Chapter 7

Waves and Vibrations

Learning Objectives

After studying this chapter, the students will be able to:

- Use the principle of superposition of waves to solve problems.
- Differentiate between constructive and destructive interference.
- Apply the principle of superposition to explain the working of noise canceling headphones.
- Illustrate experiments that demonstrate stationary waves [using microwaves, stretched strings and air columns (it will be assumed that end corrections are negligible; knowledge of the concept of end corrections is not required)]
- Explain the formation of a stationary wave using graphical representation.
- Explain the formation of harmonics in stationary waves.
- Describe an experiment that demonstrates diffraction [including the qualitative effect of the gap width relative to the wavelength of the wave; for example diffraction of water waves in a ripple tank]
- Explain beats [as the pulsation caused by two waves of slightly different frequencies interfering with each other].
- Illustrate examples of how beats are generated in musical instruments.
- Use intensity= power/area to solve problems. Use intensity $\propto$ (amplitude)^2 for a progressive wave to solve problems.
- Explain that when a source of sound waves moves relative to a stationary observer, the observed frequency is different from the source frequency [describing the Doppler effect for a stationary source and a moving observer is not required].
- Use the expression $f_o = \dfrac{f_s \cdot v}{v \pm u_s}$ for the observed frequency when a source of sound waves moves relative to a stationary observer.
- Explain the applications of the Doppler effect [such as radar, sonar, astronomy, satellite, radar speed traps and studying cardiac problems in humans].

We are well familiar with various types of waves such as water waves in the ocean and circular ripples formed on a still pool of water by rain drops. When a musician plucks a guitar string, sound waves are generated in air which reach our ears and produces sensation of music. The vast energy of the Sun, millions of kilometres away, is transferred to the Earth by light waves. In this chapter, we will discuss, formation, propagation and applications of different types of waves.

7.1 WAVES

A wave is a regular disturbance or variation that carries energy, which spreads out from the source. For example, energy is transferred from the Sun to the Earth in the form of light waves called electromagnetic waves. These waves can even travel through vacuum. However, in a medium, energy is transferred due to the regular and repeated disturbances that travel through the medium, making its particles to move up and down or back and forth (to and fro). Imagine a stone thrown into a pond of water (Fig. 7.1).

Fig. 7.1: Representing a travelling disturbance (ripple)

- The stone produces a disturbance (ripple) that travels through the water (medium).
- The water particles move up and down about their mean positions, creating a repeating pattern known as wave that spreads out.

The displacement of a particle of a wave is its distance in a specified direction from its rest / equilibrium position. If the displacement is plotted along the *y*-axis and the time in the direction of energy travel along the *x*-axis, we get a waveform as shown in Fig. 7.2.

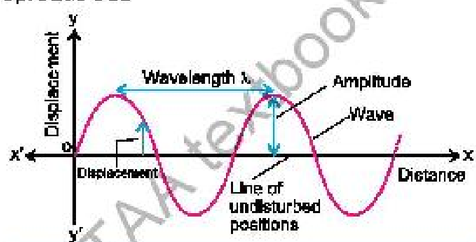


Fig. 7.2: Graphical description of a wave

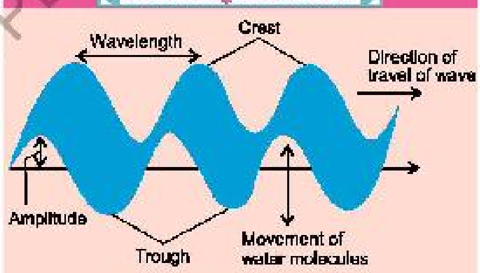
The waves can be described by the following parameters:

1\. **Amplitude (A):** The maximum displacement of the wave (or particles of the medium) from its equilibrium position.

3\. **Frequency (f):** The number of oscillations or vibrations or cycles per second.

3\. **Wavelength (λ):** The distance between two consecutive similar points on the wave that are in phase.

Interesting Information



Wavy shape of water wave in 3D

4\. **Period (T):** The time taken by the wave to complete one oscillation or cycle. it is the reciprocal of the frequency $T = 1/f$.

5\. **Speed (v):** The speed at which the wave travels. If a wave crest moves one wavelength λ in one period of oscillation T, the speed v is given by $v = \lambda / T$

as $1/T = f$, so

$$v = f\lambda \quad \text{..................} \quad (7.1)$$

6\. **Phase (θ):** The relative position of a point on the wave at a given time.

Types of Waves

Waves have various forms, each with unique characteristics. A brief detail of different types of waves is given below:

1. Mechanical Waves

These waves require a physical medium (solid, liquid, or gas) to propagate. Examples are water waves (ocean, lake, or pond ripples), sound waves (audible vibrations in air, water, or solids), seismic waves (earthquakes), etc.

2. Electromagnetic Waves

They do not require a medium to propagate and therefore, can travel through vacuum. Examples are radiowaves (wireless communication), Microwaves (cooking and heating), Infrared waves (IR or heat radiation), Visible light (sunlight, lamp light), Ultraviolet waves (UV radiation), X-rays (medical imaging), Gamma rays (high-energy radiation), etc.

3. Quantum Waves

Quantum waves are associated with particles like electrons and photons. Examples are matter waves/particle waves (electron waves in atoms) or de-Broglie waves, photon waves (light quanta), etc.

Do you know?

These wave types are essential to understand various phenomena in physics, engineering, and everyday life.

4. Surface Waves

Surface waves propagate along surfaces or interfaces between two mediums. Examples are ocean surface waves (wind-driven waves), seismic surface waves, etc.

Transverse and Longitudinal Waves

There are two main types of waves which are named as **transverse waves** and **longitudinal waves.** A transverse wave is one in which the vibrations of the particles are at right angles to the direction in which the energy of the wave is travelling whereas a longitudinal wave is one in which the direction of the vibration of the particles is along or parallel to the the direction in which the energy of the wave is travelling.

The transverse wave and longitudinal wave are illustrated in Figs. 7.3 (a and b), respectively.

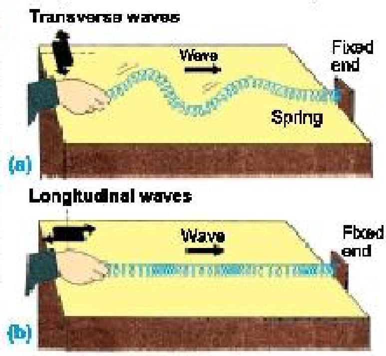


Fig. 7.3: Main types of waves

7.2 PRINCIPLE OF SUPERPOSITION OF WAVES

If a particle of the medium is simultaneously acted upon by two waves, then the resultant displacement of the particle is the algebraic sum of their individual displacements. This is called principle of superposition.

In other words, the displacements of the individual waves are added together to form a new wave pattern as shown in Fig.7.4 (a and b), respectively.

(i) If two waves, which overlap each other, have same phase, their resultant displacement will be:

$$y = y_1 + y_2$$

where y_1 = amplitude of wave 1

y_2 = amplitude of wave 2

and y = resultant amplitude

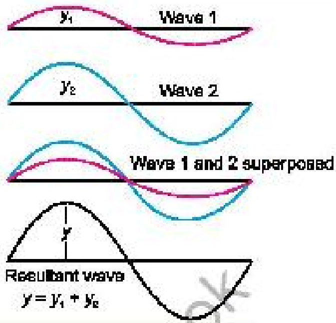
Particularly, if $y_1 = y_2$, then resultant displacement will be:

$$y = 2y_1, \text{ or } y = 2y_2$$

(ii) If two waves, which cross each other, have opposite phase, their resultant displacement will be:

$$y = y_1 + (-y_2)$$

$$y = y_1 - y_2$$



7.4 (a): Superposition of two waves of the same frequency which are exactly in phase.

Particularly, if $y_1 = y_2$, then resultant displacement will be $y = 0$.

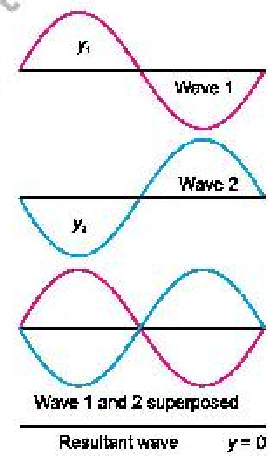
Thus, if a particle of a medium is simultaneously acted upon by n waves such that its displacement due to each of the individual n waves be y_1, y_2, ……, y_n, then the resultant displacement y of the particle, under the simultaneous action of these n waves is the algebraic sum of all the displacements, i.e.,

$$y = y_1 + y_2 + \ldots\ldots\ldots\ldots + y_n$$

This is called principle of superposition of waves. Mathematically, this can also be represented as:

$$y(x,t) = y_1(x,t) + y_2(x,t) + \ldots\ldots + y_n(x,t)$$

where $y(x,t)$ is the resultant wave, whereas $y_1(x,t)$, $y_2(x,t)$, ………, $y_n(x,t)$ are the individual waves.



7.4 (b): Superposition of two waves of the same frequency which are exactly out of phase.

In the context of waves, $y(x,t)$ represents the wave function or wave displacement at a given point x and time t. It describes the shape of the wave and its evolution over time.

The principle of superposition applies to linear waves or small amplitude waves.

Principle of superposition of waves leads to many interesting phenomena:

(i) Two waves having same frequency and travelling in the same direction (Interference).

(ii) Two waves of slightly different frequencies and travelling in the same direction (Beats).

(iii) Two waves of equal frequency travelling in opposite direction (Stationary waves).

Ponder upon!

An interference pattern formed with white light.

Applications of the Principle of Superposition

By applying the principle of superposition of waves, noise-cancelling headphones effectively eliminate unwanted noise, providing a more immersive and peaceful listening experience.

