Chapter 2

Force and Motion

Learning Objectives

After studying this chapter, the students will be able to:

- Differentiate between scalar and vector quantities
- Represent a vector in 2-D as two perpendicular components.
- Describe the product of two vectors (dot and cross-product) along with their properties.
- Derive the equations of motion [For uniform acceleration cases only. Derive from the definitions of velocity and acceleration as well as graphically]
- Solve problems using the equations of motion [For the cases of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. This also includes situations where the equations of motion need to be resolved into vertical and horizontal components for 2-D motion]
- Evaluate and analyse projectile motion in the absence of air resistance

  [This includes solving problems making use of the below facts:

  (i) Horizontal component (V_x) of velocity is constant.

  (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.

  (iii) The horizontal motion and vertical motion are independent of each other. Situations may require students to determine for projectiles:

  - How high does it go?
  - How far would it go along the level land?
  - Where would it be after a given time?
  - How long will it remain in flight?

  Situations may also require students to calculate for a projectile launched from ground height the

  - launch angle that results in the maximum range.
  - relation between the launch angles that result in the same range.]
- Predict qualitatively how air resistance affects projectile motion. [This includes analysis of both the horizontal component and vertical component of velocity and hence predicting qualitatively the range of the projectile.]
- Apply the principle of conservation of momentum to solve simple problems [Including elastic and inelastic interactions between objects in both one and two dimensions.

  Knowledge of the concept of coefficient of restitution is not required.

  Examples of applications include:

  - karate chops to break a pile of bricks
  - car crashes
  - ball & bat
  - the motion under thrust of a rocket in a straight line considering short thrusts during which the mass remains constant]
- Predict and analyse motion for elastic collisions [This includes making use of the fact that for an elastic collision, total kinetic energy is conserved and the relative speed of approach is equal to the relative speed of separation]
- Justify how the momentum of a closed system is always conserved, some change in kinetic energy may take place.

BASIC CONCEPT OF SCALARS AND VECTORS

Scalars and vectors are basic concepts in physics. Many problems in physics require to distinguish between scalar and vector quantities to apply the correct mathematical and conceptual approaches. Understanding scalars and vectors help us to grasp how Physics applies to real-world situations, such as calculating the total distance travelled (scalar) or determining the magnitude and direction of force (vector). Learning these concepts develops critical thinking and problem-solving skills. This chapter is primarily concerned with vector algebra and its application in uniform accelerated motion, in a straight line, motion of freely falling bodies in uniform gravitational field, projectile motion, and interaction between objects in one and two dimensions.

2.1 SCALARS

Scalars are physical quantities that are described solely by a magnitude (size or amount) without any mention of direction. Thus, scalars are directionless and can be fully characterized by a single number and its associated unit.

Examples:

Mass: The amount of matter in an object. For example, 2 kg.

Distance: The total length of the path travelled by an object irrespective of the direction. For example, 50 m.

Speed: The rate at which an object covers distance. For example, 40 km h^{-1}.

Time: The duration between two events taking place. For example, 20 s.

Energy: The capacity to do work. For example, 25 J.

Temperature: A measure of the average kinetic energy of particles in a substance. For example, 20°C.

2.2 VECTORS

Those physical quantities which require magnitude as well as direction for their complete specification are known as vectors.

Examples:

Displacement: The change in position of an object. It has length, a distance (magnitude) and a direction (e.g. 10 m towards west).

Velocity: The speed of an object in a particular direction (e.g., 50 km h^{-1} towards west).

Acceleration: The rate of change of velocity that occurs in either speed or direction or both (e.g., 10 m s^{-2} upward).

Force: A push or pull acting on an object, determined by its magnitude and direction (e.g; 20 N to the right)

Graphical Representation of a Vector

A good way to represent a vector quantity is to use a vector diagram, in which vectors are often represented by arrows. The length of the arrows indicates the magnitude and the head of the arrow shows the direction of the vector. Vectors are typically denoted by bold face letters (e.g; **V**, **F**) or an arrow above symbol ($\vec{A}$).

Rectangular Components of a Vector

A component of a vector is its effective value in a given direction. A vector may be considered as the resultant of its component vectors along the specified directions. It is usually convenient to resolve a vector into its components along the mutually perpendicular directions. Such components are called rectangular components.

Let there be a vector **A** represented by a line OP making an angle θ with the x-axis. Draw projection OM of vector **A** on x-axis and projection ON of vector **A** on y-axis as shown in Fig.2.1. Projection OM being along x-direction is represented by $\mathbf{A_x}$ and projection ON along y-direction is represented by $\mathbf{A_y}$. By applying head to tail rule:

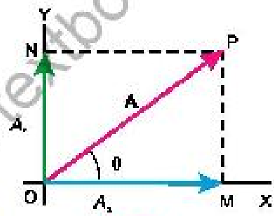


Fig. 2.1

$$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y} \quad \text{......................} \quad (2.1)$$

Thus, $\mathbf{A_x}$ and $\mathbf{A_y}$ are the comoponants of vector **A**. Since these are at right angles to each other, they are called rectangular components of **A**. Considering the right angled triangle OMP, the magnitude of $\mathbf{A_x}$ or x-component of **A** is:

$$A_x = A\cos\theta \text{ ....} (2.2)$$

And the magnitude of $\mathbf{A_y}$ or y-component of **A** is:

$$A_y = A\sin\theta \text{ ....} (2.3)$$

Determination of a Vector from its Rectangular Components

If the rectangular components of a vector as shown in Fig.(2.1) are given, we can find out the magnitude of the vector by using Pythagorean Theorem.

In the right angle ΔOMP

$$(OP)^2 = (OM)^2 + (MP)^2$$

or $$A^2 = A_x^2 + A_y^2 \text{ ....} (2.4)$$

or $$A = \sqrt{A_x^2 + A_y^2}$$

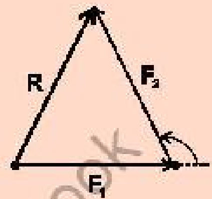
The direction θ is given by $$\tan\theta = \frac{MP}{OM} = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \text{ ....} (2.5)$$

Example 2.1 Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

Solution Let θ be the angle between two forces F_1 and F_2, where F_1 is along the x-axis. Then x-component of their resultant will be:

$$R_x = F_1 \cos 0° + F_2 \cos\theta$$
$$R_x = F_1 + F_2 \cos\theta$$



And y-component of their resultant is

$$R_y = F_1 \sin 0° + F_2 \sin\theta$$
$$R_y = F_2 \sin\theta$$

The resultant R is given by $R^2 = R_x^2 + R_y^2$

As $R = F_1 = F_2 = F$

Hence $F^2 = (F + F\cos\theta)^2 + (F\sin\theta)^2$

$$F^2 = F^2 + F^2\cos^2\theta + 2F^2\cos\theta + F^2\sin^2\theta$$

or $0 = 2F^2\cos\theta + F^2(\cos^2\theta + \sin^2\theta)$

or $0 = 2F^2\cos\theta + F^2 . 0 = F^2(2\cos\theta + 1)$

As $F \neq 0$, so $2\cos\theta + 1 = 0$

or $\cos\theta = -0.5$

or $\theta = \cos^{-1}(-0.5) = 120°$

2.3 PRODUCT OF TWO VECTORS

There are two types of vector multiplications. The product of these two types are known as scalar product and vector product.

> If the product of two vectors results in a scalar quantity, it is called scalar product while if the product of two vectors results in a vector quantity, it is called vector product.

Scalar or Dot Product

The scalar product of two vectors **A** and **B** is written as **A.B** and is defined as:

$$\mathbf{A.B} = AB\cos\theta \quad\ldots\ldots\ldots\ldots\ldots (2.6)$$

where A and B are the magnitudes of vectors **A** and **B** and θ is the angle between them.

