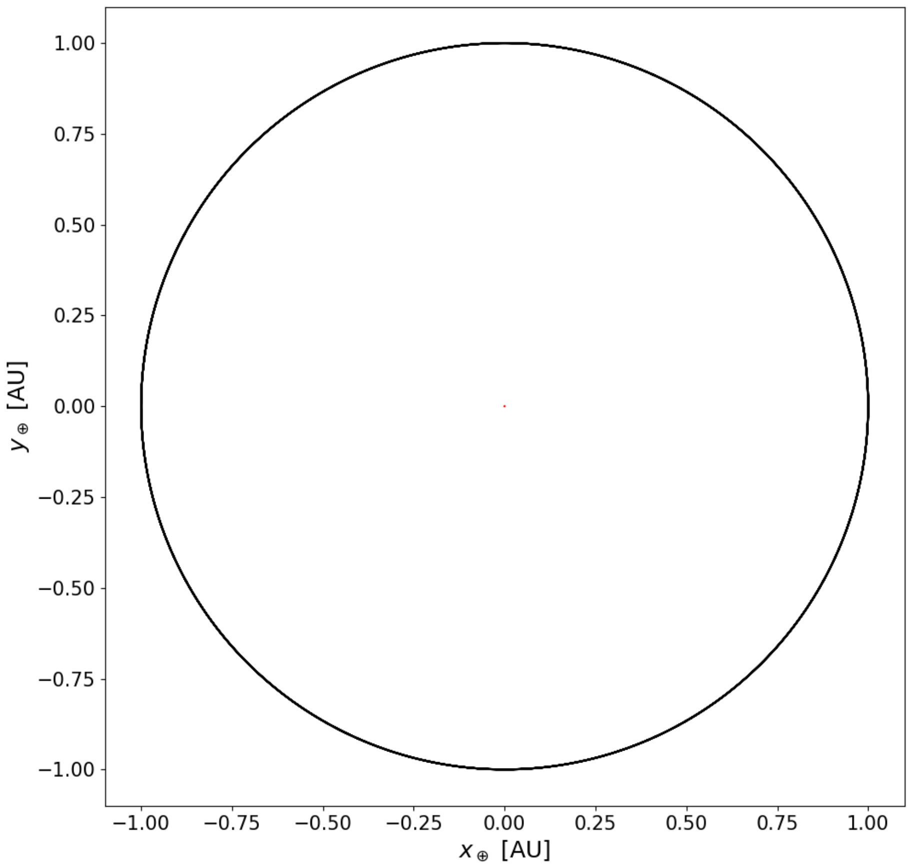

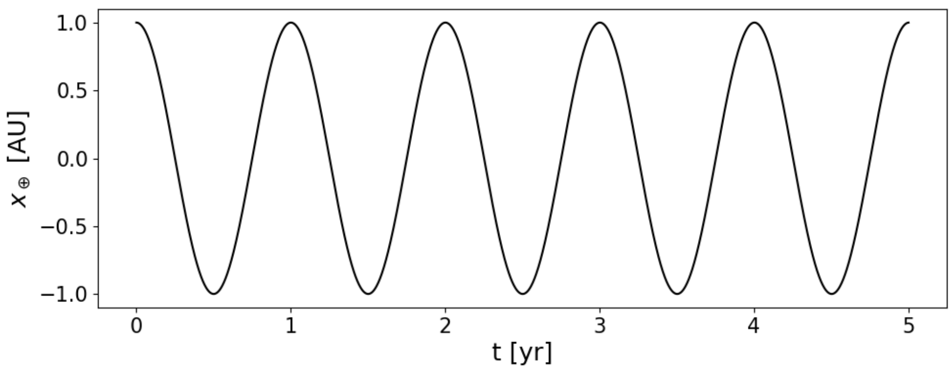

Figure 1: Validation test for the N-body simulation program showing the solution to the two-body problem of the Earth orbiting the Sun over a period of five years. As initial conditions, the Sun was placed stationary at the origin of the coordinate system, while the Earth was placed at coordinates (x_0, y_0, z_0) with all but x_0 equal to 0 AU, and with velocity $(0, \sqrt{GM_{Sun}/x_0}, 0)$, where G is the gravitational constant and M_{Sun} is the mass of the Sun. The fact that the initial z coordinates and velocities are zero makes this problem two-dimensional. Calculation of the total energy of the system at each time step showed that this is well conserved with a relative error from the initial energy that does not exceed 1.5×10^{-7} during the integration. The N-body system in three dimensions is solved using the equations of motion $\vec{a_i} = \sum_{j\neq i} Gm_j\vec{r_{ij}}/r_{ij}^3$, where $\vec{a_i}$ is the acceleration of body i, m_j is the mass of body j, $\vec{r_{ij}}$ is the vector from body i to body j, and r_{ij} is its magnitude. The total energy of the system is calculated as a sum of the total potential and total kinetic energies, where the total potential energy is given by $U = {}^1/_2 \sum_i \sum_{j\neq i}(-Gm_jm_i)/r_{ij}$.