1. The headphones contain one or more microphones that capture ambient noise (like background chatter or engine rumble or any environmental noise).
2. The microphone sends the sound signals to an amplifier and a processing unit in the headphones.
3. The processing unit generates an "anti-noise" signal, which is the exact opposite of the ambient noise (in terms of amplitude and phase).
4. The anti-noise signal is then played through the headphones, along with the desired audio (like music or voice).
5. When the anti-noise signal meets the ambient noise, the two waves cancel each other out resulting in a much quieter listening experience.

Though the above example is an oversimplification of the situation, as noise-cancelling headphones use complex algorithms and multiple microphones to achieve optimal noise cancellation, the basic principle of superposition remains a fundamental concept in understanding how they work.

7.3 INTERFERENCE AND ITS TYPES

Superposition of two waves having the same frequency and travelling in the same direction results in phenomenon called interference.

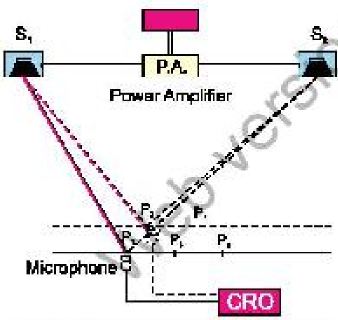


Fig. 7.5(a): An experimental setup to observe interference of sound waves.

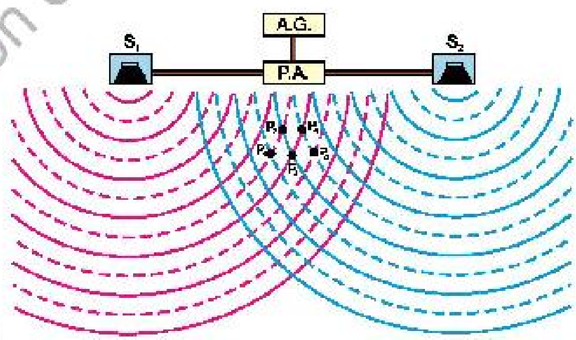


Fig. 7.5 (b): Interference of sound waves

An experimental setup to observe interference effect of sound waves is shown in Fig. 7.5(a). Two loud speakers S_1 and S_2 act as two sources of harmonic sound waves of a fixed frequency produced by an Audio Generator (AG). Since the two speakers are driven from the same generator, therefore, they vibrate in phase. Such sources of waves are called coherent sources. A microphone attached to a sensitive Cathode Ray Oscilloscope (CRO) acts as a detector of sound waves. The CRO is a device to display the input signal into waveform on its screen. The microphone is placed at various points, turn by turn, in front of the loud speakers as shown in Fig.7.5(b).

Constructive Interference

At points P_1, P_3 and P_5, we find that a compression meets a compression and a rarefaction meets a rarefaction. So, the displacement of two waves are added up at these points according to the principle of superposition and a large resultant displacement is seen on the CRO screen (Fig.7.5-c).

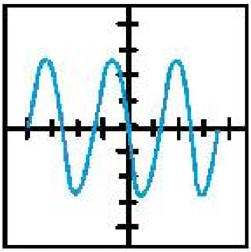


Fig. 7.5(c):
Constructive Interference: Large displacement is displayed on the CRO screen

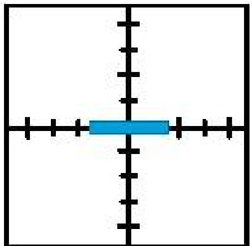
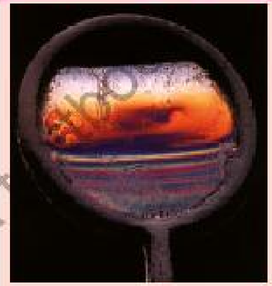


Fig. 7.5(d):
Destructive Interference: Zero displacement is displayed on the CRO screen

Interesting fact!



Interference pattern produced by a thin soap film illuminated by white light.

From Fig. 7.5 (b), we find that the path difference ΔS between the waves at the point P_1, is:

$$\Delta S = S_2P_1 - S_1P_1$$

$$\Delta S = 4\frac{1}{2}\lambda - 3\frac{1}{2}\lambda = \lambda$$

Similarly, at points P_3 and P_5, path difference is zero and λ, respectively. Here, λ is the wavelength which is the distance between any two successive solid or dashed lines.

> Whenever the path difference is an integral multiple of wavelength, the two waves are added up. This effect is called constructive interference.

Therefore, the path difference (condition) for constructive interference can be written as:

$$\Delta S = n\lambda \qquad \text{where} \quad n = 0, \pm1, \pm2, \pm3, \ldots\ldots$$

Destructive Interference

At points P_2 and P_4, a compression meets a rarefaction, so that they cancel each other's effect according to the principle of superposition. The resultant displacement becomes zero, as shown in Fig. 7.5(d).

The path difference ΔS between the waves at points P_2 and P_4 is:

$$\Delta S = S_2P_2 - S_1P_2$$

$$\Delta S = 4\lambda - 3\frac{1}{2}\lambda = \frac{1}{2}\lambda$$

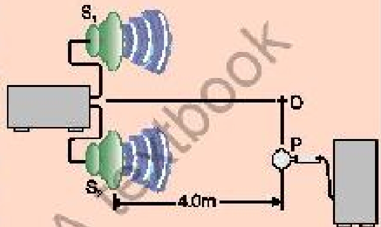
Similarly, at P_4 the path difference is $\frac{1}{2}\lambda$.

> So, at points where the displacements of two waves cancel each other's effect, the path difference is an odd integral multiple of half the wavelength. This effect is called destructive interference.

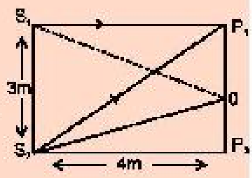
Therefore, the path difference (condition) for destructive interference can be written as:

$$\Delta S = (2n + 1)\,\lambda/2 \qquad \text{where } n = 0, \pm1, \pm2, \pm3, \ldots\ldots\ldots$$

Example 7.1 Two speakers are arranged as shown in the figure. The distance between them is 3.0 m and they emit a constant tone of 344 Hz. A microphone P is moved along a line parallel to and 4.0 m from the line connecting the two speakers. It is found that tone of maximum loudness is heard and displayed on the CRO when microphone is on the centre of the line and directly opposite to each speaker. Calculate the speed of sound.



Solution



Distance between speakers S_1S_2 $= 3.0$ m

Tone frequency f $= 344$ Hz

Distance between speakers and line of motion of P $S_2P = 4.0$ m

Speed of sound $v = ?$

For tone of maximum loudness or the condition for constructive interference, the path difference must be $0, \pm1\lambda, \pm2\lambda, \pm3\lambda, \ldots$

At middle point 'O' the path difference between two sound waves is zero ($S_1O = S_2O$), thus at that point 'O' constructive interference takes place.

For the next point P of constructive interference, the path difference between waves should be λ. So, λ = Path difference $= S_2P_1 - S_1P_1$

Now, we calculate values of S_2P_1 from right angle triangle $S_1S_2P_1$

$$S_2P_1 = \sqrt{(S_1S_2)^2 + (S_1P_1)^2} \qquad \text{(By Pythagoras Theorem)}$$

$$S_2P_1 = \sqrt{(3\text{ m})^2 + (4\text{ m})^2} = \sqrt{(9+16)}\text{ m} = \sqrt{25}\text{ m} = 5\text{ m}$$

Therefore $\lambda = S_2P_1 - S_1P_1$ or $\lambda = 5\text{ m} - 4\text{ m} = 1\text{ m}$

This is the path difference for constructive interference.

As $v = f\lambda$

Putting the values, we have $v = 344\text{ m s}^{-1} \times 1\text{ m} = 344\text{ m s}^{-1}$

Example 7.2 The wavelength of a signal from a radio transmitter is 1500 m and the frequency is 200 kHz. What is the wavelength for a transmitter operating at 1000 kHz and with what speed the radiowaves travel?

Solution

$\lambda_1 = 1500\ m = 1.5 \times 10^3\ m$, $\quad f_1 = 200\ kHz = 2.0 \times 10^5\ Hz$

$f_2 = 1000\ kHz = 1 \times 10^6\ Hz$, $\lambda_2 = ?$, $v = ?$, As $v = f\lambda$

Since, the speed of both the signals is same, so

$$v = f_1\lambda_1$$

$$v = 2.0 \times 10^5\ Hz \times 1500\ m$$

$$v = 3.0 \times 10^8\ m\ s^{-1}$$

Also

$$v = f_2\lambda_2$$

$$\lambda_2 = \frac{v}{f_2}$$

$$\lambda_2 = \frac{3 \times 10^8\ m\ s^{-1}}{1 \times 10^6\ Hz}$$

$$\lambda_2 = 3 \times 10^2 m$$

For your information

Monochromatic Light
Sodium chloride in a flame gives out pure yellow light. This light is not a mixture of red and green.

7.4 STATIONARY WAVES & THEIR FORMATION

Stationary waves, also known as standing waves, are the waves that oscillate in a fixed position, without moving or propagating. They are formed by the superposition of two waves with the same frequency and amplitude, travelling in opposite directions. The resulting wave pattern remains stationary, with nodes (points of zero amplitude) and antinodes (points of maximum amplitude) at fixed positions. Examples include waves on a string, and sound waves in a pipe. The term "standing wave" describes that the wave pattern remains fixed in space, oscillating between positive and negative values, without moving forward or backward.

