➢ Task 1:

a) First order logic representation:

$$extends(a,b)$$
$$\wedge\ extends(c,d)$$
$$\wedge\ extends(d,e)$$
$$\wedge\ \forall X \forall Y.(extends(X,Y) \rightarrow subtype(X,Y))$$
$$\wedge\ \forall X \forall Y.(\exists Z.(extends(X,Z) \wedge (subtype(Z,Y)) \rightarrow subtype(X,Y))$$

b) Model (Logical consequences):

$$extends(a,b)$$
$$\wedge\ extends(c,d)$$
$$\wedge\ extends(d,e)$$
$$\wedge\ subtype(a,b)$$
$$\wedge\ subtype(c,d)$$
$$\wedge\ subtype(d,e)$$
$$\wedge\ subtype(c,e)$$

➢ Task 2:

The model that represent the prolog program:

$$extends(class(a),class(b))$$
$$\wedge extends(class(c),class(d))$$
$$\wedge extends(class(d),class(e))$$
$$\wedge subtype(class(a),class(b))$$
$$\wedge subtype(class(c),class(d))$$
$$\wedge subtype(class(d),class(e))$$
$$\wedge subtype(class(c),class(e))$$

➢ Task 3:

a) The model:

To get the model we need to create the FOL representation first:

$$natural(0)$$
$$\wedge\ \forall X(natural(X)\ \rightarrow natural(s(X))$$

Then we get this model:

$$natural(0)$$
$$s(s(s(s(s(s(s(s(s(s(s(s(s(s(s(s.........$$

b) Difficulties in a: is a cyclic substitution occurrence

c) Comparison with Task 2:

A function symbol occurrence in the content of a head will produce a (cyclic substitution occurrence) which will lead to infinite loop

➢ Task 4:

a) likes(calvin,hobbes)=likes(X,Y)

$$\sigma_a = \{X \leftarrow calvin,\ Y \leftarrow hobbes\}$$

b) likes(calvin,hobbes)=likes(X,susie)

This unification cannot be done because $X \leftarrow calvin$ is okay but the other argument $hobbes \neq susie$

c) father(Jim, father(X))=grandfather(john, jane)

This unification don't succeed because the two predicates are different and can't be unified.

d) append([A,B,C], [D,E,F], G)=append([h,i,j], [k,l,m], [N|O])

$$\sigma_d = \{A \leftarrow h,\ B \leftarrow i, C \leftarrow j, D \leftarrow k, E \leftarrow l, F \leftarrow m, G \leftarrow [N|O]\}$$

e) [a,[b|H]|C]=[a,b,c,d]

$$\sigma_e = \{H \leftarrow c,\ C \leftarrow d\}$$

f) [[X,Y],e|[y,z]]=[A,B,C,D]

Cannot be unified because it has a different number of argument

➢ Task 5:

a) $f(X,Y)\{X \leftarrow 'Z'\} \equiv f('Z',Y)$

b) $g(X,Y)\{X \leftarrow 2, Y \leftarrow g(X)\} \equiv g(2,g(2))$

c) $h(X,Y)\{X \leftarrow h(Z,Y), Y \leftarrow h(Z), Z \leftarrow 3)\} \equiv h(h(3,h(3)),h(3))$

➢ Task 6:

a) {Xh(Z,Y)} {Yh(Z), Z3}

$$\sigma_1\sigma_2 = \{X \leftarrow h(3,h(3)), Y \leftarrow h(Z), Z \leftarrow 3\}$$

b) {Xh(Z,Y)} {X h(Z), Z 3}

$$\sigma_3\sigma_4 = \{h(Z) \leftarrow h(3,Y), Z \leftarrow 3, X \leftarrow h(Z)\}$$

c) {Xh(Z,Y)} {Y h(Z), Z X}

$$\sigma_5\sigma_6 = \{X \leftarrow h(X,h(X)), Y \leftarrow h(Z), Z \leftarrow X\}$$