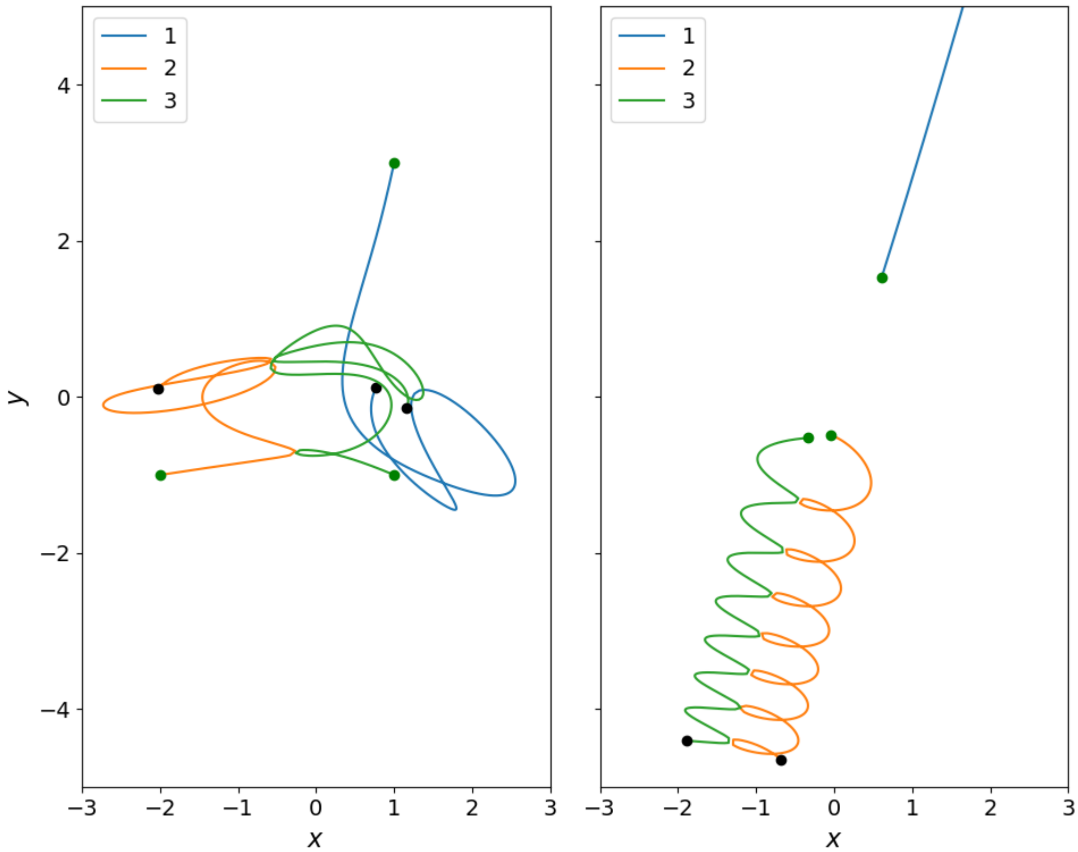

Figure 2: Solution to the three-body Pythagorean problem, known as Burrau's problem, for times between $t = 0$ and $t = 10$ on the left, and $t = 60$ to $t = 70$ on the right. The starting and ending positions of the particles are shown as green and black dots, respectively. Figure 3 shows the total energy relative error at each time step. The variables x, y and t (as well as mass in the equations of motion) are dimensionless. This can be achieved in practice by setting $G = 1$ in the equations of motion which is equivalent to using dimensionless equations of motion where the G factor in the original equations has been replaced by $T^2GM/L^3 = 1$, where T, M, and L are the time, mass, and length units used, so that the dimensionless quantities are $x = X/L$, $y = Y/L$, $m = \mu/M$ and $t = \tau/T$, where X, Y, μ and τ are distances, mass and time. The problem consists of starting the three bodies stationary at the vertices of a Pythagorean triangle with sides 3, 4, 5, where the masses are also 3, 4, 5, and letting the system evolve. The solution obtained here matches that published in the literature, with the final configuration, shown on the right in the figure, being that of body 1 ejected from the system while the other two form a binary.