Let us consider the superposition of two waves moving along a string in opposite directions. Figures 7.6 (a) and (b) show the profile of two such waves at instants $t = 0, T/4, T/2, 3/4\ T$ and T, where T is the time period of the wave. We are interested in finding out the displacements of the points 1, 2, 3, 4, 5, 6 and 7 at these instants as the waves superpose. It is obvious that the points 1,2,..,7 are distant $\lambda/4$ apart, λ being the wavelength of the waves. We can determine the resultant displacement of these points by applying the principle of superposition.

Figure 7.6 (c) shows the resultant displacement of the points 1,3,5 and 7 at the instants $t = 0$, $T/4$, $T/2$, $3T/4$ and T. It can be seen that the resultant displacement of these points is always zero. These points of the medium are known as nodes. Here, the distance between two consecutive nodes is $\lambda/2$.

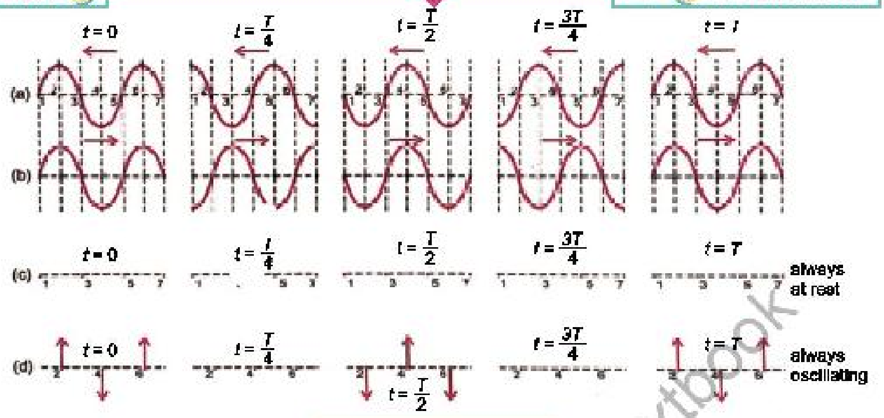


Fig. 7.6: Stationary waves

Figure 7.6 (d) shows the resultant displacement of the points 2,4 and 6 at the instants $t = 0, T/4, T/2, 3T/4$ and T. The figure shows that these points are moving with an amplitude which is the sum of the amplitudes of the component waves represented by arrows. These points are known as antinodes. They are situated midway between the nodes and are $\lambda/2$ apart. The distance between a node and the next antinode is $\lambda/4$. Such a pattern of nodes and antinodes is known as a stationary or standing wave.

Energy in a wave transfers because of the motion of the particles of the medium. The nodes always remain at rest, so energy cannot flow past these points. Hence, energy remains "standing" in the medium between nodes, although it alternates between potential and kinetic forms at the antinodes. When the antinodes are all at their extreme displacements, the energy stored is wholly potential and when they are simultaneously passing through their equilibrium positions, the energy is wholly kinetic.

7.5 STATIONARY WAVES ON A STRETCHED STRING

Consider a string of length ℓ which is kept stretched by clamping its ends so that the tension in the string is F.

(a) String Plucked at its Middle Point

If the string is plucked at its middle point, two transverse waves will originate from this point. One of them will move towards the left end of the string and the other towards the right end. When these waves reach the two clamped ends, they are reflected back thus giving rise to stationary waves. As the two ends of the string are clamped, no motion will take place there. So, nodes will be formed at the two ends and one mode of vibration of the string will be as shown in Fig.7.7(a) with the two ends as nodes with one antinode in between. Visually, the string seems to vibrate in one loop. As the distance between two

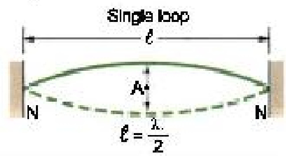


Fig. 7.7(a): First mode of vibration

consecutive nodes is one half of the wavelength of the waves set up in the string, so in this mode of vibration, the length ℓ of the string is

$$\ell = \lambda_1/2$$

$$\lambda_1 = 2\ell \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7.2)$$

where λ_1 is the wavelength of the waves set up in this mode. The speed v of the waves in the string depends upon the tension F of the string and m, the mass per unit length of the string. It is independent of the point from where string is plucked to generate wave. It is given by

$$v = \sqrt{\frac{F}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.3)$$

Knowing the speed v and wavelength λ_1, the frequency f_1 of the waves is:

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2\ell}$$

Substituting the value of v,

$$f_1 = \frac{1}{2\ell}\sqrt{\frac{F}{m}} \quad \ldots\ldots\ldots\ldots\ldots (7.4)$$

Thus, in the first mode of vibration shown in Fig.7.7 (a), waves of frequency f_1 only will be set up in the given string.

(b) String Plucked at Quarter Length

If the same string is plucked from one quarter of its length, again stationary waves will be set up with nodes and antinodes as shown in Fig. 7.7 (b). Note that now the string vibrates in two loops. This particular configuration of nodes and antinodes has developed because the string was plucked from the position of an antinode. As the distance between two consecutive nodes is half the wavelength, so the length ℓ of string is equal to the wavelength of the waves set up in this mode. If λ_2 is the measure of wavelength of these waves, then,

$$\ell = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

$$\ell = 2\frac{\lambda_2}{2}$$

$$\lambda_2 = \ell \quad \ldots\ldots\ldots\ldots\ldots (7.5)$$

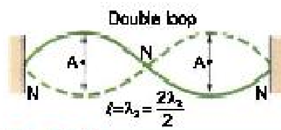


Fig. 7.7(b):
Second mode of vibration

Comparison of Eq. (7.5) with Eq. (7.1) shows that the wavelength in this case is half of that in the first case. Equation (7.2) shows that the speed of the waves depends on tension and mass per unit length of the string, it is independent of the point where the string is plucked. So, speed v remains the same in both cases.

If f_2 is frequency of vibration of string in its second mode, then

$$f_2 = \frac{v}{\lambda_2}$$

Since $\lambda_2 = \ell$

Therefore $f_2 = \dfrac{v}{\ell}$

Multiplying and dividing by 2, we have

$$f_2 = \frac{2v}{2\ell}$$

We know that; $f_1 = \dfrac{v}{2\ell}$

So $f_2 = 2f_1$

Brain teaser

A guitar string is plucked at its centre. What harmonic is produced?

Thus, when the string vibrates in two loops, its frequency becomes double than when it vibrates in one loop.

(c) String Plucked at an Arbitrary Point

Let the string resonates in n number of loops with $(n + 1)$ nodes and n antinodes. Thus, we can say that if the string is made to vibrate in n loops, the frequency of stationary waves set up on the string will be:

$$f_n = n\left(\frac{v}{2\ell}\right)$$

$$f_n = nf_1 \quad \text{..........................(7.6)}$$

and the corresponding wavelength is, $\lambda_n = \dfrac{2\ell}{n}$ ....(7.7)

where $n = 1,2,3,...$

It is clear that as the string vibrates in more than one loop, its frequency f goes on increasing and the wavelength λ gets correspondingly shorter. However, the product of the frequency f and wavelength λ is always equal to v, the speed of waves.

The above discussion clearly establishes that;

1. The stationary waves have a discrete set of frequencies f_1, $2f_1$, $3f_1$, ...,nf_1 which is known as harmonic series. The lowest characteristic frequency of vibration is the fundamental frequency f_1 corresponds to the first harmonic. The frequency $f_2 = 2f_1$ corresponds to the second harmonic and so on.
2. In other words, quantum jumps in frequency exist between the resonance frequencies. This phenomenon is known as the Quantization of frequencies. It means $f_n = nf_1$ where $n = 1, 2, 3, ...$(Integral multiples). The stationary waves can be set up on the string only with the frequencies of harmonic series determined by the tension, length and mass per unit length of the string. Waves which are not in harmonic series are quickly damped out.
3. The frequency of a string on a musical instrument can be changed either by varying the tension or by changing the length. For example, the tension in guitar and violin

strings is varied by tightening the pegs on the neck of the instrument. Once the instrument is tuned, the musicians vary the frequency by moving their fingers along the neck, thereby changing the length of the vibrating portion of the string.

Harmonics

In the above example, the set of all the possible standing waves, having frequencies f_1, $2f_1$, $3f_1$, ..., nf_1, are called harmonics of the system. The lowest or fundamental frequency of all the harmonics is called the fundamental or first harmonic. Subsequent frequencies are called as second harmonic, third harmonic, etc.

Example 7.3 A stationary wave is established in a string which is 120 cm long and fixed at both ends. The string vibrates in four segments; at a frequency of 120 Hz. Determine its wavelength and the fundamental frequency?

Solution $\ell = 120\text{ cm} = \frac{120}{100}\text{ m} = 1.2\text{ m}$

$n = 4$

$f_4 = 120\text{ Hz}$

$f_1 = ?$

$\lambda = ?$

(i) As the string vibrates in four segments and the distance between two consecutive nodes is $\lambda/2$, so the wavelength of the string is:

$$\ell = n\frac{\lambda_n}{2}$$

$$\lambda_n = \frac{2\ell}{n}$$

$$\lambda_4 = \frac{2 \times 1.2\text{ m}}{4}$$

$$\lambda_4 = 0.6\text{ m}$$

(ii) If string vibrates in n loops, then frequency of stationary waves will be:

$$f_n = nf_1$$

$$f_4 = 4f_1$$

$$120\text{ Hz} = 4f_1$$

$$f_1 = \frac{120\text{ Hz}}{4}$$

$$f = 30\text{ Hz}$$

7.6 STATIONARY WAVES IN AIR COLUMNS

Stationary waves can be set up in other media also, such as air column inside a pipe or tube. A common example of vibrating air column is in the organ pipe.