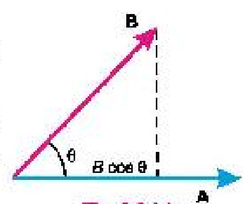


Fig. 2.2 (a)

For physical interpretation of dot product of two vectors **A** and **B**, these are first brought to a common origin (Fig.2.2-a) then,

A.B = A (projection of **B** on **A**)

or

A. B = **A** (magnitude of component of **B** in the direction of **A**), Fig.2.2(b)

$= A\ (B \cos\theta) \quad = AB \cos\theta$

Similarly **B. A** = $B\ (A \cos\theta) = BA \cos\theta$

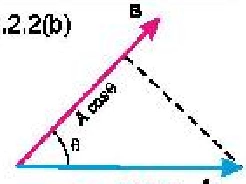


Fig. 2.2 (b)

This type of product when we consider the work done by a force **F** whose point of application moves a distance d in a direction making an angle θ with the line of action of **F**, as shown in Fig. 2.3.

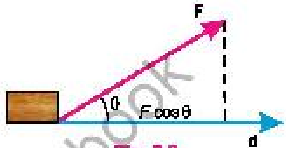


Fig. 2.3

Work done = (Effective component of force in the direction of motion) x Distance moved

$= (F \cos\theta)\ d \ = Fd \cos\theta$

Using vector notation:

F.d $= Fd \cos\theta$ = Work done

Characteristics of Scalar Product

1. Since **A.B** = $AB \cos\theta$ and **B.A** = $BA \cos\theta = AB \cos\theta$, hence, **A.B = B.A.** The order of multiplication is irrelevant. In other words, scalar product is commutative.
2. The scalar product of two mutually perpendicular vectors ($\theta = 90°$) is zero.

   A.B $= AB \cos 90° = 0$

3. The scalar product of two parallel vectors is equal to the product of their magnitudes. Thus, for parallel vectors ($\theta = 0°$).

   A.B $= AB \cos 0° = AB$

   For antiparallel vectors ($\theta = 180°$)

   A.B $= AB \cos 180° = -AB$

4. The self product of a vector **A** is equal to square of its magnitude.

   A.A $= AA \cos 0° = A^2$

5. Scalar product of two vectors **A** and **B** in terms of their rectangular components

A.B $= A_xB_x + A_yB_y + A_zB_z$ ………………(2.7)

Equation (2.6) can be used to find the angle between two vectors. Since,

A.B $= AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$

Therefore $\cos\theta = \dfrac{A_xB_x + A_yB_y + A_zB_z}{AB}$ ………… (2.8)

Vector or Cross Product

The vector product of two vectors **A** and **B**, is a vector which is defined as:

$$\mathbf{A} \times \mathbf{B} = AB \sin\theta\, \hat{n} \quad \text{..................(2.9)}$$

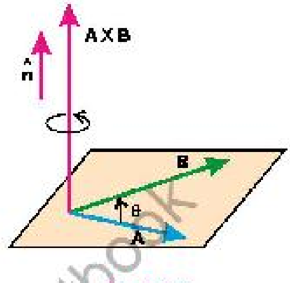


Fig. 2.4(a)

where $\hat{n}$ is a unit vector perpendicular to the plane containing **A** and **B** as shown in Fig. 2.4 (a). Its direction can be determined by right hand rule. For that purpose, place together the tail of vectors **A** and **B** to define the plane of vectors **A** and **B**. The direction of the product vector is perpendicular to this plane. Rotate the First vector **A** into **B** through the smaller of the two possible angles and curl the fingers of the right hand in the direction of rotation, keeping the thumb erect. The direction of the product vector will be along the erect thumb, as shown in the Fig 2.4 (b). Because of this direction rule, **B x A** is a vector opposite in sign to **A x B** (Fig. 2.4-c). Hence,

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \quad \text{.............. (2.10)}$$

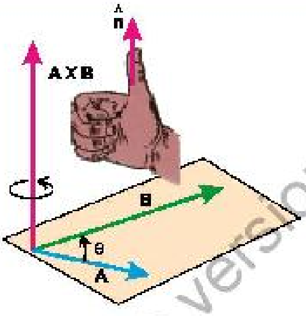


Fig. 2.4(b)

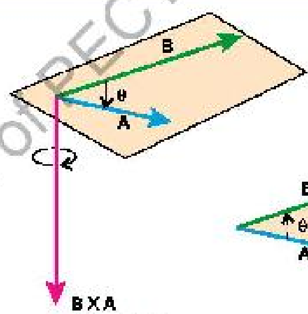


Fig. 2.4(c)

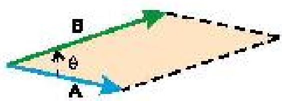


Fig. 2.4(d)

Characteristics of Cross Product

1. Since **A x B** is not the same as **B x A**, the cross product is non commutative. so,

   $$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

2. The cross product of two perpendicular vectors ($\theta = 90°$) has maximum magnitude.

   $$\mathbf{A} \times \mathbf{B} = AB \sin 90° \hat{n} = AB\, \hat{n}$$

3. The cross product of two parallel or anti-parallel vectors is a null vector, because for such vectors $\theta = 0°$ or $180°$. Hence,

   $$\mathbf{A} \times \mathbf{B} = AB \sin 0° \hat{n} = 0 \quad \text{or} \quad \mathbf{A} \times \mathbf{B} = AB \sin 180° \hat{n} = 0$$

   As a consequence $\mathbf{A} \times \mathbf{A} = \mathbf{0}$ ($\theta = 0°$)

4. The magnitude of **A x B** is equal to the area of the parallelogram formed with **A** and **B**

as two adjacent sides (Fig.2.4-d).

Examples of Vector Product

i. When a force **F** is applied on a rigid body at a point whose position vector is **r** from any point on the axis about which the body rotates, then the turning effect of the force called the torque τ is given by the vector product of **r** and **F**.

$$\tau = \mathbf{r} \times \mathbf{F}$$

ii. The force **F** on a particle of charge q and velocity **v** in a magnetic field of strength **B** is given by vector product of **v** and **B**.

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

2.4 EQUATIONS OF MOTIONS

Equations of motion can be used to describe the motion of an object in terms of its three kinematic variables: velocity v, position S and time t. There are three ways to pair these variables up: velocity-time, position-time and velocity-position. In this order, they are called first equation of motion, second equation of motion and third equation of motion, respectively.

These equations of motion can only be applied to those objects, which are moving in a straight line with constant acceleration.

Derivation of First Equation of Motion

Suppose a body is moving with uniform acceleration along a straight line with an initial velocity v_i . Suppose its velocity changes from initial value v_i to a final value v_f in time interval t. Then the acceleration produced in the body during this time interval is given as:

$$a = \frac{v_f - v_i}{t}$$

Rearranging, we can write

$$v_f - v_i = at$$

$$v_f = v_i + at \ldots\ldots\ldots\ldots(2.11)$$

This is the first equation of motion. It correlates the final velocity attained by a body with initial velocity and the time interval t, when moving with constant acceleration a.

Derivation of First Equation of Motion By Graphical Method

First equation of motion can be derived using velocity-time graph for an object moving with initial velocity v_i final velocity v_f and constant acceleration a.

Let the velocity of a body at point A be v_i which changes to v_f at point B in time interval t as shown in Fig.2.5. A perpendicular BD is drawn from point B to x-axis and another perpendicular BE from B on y-axis, such that

OA = v_i = Initial velocity of the body

OE = DB = v_f = Final velocity of the body

From the graph, it can observed that;

DB = DC + CB

DB = OA + CB (As OA = DC)

Therefore v_f = CB + v_i ....(2.12)

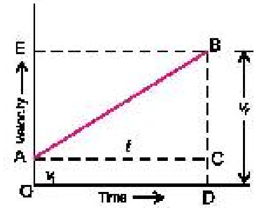


Fig. 2.5: Velocity-Time Graph

The value of CB in the above equation can be determined by taking the slope of line AB, which is equal to acceleration *a*.

$$a = \frac{CB}{AC}$$

As AC = t

So $a = \frac{CB}{t}$

or BC = at ....(2.13)

Combining Eqs. (2.12) and (2.13), we have

$$v_f = v_i + at$$

This is the first equation of motion.