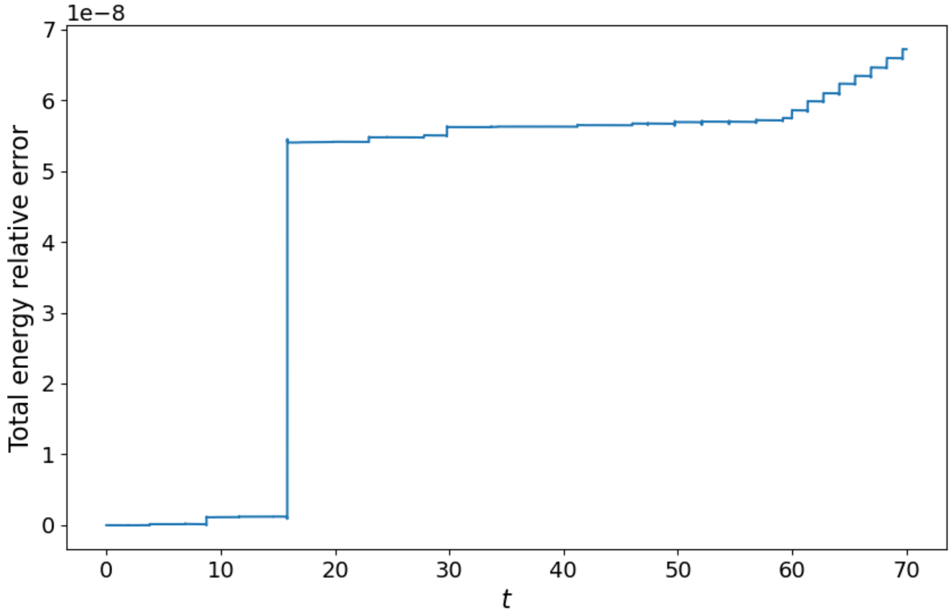

Figure 3: Relative error of the total energy (from the initial) for the Pythagorean system in shown in Figure 2. The relative and absolute error tolerances of the solver used were set to 1×10^{-13} and 1×10^{-15}, respectively. The figure shows that at the end of the integration the relative error had grown to about 7×10^{-8}, primarily due to a large jump at around $t = 16$, even with the very low error tolerance limits. This jump in energy relative error corresponds to the distance of closest approach of the bodies, which was quoted by one source (doi:10.1086/110355) as 4×10^{-4} at $t = 15.830$ between bodies 2 and 3. A wrong solution is obtained if the relative and absolute error tolerances of the solver are left at their default values, usually of around 1×10^{-6}. The system also shows sensitive dependence on initial conditions.

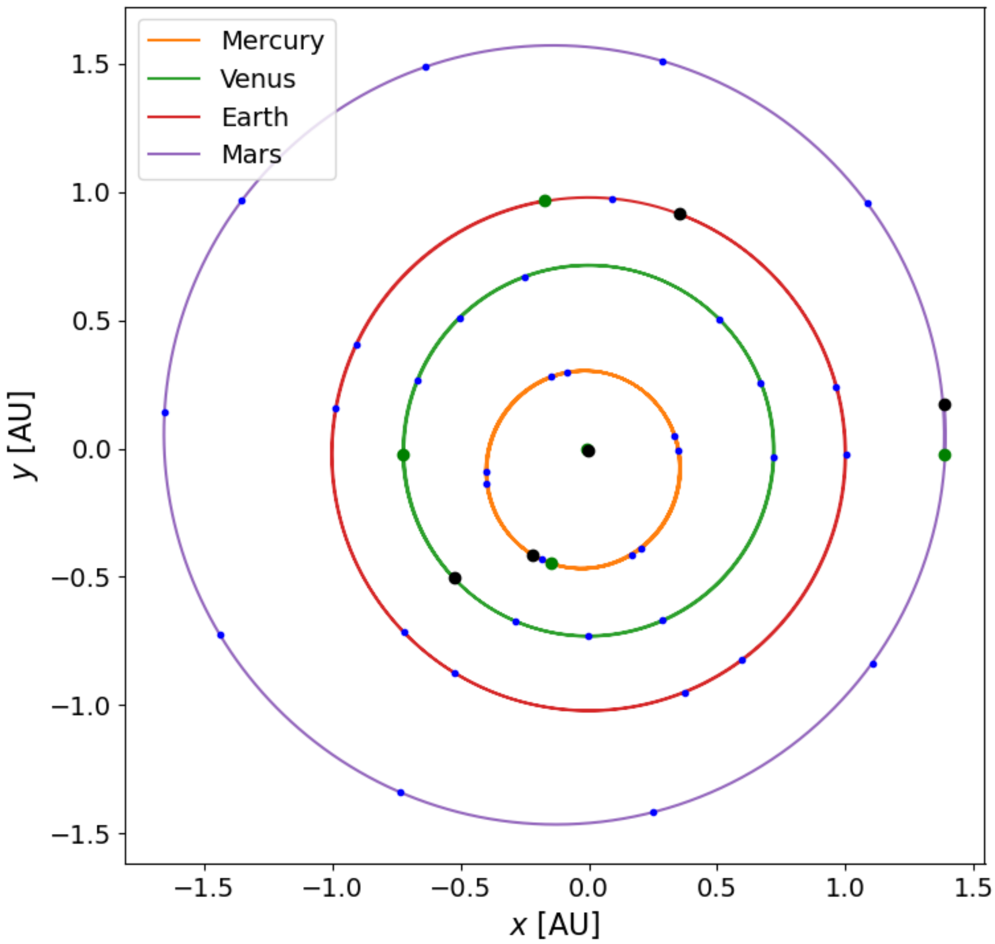

Figure 4: Trajectory plots for Mercury – Mars in the x-y plane of the Earth ecliptic (epoch J2000.0) reference frame, with the coordinate center at the Solar System Barycenter. The Sun is shown as a black dot inside Mercury's orbit. The solid lines are integrated trajectories from a simulation of the Solar System including all the planets and Pluto, and the two largest planetary satellites, Ganymede and Titan, over 700 days which is just over the period of Mars. The software interfaced with the JPL Horizons system (https://ssd.jpl.nasa.gov/horizons/) via its HTTP API to download ephemerides for the Solar System bodies and set the initial conditions for the integration from them. Position and velocity data were downloaded for the period 1st January 2000 to 1st December 2001 at intervals of 70 days. All the dots on the figure are observed positions at those times. The starting and ending positions are shown as green and black dots, respectively. There is agreement between integrated and observed trajectories, and Figure 5 shows more detailed comparison by considering the position relative error of the Mars trajectory.