Organ pipe

An organ pipe is a wind instrument in which sound is produced, due to setting up of stationary waves in air column. It consists of a hollow long tube with both ends open or with one end open and the other closed. The relationship between the incident wave and the reflected wave depends upon whether the reflecting end of the pipe is open or closed.

(i) If the reflecting end is open, the air molecules have complete freedom of motion and this behaves as an antinode.

(ii) If the reflecting end is closed, then it behaves as a node because the movement of the molecules is restricted.

Modes of Vibrations

Stationary longitudinal waves occur in a pipe as discussed by the following two cases:

Case (1): Modes of vibrations in an organ pipe open at both ends

Let us consider an organ pipe of length ℓ which is open at both ends. As at the open end, an air molecule has complete freedom of motion so it acts as antinode as shown in Fig.7.8. In this figure, longitudinal waves set up inside the pipe have been represented by transverse curved lines which represent the displacement and amplitude of vibration of air particles at various points along the axis of pipe.

(a) Fundamental mode of vibration

In this case, as shown in Fig.7.8 (a), there is only one node N at the middle of the pipe. As both ends of pipe are open, there are two antinodes at both the ends. If λ_1 is the wavelength of sound, then

$$\ell = \frac{\lambda_1}{4} - \frac{\lambda_1}{4}$$

$$\ell = \frac{\lambda_1}{2}$$

or $\lambda_1 = 2\ell$ ....(7.8)

If f_1 is the frequency of sound, then the velocity of sound is:

$$v = f_1 \lambda_1$$

$$f_1 = v / \lambda_1$$

Putting the value of λ_1, we have

$f_1 = v / 2\ell$ ....(7.9)

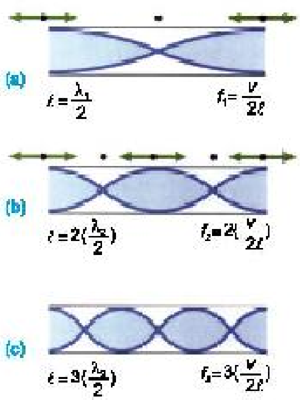


Fig. 7.8: Stationary longitudinal waves in a pipe open at both ends.

This frequency is called fundamental frequency or first harmonic.

(b) Second mode of vibration

If there are three antinodes and two nodes, frequency will be twice that of fundamental frequency. It is second mode of vibration as shown in Fig.7.8 (b). In this case, there are three antinodes and two nodes.

If λ_2 is the wavelength of sound, then

$$\ell = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4}$$

$$\ell = (1+2+1)\ \frac{\lambda_2}{4}$$

or $$\lambda_2 = \ell$$

If f_2 is the frequency of sound, then speed v of sound becomes:

$$v = f_2 \lambda_2$$

or $$f_2 = \frac{v}{\lambda_2}$$

Putting the value of λ_2, we have

$$f_2 = \frac{v}{\ell}$$

or $$f_2 = 2f_1 \qquad (\because \frac{v}{2\ell} = f_1)$$

Brain teaser

A stationary wave is formed on a string with a frequency of 100 Hz. If the string is 2 m long, how many nodes and antinodes are formed?

(c) nth mode of vibration

Similarly, frequency for air column vibrating in n loops is:

$$f_n = n(v/2\ell)$$

$$f_n = nf_1$$

and wavelength is

$$\lambda_n = \frac{2\ell}{n} \qquad \text{............... (7.10)}$$

where $n = 1,2,3,4,5, \ldots\ldots\ldots$

So, the longitudinal stationary waves have a discrete set of frequencies f_1, $2f_1$, $3f_1$, ...,nf_1, which is known as harmonic series. The frequency f_1 is known as fundamental frequency and the others are called harmonics.

Case (2): Modes of vibration in an organ pipe closed at one end

Let us consider an organ pipe of length ℓ which is closed at one end. Then at the closed end, we get a node while at the open end, we get an antinode as shown in the Fig. 7.9.

(a) Fundamental mode of vibration:

Fundamental mode of vibration has one node and one antinode as shown in Fig.7.9 (a). If λ_1 is the wavelength of fundamental mode, then length of the pipe is:

$$\ell = \frac{\lambda_1}{4}$$

or $$\lambda_1 = 4\ell \qquad \text{.............(7.11)}$$

So, the speed v becomes:

$$v = f_1 \lambda_1$$

$$f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{4\ell} \qquad (\because \lambda_1 = 4\ell)$$

The frequency f_1 is called fundamental frequency.

(b) Second Mode of Vibration:

Second mode of vibration contains two nodes and two anti-nodes as shown in Fig.7.9 (b).

If λ_3 is the wavelength, then length of the pipe is:

$$\ell = \frac{\lambda_3}{4} + \frac{\lambda_3}{2}$$

$$\ell = \frac{3}{4}\lambda_3$$

$$\lambda_3 = \frac{4\ell}{3} \quad \ldots\ldots\ldots\ldots(7.12)$$

$$f_3 = \frac{v}{\lambda_3}$$

Putting value of λ_3, we have $\quad v = f_3 \lambda_3$

$$f_3 = \frac{v}{4\ell/3}$$

or $$f_3 = \frac{3v}{4\ell}$$

so $$f_3 = 3f_1 \qquad [\because \frac{v}{4\ell} = f_1]$$

This is called second harmonic.

(c) nth mode of vibration

If air column vibration in n loops, then frequency f_n is:

$$f_n = n\left(\frac{v}{4\ell}\right)$$

$$f_n = nf_1 \qquad \text{where } n = 1, 3, 5, \ldots$$

and the wavelength λ_n is:

$$\lambda_n = \frac{4\ell}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7.13)$$

By studying both cases, we conclude that the pipe which is open at both ends is richer in harmonics than that of closed at one end.

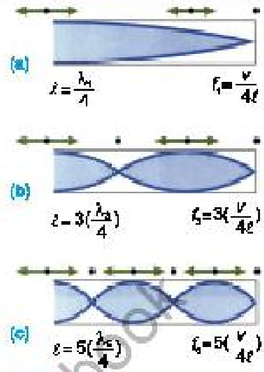
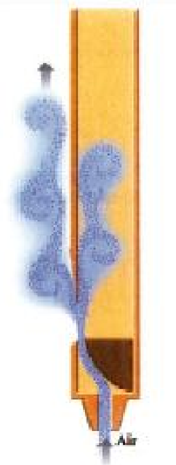


Fig. 7.9: Stationary longitudinal waves in a pipe closed at one end. Only odd harmonics are present.

Scientific Fact



In an organ pipe, the primary driving mechanism is wavering, sheet like jet of air from flute-slit, which interacts with the upper lip and the air column in the pipe to maintain a steady oscillation.

Example 7.4 An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic, when it is:

(a) open at both ends

(b) closed at one end

(Speed of sound = $350\ m\ s^{-1}$)

Solution $\ell = 50\ cm = \frac{50}{100}\ m = 0.5\ m$

$v = 350\ m\ s^{-1}$

Ponder upon!

Open pipes produce all harmonics, while closed pipes produced only odd harmonics.

a) When pipe is open at both ends:

Fundamental frequency $f_1 = ?$

Next harmonic frequency $f_2 = ?$

The frequency for *n*th harmonic in an open organ pipe is:

$$f_n = n\frac{v}{2\ell} \quad \text{when } n = 1,2,3,\ldots\ldots\ldots$$

so the fundamental frequency is,

$$f_1 = \frac{1 \times 350\ m\ s^{-1}}{2 \times 0.5\ m} \quad \text{put } n = 1$$

$$f_1 = 350\ Hz$$

Next harmonic frequency i.e., n = 2 is:

$$f_2 = \frac{2v}{2\ell}$$

$$f_2 = \frac{v}{\ell} = \frac{350\ m\ s^{-1}}{0.5\ m} = 700\ s^{-1}$$

$$f_2 = 700\ Hz$$

b) When pipe is closed at one end:

Fundamental frequency $f_1 = ?$

Next harmonic frequency $f_3 = ?$

When the pipe is closed at one end, then frequency for nth harmonic is

$$f_n = n\frac{v}{4\ell} \quad \text{when } n = 1,3,5,7,\ldots\ldots\ldots$$

So fundamental frequency is:

$$f_1 = \frac{1 \times 350\ m\ s^{-1}}{4 \times 0.5\ m} \quad \text{putting } n = 1$$

$$f_1 = 175\ Hz$$

Next harmonic frequency i.e., $n = 3$ is:

$$f_3 = \frac{3v}{4\ell}$$

$$f_3 = \frac{3 \times 350 \text{ m s}^{-1}}{4 \times 0.5 \text{ m}}$$

$$f_3 = 525 \text{ Hz}$$

Example 7.5 A church organ consists of pipes, each open at one end, of different lengths. The minimum length is 30 mm and the longest is 4 m. Calculate the frequency range of the fundamental notes. (Speed of sound = 340 m s^{-1})

Solution $$\ell_{min} = 30 \text{ mm} = \frac{30}{1000} \text{ m} = 30 \times 10^{-3} \text{ m}$$

$$\ell_{max} = 4 \text{ m}$$

$$v = 340 \text{ m s}^{-1}$$

Frequency range = ?

For an organ pipe open at one end only:

$$f_n = \frac{nv}{4\ell}$$

(i) Minimum length

For fundamental frequency, put n =1

$$f_{1,max} = \frac{nv}{4\ell_{min}}$$

$$f_{1,max} = \frac{1 \times 340 \text{ m s}^{-1}}{4 \times 30 \times 10^{-3} \text{ m}}$$

$$f_{1,max} = 2833.33 \text{ Hz}$$

(ii) Maximum length:

For fundamental frequency, put n = 1

$$f_{1,min} = \frac{nv}{4\ell_{max}}$$

$$f_{1,min} = \frac{1 \times 340 \text{ m s}^{-1}}{4 \times 4 \text{ m}}$$

$$f_{1,min} = 21.25 \text{ Hz}$$

So, the fundamental frequency range is approximately from 21 Hz to 2833 Hz.