Derivation of Second Equation of Motion

Suppose a body is moving with uniform acceleration *a* along a straight line with an initial velocity v_i, which become v_f, after time interval t. Let it covers a distance S in a particular direction during time t, then using the definition of velocity as rate of change of displacement, we can write

Velocity = Displacement / Time

or Displacement = Velocity x Time

If velocity of the body is not constant, we can use average velocity instead of velocity.

Thus Displacement = Average velocity × Time

$$\text{Displacement} = \frac{\text{(Initial velocity + (Final velocity)}}{2} \times \text{Time}$$

$$S = \frac{(v_i + v_f)}{2} \times t$$

Using first equation of motion, $S = \frac{(v_i + v_i + at)}{2} \times t$

$$S = \frac{(2v_i + at)}{2} \times t$$

$$2S = 2v_it + at^2$$

$$S = v_it + \frac{1}{2}at^2 \quad \text{..........................(2.14)}$$

This is the second equation of motion.

Derivation of Second Equation of Motion by Graphical Method

Second equation of motion can be derived using velocity-time graph for a body moving with initial velocity v_i which attains a final value v_f in time interval t. While moving with constant acceleration a, it covers a displacement S in time t.

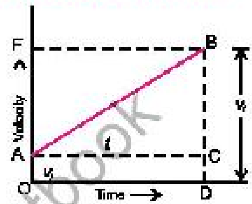


Fig. 2.6: Velocity-Time Graph

It can be seen from the graph that distance travelled by the body is, $S = v \times t$.

Also $\quad$ S = Area of the figure OABD

S = (Area of the rectangle OACD) + (Area of the triangle ABC)

$$S = (OA \times OD) + \frac{1}{2}(AC \times BC)$$

As $\quad$ $OA = v_i$ and $OD = AC = t$. So, the above equation becomes:

$$S = v_i \times t + \frac{1}{2}(t \times BC)$$

Here $BC = at$ (From graphical representation of first equation of motion). By putting this value in the above equation, we have

$$S = v_it + \frac{1}{2}(t \times at)$$

$$S = v_it + \frac{1}{2}at^2$$

This is the second equation of motion.

Derivation of third equation of motion

Consider a body moving along a straight line with an initial velocity v_i which attains a final value v_f in time t. Let the displacement of the body be S during this time interval. Then, we can write:

$$\text{Displacement} = \left(\frac{\text{Initial velocity} + \text{Final velocity}}{2}\right) \times \text{Time}$$

$$S = \frac{(v_i + v_f)}{2} \times t$$

$$2S = (v_i + v_f) \times t \quad \text{.............................. (2.15)}$$

Using the first equation of motion:

$$v_f = v_i + at$$

or

$$t = \frac{v_f - v_i}{a}$$

Putting the value of t in Eq. (2.15)

$$2S = (v_f + v_i)\left(\frac{v_f - v_i}{a}\right)$$

$$2aS = v_f^2 - v_i^2$$

This is the third equation of motion.

Derivation of third Equation of Motion by Graphical method

In the speed-time graph shown in the figure, the total distance S travelled by a body is given by the area OABD under the graph, such that

$$S = \frac{1}{2} \text{ (Sum of parallel sides) x Height}$$

$$S = \frac{1}{2}(\text{OA} + \text{BD}) \times \text{OD}$$

Since $\quad \text{OA} = v_i, \;\; \text{BD} = v_f \;\; \text{and} \;\; \text{OD} = t$

The above equation becomes:

$$S = \frac{1}{2}(v_f + v_i) \times t$$

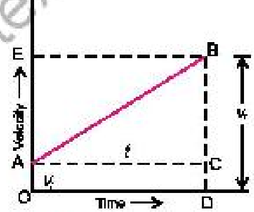


Fig. 2.7: Velocity-Time Graph

From first equation of motion,

$$t = \frac{v_f - v_i}{a}$$

Putting t in above equation

$$S = \frac{1}{2}(v_f + v_i)\frac{(v_f - v_i)}{a}$$

or

$$S = \frac{1}{2}(v_f + v_i)\frac{(v_f - v_i)}{a}$$

$$2aS = v_f^2 - v_i^2 \quad \text{......................(2.16)}$$

This is the third equation of motion.

The equations of motion are useful in solving the problems relating to linear motion with uniform acceleration, when an object moves along a straight line. If its direction of motion does not change, then all the vector quantities can be manipulated like scalars. In such cases, initial velocity is taken as positive. A negative sign is assigned to quantities where direction is opposite to that of initial velocity. In the absence of air resistance, all objects in free fall at the surface of the Earth, move towards the Earth with a uniform acceleration. This acceleration is known as acceleration due to gravity, denoted by g and its average value at the Earth surface is taken as 9.8 m s^{-2} in the downward direction. The equation for uniformly accelerated motion can also be applied to free fall motion of the object by replacing a by g.

Example 2.2 A car travelling at 10 m s^{-1} accelerates uniformally at 2 m s^{-2}. Calculate its velocity after 5 s.

Solution
$v_i = 10\,m\,s^{-1}$

$a = 2\,m\,s^{-2}$

$t = 5\,s$

$v_f = ?$

Using first equation of motion, we can write

$$v_f = v_i + at$$

$$v_f = 10\,m\,s^{-1} + 2\,m\,s^{-2} \times 5\,s$$

$$v_f = 10\,m\,s^{-1} + 10\,m\,s^{-1}$$

$$v_f = 20\,m\,s^{-1}$$

Example 2.3 A car travels with initial velocity of 15 m s^{-1}. It accelerates at a rate of 2 m s^{-2} for 4 seconds. Find the displacement of the car.

Solution

$v_i = 15\,m\,s^{-1}$

$a = 2\,m\,s^{-2}$

$t = 4\,s$

Displacement $S = ?$

By using 2nd equation of motion

$$S = v_i t + \frac{1}{2} a t^2$$

Putting the values

$$S = (15\,m\,s^{-1} \times 4\,s) + \frac{1}{2}(2\,ms^{-2})(4\,s)^2$$

$$S = 76\,m$$

Example 2.4 In a short distance race, a contestant in a car starts from rest and reaches the velocity of 300 km h^{-1}, after covering a distance of 0.45 km at a constant acceleration. Find this constant acceleration.

Solution Initial velocity = $v_i = 0$

Final velocity = $v_f = 300\,km\,h^{-1}$

$$v_f = \frac{300 \times 1000}{60 \times 60}\ ms^{-1}$$

$$= 83.33\ \text{m s}^{-1}$$

Distance covered = S = 0.45 km = 0.45 x 1000 m = 450 m

Using third equation of motion, we have

$$v_f^2 - v_i^2 = 2aS$$

$$(83.33\ \text{m s}^{-1})^2 - (0)^2 = 2 \times a \times 450\ \text{m}$$

$$a = \frac{6943.88\ \text{m}^2\ \text{s}^{-2}}{900\ \text{m}}$$

$$a = 7.72\ \text{m s}^{-2}$$

2.5 MOTION UNDER GRAVITY

A body falling freely under the action of gravity is the most familiar example of uniformly accelerated rectilinear motion. According to Galilo, all bodies fall freely (in vacuum) under the acceleration due to gravity, denoted by 'g'. Its experimental value is 9.8 m s^{-2} in SI units. This means that different bodies, when allowed to fall from the same height, strike the ground with the same velocity. As regards the sign of g, it is taken positive for a falling body (when initial velocity is zero) and negative for a body projected vertically upward (when initial velocity is not zero).

