relativity reduce to?

11.5 According to Einstein's equation; $E = mc^2$, is it possible to create a single electron from energy? Explain.

Comprehensive Questions

11.1 What is meant by the "frame of reference"? Distinguish between inertial frame of reference and non inertial frame of reference by giving examples.

11.2 Describe the Einstein's mass-energy equation; why cannot we observe its effects in everyday life? What are its significant consequences? Give examples.

11.3 State the Einstien's concept about the space-time. Describe the view of gravity according to this concept.

Numerical Problems

11.1 An electron is accelerated to a speed of 0.995 c which passes down an evacuated tube 500 m long. How long will the tube appear to the electron? (Ans: 50 m)

11.2 A neutron, being not a stable particle, disintigerates in 20 minutes on the average. How long will it seem to exist if shoots out from a nucleus with a speed of 0.8 c? (Ans: 33.3 min)

11.3 A spaceship is measured 100 m long while it is at rest with respect to an observer, if this spaceship now flies by the observer with a speed of 0.99 c, what length will the observer find for the spaceship? (Ans: 14 m)

11.4 The rest mass of an electron is 9.11 x 10^{-31} kg. Calculate the corresponding rest-mass energy. (Ans: 8.2 x 10^{-14} J or 0.51 MeV)

11.5 An electron is accelerated to a speed v = 0.85 c. Calculate its total energy and kinetic energy in electron volt. (Ans: 0.97 Mev, 0.459 MeV)

11.6 At what speed, would the mass of a proton in a particle accelerator be tripled? (Ans:0.943 c)

11.7 The period of pendulum is measured to be 3 s in an inertial frame of reference. What will be the period measured by an observer in a spaceship with a constant speed of 0.95 c with respect to the pendulum? (Ans: 9.6 s)

11.8 Hypothetically, if a ball of mass 0.5 kg is projected with a velocity of 0.9 c, what will be its mass in flight? (Ans: 1.15 kg)