Brain teaser

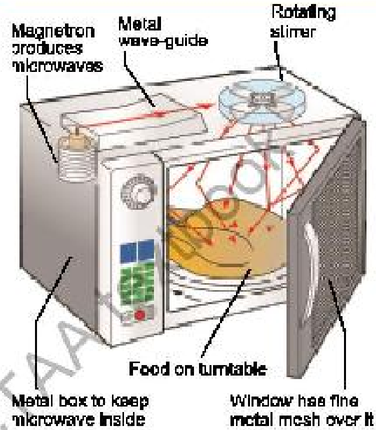
A flute player notices that the flute is producing a pitch which is slightly sharp. What could be the cause of this problem?

7.7 EXPERIMENT DEMONSTRATING STATIONARY WAVES USING MICROWAVES

Microwaves are a form of electromagnetic radiations. They are called "micro" waves because their wavelengths are typically of the order of millimetres or centimetres, much shorter than radiowaves. Stationary waves, also known as standing waves, can be produced by microwaves when they are confined to a specific region or cavity such as wave guides or resonant chambers. In these structures, microwaves can bounce back and forth, creating a standing wave pattern with nodes and antinodes. It occurs when the microwave frequency matches the resonant frequency of the cavity.

The stationary waves can be created using microwaves by the following simple method as shown in Fig. 7.10.

Structure of Microwave Oven



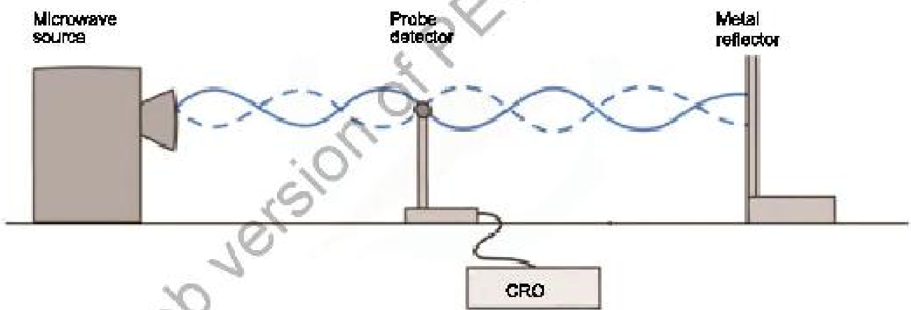


Fig. 7.10: Experimental setup for stationary waves using microwave

The experiment setup consists of a microwave source (transmitter), a probe detector and a metal reflector (a metallic plate for the reflection of microwave). Three of the mentioned are placed in line.

The waves coming out of microwave source are moving towards the metal plate and then reflected back. The reflected wave and incident wave superpose and create a stationary wave pattern. This can be detected by a probe detector placed between transmitter and metallic plate. The intensity of the signal can be observed by the detector. You can move the plate or detector to observe antinode and node. By finding the distance from one antinode to the next antinode, the wavelength of stationary wave can be found.

7.8 DIFFRACTION OF WAVES

Diffraction of waves is the bending of waves around the sharp edges or corners of obstacles or the spreading of waves beyond a barrier. It occurs when a wave encounters a physical barrier or an opening (a slit) that is comparable in size to the wavelength of the wave. The longer the wavelength, the greater the spreading and vice versa.

Diffraction can be observed in various types of waves, including water waves, sound waves, light waves and electromagnetic waves.

Some examples of the phenomenon of diffraction include:

- Hearing of sound waves around corners or through door way from where they were generated as sound waves bend around the corners.
- Diffraction of X-rays by crystals as the spacing between the regular arrays of atoms is of the order of X-rays wavelength.

Diffraction is a fundamental aspect of wave behaviour and has many practical applications in various fields.

The Ripple Tank

Figure 7.11 shows a ripple tank. It is a very useful apparatus not only to generate water waves, but also to demonstrate wave properties (such as reflection, diffraction and refraction).

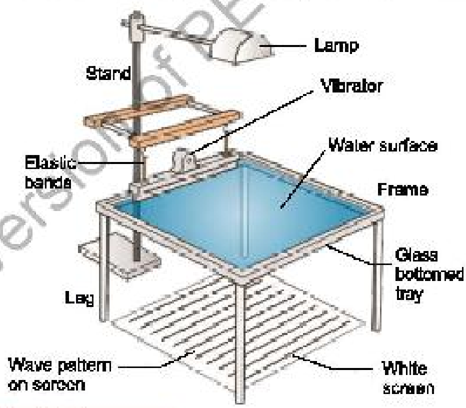


Fig. 7.11: A ripple tank

Ripple tank contains water, vibrator (e.g. a motorized oscillating needle), obstacles (e.g. a small rectangular block or a semicircular barrier) and gap widths of different sizes. It creates a series of concentric circles

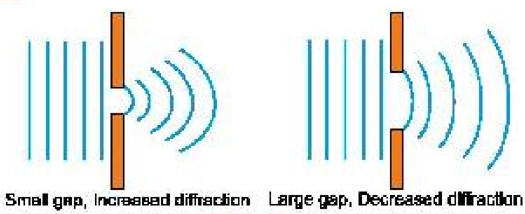


Fig. 7.12

or parallel waves using the vibrator. An obstacle is placed for creating a gap with a specific width. The experiment can be repeated with different gap widths.

It is observed that when the gap width is small compared to the wavelength, diffraction is significant and the waves bend around the obstacle, creating a semicircular pattern. As the gap width increases relative to wavelength, diffraction decreases and the waves pass through the gap with less bending.

For your information

Diffraction of white light is shown by a fine diffraction grating.

This experiment demonstrates the qualitative effect of gap width on diffraction in a ripple tank, illustrating how the relationship between gap width and wavelength affects wave behaviour.

Example 7.6

In a ripple tank, a wave generator produces 500 pulses in 10 s. Find the frequency at time period of the pulses produced?

Solution $n = 500$ pulses, $t = 10\text{ s}$, $f = ?$

As Frequency $= \dfrac{\text{Number of pulses}}{\text{Time}}$

$$f = \frac{500}{10\text{ s}} = 50\text{ Hz}$$

We know that; $T = \dfrac{1}{f} = \dfrac{1}{50\text{ Hz}}$

$$T = 0.02\text{ s}$$

Interesting information

The fine rulings, each 0.5 µm wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms coloured lanes that are composite of the diffraction patterns from the rulings.

7.9 BEATS

When two waves of slightly different frequencies, travelling in the same direction overlap each other then there is a periodic variation of sound between maximum and minimum loudness which is called as beats.

Tuning forks give out pure notes (single frequency). If two tuning forks A and B of the same frequency say 32 Hz are sounded separately, they will give out pure notes. If they are sounded simultaneously, it will be difficult to differentiate the notes of one tuning fork from that of the other. The sound waves of the two will be superposed on each other and will be heard by the human ear as a single pure note.

If the tuning fork B is loaded with some wax or plasticine, its frequency will be lowered slightly, say it becomes 30 Hz. If now the two tuning forks are sounded together, a note of alternately increasing and decreasing intensity will be heard. This note is called **beat note** or a beat which is due to interference between the sound waves from tuning forks A and B.

Fig. 7.13 (a) shows the waveform of the note emitted from a tuning fork A. Similarly, Fig. 7.13(b) shows the waveform of the note emitted by tuning fork B. When both the tuning forks A and B are sounded together, the resultant waveform is shown in Fig. 7.12(c). It shows how do the beat note occur. At some instant X, the displacement of the two waves is in the same direction. The resultant displacement is large and a loud sound is heard.

After 1/4 s the displacement of the wave due to one tuning fork is opposite to the displacement of the wave due to the other tuning fork resulting in minimum displacement at Y, hence, faint sound or no sound is heard.

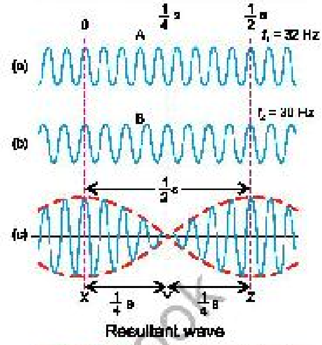


Fig. 7.13: Formation of beats

Another 1/4 s later, the displacements are again in the same direction and a loud sound is heard again at Z.

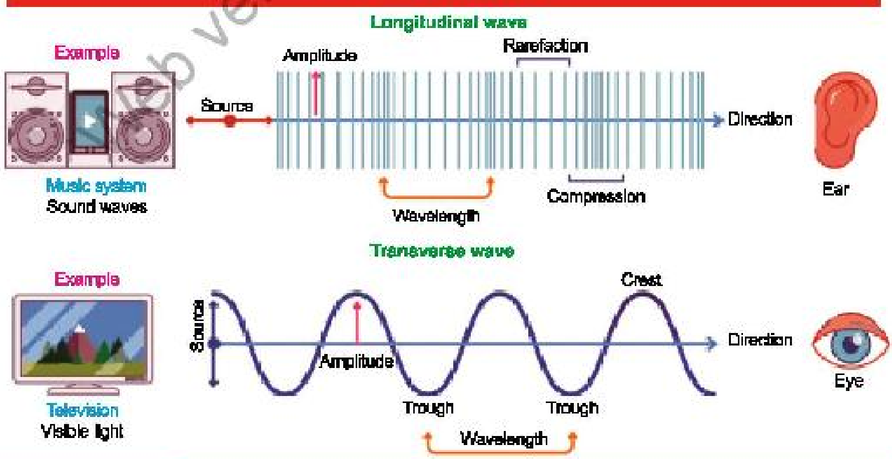
Thus, during one second, we observe two faintest sounds or two loudest. As the difference of the frequency of the two tuning forks is also 4 Hz so, we find that the number of beats per second is equal to the difference between the frequencies of the tuning forks.

$$f_A = 32\text{ Hz}, \quad f_B = 30\text{ Hz} \qquad f_A > f_B$$

$$\text{No. of beats} = f_A - f_B = 32\text{ Hz} - 30\text{ Hz} = 2$$

However, when the difference between the frequencies of the two sounds is more than 10 Hz, it becomes difficult to recognize the beats.

