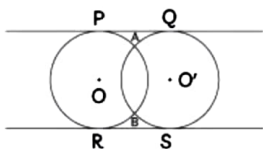


Previous Year Questions 2024

Q1: The maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is: (CBSE 2024)

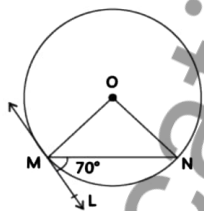
- (a) 4
- (b) 3
- (c) 2
- (d) 1

Ans: (c)



Here, circle with centre O and O' are intersecting at two distinct points A and B. So, in this situation PQ, RS are the tangents which can be drawn.

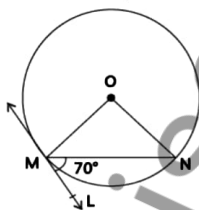
Q2: In the given figure, O is the centre of the circle. MN is the chord and the tangent ML at point M makes an angle of 70° with MN. The measure of $\angle MON$ is: (CBSE 2024)



- (a) 120°
- (b) 140°
- (c) 70°
- (d) 90°

Ans: (b)

$OM \perp ML$ [as tangent from centre is \perp at point of contact]



$$\angle OML = 90^\circ$$

$$\text{and } \angle NML = 70^\circ$$

$$\Rightarrow \angle OMN = 90^\circ - 70^\circ = 20^\circ$$

$$\because OM = ON = \text{Radii of same circle}$$

$$\therefore \angle OMN = \angle ONM = 20^\circ$$

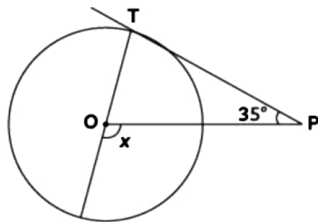
In $\triangle OMN$,

$$\angle OMN + \angle ONM + \angle MON = 180^\circ$$

$$\Rightarrow 20^\circ + 20^\circ + \angle MON = 180^\circ$$

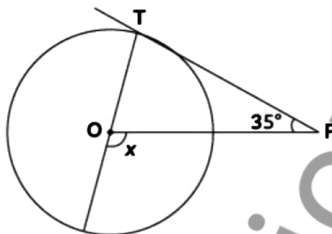
$$\Rightarrow \angle MON = 140^\circ$$

Q3: In the given figure, if PT is tangent to a circle with centre O and $\angle TPO = 35^\circ$, then the measure of $\angle x$ is (CBSE 2024)



- (a) 110°
- (b) 115°
- (c) 120°
- (d) 125°

Ans: (d)



$\angle OTP = 90^\circ$ [Line from centre is \perp to tangent at point of contact]

$\angle x = \angle TPO + \angle OTP$ [Exterior Angle Prop.]

$$x = 35^\circ + 90^\circ = 125^\circ$$

Previous Year Questions 2023

Q4: In the given figure, PT is a tangent at T to the circle with centre O. If $\angle TPO = 25^\circ$, then x is equal to:



- (a) 25°
- (b) 65°
- (c) 90°
- (d) 115° (CBSE 2023)

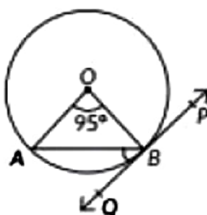
Ans: (d)

Since tangent is perpendicular to radius at the point of contact.

$$\therefore \angle PTO = 90^\circ$$

Hence, by the exterior angle formula, in $\triangle OTP$, we get $x = 90^\circ + 25^\circ$
 $= 115^\circ$

Q5: In the given figure, PQ is tangent to the circle centred at O. If $\angle AOB = 95^\circ$, then the measure of $\angle ABQ$ will be (2023)



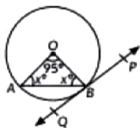
- (a) 47.5°
- (b) 42.5°
- (c) 85°
- (d) 95°

Ans: (a)

We have $\angle AOB = 95^\circ$

In $\triangle AOB$, $\angle OAB = \angle OBA$

Now, $\angle OAB + 95^\circ + \angle OBA = 180^\circ$ (Angle sum property of a triangle)



$$\Rightarrow \angle OAB = \frac{85^\circ}{2} = 42.5^\circ$$

$\therefore \angle OAB = \angle OBA = 42.5^\circ$ [From (i)]

Now, OB is perpendicular to the tangent line PQ

$\angle OBQ = 90^\circ$

OA = OB (Radius of circle)

So $\angle OAB = \angle OBA$

$95 + 2x = 180$ (Sum of angles of a triangle is 180)

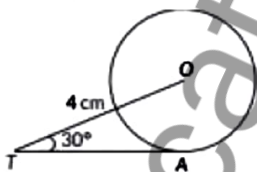
$$2x = 85$$

$$\Rightarrow x = 42.5$$

$$\angle ABQ = 90 - 42.5$$

$$= 47.5$$

Q6: In the given figure. TA is a tangent to the circle with centre O such that OT = 4 cm, $\angle OTA = 30^\circ$, then length of TA is (2023)



(a) $2\sqrt{3}$ cm

(b) 2cm

(c) $2\sqrt{2}$ cm

(d) $\sqrt{3}$ cm

Ans: (a)

Draw $OA \perp TA$.