The equations of motion for a freely falling body, on putting $a = g$, become

$$v_f = v_i + gt$$

$$S = h = v_i t + \frac{1}{2}gt^2$$

$$v_f^2 - v_i^2 = 2gh$$

Example 2.5 An iron ball of mass 1 kg is dropped from a tower. The ball reaches the ground in 3.34 s. Find: (a) the velocity of the ball on striking the ground, (b) the height of the tower.

Solution Since the ball is falling under the action of gravity, we shall putt $a = g$ in equations of motion.

Mass of the ball	m = 1 kg
Time taken to reach ground	t = 3.34 s
Initial velocity	$v_i = 0$
Final velocity	v_f = ?
Acceleration	$a = g = 9.8$ m s^{-2}

(a) Using first equation of motion:

$$v_f = v_i + gt$$
$$v_f = 0 + (9.8\text{ m s}^{-2})(3.34\text{ s})$$
$$v_f = 32.7\text{ m s}^{-1}$$

(b) Using third equation of motion:

$$v_f^2 - v_i^2 = 2gh$$
$$(32.7\text{m s}^{-1})^2 - (0)^2 = 2 \times 9.8\text{ m s}^{-2} \times h$$
$$h = \frac{1069.29\text{ m}^2\text{ s}^{-2}}{19.6\text{ m s}^{-2}}$$
$$h = 54.56\text{ m}$$

2.6 PROJECTILE MOTION

Uptill now we have been studying the motion of a particle along a straight line i.e., motion in one dimension. Now we consider the motion of a ball, when it is thrown horizontally from certain height. It is observed that the ball travels forward as well as falls downward, until it strikes something such as ground. Suppose that the ball leaves the hand of the thrower at point A (Fig 2.8-a) and that its velocity at that instant is completely horizontal. Let this velocity be v_x. According to Newton's first law of motion, there will be no acceleration in horizontal direction, unless a horizontally directed force acts on the ball. Ignoring the air friction, only force acting on the ball during flight is the force of gravity. There is no horizontal force acting on it. So, its horizontal velocity will remain unchanged and will be v_x, until the ball hits the ground. The horizontal motion of ball is simple. The ball moves with constant horizontal velocity component. Hence, horizontal distance x is given by

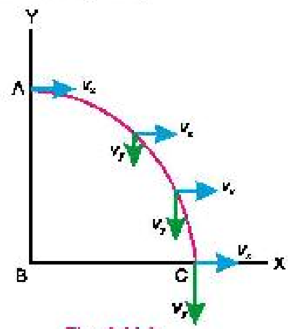


Fig. 2.8(a)

$$x = v_x \times t \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.17)$$

The vertical motion of the ball is also not complicated. It will accelerate downward under the force of gravity and hence a = g. This vertical motion is the same as for a freely falling body. Since initial vertical velocity is zero, hence, vertical distance y, using Eq. 2.14 is given by

$$y = \frac{1}{2}gt^2$$

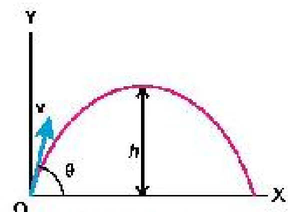


Fig. 2.8(b)

It is not necessary that an object should be thrown with some initial velocity in the horizontal direction. A football kicked off by a player; a ball thrown by a cricketer and a missile fired from a launching pad, all projected at some angles with the horizontal, are called projectiles.

Projectile motion is two dimensional motion under constant acceleration due to gravity.

In such cases, the motion of a projectile can be studied easily by resolving it into horizontal and vertical components which are independent of each other. Suppose that a projectile is fired in a direction angle θ with the horizontal by velocity $\mathbf{v}_i$ as shown in Fig.2.8(b). Let components of velocity $\mathbf{v}_i$ along the horizontal and vertical directions be $v_i\cos\theta$ and $v_i\sin\theta$, respectively. The horizontal acceleration is $a_x = 0$ because we have neglected air resistance and no other force is acting along this direction, whereas the vertical acceleration is $a_y = g$. Hence, the horizontal component v_x remains constant and at any time t, we have

$$v_{fx} = v_{ix} = v_i \cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.18)$$

Now we consider the vertical motion. The initial vertical component of the velocity is $v_i\sin\theta$ in the upward direction.

The vertical component v_{fy} at any instant t can be determined by considering the upward motion of projectile as free fall motion ($a_y = -g$). Using 1st equation of motion:

$$v_{fy} = v_i \sin\theta - gt \quad \ldots\ldots\ldots \quad (2.19)$$

The magnitude of velocity at any instant is:

$$v = \sqrt{v_{fx}^2 + v_{fy}^2} \quad \ldots\ldots\ldots \quad (2.20)$$

The angle ϕ which this resultant velocity makes with the horizontal can be found from

$$\tan\phi = \frac{v_{fy}}{v_{fx}} \quad \ldots\ldots\ldots \quad (2.21)$$

Interesting Information

A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

In projectile motion one may wish to determine the height to which the projectile rises, the time of flight and horizontal range. These are described below.

Height of the Projectile

In order to determine the maximum height the projectile attains, we use the equation of motion:

$$2aS = v_f^2 - v_i^2$$

As body moves upward, $a = -g$, the initial vertical velocity $v_{iy} = v_i \sin\theta = v_i$ as $v_{fy} = 0 = v_f$, because the body comes to rest after reaching the highest point. Since

$$S = height = h$$

$$-2gh = 0 - v_i^2 \sin^2\theta$$

or $$h = \frac{v_i^2 \sin^2\theta}{2g} \quad .... (2.22)$$

The height of projectile will be reduced in presence of air resistance. In the presence of air resistance, the upward velocity of the projectile will decrease and hence its height will also decrease during time t.

Time of Flight

The time taken by body to cover the distance from the place of its projection to the place where it hits the ground is called the time of flight.

This can be obtained by taking $S = h = 0$, because body goes up and comes back to the same level, thus covering no vertical distance. If the body is projecting with velocity v_i making angle θ with the horizontal, then its vertical component will be $v_i \sin\theta$. Hence, the equation of motion is:

$$S = v_i t + \frac{1}{2} g t^2$$

$$0 = v_i \sin\theta\, t - \frac{1}{2} g t^2$$

$$t = \frac{2 v_i \sin\theta}{g} \quad .... (2.23)$$

where t is the time of flight of the projectile when it is projected from the ground.

Range of the projectile

Maximum distance which a projectile covers in the horizontal direction is called the range of the projectile.

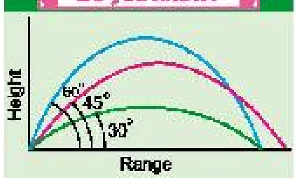
To determine the range R of the projectile, we multiply the horizontal component of the velocity of projection with total time taken by the body to hit the ground after leaving the point of projection. Thus,

$$R = v_{ix} \times t$$

or $$R = \frac{v_i \cos\theta \times 2 v_i \sin\theta}{g}$$

or $$R = \frac{v_i^2}{g} 2 \sin\theta \cos\theta$$

Do you know?



For an angle less than 45°, the height reached by the projectile and the range both will be less. When the angle of projectile is larger than 45°, the height attained will be more but the range is again less.

As $2 \sin\theta \cos\theta = \sin 2\theta$, thus, the range of the projectile depends upon the velocity of projection and the angle of projection.

Therefore $$R = \frac{v_i^2}{g} \sin 2\theta \quad \text{.................. (2.24)}$$

For maximum range R, the factor $\sin 2\theta = 1$, so

$$2\theta = \sin^{-1}(1) \quad \text{or} \quad 2\theta = 90^\circ \quad \text{or} \quad \theta = 45^\circ$$

Air resistance will slow down projectile forward motion, reducing its velocity v_i. The reduction in v_i will result in a decrease in the range of projectile.

Furthermore, air resistance is not constant throughout the flight of the object. As the object slows down, the air resistance experienced by it also decreases. This means that the object retards more slowly and accelerates more slowly as it falls down. This results in a trajectory that is not perfectly parabolic but is skewed, with steeper descent than ascent.