Pictorial Comparison



The difference between the frequencies of the two waves is termed as beat frequency f_{beat}.

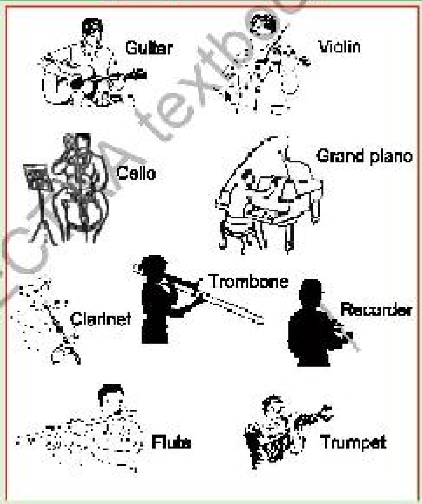
One can use beats to tune a string instrument, such as piano or violin, by beating a note against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beats are heard.

Tuning Musical Instruments

Here are some examples of how beats are generated in musical instruments:

1. **Guitar:** When playing two strings with slightly different tunings, beats are created. For example, playing a standard tuned string and a string tuned a few cents higher or lower.
2. **Piano:** Playing two keys white and black, adjacent to each other, creates beats.
3. **Violin:** When playing two strings with slightly different bow pressures or speeds, beats are generated.
4. **Drums:** When two drums with slightly different tunings are played simultaneously, beats are created.
5. **Flute:** When playing two notes with slightly different embouchure (lip and facial muscles) positions, beats are generated.
6. **Organ:** When playing two pipes with slightly different tunings, beats are created.

Do you know?



The sound produced by most of string and wind musical instruments is due to the formation of stationary waves or standing waves in these instruments.

7. **Synthesizer:** Generating two oscillators with slightly different frequencies creates beats.

In each of these examples, the slight difference in frequency between the two sound sources creates a periodic increase and decrease in amplitude, resulting in a "beat" or pulsation effect.

Musicians often use beats intentionally to create interesting rhythmic effects, add texture, or produce a sense of tension and release. However, in some cases, beats can be unwanted and may require adjustments to tuning, pitch, or playing technique to minimize their impact.

Example 7.7 Two tuning forks exhibit beats at a beat frequency of 3 Hz. The frequency of one fork is 256 Hz. Its frequency is then lowered slightly by adding a bit of wax to one of its prongs. The two forks then exhibit a beat frequency of 1 Hz. Determine the frequency of the second tuning fork.

Solution Frequency of first tuning fork $= f_1 = 256$ Hz

Beat frequency before loading = 3 Hz

Beat frequency after loading = 1 Hz

Frequency of second tuning fork $= f_2 = ?$

As $f_1 - f_2 = \pm n$

Then $f_2 = f_1 \pm n$

Putting the values, we have

$$f_2 = 256\text{ Hz} \pm 3\text{ Hz}$$

Either $f_2 = 256\text{ Hz} + 3\text{ Hz}$;

or $f_2 = 256\text{ Hz} - 3\text{ Hz}$

$$f_2 = 259\text{ Hz} \quad \text{or} \quad 253\text{ Hz}$$

Let us consider 259 Hz as correct answer (i.e., frequency of second tuning fork). When first fork is loaded with wax, the frequency of first fork must fall below 256 Hz i.e., 255 Hz, 254 Hz and thus the number of beats produced per second will increase and will be greater than 3 beats. Since the number of beats per second decreases on loading first fork is one, therefore 259 Hz is not correct frequency of second tuning fork.

Thus, Correct frequency $= f_2 = 253$ Hz

Do you know?

In 1711, F. J. Shore, who was a royal trumpeter and lutenist invented tuning forks.

7.10 INTENSITY (I) OF A WAVE

Intensity is defined as the amount of energy transmitted per unit area per unit time in the direction of propagation of progressive wave.

It is a measure of the power of a wave and is usually denoted by the symbol "I". It is measured in units of watts per square metre (W m^{-2}).

A progressive wave or travelling wave is one that travels through a medium in a consistent direction and transferring energy from one point to another. It is a wave that propagates or moves forward, as opposed to a stationary or standing wave. Examples of progressive waves include water waves, sound waves, light waves, etc.

By definition, the intensity of a wave is:

$$I = \frac{E}{A \times t}$$

$$I = \frac{E/t}{A}$$

$$I = \frac{P}{A} \quad (\because E/t = P)$$

Here

I = Intensity of wave in (W m^{-2})

E = Energy in joules (J)

t = Time in seconds (s)

P = Power in watts (W)

We know that in mechanical waves, such as sound waves, water waves, or waves on a vibrating string, energy is stored as kinetic energy and potential energy of the medium's particles. How much energy is stored depends upon the displacement (amplitude) of the particles from the mean position. Therefore, the intensity I of waves is proportional to the square of the amplitude A, i.e.,

$$I \propto A^2$$

or $$I = kA^2 \quad \text{......} (7.14)$$

Here k is the constant of proportionality and depends upon the physical properties of the wave and the medium.

Example 7.8

(a) A wave has an intensity of 0.5 W m^{-2} at a distance of 3.0 m from the source. What is the power of the wave?

(b) Two progressive waves have intensities of 0.5 W m^{-2} and 0.25 W m^{-2}. Find total intensities of two waves.

Solution

(a)

$$I = 0.5 \text{ W m}^{-2}$$
$$r = 3.0 \text{ m}$$
$$P = ?$$
$$\because I = \frac{P}{A}$$
$$\because I = \frac{P}{4\pi r^2}$$

Putting the values, $$0.5 \text{ W m}^{-2} = \frac{P}{4 \times 3.14 (3.0 \text{ m})^2}$$

$$P = 0.51 \times 13.04$$
$$P = 56.52 \text{ W}$$

(b)

$$I_1 = 0.5 \text{ W m}^{-2}$$
$$I_2 = 0.25 \text{ W m}^{-2}$$

Tidbits

Frequency and amplitude of a travelling wave are independent of each other. That is why you can turn up the volume of a song (increase amplitude) without changing its pitch (which depends on frequency).

$$I = ?$$
$$I = I_1 + I_2$$
$$I = 0.5\ \text{W m}^{-2} + 0.25\ \text{W m}^{-2}$$
$$I = 0.75\ \text{W m}^{-2}$$

Brain teaser

Can you find the decibel level of a travelling wave whose intensity is 10 W m^{-2}?

Example 7.9 A speaker is emitting sound waves with a power of 50 watts. If the sound waves are spreading out evenly in all directions and the intensity of the sound waves is measured at a distance of 5 m from the speaker, what is the intensity of the sound waves if the area of the sphere (the surface area of a sphere) at that distance is approximately 314 m^2?

Solution Power P = 50 W

Area A = 314 m^2

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}}$$
$$I = \frac{P}{A}$$
$$I = \frac{50\ \text{W}}{314\ \text{m}^2}$$
$$I \approx 0.159\ \text{W m}^{-2}$$

Tidbit

Sound cannot travel through vacuum, as there are no particles to transmit the sound wave.

7.11 DOPPLER EFFECT

The apparent change in the frequency (or pitch) of waves due to the relative motion between the source and observer (listener) is called Doppler Effect.

This effect was first observed by John Doppler while he was observing the frequency of light emitted from a star. In some cases, the frequency of emitted light was found to be slightly different from that emitted from a similar source on the Earth. He found that the change of frequency of light depends upon motion of star relative to Earth.

This effect can be observed with sound waves also. For example, when an observer is standing on a railway platform, the pitch of whistle of an engine coming towards the platform appears to become higher to an observer standing on the platform. However, the pitch of whistle of an engine going away from the platform appears to become lower to an observer standing on the platform.

Consider a source of sound S at rest which emits sound waves having wavelength λ. Let speed of the sound for a stationary observer (i.e., listener) is v then the number of waves received by observer in one second i.e., frequency f is:

$$f = \frac{v}{\lambda}$$

Case 1: When source of sound moves towards the stationary observer

When the source moves towards the stationary observer A with velocity u_s then waves are compressed and their wavelength is decreased as shown in Fig. 7.14. In this case, the waves are compressed by an amount given as

$$\Delta\lambda = \frac{u_s}{f}$$

The compression of the waves is due to the fact that same number of waves are contained in a shorter space depending upon the velocity of the source. The wavelength observed by the observer A is then,

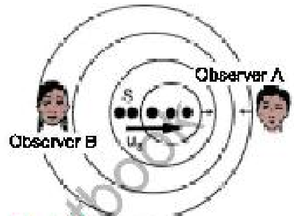


Fig. 7.14

A source moving with velocity u_s towards a stationary observer A and away from stationary observer B respectively. Observer A hears an increased and observer B hears a decreased frequency.

$$\lambda_A = \lambda - \Delta\lambda$$

or

$$\lambda_A = \frac{v}{f} - \frac{u_s}{f}$$

$$\lambda_A = \frac{v - u_s}{f}$$

Here $\Delta\lambda$ is the decrease in wavelength in one second and is called Doppler shift.