In $\triangle OTA$ $\angle OAT = 90^\circ$ [\because Tangent to a circle is perpendicular to the radius passing through the point of contact]

and $\angle OTA = 30^\circ$

$$\therefore \frac{TA}{OT} = \cos 30^\circ \Rightarrow TA = 4 \cos 30^\circ = 4 \times \frac{\sqrt{3}}{2}$$

$$TA = 2\sqrt{3} \text{ cm}$$

Q7: In figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90^\circ$, then length of PQ is (2023)

(a) 3 cm

(b) 4 cm

(c) 2 cm

(d) $2\sqrt{2}$ cm

Ans: (b)

Join OR.

We know that tangent to a circle is \perp to radius at the point of contact. So, $QQ \perp PQ$ and

$QR \perp PR$.

Also, $\angle QPR = 90^\circ$

Now, in quadrilateral OQPR,

$$\angle QOR = 360^\circ - (90^\circ + 90^\circ + 90^\circ)$$

$$= 90^\circ$$

Also, $PQ = PR$ [\because Tangents drawn from an external point are equal]

\therefore PQQR is a square.

Hence, $PQ = OQ = 4 \text{ cm}$

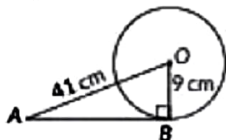
Q8: The length of tangent drawn to a circle of radius 9 cm from a point 41 cm from the centre is (2023)

- (a) 40 cm
- (b) 9 cm
- (c) 41 cm
- (d) 50 cm

Ans: (a)

$OB \perp AB$ [\because As tangent to a circle is perpendicular to the radius through the point of the contact]

In $\triangle OAB$,



$$OA^2 = OB^2 + AB^2 \text{ [By Pythagoras theorem]}$$

$$\Rightarrow (41)^2 = 9^2 + AB^2$$

$$\Rightarrow AB^2 = 41^2 - 9^2$$

$$= (41 - 9)(41 + 9)$$

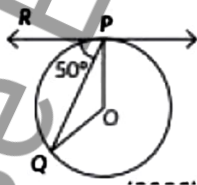
$$= (32)(50)$$

$$= 1600$$

$$\Rightarrow AB = \sqrt{1600}$$

$$= 40 \text{ cm}$$

Q9: In the given figure. O is the centre of the circle and PQ is the chord. If the tangent PR at P makes an angle of 50° with PQ, then the measure of $\angle POQ$ is (2023)



- (a) 50°
- (b) 40°
- (c) 100°
- (d) 130°

Ans: (c)

PR is tangent which touches circle at point P.

$$\text{So, } \angle OPR = 90^\circ$$

$$\angle OPQ = 90^\circ - \angle RPQ = 90^\circ - 50^\circ = 40^\circ$$

In $\triangle POQ$,

$$OP = OQ \text{ (Radii of circle)}$$

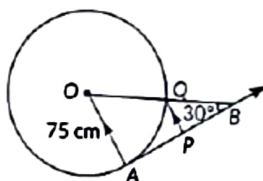
$$\text{So, } \angle OQP = \angle OPQ = 40^\circ$$

$$\Rightarrow \angle POQ = 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

Q10: Case Study: The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is the centre of the circle and $\angle ABO = 30^\circ$. PQ is parallel to OA.



Based on above information

(a) Find the length of AB.

(b) Find the length of OB.

(c) Find the length of AP.

OR

Find the value of PQ. (2023)

Ans:

(a): Given, $\angle ABO = 30^\circ$, $OA = 75$ cm

In $\triangle OAB$, $\tan 30^\circ = \frac{OA}{AB}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{AB} \Rightarrow AB = 75\sqrt{3} \text{ cm}$$

(b) In $\triangle OAB$, $\sin 30^\circ = \frac{OA}{OB}$

$$\Rightarrow \frac{1}{2} = \frac{75}{OB} \Rightarrow OB = 150 \text{ cm}$$

(c) In $\triangle OAB$, $PQ \parallel OA$

$$\frac{QB}{QO} = \frac{BP}{AP}$$

$$\Rightarrow \frac{150 - 75}{75} = \frac{AB}{AP} - 1 \Rightarrow 2 = \frac{AB}{AP} = \frac{75\sqrt{3}}{AP}$$

$$\Rightarrow AP = 75 \times \frac{\sqrt{3}}{2} \Rightarrow AP = \frac{75\sqrt{3}}{2} \text{ cm}$$

OR

$OA = OQ = 75$ cm

(\therefore Radius)