Example 2.6 A ball is thrown with a speed of 30 m s^{-1} in a direction 30° above the horizon. Determine the height to which it rises, the time of flight and the horizontal range.

Solution Initially

$$v_{ix} = v_i \cos\theta = 30 \text{ m s}^{-1} \times \cos 30^\circ = 25.98 \text{ m s}^{-1}$$

$$v_{iy} = v_i \sin\theta = 30 \text{ m s}^{-1} \times \sin 30^\circ = 15 \text{ m s}^{-1}$$

As the time of flight, is

$$t = \frac{2v_i \sin\theta}{g}$$

$$= \frac{2 \times 30 \text{ m s}^{-1}}{9.8 \text{ m s}^{-2}} \times (0.5)$$

So $$t = \frac{2 \times 15 \text{ m s}^{-1}}{9.8 \text{ m s}^{-2}} = 3.1 \text{ s}$$

Height $$h = \frac{v_i^2 \sin^2\theta}{2g}$$

So $$h = \frac{(30 \text{ m s}^{-1})^2 (0.5)^2}{19.6 \text{ m s}^{-2}}$$

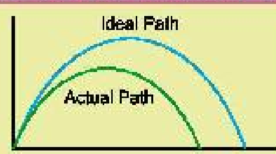
$$h = 11.5 \text{ m}$$

Range $$R = \frac{v_i^2}{g} \sin 2\theta = \frac{v_i^2}{g} \sin 60^\circ$$

So $$R = \frac{(30 \text{ m s}^{-1})^2 \times 0.866}{9.8 \text{ m s}^{-2}}$$

$$= 79.53 \text{ m}$$

For your information



In the presence of air friction the trajectory of a high speed projectile falls short of a parabolic path.

2.7 MOMENTUM

We are aware of the fact that moving object possesses a quality by virtue of which it exerts a force on anything that tries to stop it. The faster the object is travelling, the harder it is to stop it. Similarly, if two objects move with the same velocity, then it is more difficult to stop the massive of the two. Newton referred to this property as "momentum", a vector quantity defined as the product of an object's mass and velocity. This term is now called linear momentum **p** of the body and is defined by the relation:

$$\mathbf{p} = m\,\mathbf{v} \quad \text{..........(2.25)}$$

In this expression, $\mathbf{v}$ is the velocity of the mass m. Linear momentum is, therefore, a vector quantity and has the direction of velocity. The SI unit of momentum is kilogram metre per second (kg m s^{-1}). It can also be expressed as newton second (N s).

Momentum and Newton's Second Law of Motion

Consider a body of mass m moving with an initial velocity $\mathbf{v}_i$. Suppose an external force **F** acts upon it for time t after which velocity becomes $\mathbf{v}_f$. The acceleration a produced by this force is given by

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

By Newton's second law, the acceleration is given as:

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Equating the two expressions of acceleration, we have

$$\frac{\mathbf{F}}{m} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

or

$$\mathbf{F} \times t = m\mathbf{v}_f - m\mathbf{v}_i \quad \text{........... (2.26)}$$

where $m\mathbf{v}_i$ is the initial momentum and $m\mathbf{v}_f$ is the final momentum of the body.

The equation (2.26) shows that change in momentum is equal to the product of force and the time for which force is applied. This form of the second law is more general than the form $\boldsymbol{F} = m\boldsymbol{a}$, because it can easily be extended to account for changes as the body accelerates when its mass also changes. For example, as a rocket accelerates, it loses mass because its fuel is burnt and ejected to provide greater thrust.

From Eq. (2.26)

$$\mathbf{F} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{t}$$

Thus, second law of motion can also be stated in terms of momentum as:

Point to ponder!

Which hurt you in the above situations (a) or (b) and think why?

Point to ponder!

Can a moving object experience impulse?

Do you know?

Your hair acts like a crumple zone on your skull. A force of 5 N might be enough to fracture your naked skull (cranium), but with a covering of skin and hair, a force of 50 N would be needed.

Time rate of change of momentum of a body is equal to the applied force.

Impulse

Sometimes we wish to apply the concept of momentum to cases where the applied force is not constant, it acts for very short time. For example, when a bat hits a cricket ball, the force certainly varies from instant to instant during the collision. In such cases, it is more convenient to deal with the product of force and time ($F \times t$) instead of either quantity alone. The product of average force F that acts during time t is called impulse given by

$$\text{Impulse} = F \times t = m\mathbf{v}_f - m\mathbf{v}_i \text{ .... } (2.27)$$

Example 2.7 A 1500 kg car has its velocity reduced from 20 m s^{-1} to 15 m s^{-1} in 3.0 s. How large was the average retarding force?

Solution Using the Eq. (2.27)

$$F \times t = m\mathbf{v}_f - m\mathbf{v}_i$$

$$F \times 3.0\text{ s} = 1500\text{ kg} \times 15\text{ m s}^{-1} - 1500\text{ kg} \times 20\text{ m s}^{-1}$$

or $$F = -2500\text{ kg m s}^{-2}$$

$$= -2500\text{ N} = -2.5\text{ kN}$$

The negative sign indicates that the force is retarding one.

Law of Conservation of Momentum

Let us consider an isolated system. It is a system on which no external agency exerts any force. For example, the molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion, but being enclosed by glass vessel, no external agency can exert a force on them.

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2, moving along the same straight line, in the same direction, with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Both the balls collide and after collision, ball of mass m_1 moves with velocity $\mathbf{v}_1'$ vand m_2 moves with velocity $\mathbf{v}_2'$ in the same direction as shown in Fig. (2.9).

To find the change in momentum of mass m_1,

Using Eq. (2.27) as:

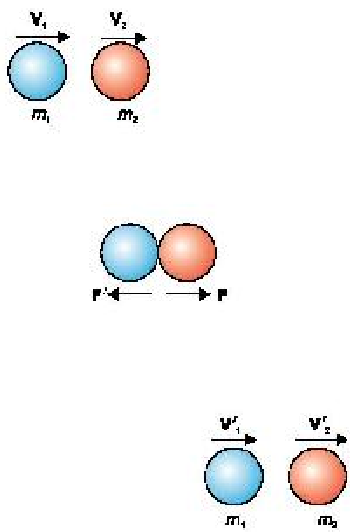


Fig. 2.9

$$\mathbf{F} \times t = m_1\mathbf{v}_1' - m_1\mathbf{v}_1$$

Similarly, for the ball of mass m_2, we have

$$\mathbf{F}' \times t = m_2\mathbf{v}_2' - m_2\mathbf{v}_2$$

Adding these two expressions, we have

$$(\mathbf{F} + \mathbf{F}')t = (m_1\mathbf{v}_1' - m_1\mathbf{v}_1) + (m_2\mathbf{v}_2' - m_2\mathbf{v}_2)$$

Since the action force **F** is equal and opposite to the reaction force **F′**, we have **F′** = – **F**, or **F**+**F′** = 0 so the left hand side of the equation is zero. Hence,

$$0 = (m_1\mathbf{v}_1' - m_1\mathbf{v}_1) + (m_2\mathbf{v}_2' - m_2\mathbf{v}_2)$$

In other words, change in momentum of 1st ball + change in momentum of the 2nd ball is zero.

or $\quad (m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = (m_1\mathbf{v}_1' + m_2\mathbf{v}_2')$ ....(2.28)

Do you know?

When a moving car stops quickly, the passengers move forward towards the windshield. Seatbelts change the forces of motion and prevent the passengers from moving. Thus, the chance of injury is greatly reduced.

Which means that total initial momentum of the system before collision is equal to the total final momentum of the system after collision. Consequently, the total change in momentum of the isolated two ball system is zero.