Thus, the number of waves received by observer A in one second (i.e., changed or apparent frequency) is

$$f_A = \frac{v}{\lambda_A}$$

Putting the value of λ_A, we have

$$f_A = \left[\frac{v}{(v - u_s)/f}\right]$$

$$f_A = \left[\frac{v}{v - u_s}\right]f$$

As $$\frac{v}{v - u_s} > 1$$

Therefore $$f_A > f$$

Thus, the apparent frequency of sound heard by the observer increases which in turn will increase the pitch of sound.

Case 2: When source of sound moves away from the stationary observer

When the source moves away from the stationary observer B with velocity u_s, then waves are expanded and their wavelength is increased. In this case, the waves expanded by an amount:

$$\lambda_3 = \frac{u_s}{f}$$

The expansion of the waves is due to the fact that same number of waves are now contained in a large distance. The wavelength observed by the observed B is then,

$$\lambda_B = \lambda + \Delta\lambda$$

where $\Delta\lambda$ is the increase in wavelength in one second and is called Doppler shift.

Thus, the number of waves received by observer B in one second (i.e., changed or apparent frequency) is:

$$f_B = \frac{v}{\lambda_B}$$

Putting the value of λ_B, we have

$$f_B = \left[\frac{v}{(v+u_s)/f}\right]$$

$$f_B = \left[\frac{v}{(v+u_s)}\right]f$$

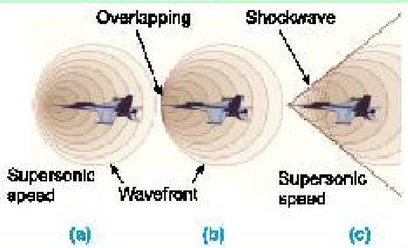
As $$\frac{v}{v+u_s} < 1$$

Therefore $$f_B < f$$

Thus, the apparent frequency of sound heard by the observer decreases which in turn will decrease the pitch of sound.

Info Corner	
Hearing Ranges	
Organisms	Frequencies in Hz
Dolphin	150 – 150,000
Bat	1000 – 120,000
Cat	60 – 70,000
Dog	15 50,000
Human	20 – 20,000

Do you know?

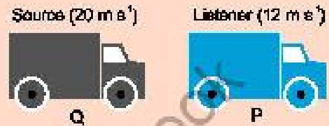


As a plane accelerates, it builds up a front of air pressure by pushing air in front of it. When it passes the speed of sound, the pressure trails behind like a boat's wake, forming a sonic shockwave.

Example 7.10 Two trucks P and Q travelling along a motorway in the same direction. The loading truck P travels at a steady speed of 12 m s^{-1}, the other truck Q, travelling at a steady speed of 20 m s^{-1}, sound its horn to emit a steady note which P's driver estimate, has a frequency of 830 Hz. What frequency does Q's own driver hear?

(Speed of sound = 340 m s^{-1})

Solution



u_p = 12 m s^{-1}

u_Q = 20 m s^{-1}

v = 340 m s^{-1}

f_p = 830 Hz

f_Q = ?

Speed of Q relative to P = $u_s = u_Q - u_p$ = 20 m s^{-1} – 12 m s^{-1} = 8 m s^{-1}

$$\because \quad f = \left(\frac{v}{v - u_s}\right) f'$$

$$\therefore \quad f_p = \left(\frac{v}{v - u_s}\right) f_Q$$

Putting the value, we have

$$830\text{ Hz} = \left(\frac{340\text{ m s}^{-1}}{340\text{ m s}^{-1} - 8\text{ m s}^{-1}}\right) f_Q$$

$$830\text{ Hz} = \left(\frac{340\text{ m s}^{-1}}{332\text{ m s}^{-1}}\right) f_Q$$

$$\text{or} \quad f_Q = \left(\frac{830\text{ Hz} \times 332\text{ m s}^{-1}}{340\text{ m s}^{-1}}\right)$$

$$f_Q = 810.47\text{ Hz}$$

Example 7.11 A train sounds its horn before it sets off from the station and an observer waiting on the platform estimates its frequency at 1200 Hz. The train then moves off and accelerates steadily. Fifty seconds after departure, the driver sounds the horn again and the platform observer estimates the frequency at 1140 Hz. Calculate the train speed 50 s after departure. How far from the station is the train after 50 s.

(Speed of sound = 340 m s^{-1})

Solution

Original frequency of horn = f = 1200 Hz

Apparent frequency = f' = 1140 Hz

Speed of sound = v = 340 m s^{-1}

Time $= t = 50\ s$

Speed of source (i.e., train) $= u_s = ?$

Distance covered by the train $= S = ?$

(i)
$$f' = \left(\frac{v}{v + u_s}\right) f$$

Putting the values, we have

$$1140\ Hz = \left(\frac{340\ m\ s^{-1}}{340\ m\ s^{-1} + u_s}\right) \times 1200\ Hz$$

$$340\ m\ s^{-1} + u_s = \frac{340\ m\ s^{-1} \times 1200\ Hz}{1140\ Hz}$$

$$u_s = 357.89 - 340$$

$$u_s = 17.89\ m\ s^{-1}$$

(ii)
$$S = v_{av}\, t$$

$$S = \left(\frac{v_i + v_f}{2}\right) t$$

$$S = \left(\frac{0 + 17.89\ m\ s^{-1}}{2}\right) 50\ s$$

$$S = 448\ m$$

Do you know?

Bats navigate and find food by echo location.

7.12 APPLICATIONS OF DOPPLER EFFECT

Doppler effect is also applicable to electromagnetic waves. One of its important applications is the radar system, which uses radio waves to determine the elevation and speed of an aeroplane. **RADAR (RAdio Detection And Ranging)** is a device, which transmits and receives radio waves. If an aeroplane approaches towards the radar, then the wavelength of the wave reflected from aeroplane would be shorter and if it moves away, then the wavelength would be larger as shown in Fig.7.15 (a) & Fig.7.15 (b), respectively. Similarly, speed of satellites moving around the Earth can also be determined by the same principle.

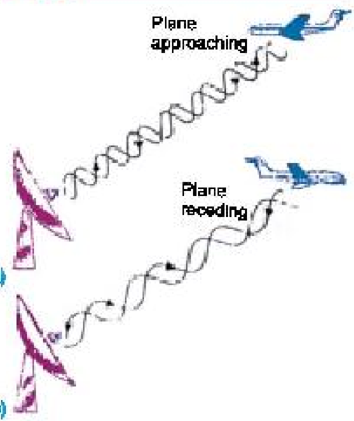


Fig. 7.15: A frequency shift is used in a radar to detect the motion of an aeroplane.

SONAR is an acronym derived from "Sound Navigation And Ranging". It is the general name for sonic or ultrasonic underwater echo-ranging and echo-sounding system. Sonar is the name of a technique for detecting the presence of objects under water by acoustical echo. In **Sonar**, "Doppler detection" relies upon the relative speed of the target and the detector to provide an indication of the target speed. It employs the Doppler effect, in which an apparent change in frequency occurs when the source and the observer are in relative motion to one another. Its known military applications include the detection and location of submarines, control of antisubmarine weapons, mine hunting and depth measurement of sea.

In Astronomy, astronomers use the Doppler effect to calculate the speeds of distant stars and galaxies. By comparing the line spectrum of light from the star with light from a laboratory source, the Doppler shift of the star's light can be measured. Then, the speed of the star can be calculated.

(i) Stars moving away from the Earth show a red shift as shown in Fig.7.16(b). The emitted waves have a longer wavelength than if the star had been at rest. So, the spectrum is shifted towards longer wavelength, i.e., towards the red end of the spectrum. Astronomers have also discovered that all the distant galaxies are moving away from us and by measuring their red shifts, they have estimated their speeds.

(ii) Stars moving towards the Earth show a blue shift as shown in Fig.7.16(c). This is because the wavelength of light emitted by the star are shorter than if the star had been at rest. So, the spectrum is shifted towards shorter wavelength, i.e., to the blue end of the spectrum.

Another important application of the Doppler shift using electromagnetic waves is the **radar speed trap**. Microwaves are emitted from a transmitter in short bursts. Each burst is reflected off by any car in the path of microwaves in between sending out bursts. The transmitter is open to detect reflected microwaves. If the reflection is caused by a moving obstacle, the reflected microwaves are Doppler shifted. By measuring the Doppler shift, the speed at which the car moves is

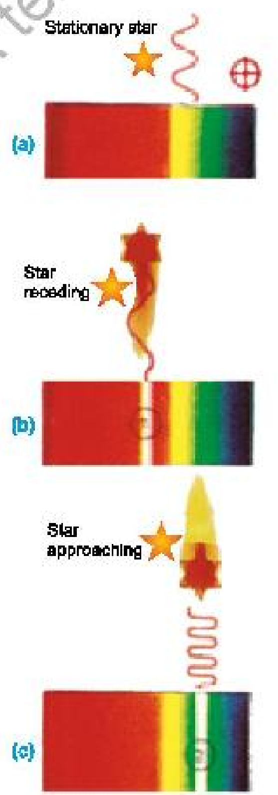


Fig. 7.16

calculated by computer programme.

Satellite Navigation uses Doppler shift to determine satellite velocity and position, enabling accurate location tracking.

Satellite Communication also uses Doppler shift compensation ensuring stable communication signals.

Doppler radar detects wind velocity and precipitation patterns. **Doppler shift** helps measure Earth's surface velocity and deformation.