In $\triangle OAB$,

We have, $PQ \parallel OA$

In $\triangle BQP$ and $\triangle BOA$

$\angle BQP = \angle BOA$ (corresponding angles)

$\angle B = \angle B$ (common)

$\therefore \triangle BQP \sim \triangle BOA$ (By AA similarity)

$$\therefore \frac{BQ}{BO} = \frac{QP}{OA} = \frac{BP}{BA}$$

$$\Rightarrow \frac{PQ}{75} = \frac{AB - AP}{AB}$$

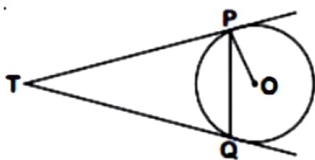
$$\Rightarrow \frac{PQ}{75} = 1 - \frac{AP}{AB}$$

$$\Rightarrow \frac{PQ}{75} = 1 - \frac{75\sqrt{3}}{2 \times 75\sqrt{3}}$$

$$\Rightarrow \frac{PQ}{75} = \frac{1}{2} \therefore PQ = \frac{75}{2} = 37.5$$

$$\Rightarrow PQ = 37.5 \text{ cm}$$

Q11: Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$. (2023)



Ans:

We know that the lengths of tangents drawn from an external point to a circle are equal.

$$\therefore TP = TQ \dots (1)$$

$$\therefore \angle TQP = \angle TPQ \text{ (angles of equal sides are equal)} \dots (2)$$

Now, PT is tangent, and OP is the radius.

$\therefore OP \perp TP$ (Tangent at any point of a circle is perpendicular to the radius through the point of contact)

$$\therefore \angle OPT = 90^\circ$$

$$\text{or, } \angle OPQ + \angle TPQ = 90^\circ$$

$$\text{or, } \angle TPQ = 90^\circ - \angle OPQ \dots (3)$$

In $\triangle TPQ$,

$$\angle TPQ + \angle PQT + \angle QTP = 180^\circ \text{ (Sum of angles of a triangle is } 180^\circ)$$

$$\text{or, } 90^\circ - \angle OPQ + \angle TPQ + \angle QTP = 180^\circ$$

$$\text{or, } \angle (90^\circ - \angle OPQ) + \angle TPQ + \angle QTP = 180^\circ \text{ [from (2) and (3)]}$$

$$\text{or, } 180^\circ - 2\angle OPQ + \angle TPQ = 180^\circ$$

$$\text{or, } 2\angle OPQ = \angle TPQ - \text{proved}$$

Q12: In the given figure, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^\circ$. If $AD = 17$ cm, $AB = 20$ cm and $DS = 3$ cm, then find the radius of the circle. (2023)

Ans: Given, $\angle B = 90^\circ$, $AD = 17$ cm, $AB = 20$ cm, $DS = 3$ cm

Now, $DS = DR$ and $AR = AQ$ [\because Tangents drawn from an external point to the circle are equal]

$$\therefore DR = 3 \text{ cm}$$

$$AR = AD - DR = 17 - 3 = 14 \text{ cm}$$

$$\therefore AQ = 14 \text{ cm}$$

$$\text{Now, } BQ = AB - AQ = 20 - 14 = 6 \text{ cm}$$

$OQ \perp BQ$, $OP \perp BP$ [\because Tangent at any point of a circle is perpendicular to the radius through the point of contact]

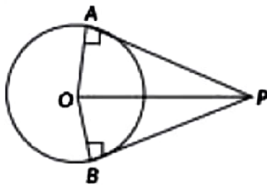
\therefore Quadrilateral BQOP is a square

$$\therefore BQ = OQ = r = 6 \text{ cm}$$

Hence, the radius of the circle = 6 cm.

Q13: From an external point, two tangents are drawn to a circle. Prove that the line joining the external point to the centre of the circle bisects the angle between the two tangents. (CBSE 2023)

Ans: Let P lie an external point, O be the centre of the circle and PA and PB are two tangents to the circle as shown in figure.



In $\triangle QAP$ and $\triangle OBP$.

OA = OB [Radius of the circle]

OP = OP [common]

PA = PB

\therefore Tangents drawn from an external point to a circle are equal]

So, $\triangle OAP = \triangle OPB$

So, $\angle APO = \angle BPO$

Hence, OP bisects $\angle APB$

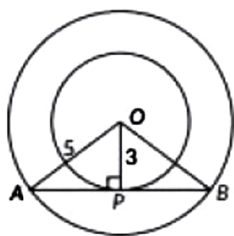
Q14: Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. (CBSE 2023)

Ans: Let the centre of the two concentric circle is O and AB be the chord of the larger circle which touches the smaller circle at point P as shown in figure.