For such a group of objects, if one object within the group experiences a force, there must exist an equal but opposite reaction force on other object in the same group. As a result, the change in momentum of the group of objects as a whole is always zero. This can be expressed in the form of law of conservation of momentum, which states that:

The total linear momentum of an isolated system remains constant.

In applying the conservation law, we must notice that the momentum of a body is a vector quantity.

Example 2.8 Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities of 6.0 m s^{-1} and 4 m s^{-1}, respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 m s^{-1}?

Solution As both the balls are moving towards one another, so their velocities are of opposite sign. Let us suppose that the direction of motion of 2 kg ball is positive and that of the 3 kg is negative.

The momentum of the system before collision is:

$$m_1v_1 + m_2v_2 = 2\,\text{kg} \times 6\,\text{m s}^{-1} + 3\,\text{kg} \times (-4\,\text{m s}^{-1})$$
$$= 12\,\text{kg m s}^{-1} - 12\,\text{kg m s}^{-1} = 0$$

Do you know?

A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

Momentum of the system after collision = $m_1v_1' + m_2v_2'$

$$= 2\ \text{kg} \times v_1' + 3\ \text{kg} \times (-3)\ \text{m s}^{-1}$$

From the law of conservation of momentum

$$\begin{bmatrix}\text{Momentum of the system} \\ \text{before collision}\end{bmatrix} = \begin{bmatrix}\text{Momentum of the system} \\ \text{after collision}\end{bmatrix}$$

$$0 = 2\,\text{kg} \times v_1' - 9\ \text{kg m s}^{-1}$$

$$v_1' = 4.5\ \text{m s}^{-1}$$

2.8 ELASTIC AND INELASTIC COLLISIONS

When a tennis ball is dropped on the floor vertically, it may not rebound to its initial height. It is because, a portion of *K.E.* is lost, partly due to friction as the molecules in the ball move past one another when the ball distorts and partly due to its change into heat and sound energies. Similar is the case when two tennis balls collide with certain velocities, their final kinetic energy may be less than the total initial kinetic energy.

A collision in which the *K.E.* of the system is not conserved, is called inelastic collision.

Under certain special conditions, no kinetic energy is lost in the collision or impact on hitting the floor. Such type of collision is said to be elastic collision.

For example, when a hard ball is dropped onto a marble floor, it rebounds to very nearly the initial height. It loses negligible amount of energy in the collision with the floor. It is to be noted that momentum and total energy are conserved in all types of collisions. However, the *K.E.* is conserved only if it is an elastic collision.

Elastic Collisions in One Dimension

Consider two smooth, non-rotating balls of masses m_1 and m_2 moving initially with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after the collision be $\mathbf{v}'_1$ and $\mathbf{v}'_2$ respectively, as shown in Fig.(2.10).

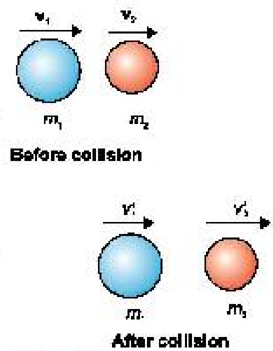


Fig. 2.10

We take the positive direction of the velocity and momentum to the right. By applying the law of conservation of momentum we have

$$(m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = (m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2)$$

$$m_1(\mathbf{v}_1 - \mathbf{v}'_1) = m_2(\mathbf{v}'_2 - \mathbf{v}_2) \quad \text{.........................} (2.29)$$

As the collision is elastic, so the *K.E.* is conserved. From the conservation of *K.E.*, we have $$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

or $$m_1(v_1^2-v_1'^2) = m_2(v_2'^2 - v_2^2)$$

or $$m_1(v_1+v_1')(v_1-v_1') = m_2(v_2'+v_2)(v_2'-v_2) \quad \text{.........(2.30)}$$

Dividing Eq. (2.29) by (2.30)

$$(v_1+v_1') = (v_2'+v_2) \quad \text{...........(2.31)}$$

or $$(v_1 - v_2) = (v_2' - v_1') = -(v_1' - v_2')$$

We note that, before collision $(v_1 - v_2)$ is the velocity of first ball relative to the second ball. Similarly $(v'_1 - v'_2)$ is the velocity of the first ball relative to the second ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation.

In equations (2.29) and (2.30) m_1, m_2, v_1 and v_2 are known quantities. We solve these equations to find the values of v_1' and v_2' which are unknown. The results are

$$v_1' = \frac{m_1-m_2}{m_1+m_2}v_1' + \frac{2m_2}{m_1+m_2}v_2 \quad \text{...........(2.32)}$$

$$v_2' = \frac{2m_1}{m_1+m_2}v_1' + \frac{m_2-m_1}{m_1+m_2}v_2 \quad \text{...........(2.33)}$$

case (I)

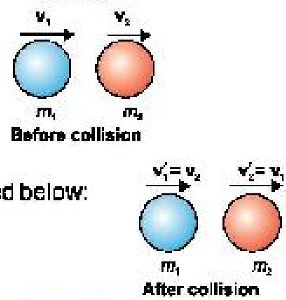


Fig. 2.11

There are some cases of special interest, which are discussed below:

(i) When $m_1 = m_2$

From Eq. (2.32) and (2.33), we find that

$$v_1' = v_2$$

and $$v_2' = v_1 \qquad \text{(as shown in Fig. 2.11)}$$

case (II)

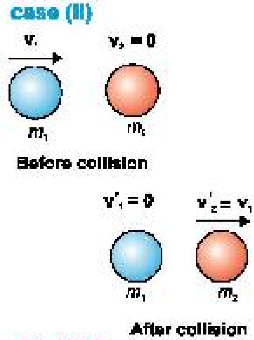


Fig. 2.12

(ii) When $m_1 = m_2$ and $v_2 = 0$

In this case, the mass m_2 be at rest, and $v_2 = 0$, then Eqs. (2.32) and (2.33) give

$$v_1' = 0 \quad ; \quad v_2' = v_1$$

When $m_1 = m_2$ then ball of mass m_1 after collision will come to a stop and m_2 will take off with the velocity that m_1 originally had, as shown in Fig.(2.12). Thus when a billiard ball m_1, moving on a table collides with exactly similar ball m_2 at rest, the ball m_1 stops while m_2 begins to move with the same velocity, with which m_1 was moving initially.

(iii) When a light body collides with a massive body at rest

In this case initial velocity $\mathbf{v}_2 = 0$ and $m_2 >> m_1$. Under these conditions m_1 can be neglected as compared to m_2. From Eq. (2.33) and (2.32), we have $\mathbf{v}_1' = -\mathbf{v}_1$ and $\mathbf{v}_2' = 0$.

The result is shown in Fig.(2.13). This means that m_1 will bounce back with the same velocity while m_2 will remain stationary. This fact is used of by the squash player.

case (iii)

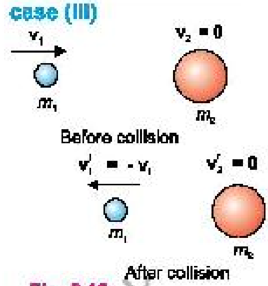


Fig. 2.13

(iv) When a massive body collides with light stationary body

In this case, $m_1 >> m_2$ and $\mathbf{v}_2 = 0$, so m_2 can be neglected in Eqs.(2.32) and (2.33). This gives $\mathbf{v}_1' \approx \mathbf{v}_1$ and $\mathbf{v}_2' \approx 2\mathbf{v}_1$. Thus, after the collision, there is practically no change in the velocity of massive body, but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body, as shown in Fig.(2.14).

case (iv)

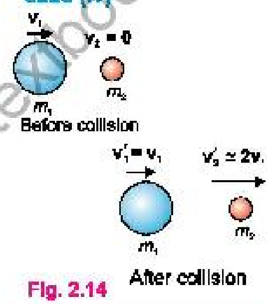


Fig. 2.14

2.9 INELASTIC COLLISION IN ONE DIMENSION

Consider two bodies having masses m_1 and m_2, moving with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, along the same line such that $\mathbf{v}_1 > \mathbf{v}_2$. In such a case m_1 is regarded as projectile and m_2 as target. After time t both the bodies make inelastic collision and stick together. Let their combined mass become $m_1 + m_2$ which moves with final velocity $\mathbf{v}_f$ after collision.