Doppler echocardiography measures blood flow velocity and detects cardiac abnormalities, such as valve stenosis or regurgitation. Doppler echocardiography optimizes pacemaker settings.

Doppler ultrasound measures blood flow and calculates cardiac output. Doppler ultrasound detects vascular stenosis or occlusion.

Example 7.12 The wavelength of one of its lines of the absorption spectrum of a faint galaxy is identified as Ca-α line found to be 478 nm. The wavelength of same line is observed and measured as 397 nm in the laboratory.

(a) Is the galaxy moving towards or away from the Earth?

(b) Compute the speed of the galaxy relative to the Earth.

Solution

Laboratory measured original wavelength

$$\lambda = 397\text{ nm} = 397 \times 10^{-9}\text{ m}$$

Changed or Apparent wavelength $\lambda' = 478\text{ nm} = 478 \times 10^{-9}\text{ m}$

Speed of light $c = 3 \times 10^{8}\text{ m s}^{-1}$

(a) $\because \quad v = f\lambda$

$\therefore \quad c = f\lambda \qquad (\because v = c)$

$\Rightarrow \quad f = \dfrac{c}{\lambda}$

$$f = \frac{3 \times 10^{8}\text{ m s}^{-1}}{397 \times 10^{-9}\text{ m}}$$

$$f = 7.56 \times 10^{-3} \times 10^{8} \times 10^{9}\text{ s}^{-1}$$

Laboratoy frequency $f = 7.56 \times 10^{14}\text{ Hz}$

Apparent frequency $f' = \dfrac{c}{\lambda'}$

$$= \frac{3 \times 10^{8}\text{ m s}^{-1}}{478 \times 10^{-9}\text{ m}}$$

$$f' = 6.28 \times 10^{14}\text{ Hz}$$

$\because \quad \lambda' > \lambda \quad \text{or} \quad f' < f$

$\therefore$ The galaxy is moving away from the Earth.

(b)
$$f' = \left(\frac{v}{v + u_s}\right) f$$
$$f' = \left(\frac{c}{c + u_s}\right) f \qquad (\because v = c)$$

Putting the values, we have

$$6.28 \times 10^{14}\,\text{Hz} = \left(\frac{3 \times 10^8\,\text{m s}^{-1}}{3 \times 10^8\,\text{m s}^{-1} + u_s}\right) 7.56 \times 10^{14}\,\text{Hz}$$
$$3 \times 10^8 + u_s = \frac{3 \times 10^8\,\text{m s}^{-1} \times 7.56 \times 10^{14}\,\text{Hz}}{6.28 \times 10^{14}\,\text{Hz}}$$
$$u_s = \frac{22.68 \times 10^8}{6.28} - 3 \times 10^8\,\text{m s}^{-1}$$
$$u_s = 6.12 \times 10^7\,\text{m s}^{-1}$$

Galactic Motion

A galaxy is moving away from us at 20% of the speed of light (i.e., 0.2 c). We observe a spectral line from this galaxy that is normally emitted at a wavelength of 500 nm.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

7.1 The simple wave speed equation is represented by:

(a) $v = f\lambda$ (b) $v = S/t$ (c) $v = r\omega$ (d) $v = \Delta d/\Delta t$

7.2 The principle of superposition in waves is stated as:

(a) the displacement of a wave is the sum of the displacements of its individual components

(b) the velocity of a wave is the product of its individual components

(c) the frequency of a wave is the difference of its individual components

(d) the amplitude of a wave is the ratio of its individual components

7.3 A node in a stationary wave is:

(a) a point of maximum displacement (b) a point of intermediate displacement

(c) a point of zero displacement (d) a point of infinite displacement

7.4 An antinode in a stationary wave is:

(a) a point of maximum displacement (b) a point of minimum displacement

(c) a point of zero displacement (d) a point of infinite displacement

7.5 Stationary waves are defined as:

(a) waves that move with a constant velocity
(b) waves that move with a changing velocity
(c) waves that oscillate in a fixed position
(d) waves that propagate through a medium

7.6 Harmonics are:

(a) integer multiples of a fundamental frequency
(b) Integer submultiples of a fundamental frequency
(c) random frequencies
(d) non-integer multiples of a fundamental frequency

7.7 The result of constructive interference between two waves is represented as:

(a) a decrease in amplitude (b) an increase in amplitude
(c) no change in amplitude (d) a shift in phase

7.8 If the amplitude of the wave is tripled, then the amount of energy is increased by:

(a) 3 times (b) 6 times (c) 9 times (d) 12 times

7.9 What type of waves do headphones use to produce sound?

(a) Electromagnetic waves (b) Mechanical waves
(c) Pressure waves (d) Longitudinal waves

7.10 The typical frequency range of microwaves is:

(a) $10^3 - 10^5$ Hz (b) $10^5 - 10^7$ Hz (c) $10^7 - 10^9$ Hz (d) $10^9 - 10^{11}$ Hz

7.11 The bending of waves around an obstacle is called as:

(a) refraction (b) reflection (c) diffraction (d) interference

7.12 The Doppler Effect used in astronomy is for:

(a) measuring the diameters of stars
(b) determining velocity of galaxies
(c) analyzing properties of black holes
(d) studying behaviour of electromagnetic waves

Short Answer Questions

7.1 What are the conditions for interference to occur?

7.2 Differentiate between constructive and destructive interference of waves.

7.3 What are coherent waves and coherent sources? Give examples.

7.4 Distinguish between longitudinal and transverse waves.

7.5 Is it possible for two identical waves travelling in the same direction along a string to give rise to a stationary wave? How is it so?

7.6 How would you apply Doppler effect in studying cardiac problems in humans?

7.7 What is meant by diffraction of waves? For what purpose, the ripple tank is used?

Constructed Response Questions

7.1 Which measurement of a wave is the most important when determining the wave's intensity?

7.2 Can you apply Doppler effect to light waves? Describe briefly.

7.3 Can you compare the compressions and rarefactions of the longitudinal wave with the peaks and troughs of the transverse wave? Discuss.

7.4 How should a source of sound move with respect to an observer so that the frequency of its sound does not change? Write two examples.

7.5 Why is it difficult to recognize beats when the frequency difference is greater than 10 Hz? Exemplify.

Comprehensive Questions

7.1 State and explain the principle of superposition of waves. Apply this principle to elaborate the working of noise canceling headphones.

7.2 What are standing waves? Illustrate a detailed experiment that demonstrates the standing waves using stretched strings.

7.3 Find the frequencies of the harmonics produced in an organ pipe when it is open at both ends and when it is closed at one end.

7.4 Define and exemplify diffraction of waves. Describe this phenomenon by ripple tank experiment.

7.5 What is meant by the term beats? Prove that number of beats per second is equal to the difference between the frequencies of vibrating tuning forks.

7.6 What do you understand by progressive waves? Discuss the intensity of progressive waves.

7.7 Keeping in mind "Doppler effect", analyze the following cases:

(a) when source of sound moves away from the stationary observer.

(b) when source of sound moves towards the stationary observer

Numerical Problems

7.1 The speed of a wave on a typical string is 24 m s^{-1}. What driving frequency will it resonate if its length is 6.0 m? (Ans: 2 Hz)

7.2 The lowest resonance frequency for a guitar string of length 0.75 m is 400 Hz. Calculate the speed of a transverse wave on the string. (Ans: 600 m s^{-1})

7.3 A tuning fork A produces 4 beats per second with another tuning fork B. It is found that by loading B with some wax, the beat frequency increases to 6 beats per second. If the frequency of A is 320 Hz, determine the frequency of B when loaded.

(Ans: 314 Hz)

7.4 A steel wire hangs vertically from a fixed point, supporting a weight of 80 N at its lower end. The diameter of the wire is 0.50 mm and its length from the fixed point to the weight is 1.5 m. Calculate the fundamental frequency emitted by the wire when it is plucked. Density of steel wire is 7.8×10^{3} kg m^{-3}. (Ans: 76.2 Hz)

7.5 Average intensity of sunlight on the surface of the Earth is nearly 500 W m^{-2}. Determine the amount of energy that falls on a solar panel having an area of 0.50 m^{2} in four hours. (Ans: 3.6×10^{6} J)

7.6 (a) If the intensity of a wave is 16 W m^{-2} and the amplitude is 2 m, what is the value of constant k? (Ans: 4 W m^{-4})

(b) If the intensity of a wave is 25 W m^{-2} and the constant k is 5 W m^{-4}, what is the amplitude? (Ans: 2.24 m)

7.7 (a) A sound system produces 200 watts of power. If the sound is directed at a crowd with an area of 150 m^{2}, what is the intensity of the sound?

(Ans: 1.33 W m^{-2})

(b) A light bulb emits 100 watts of power. If the light is spread out evenly over a sphere with a surface area of 400 m^{2}, what is the intensity of the light?

(Ans: 0.25 W m^{-2})

7.8 A radio antenna broadcasts 500 watts of power. If the signal is received at a distance of 10 km, what is the intensity of the signal? (Ans: 4×10^{-7} W m^{-2})

7.9 An organ pipe has a length of 1 m. Determine the frequencies of the fundamental and the first two harmonics:

(a) if the pipe is open at both ends and (b) if the pipe is closed at one end.

(Speed of sound in air is 340 m s^{-1})

(Ans: 170 Hz, 340 Hz, 510 Hz; 85 Hz, 255 Hz, 425 Hz, respectively)

7.10 A train is approaching a station at 90 km h^{-1}, sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle as heard by a listener sitting on the platform? What will be the apparent frequency heard by the same listener if the train moves away from the station with the same speed? (Speed of sound is 340 m s^{-1}) (Ans: 1079.4 Hz, 931.5 Hz, respectively)