\therefore AB is a tangent to the smaller circle at point P

$\Rightarrow OP \perp AB$

By Pythagoras theorem, in $\triangle OPA$



$$OA^2 = AP^2 + OP^2$$

$$\Rightarrow 5^2 = AP^2 + 3^2$$

$$\Rightarrow AP^2 = 5^2 - 3^2 = 25 - 9$$

$$\Rightarrow AP^2 = 16 \Rightarrow AP = 4\text{cm}$$

$$\therefore AB = 2AP = 8\text{cm}$$

In $\triangle OPB$ Since, $OP \perp AB$

AP = PB [\because Perpendicular drawn from the centre of the circle bisects the chord]

$$\therefore AS = 2AP = 2 \times 4 = 8\text{ cm}$$

\therefore The length of the chord of the larger circle is 8 cm.

Q15: Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre. (2023)

Ans: Let PA and PB are two tangents on a circle from point P as shown in the figure.

Let is known that tangent to a circle is perpendicular to the radius through the point of contact.

$$\angle OAP = \angle OBP = 90^\circ$$

In quadrilateral AOBP,

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$$

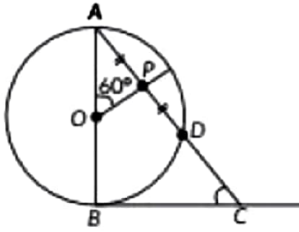
$$90^\circ + \angle APB + 90^\circ + \angle BOA = 360^\circ \quad [\text{Using (i)}]$$

$$\angle APB + \angle BOA = 360^\circ - 180^\circ$$

$$\therefore \angle APB + \angle BOA = 180^\circ$$

Previous Year Questions 2022

Q16: In Fig, AB is the diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60^\circ$, then find $m\angle C$. (2022)



Ans: Since, OP bisects the chord AD, therefore $\angle OPA = 90^\circ$...[\because The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]

Now, In $\triangle AOP$,

$$\angle A = 180^\circ - 60^\circ - 90^\circ$$

$$= 120^\circ - 90^\circ$$

$$= 30^\circ$$

Also, we know that the tangent at any point of a circle is perpendicular to the radius through the point of contact

$$\therefore \angle ABC = 90^\circ$$

Now, In $\triangle ABC$,

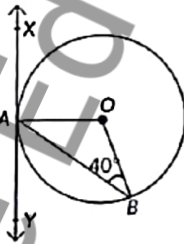
$$\angle C = 180^\circ - \angle A - \angle B$$

$$= 180^\circ - 30^\circ - 90^\circ$$

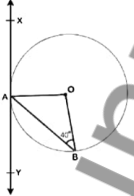
$$= 150^\circ - 90^\circ$$

$$= 60^\circ$$

Q17: In Fig, XAY is a tangent to the circle centred at O. If $\angle ABO = 40^\circ$. Then find $\angle BAY$ and $\angle AOB$ (2022)



Ans:



Given, $\angle ABO = 40^\circ$

$\angle XAO = 90^\circ$... (Angle between radius and tangent)

$OA = OB$... (Radii of same circle)

$$\Rightarrow \angle OAB = \angle OBA$$

$$\therefore \angle OAB = 40^\circ$$

Now, applying the linear pair of angles property, we get

$$\angle BAY + \angle OAB + \angle XAO = 180^\circ$$

$$\Rightarrow \angle BAY + 40^\circ + 90^\circ = 180^\circ$$

$$\Rightarrow \angle BAY + 130^\circ = 180^\circ$$

$$\Rightarrow \angle BAY = 180^\circ - 130^\circ$$

$$\Rightarrow \angle BAY = 50^\circ$$

Now, In $\triangle AOB$,

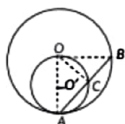
$$\angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$\text{or, } \angle AOB + 40^\circ + 40^\circ = 180^\circ$$

$$\text{or, } \angle AOB = 180^\circ - 80^\circ = 100^\circ$$

Hence proved.

Q18: In Figure, two circles with centres at O and O' of radii 2r and r, respectively, touch each other internally at A. A chord AB of the bigger circle meets the smaller circle at C. Show that C bisects AB. (2022)



Ans: Given: Two circles with centres O and O' of radii 2r and r respectively, touch each other internally at A, AB is the chord of bigger circle touches the smaller circle at C.

To prove: C bisects AB i.e. AC = CB

Here, for smaller circle (O' r)

$$\angle ACO = 90^\circ \text{ (Angle in a semicircle is } 90^\circ)$$

$$\therefore OC \perp AC$$

Now, in bigger circle (O, 2r)

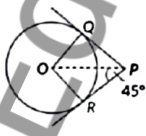
Since, AB is a chord and $OC \perp AB$.

$$AC = CB$$

[\because Perpendicular drawn from centre of the circle to a chord bisects the chord]

Hence, C bisects the chord AB.