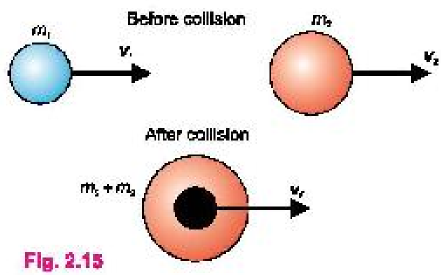


Fig. 2.15

Since the collision is perfectly inelastic, the total momentum of balls is conserved. Using law of conservation of momentum.

Total momentum of system before collision = Total momentum of the system after collision

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}_f$$

$$\mathbf{v}_f = \frac{m_1}{m_1 + m_2}\mathbf{v}_1 + \frac{m_2}{m_1 + m_2}\mathbf{v}_2 \quad ....(2.34)$$

Which gives the common velocity of the body after inelastic collision.

In a special case when the target m_2 is at rest, $v_2 = 0$, the above equation becomes:

$$\mathbf{v}_f = \frac{m_1}{m_1 + m_2} \mathbf{v}_1$$

It shows that velocity of m_1 is reduced by the mass ratio i.e., $\frac{m_1}{m_1 + m_2}$.

Example 2.9 A 70 g ball collides with another ball of mass 140 g. The initial velocity of the first ball is 9 m s^{-1} to the right while the second ball is at rest. If the collision were perfectly elastic. What would be the velocity of the two balls after the collision?

Solution

$m_1 = 70\ \text{g}$ $\quad v_1 = 9\ \text{m s}^{-1}$ $\quad v_2 = 0$

$m_2 = 140\ \text{g}$ $\quad v'_1 = ?$ $\quad v'_2 = ?$

We know that;

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$= \left(\frac{70\ \text{g} - 140\ \text{g}}{70\ \text{g} + 140\ \text{g}}\right) \times 9\ \text{m s}^{-1} = -3\ \text{m s}^{-1}$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1$$

$$= \left(\frac{2 \times 70\ \text{g}}{70\ \text{g} + 140\ \text{g}}\right) \times 9\ \text{m s}^{-1}$$

$$= 6\ \text{m s}^{-1}$$

2.10 ELASTIC COLLISION IN TWO DIMENSIONS

Consider the motion of two balls of mass m_1 and m_2 in a straight line with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively undergoing an elastic collision with each other as shown in Fig. 2.16

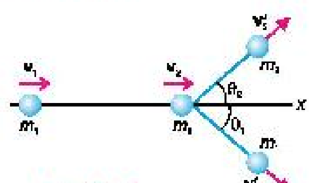


Fig. 2.16

Assume the bodies move off in different directions after collision with velocities $\mathbf{v}'_1$ and $\mathbf{v}'_2$ making angles θ_1 and θ_2 respectively with x- axis.

As, the collision is elastic, so we apply both the laws of conservation of momentum and law of conservation of kinetic energy. Momentum is a vector quantity, we resolve it into its rectangular components and apply the law of conservation of momentum along both axes.

Momentum conservation along x-axis is:

Momentum before collision = momentum after collision

$$m_1 v_1 + m_2 v_2 = m_1 v_1' \cos\theta_1 + m_2 v_2' \cos\theta_2 \quad \ldots\ldots\ldots\ldots(2.35)$$

Momentum conservation along y-axis:

Momentum before collision = Momentum after collision

$$0 = m_1 v_1' \sin\theta_1 - m_2 v_2' \sin\theta_2 \quad \ldots\ldots\ldots\ldots(2.36)$$

Conservation of Energy

Kinetic Energy before collision = Kinetic Energy after collision

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 \quad \ldots\ldots\ldots\ldots(2.37)$$

These equations (2.35 to 2.37) are useful in solving problems about elastic collisions in two dimensions.

2.11 INELASTIC COLLISION IN TWO DIMENSIONS

The macroscopic collisions are generally inelastic and do not conserve Kinetic energy.

The perfect inelastic collision is one in which the colliding objects stick together to make a single mass after collision. Its analysis can be carried out as follows:

Let us take two balls having masses m_1 and m_2 moving with velocities, $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively, in a two-dimensional xy-plane. Assume that the first body is moving along the x-axis while the second body moves in a direction, making an angle θ with x-axis. Both the bodies collide at the origin as shown in the figure 2.17.

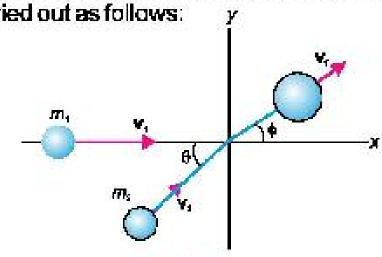


Fig. 2.17

After collision, bodies stick together, having combined mass $M = m_1 + m_2$, which moves with velocity v_f, making an angle ϕ with x-axis.

Momentum in the x-direction:

$$m_1 v_1 + m_2 v_2 \cos\theta = M v_f \cos\phi \quad \ldots\ldots\ldots\ldots(2.38)$$

Momentum in the y-direction:

$$0 + m_2 v_2 \sin\theta = M v_f \sin\phi \quad \ldots\ldots\ldots\ldots(2.39)$$

Equation 2.38 and 2.39 can be used to find the final velocity.

Kinetic Energy

Since collision is inelastic, the kinetic energy of colliding system is not conserved. The loss of kinetic energy is computed as follows:

Initial Kinetic Energy

The total initial kinetic energy $K.E_i$ of the system before the collision is:

$$(K.E)_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad ....(2.40)$$

Since $K.E.$ is a scalar quantity, so velocities involving in the formula of $K.E.$ does not require to break velocities into their components.

Final Kinetic Energy

The total final kinetic energy $K.E_f$ after the collision (when the objects stick together) is:

$$(K.E)_f = \frac{1}{2}Mv_f^2 \quad ....(2.41)$$

where v_f is magnitude of the final velocity which can be calculated from Eq. (2.41).

Energy Loss in the Collision

Since the collision is inelastic, there is a loss in kinetic energy, represented by $\Delta K.E.$

$\Delta K.E. = (K.E.)_i - (K.E.)_f$

This lost kinetic energy is transformed into other forms of energy, such as heat, sound, or in deformation.

Some examples of an Inelastic collision:

(i) When a karate chop breaks a pile of bricks, it's an example of an inelastic collision. In this type of collision, the objects involved don't bounce back after impact. Instead, some of the energy from the strike is absorbed by the bricks, converting into heat, sound, and the force needed to break them. This means the energy goes into breaking the bricks rather than causing the hand to rebound. If the Karate chop is not perfectly vertical and involves some horizontal motion, the momentum transfer and the resulting forces will have both horizontal and vertical components.

(ii) In a car crash, the collision is an inelastic nature. When the vehicles collide and absorb the impact energy, causing them to crumple and deform. This energy absorption slows down the cars, stopping them from bouncing back. Most of the kinetic energy is lost, turning into heat, sound, and damage to the vehicles.

(iii) In real-world collisions, a ball and bat show an inelastic behaviour. When the bat hits the ball, some of the kinetic energy is lost because the ball deforms, and energy is also converted into heat and sound. Even though the bat is rigid, it does not transfer energy perfectly and absorbs some energy itself. The ball compresses upon impact, which leads to further energy loss. Consequently, not all of the initial kinetic energy is conserved, making the collision overall an inelastic.