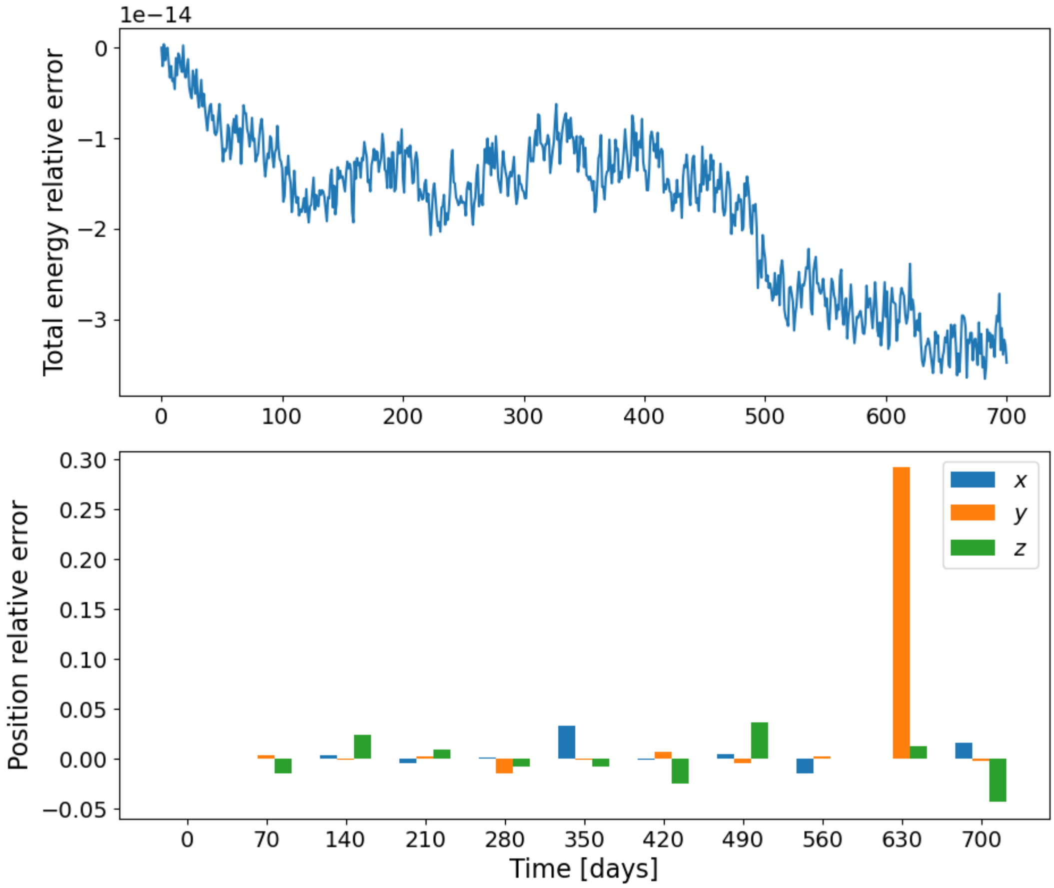

Figure 5: On the top is a plot of the total energy relative error for the system in Figure 4 – a simulation of the Solar System including all the planets and the two largest planetary satellites. On the bottom is the position relative error of the Mars trajectory, calculated at intervals of 70 days using JPL Horizons ephemeris data. The integrator was set with relative and absolute error tolerances of $1{\times}10^{-12}$ and $1{\times}10^{-14}$, respectively, with the step sizes freely chosen by the integrator which used the LSODA method. The magnitude of the energy relative error shows a slight increasing trend during the integration (due to propagating numerical errors) but stays low. On the bottom plot, if there is a bar missing then the error is too small to plot or is zero as on day zero when the position corresponds to the initial conditions taken from the ephemeris. Overall, the position relative errors show that there is good agreement between the simulated and observed positions of Mars, which should be representative of the other trajectories too, as suggested qualitatively in Figure 4.