Q19: In Figure, PQ and PR are tangents to the circle centred at O. If $\angle OPR = 45^\circ$, then prove that ORPQ is a square. (2022)



Ans: It is given that $\angle QPR = 90^\circ$

We know that the lengths of the tangents drawn from the outer point to the circle are equal.

$$PQ = PR \dots (1)$$

The radius is Perpendicular to the tangent line at the point of contact.

$$\therefore \angle PQO = 90^\circ$$

and

$$\angle ORP = 90^\circ$$

In quadrilateral OQPR:

$$\angle QPR + \angle PQO + \angle QOR + \angle ORP = 360^\circ$$

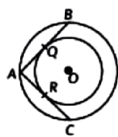
$$\Rightarrow 90^\circ + 90^\circ + \angle QOR + 90^\circ = 360^\circ$$

$$\Rightarrow \angle QOR = 360^\circ - 270^\circ = 90^\circ$$

$$\therefore \angle QPR = \angle PQO = \angle QOR = \angle ORP = 90^\circ$$

It can be concluded that PQOR is a square.

Q20: In Fig., there are two concentric circles with centre O. If ARC and AQB are tangents to the smaller circle from point A lying on the larger circle, find the length of AC if AO = 5 cm. (2022)



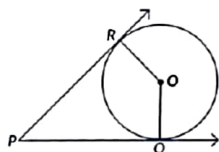
Ans: Given, AQ = 5 cm

AQ = AR = 5 cm (∵ Tangents drawn from an external point to the circle are equal)

Now, AC = AR + RC (∵ OR is a perpendicular bisector of AC AR = RC)

AC = 10 cm

Q21: In Figure, O is the centre of the circle. PQ and PR are tangent segments. Show that the quadrilateral PQOR is cyclic. (2022)



Ans: Given: PQ and PR are tangents from an external point P.

To prove: PQOR is a cyclic quadrilateral.

Proof OR and OQ are the radius of circle centred at O, and PR and PQ are tangents.

$\angle ORP = 90^\circ$ and $\angle OQP = 90^\circ$

In quadrilateral PQOR, we have

$\angle OQP + \angle QOR + \angle ORP + \angle RPQ = 360^\circ$

$90^\circ + \angle QOR + 90^\circ + \angle RPQ = 360^\circ$

$180^\circ + \angle QOR + \angle RPQ = 360^\circ$

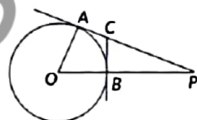
$\angle QOR + \angle RPQ = 360^\circ - 180^\circ$

So, $\angle O + \angle P = 180^\circ$

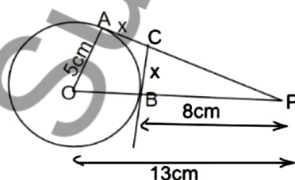
$\angle P$ and $\angle O$ are opposite angles of quadrilateral which are supplementary.

∴ PQOR is a cyclic quadrilateral.

Q22: In Figure O is centre of a circle of radius 5 cm. PA and BC are tangents to the circle at A and B respectively. If OP = 13 cm. then find the length of tangents PA and BC. (2022)



Ans:



Given, radius of circle = 5 cm

PA and BC are two tangent at point A and B

OP = 13 cm

Step1: OA is perpendicular on tangent AP (OA is radius of the circle)

In right angle triangle AOAP

$$(OP)^2 = (OA)^2 + (AP)^2$$

$$\Rightarrow (AP)^2 = (OP)^2 - (OA)^2$$

$$\Rightarrow (AP)^2 = (13)^2 - (5)^2 = 169 - 25 = 144$$

$$AP = \sqrt{144} = 12$$

$$AP = PA = 12 \text{ cm}$$

Step 2: Let length of BC be x

But AC = BC = x (tangent from an external point)

So length of PC = 12 - x and PB = OP - OB = 13 - 5 cm

(OB is the radius and length of OP is given)

OB is perpendicular on tangent CB, so $\angle OBC = \angle CBP = 90^\circ$

In right angle triangle ΔCBP

$$(CP)^2 = (BP)^2 + (BC)^2$$

$$\Rightarrow (12 - x)^2 = (8)^2 + (x)^2$$

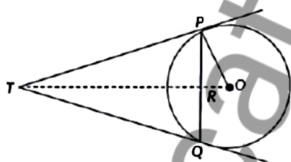
$$\Rightarrow 144 - 24x + x^2 = 64 + x^2$$

$$\Rightarrow 144 - 24x - 64 = 0$$

$$\Rightarrow 80 - 24x = 0 \Rightarrow x = 80/24 = 3.33 \text{ cm}$$

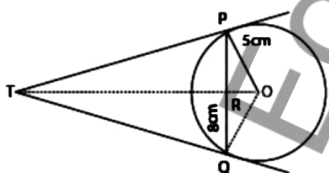
Hence, the length of BC is 3.33 cm and PA is 12 cm

Q23: In fig. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q meet at a point T. Find the length of TP. (2022)