2.12 ROCKET PROPULSION

Rockets move by expelling burning gas through engines at their rear. The ignited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines (Fig. 2.18). The rocket gains momentum equal to the momentum of the gas expelled from the engine but in opposite direction. The rocket engines continue to expel gases after the rocket has begun moving and hence rocket continues to gain more and more momentum. So, instead of travelling at steady speed the rocket gets faster and faster so long the engines are operating.

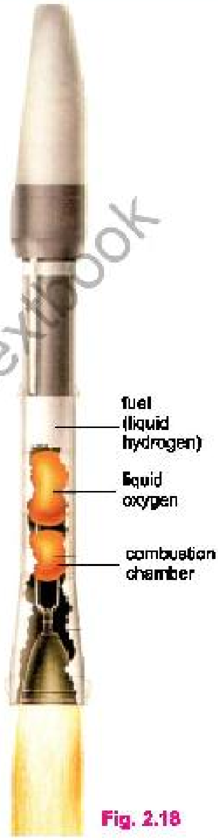


Fig. 2.18

Fuel and oxygen mix in the combustion chamber. Hot gases exhaust the chamber at a very high velocity. The gain in momentum of the gases equals the gain in momentum of the rocket. The gas and rocket push against each other and move in opposite directions.

A rocket carries its own fuel in the form of a liquid or solid hydrogen and oxygen. It can, therefore work at great heights where very little or no air is present. In order to provide enough upward thrust to overcome gravity, a typical rocket consumes about 10000 kg s^{-1} of fuel and ejects the burnt gases at speeds of over 4000 m s^{-1}. In fact, more than 80% of the launch mass of a rocket consists of fuel only. One way to overcome the problem of mass of fuel is to make the rocket from several rockets linked together.

When one rocket has done its job, it is discarded leaving others to carry the space craft further up at ever greater speed.

If m is the mass of the gases ejected per second with velocity v relative to the rocket, the change in momentum per second of the ejecting gases is $m\text{v}$. This equals the thrust produced by the engine on the body of the rocket. So, the acceleration 'a' of the rocket is

$$a = \frac{m\text{v}}{M} \quad .... (2.42)$$

where M is the mass of the rocket. When the fuel in the rocket is burned and ejected, the mass M of rocket decreases and hence the acceleration increases.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

2.1 The angle at which dot product becomes equal to cross product:
(a) 65° (b) 45° (c) 76° (d) 30°

2.2 The projectile gains its maximum height at an angle of:
(a) 0° (b) 45° (c) 60° (d) 90°

2.3 The scalar product of two vectors is maximum if they are:
(a) perpendicular (b) parallel (c) at 30° (d) at 45°

2.4 The range of projectile is same for two angles which are mutually:
(a) perpendicular (b) supplementary
(c) complementary (d) 270°

2.5 The acceleration at the top of a trajectory of projectile is:
(a) maximum (b) minimum (c) zero (d) g

2.6 SI unit of impulse is:
(a) $kg\,m\,s^{-2}$ (b) N m (c) N s (d) $N\,m^2$

2.7 The rate of change of momentum is:
(a) force (b) impulse (c) acceleration (d) power

2.8 As rocket moves upward during its journey, then its acceleration goes on:
(a) increasing (b) decreasing
(c) remains same (d) it moves with uniform velocity

2.9 Elastic collision involves:
(a) loss of energy
(b) gain of energy
(c) no gain, no loss of energy
(d) no relation between energy and elastic collision

Short Answer Questions

2.1 State right hand rule for two vectors with reference to vector product.

2.2 Define impulse and show how it is related to momentum.

2.3 Differentiate between an elastic and an inelastic collision.

2.4 Show that rate of change in momentum is equal to force applied. Also state Newton's second law of motion in terms of momentum.

2.5 State law of conservation of linear momentum. Also state condition under which it holds.

2.6 Show that range of projectile is maximum at an angle of 45°.

2.7 Find the time of flight of a projectile to reach the maximum height.

2.8 The maximum horizontal range of a projectile is 800 m. Find the value of height attained by the projectile at $\theta = 60°$.

Constructed Response Questions

2.1 Why does a hunter aiming a bird in a tree miss the target exactly at the bird?

2.2 A person falling on a heap of sand does not hurt more as compared to a person falling on a concrete floor. Why?

2.3 State the conditions under which birds fly in air.

2.4 Describe the circumstances for which velocity and acceleration of a vehicle are:

(i) v is zero but a is not zero

(ii) a is zero but v is not zero

(iii) perpendicular to one another

2.5 Describe briefly effects of air resistance on the range and maximum height of a projectile.

Comprehensive Questions

2.1 Define and explain scalar product. Write down its important characteristics.

2.2 Define and explain vector product of two vectors. Discuss important characteristics of vector product.

2.3 Derive three equations of motion by graphical method.

2.4 What is projectile motion? Explain.

2.5 Derive the following expressions for projectile motion:

(i) time of flight

(ii) height attained

(iii) range for projectile.

2.6 Explain elastic collision in one dimension. Show that magnitude of relative velocities before and after collision are equal.

2.7 Explain elastic collision in two dimensions.

2.8 Explain an inelastic collision in one and two dimensions.

Numerical Problems

2.1 The magnitude of cross and scalar products of two vectors are $4\sqrt{3}$ and 4, respectively. Find the angle between the vectors. (Ans: 60°)

2.2 A helicopter is ascending vertically at the rate of 19.6 m s^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground? (Ans: 8.0 s)

2.3 If |**A**+**B**| = |**A**-**B**|,then prove that **A** and **B** are perpendicular to each other.

(Ans: $\theta = 90°$)

2.4 A body of mass *M* at rest explodes into 3 pieces, two of which of mass *M* / 4 each are thrown off in perpendicular directions with velocities of 3 m s^{-1} and 4 m s^{-1}, respectively. Find the velocity of 3rd piece with which it will be flown away.

(Ans: 2.5 m s^{-1}, opposite to resultant velocity vector of two pieces)

2.5 A cricket ball is hit upward with velocity of 20 m s^{-1} at an angle of 45° with the ground. Find its:

(a) time of flight (b) maximum height (c) how far away it hits the ground

(Ans: 2.9 s, 41 m, 10.2 m)

2.6 A 20 g ball hits the wall of a squash court with a constant force of 50 N. If the time of impact of force is 0.50 s, find the impulse. (Ans: 25 N s)

2.7 A ball is kicked by a footballer. The average force on the ball is 240 N, and the impact lasts for a time interval of 0.25 s.

(a) Calculate change in momentum

(b) State the direction of change in momentum

[Ans: (a) 60 N s, (b) In the direction of force]

2.8 An aeroplane is moving horizontally at a speed of 200 m s^{-1} at a height of 8 km to drop a bomb on a target. Find horizontal distance from the target at which the bomb should be released. (Ans: 8.08 km)

2.9 Why does range *R* of a projectile remain the same when angle of projection is changed from θ to $\theta' = 90° - \theta$. Also show that for complementary angles of projection, the ratio R/R' is equal to 1.

2.10 A trolley of mass 1.0 kg moving with velocity 1.0 m s^{-1} collides with a similar trolley at rest:

(i) after collision, the 1st trolley comes to rest whereas the second starts moving with velocity of 1.0 m s^{-1} in the same direction. Show that it is an example of an elastic collision.

(ii) after the collision, they stick together and move away with a velocity of 0.5 m s^{-1}. Show that it is an example of an inelastic collision.

2.11 A railway wagon of mass 4 x 10^4 kg moving with velocity of 3 m s^{-1} collides with another wagon of mass 2 x 10^4 kg which is at rest. They stick together and move off together. Find their combined velocity. (Ans: 2 m s^{-1})

2.12 A car with mass 575 kg moving at 15.0 m s^{-1} smashes into the rear end of a car with mass 1575 kg moving at 5 m s^{-1} in the same direction.

(a) What is the final velocity if the wrecked car lock together?

(b) How much kinetic energy is lost in the collision?

[Ans: (a) 7.67 m s^{-1}, (b) 2.11 x 10^4 J]