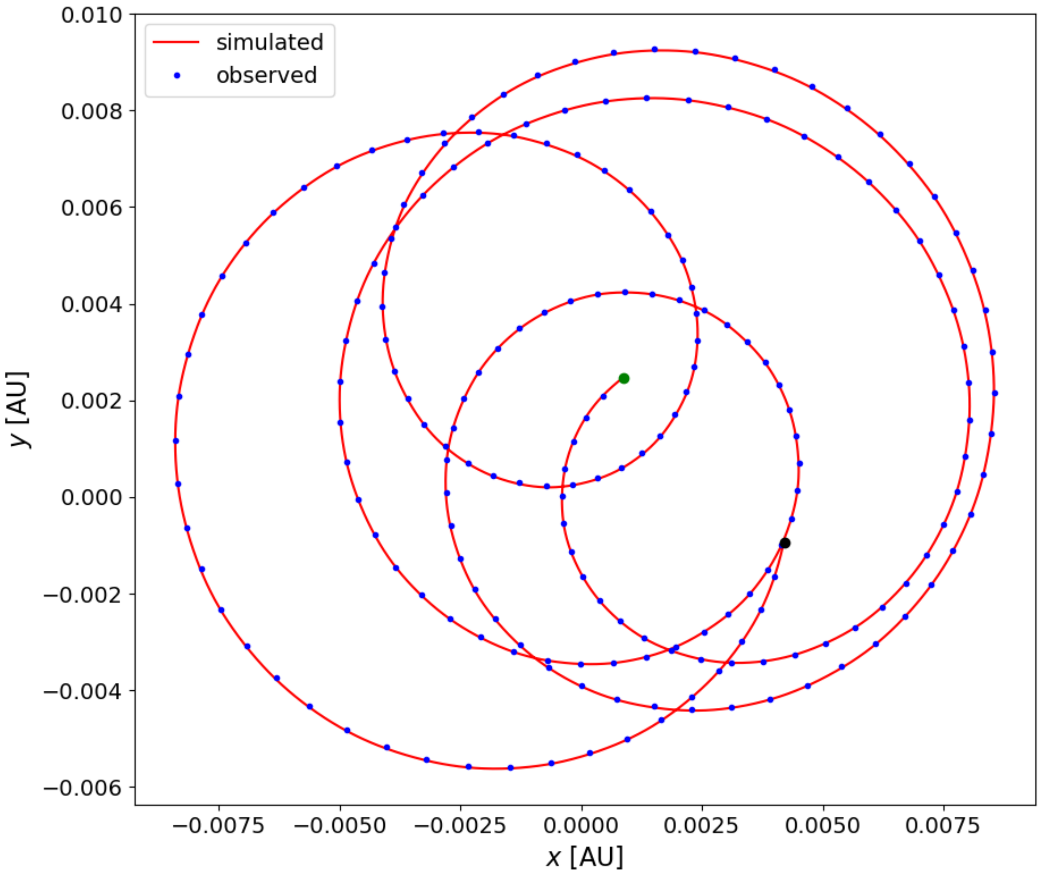

Figure 6: Solar System simulation focusing on the Sun's trajectory about the Solar System Barycenter which is at $(0, 0, 0)$. The simulated and observed positions are of the Sun's center. The initial conditions were taken from JPL Horizons ephemeris data for the date 1st January 1950 and the system was integrated over a period of 20,000 days, about 55 years, until the date 4th October 2004. As before, green and black dots correspond to the initial and final positions, respectively, while all the dots (observed positions) are spaced 100 days apart. This time the system included only the Sun, planets and Pluto, due to the long computation time. Studying a start's orbit about its system's barycenter can provide clues about other bodies in the system when direct detection is not possible, i.e., when only the star can be seen.