Ans: In the given figure,

PQ = 8 cm and OP = 5 cm



$OR \perp PQ$ and so, OR bisects PQ. [\because Perpendicular drawn from the center to the chord bisects the chord]

$$\Rightarrow PR = RQ = 4 \text{ cm}$$

In ΔPOR ,

$$OP^2 = OR^2 + PR^2$$

$$\Rightarrow 5^2 = OR^2 + 4^2$$

$$\Rightarrow OR = 3 \text{ cm}$$

In ΔTPO and ΔPRO ,

$\angle TOP = \angle ROP$ [common]

and $\angle TPO = \angle PRO$ [each 90°]

$\therefore \Delta TPO$ and ΔPRO are similar. [by AAA Similarity]

$$\Rightarrow \frac{TP}{PO} = \frac{RO}{PR}$$

$$\Rightarrow \frac{TP}{5} = \frac{3}{4}$$

$$\therefore TP = \frac{20}{3} \text{ cm}$$

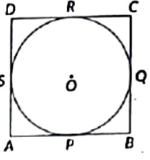
[\because Tangents drawn from an external point to a circle are equal in length]

Q24: Prove that a parallelogram circumscribing a circle is a rhombus. (2022)

Ans: Given : A parallelogram ABCD circumscribing a circle with centre O.

To prove : ABCD is a rhombus.

Proof: We know that the tangents drawn to a circle from an external Point are equal in length.



$$\Rightarrow AP = AS \text{ [Tangents drawn from A] } \dots(i)$$

$$\Rightarrow BP = BQ \text{ [Tangents drawn from B] } \dots(ii)$$

$$\Rightarrow CR = CQ \text{ [Tangents drawn from C] } \dots(iii)$$

$$\Rightarrow DR = DS \text{ [Tangents drawn from D] } \dots(iv)$$

Adding (i), (ii), (iii) and (iv) we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$= (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

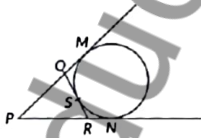
$$\Rightarrow 2AB = 2BC \text{ [Opposite sides of the given parallelogram are equal } \therefore AB = DC \text{ and } AD = BC]$$

$$AB = BC = DC = AD$$

$$AB = BC = DC = AD$$

Hence, ABCD is a rhombus.

Q25: In fig, If a circle touches the side QR of ΔPQR at S and extended sides PQ and PR at M and N, respectively, then



Prove that $PM = \frac{1}{2}(PQ + QR + PR)$ (2022)

Ans: Given: A circle is touching a side QR of ΔPQR at point S.

PQ and PR are produced at M and N respectively.

$$\text{To prove: } PM = \frac{1}{2} (PQ + QR + PR)$$

Proof: $PM = PN$...(i) (Tangents drawn from an external point P to a circle are equal)

$QM = QS$...(ii) (Tangents drawn from an external point Q to a circle are equal)

$RS = RN$...(iii) (Tangents drawn from an external point R to a circle are equal)

$$\text{Now, } 2PM = PM + PM$$

$$= PM + PN \dots[\text{From equation (i)}]$$

$$= (PQ + QM) + (PR + RN)$$

$$= PQ + QS + PR + RS \dots[\text{From equations (i) and (ii)}]$$

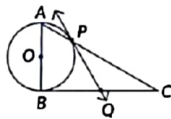
$$= PQ + (QS + SR) + PR$$

$$= PQ + QR + PR$$

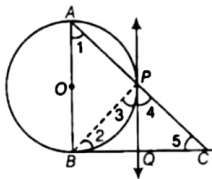
$$\therefore PM = \frac{1}{2} (PQ + QR + PR)$$

Hence proved.

Q26: In figure, a triangle ABC with $\angle B = 90^\circ$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P. Prove that the tangent drawn at point P bisects BC. (2022)



Ans:



According to the question,

In a right angle $\triangle ABC$ in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P.

Also PQ is a tangent at P

To Prove: PQ bisects BC i.e. $BQ = QC$

Proof: $\angle APB = 90^\circ$...[Angle in a semicircle is a right-angle]

$\angle BPC = 90^\circ$...[Linear Pair]

$\angle 3 + \angle 4 = 90^\circ$...[1]

Now, $\angle ABC = 90^\circ$

So in $\triangle ABC$

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$

$90^\circ + \angle 1 + \angle 5 = 180^\circ$

$\angle 1 + \angle 5 = 90^\circ$...[2]

Now, $\angle 1 = \angle 3$...[Angle between tangent and the chord equals angle made by the chord in alternate segment]

Using this in [2] we have

$\angle 3 + \angle 5 = 90^\circ$...[3]

From [1] and [3] we have

$\angle 3 + \angle 4 = \angle 3 + \angle 5$

$\angle 4 = \angle 5$

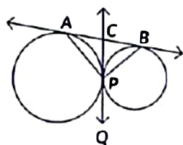
$QC = PQ$...[Sides opposite to equal angles are equal]

But also, $PQ = BQ$...[Tangents drawn from an external point to a circle are equal]

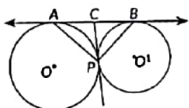
So, $BQ = QC$

i.e. PQ bisects BC.

Q27: In the figure, two circles touch externally at P. A common tangent touches them at A and B, and another common tangent is at P, which meets the common tangent AB at C. Prove that $\angle APB = 90^\circ$. (2022)



Ans: Let common tangent at P meets the tangent AB at C. Since, tangents drawn from an external point to a circle are equal



$\therefore AC = CP$

and $BC = CP$

$\Rightarrow \angle CAP = \angle CPA = x$ (say) ... (i)

and $\angle CBP = \angle CPB = y$ (say) ... (ii)

Now, $\angle ACP + \angle BCP = 180^\circ$ [Linear pair] ... (*)

In $\triangle ACP$, $\angle ACP + \angle CPA + \angle CAP = 180^\circ$... (iii)

and in $\triangle BCP$, $\angle BCP + \angle CPB + \angle CBP = 180^\circ$... (iv)

Adding (iii) and (iv), we get

$\angle ACP + x + x + \angle BCP + y + y = 360^\circ$

$\angle ACP + \angle BCP + 2x + 2y = 360^\circ$ [Using (i) & (ii)]

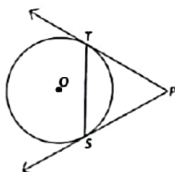
$= 2(x + y) = 360^\circ - 180^\circ = 180^\circ$ [Using (*)]

$\Rightarrow x + y = 90^\circ$

i.e., $\angle CPA + \angle CPB = 90^\circ \Rightarrow \angle APB = 90^\circ$

Previous Year Questions 2021

Q28: In the given figure, PT and PS are tangents to a circle with centre O, from a point P such that $PT = 4$ cm and $\angle TPS = 60^\circ$. Find the length of the chord TS. (2021)



Ans: Given TP and SP are tangents from an external point P.

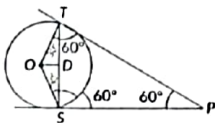
$PT = PS = 4$ cm (Tangents drawn from an external point to the circle are equal)

$\angle PTS = \angle PST$

(\because Angles opposite to equal sides are equal) In $\triangle TPS$, by angle sum property

$\angle TPS = \angle PTS = \angle PST = 60^\circ$

$\Rightarrow \triangle TPS$ is an equilateral triangle.



$\therefore TP = PS = TS = 4$ cm

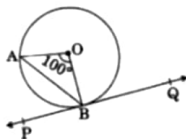
$\angle OSP = 90^\circ$ and $\angle TSP = 60^\circ$

$\therefore \angle OSD = 30^\circ$

Now, $\frac{DS}{OS} = \cos 30^\circ \Rightarrow \frac{2}{OS} = \frac{\sqrt{3}}{2} \Rightarrow OS = \frac{4\sqrt{3}}{3}$ cm

Previous Year Questions 2020

Q29: In figure, PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100^\circ$, then $\angle ABP$ is equal to (2020)



(a) 50°

(b) 40°

(c) 60°

(d) 80°

Ans: (a)

Given that

$$\angle AOB = 100^\circ$$

Since $OA = OB$

$$\text{So } \angle OAB = \angle OBA = 40^\circ$$

Since PQ is tangent on the circle. So OB is perpendicular to PQ .

So,

$$\angle OBP = 90^\circ$$

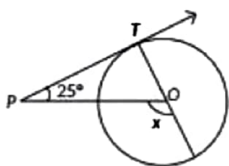
$$\angle OBA + \angle ABP = 90^\circ$$

$$\angle ABP = 90^\circ - \angle OBA$$

$$\therefore \angle ABP = 90^\circ - 40^\circ$$

$$\therefore \angle ABP = 50^\circ$$

Q30: In the given figure, PT is a tangent at T to the circle with centre O . If $\angle TPO = 25^\circ$, then x is equal to (2020)



(a) 25°

(b) 65°

(c) 90°

(d) 115°

Ans: (d)

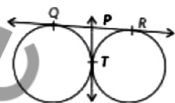
Since $\angle TPO = 25^\circ$ and $\angle OTP = 90^\circ$

$$x = \angle OTP + \angle TPO$$

$$= 90^\circ + 25^\circ = 115^\circ$$

[\because Radius is perpendicular to the tangent T]

Q31: In the given figure, QR is a common tangent to the given circles, touching externally at the point T . The tangent at T meets QR at P . If $PT = 3.8$ cm, then the length of QR (in cm) is (2020)



(a) 3.8

(b) 7.6

(c) 5.7

(d) 1.9

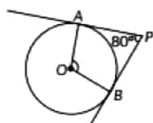
Ans: (b)

It is known that the length of the tangents drawn from an external point to a circle are equal.

$$\therefore QP = PT = 3.8 \text{ cm and } PR = PT = 3.8 \text{ cm}$$

$$\text{Now, } QR = QP + PR = 3.8 \text{ cm} + 3.8 \text{ cm} = 7.6 \text{ cm}$$

Q32: In Figure, if tangents PA and PB from an external point P to a circle with centre O are inclined to each other at an angle of 80° then $\angle POA$ is equal to (2020)



- (a) 50°
- (b) 60°
- (c) 80°
- (d) 100°

Ans: (a)

A tangent at any point of a circle is perpendicular to the radius at the point of contact.