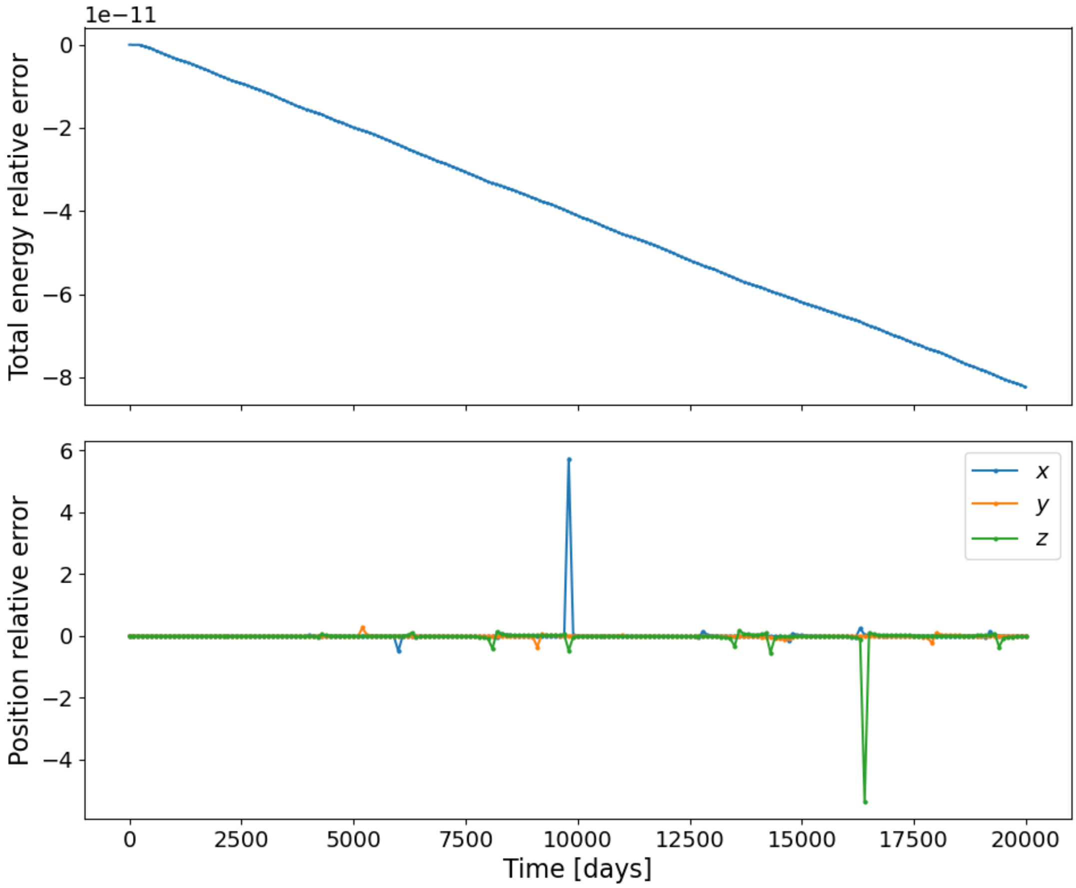

Figure 7: The plot on the top shows the total energy relative error for the system in Figure 6, which consisted of the Sun and planets, while the plot on the bottom is the position relative errors of the Sun's center, calculated at every 100 days using JPL Horizons ephemeris data. The relative and absolute error tolerances for the solver were set at 1×10^{-12} and 1×10^{-14}, respectively, as for the system in Figure 4. The top plot shows clearly the increase of the magnitude of the energy relative error which is a result of the propagating numerical errors. Despite this the energy relative error is still small at the end of the integration, about -8×10^{-11}, and the effect on the position relative errors of the Sun cannot be observed, as can be seen from the bottom plot (the two spikes could be taken as outliers).

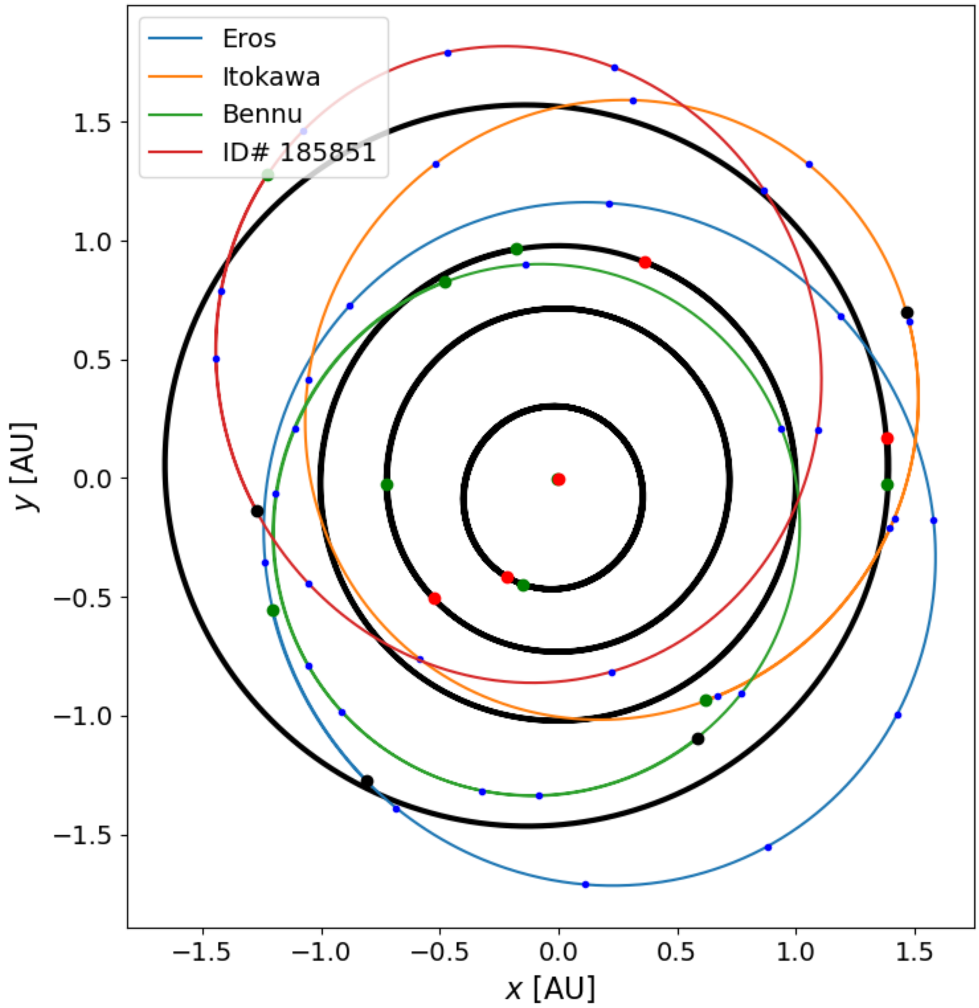

Figure 8: Simulation showing the trajectories of four near-Earth objects, three of which have been classified as a potentially hazardous asteroid (PHA). The black lines are the trajectories of the planets Mercury – Mars, and the Sun is in the center. The JPL Horizons system provides ephemerides for over 27,000 near-Earth objects (NEOs). Of those, four have their gravitational parameter, GM, value listed on the system, which is required to run this simulation. Their trajectories have been integrated here for a period of 700 days, 1st January 2000 – 1st December 2001, and the dots on the plot (observed positions) are spaced 70 days apart. The integration included the Sun, planets and the four NEOs.

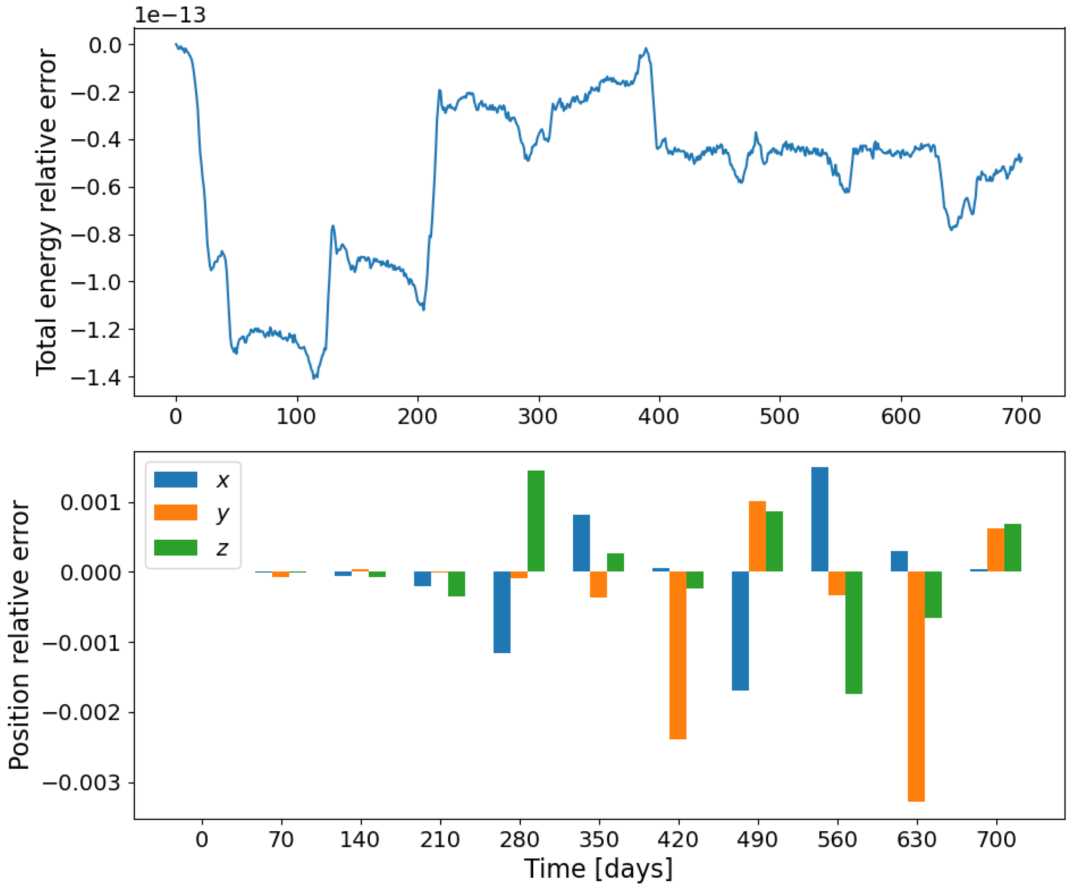

Figure 9: The plot on the top is the total energy relative error for the system in Figure 8 – a simulation of the Solar System and four NEOs. The plot on the bottom is the position relative error of the asteroid *Itokawa* calculated at every 70 days from JPL Horizons ephemeris data. The integrator parameters were set as for the other Solar System simulations in Figures 4 and 6. The errors in both plots are relatively low as expected.