In $\triangle OAP$ and in $\triangle OBP$:

- $OA = OB$ (radii of the circle are always equal)
- $AP = BP$ (length of the tangents)
- $OP = OP$ (common)

Therefore, by SSS congruency $\triangle OAP \cong \triangle OBP$.

SSS congruence rule: If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

If two triangles are congruent, then their corresponding parts are equal.

Hence:

- $\angle POA = \angle POB$
- $\angle OPA = \angle OPB$

Therefore, OP is the angle bisector of $\angle APB$ and $\angle AOB$.

Hence, $\angle OPA = \angle OPB = \frac{1}{2} (\angle APB)$

$$= \frac{1}{2} \times 80^\circ$$

$$= 40^\circ$$

By the angle sum property of a triangle, in $\triangle OAP$:

$$\angle A + \angle POA + \angle OPA = 180^\circ$$

$OA \perp AP$ (Theorem 10.1: The tangent at any point of a circle is perpendicular to the radius through the point of contact.)

Therefore, $\angle A = 90^\circ$

$$90^\circ + \angle POA + 40^\circ = 180^\circ$$

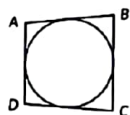
$$130^\circ + \angle POA = 180^\circ$$

$$\angle POA = 180^\circ - 130^\circ$$

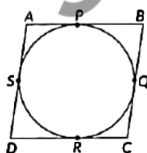
$$\angle POA = 50^\circ$$

Thus, option (A) 50° is the correct answer.

Q33: In figure, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = BC + AD$. (2020)



Ans: Let the circle touches the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively. Since, lengths of tangents drawn from an external point to the circle are equal.



$$AP = AS \quad \dots(1) \quad (\text{Tangents drawn from A})$$

$$BP = BQ \quad \dots(2) \quad (\text{Tangents drawn from B})$$

$CR = CQ$... (3) (Tangents drawn from C)

$DR = DS$... (4) (Tangents drawn from D)

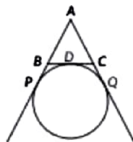
Adding (1), (2), (3) and (4), we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + PB) + (CR + RD) = (AS + SD) + (BQ + QC)$$

$$\Rightarrow AB + CD = AD + BC$$

Q34: In figure, find the perimeter of $\triangle ABC$ if $AP = 12$ cm. (2020)



Ans:

Step 1: Identify the Tangents

From the problem, we know that AP and AQ are tangents to the circle from point A , and BC is also a tangent. According to the properties of tangents from an external point, the lengths of the tangents drawn from the same external point to a circle are equal.

Step 2: Set Up the Equations

Since $AP = 12$ cm, we can conclude that:

- $AP = AQ = 12$ cm (Equation 1)

Step 3: Identify Other Tangents

From point B , the tangents BD and BP are equal:

- $BD = BP$ (Equation 2)

From point C , the tangents CD and CQ are equal:

- $CD = CQ$ (Equation 3)

Step 4: Express Perimeter of Triangle ABC

The perimeter of triangle ABC can be expressed as:

$$\text{Perimeter} = AB + BC + AC$$

Step 5: Substitute for BC

Since BC is composed of the tangents from B and C :

$$BC = BD + CD$$

Thus, we can rewrite the perimeter as:

$$\text{Perimeter} = AB + (BD + CD) + AC$$

Step 6: Express AB and AC in Terms of Tangents

From the properties of tangents:

- $AP = AB + BP$
- $AQ = AC + CQ$

Substituting BP and CQ with BD and CD respectively, we have:

- $AP = AB + BD$ (Equation 4)
- $AQ = AC + CD$ (Equation 5)

Step 7: Substitute Equations into Perimeter Now, substituting the expressions from Equations 4 and 5 into the perimeter equation: $\text{Perimeter} = (AP - BD) + (BD + CD) + (AQ - CD)$

Step 8: Simplify the Expression

Since $AP = AQ$ and both are equal to 12 cm:

$$\text{Perimeter} = (12 - BD) + (BD + CD) + (12 - CD)$$

This simplifies to:

$$\text{Perimeter} = 12 + 12 = 24 \text{ cm}$$