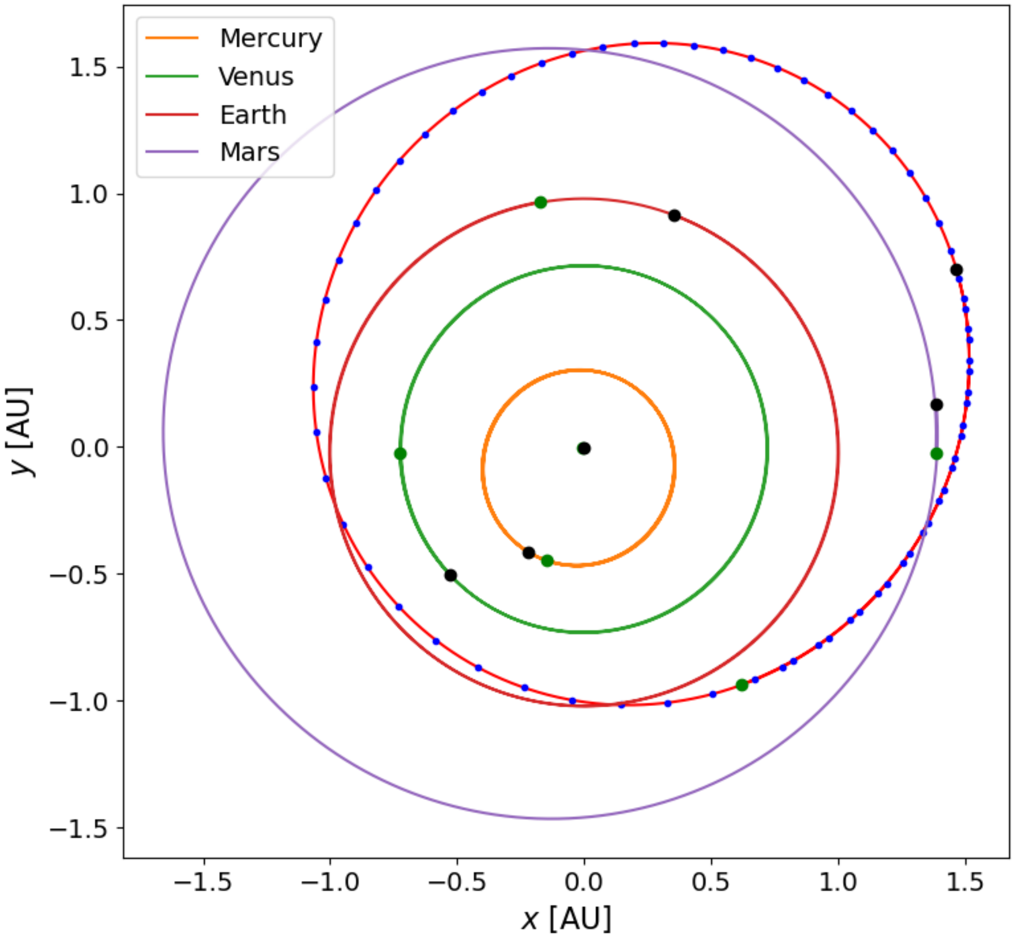

Figure 10: Plot of the trajectory of the asteroid Itokawa over the same period as in Figure 8 but integrated using an approximation. If it can be assumed that a body is small enough that its gravitational field has no effect on the motion of larger bodies in the system over a certain period, then the trajectory of the small body can be solved from knowledge of the positions of the larger bodies in the system. The system of coupled differential equations, $\vec{a_i} = \sum_{j \neq i} G m_j \vec{r_{ij}} / r_{ij}^3$, effectively reduces to one equation for the small body. This has the potential to reduce computation time significantly. For this simulation, ephemeris data for the Sun and planets were downloaded via the Horizons API for the same time period as in Figure 8 but with points spaced only 0.1 days apart. The trajectory of the asteroid Itokawa was then integrated from the initial conditions using this position data. On the figure, the solid line trajectories of the planets are from observed ephemeris data, while the red solid line with blue dots is the integrated trajectory of the asteroid. The blue dots represent the observed trajectory of the asteroid at regular intervals for comparison with the integrated trajectory.

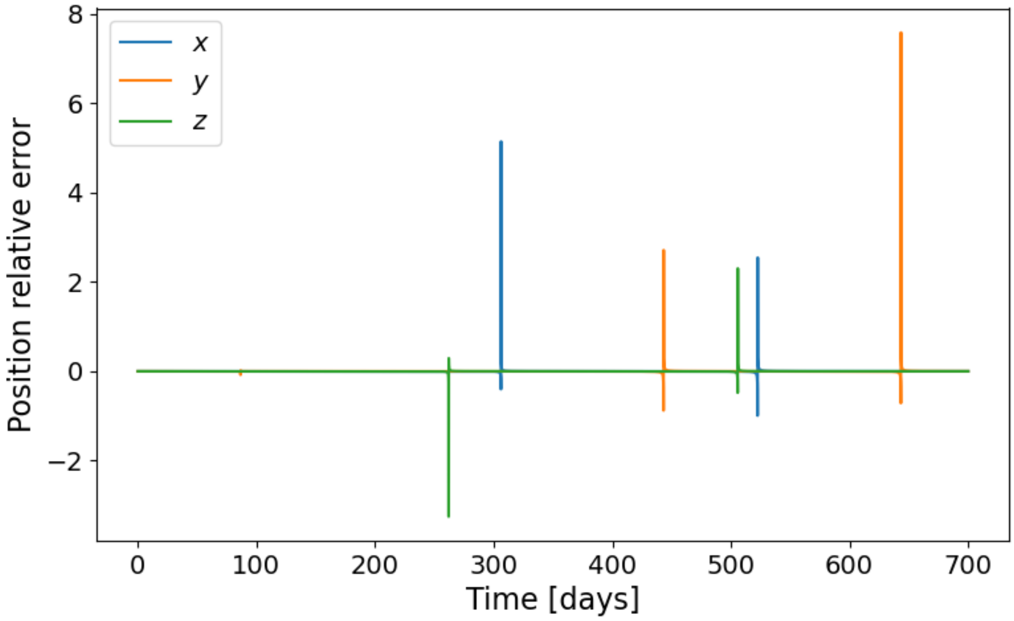

Figure 11: Plot of position relative error for the asteroid Itokawa from the system in Figure 10 which was integrated using an approximation where the effect of the small body on the larger bodies was ignored. The relative error is calculated at every 0.1 days using observed ephemeris data. Overall, the simulated and observed positions agree well, except at a few outlier positions. In order to use all the position data available, the step sizes for the integrator had to be fixed to match up with gaps in the data. This had the effect that the computation time was not significantly faster compared with the full simulation (Figure 8) which also included three additional NEOs, because the integrator was not able to optimise the step sizes based on the required error tolerances. In this case, the error tolerances were left at their default values, while the step sizes were fixed.