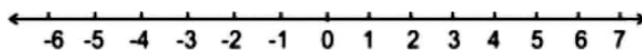


## NUMBER SYSTEMS

1. Rational Numbers
2. Irrational Numbers
3. Real Numbers and their Decimal Expansions
4. Operations on Real Numbers
5. Laws of Exponents for Real Numbers



- Natural numbers are - 1, 2, 3, ..... denoted by N.
- Whole numbers are - 0, 1, 2, 3, ..... denoted by W.
- Integers - ..... -3, -2, -1, 0, 1, 2, 3, ..... denoted by Z.
- Rational numbers - All the numbers which can be written in the form  $r/s$   $p/q$ , are called rational numbers where p and q are integers.
- Irrational numbers - A number s is called irrational, if it cannot be written in the form  $p/q$  where p and q are integers and
- The decimal expansion of a rational number is either terminating or non-terminating recurring. Thus we say that a number whose decimal expansion is either terminating or non-terminating recurring is a rational number.
- The decimal expansion of an irrational number is non terminating non-recurring.
- All the rational numbers and irrational numbers taken together.
- Make a collection of real number.
- A real no is either rational or irrational.
- If r is rational and s is irrational then  $r+s$ ,  $r-s$ ,  $r.s$  are always irrational numbers but  $r/s$  may be rational or irrational.
- Every irrational number can be represented on a number line using Pythagoras theorem.
- Rationalization means to remove square root from the denominator.

$\frac{3+\sqrt{5}}{\sqrt{2}}$  to remove we will multiply both numerator & denominator by  $\sqrt{2} \frac{1}{a \pm \sqrt{b}}$  its

rationalization factor  $a \mp \sqrt{b}$

## POLYNOMIALS

1. Polynomials in one Variable
2. Zeroes of a Polynomial
3. Remainder Theorem
4. Factorisation of Polynomials
5. Algebraic Identities

- **Constants:** A symbol having a fixed numerical value is called a constant.
- **Variables:** A symbol which may be assigned different numerical values is known as variable.
- **Algebraic expressions:** A combination of constants and variables. Connected by some or all of the operations  $+$ ,  $-$ ,  $\times$  and is known as algebraic expression.
- **Terms:** The several parts of an algebraic expression separated by '+' or '-' operations are called the terms of the expression.
- **Polynomials:** An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

(i)  $5x^2 - 4x^2 - 6x - 3$  is a polynomial in variable  $x$ .

(ii) (ii)  $5 + 8x^{\frac{3}{2}} + 4x^{-2}$  is an expression but not a polynomial.

Polynomials are denoted by  $p(x)$ ,  $q(x)$  and  $r(x)$  etc.

- **Coefficients:** In the polynomial  $x^3 + 3x^2 + 3x + 1$ , coefficient of  $x^3$ ,  $x^2$ ,  $x$  are 1, 3, 3 respectively and we also say that +1 is the constant term in it.
- **Degree of a polynomial in one variable:** In case of a polynomial in one variable the highest power of the variable is called the degree of the polynomial.
- **Classification of polynomials on the basis of degree.**

Degree	Polynomial	Example
(a) 1	Linear	$x + 1$ , $2x + 3$ etc.
(b) 2	Quadratic	$ax^2 + bx + c$ etc.
(c) 3	Cubic	$x^3 + 3x^2 + 1$ etc. etc.
(d) 4	Biquadratic	$x^4 - 1$

Classification of polynomials on the basis of no. of terms

No. of terms                      Polynomial & Examples.

(i)      1                      Monomial -  $5, 3x, \frac{1}{3}y$  etc.

(ii)     2                      Binomial -  $(3+6x), (x-5y)$  etc.

(iii)    3                      Trinomial-  $2x^2 + 4x + 2$  etc. etc.

- **Constant polynomial:** A polynomial containing one term only, consisting a constant term is called a constant polynomial the degree of non-zero constant polynomial is zero.
- **Zero polynomial:** A polynomial consisting of one term, namely zero only is called a zero polynomial. The degree of zero polynomial is not defined.
- **Zeroes of a polynomial:** Let  $p(x)$  be a polynomial. If  $p(\alpha)=0$ , then we say that is a zero of the polynomial of  $p(x)$ .
- **Remark:** Finding the zeroes of polynomial  $p(x)$  means solving the equation  $p(x)=0$ .
- **Remainder theorem:** Let  $f(x)$  be a polynomial of degree  $n \geq 1$  and let  $a$  be any real number. When  $f(x)$  is divided by  $(x-a)$  then the remainder is  $f(a)$
- **Factor theorem:** Let  $f(x)$  be a polynomial of degree  $n > 1$  and let  $a$  be any real number.
  - (i) If  $f(a) = 0$  then  $(x-a)$  is factor of  $f(x)$
  - (ii) If  $(x-a)$  is factor of  $f(x)$  then  $f(a) = 0$
- **Factor:** A polynomial  $p(x)$  is called factor of  $q(x)$  divides  $q(x)$  exactly.
- **Factorization:** To express a given polynomial as the product of polynomials each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorization.

Some algebraic identities useful in factorization:

(i)  $(x+y)^2 = x^2 + 2xy + y^2$

(ii)  $(x-y)^2 = x^2 - 2xy + y^2$

(iii)  $x^2 - y^2 = (x-y)(x+y)$

(iv)  $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v)  $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi)  $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(vii)  $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

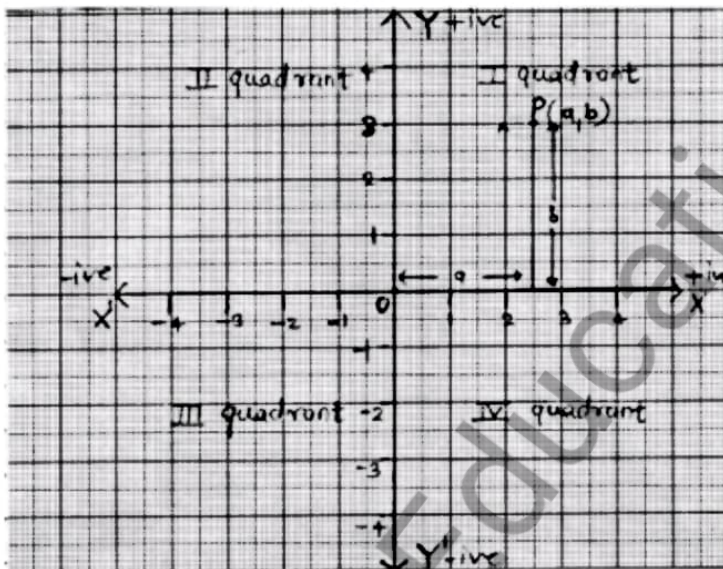
(viii)  $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 = 3xyz$  if  $x + y + z = 0$

## Coordinate Geometry

1. Cartesian System
2. Plotting a Point in the Plane with given Coordinates

- **Coordinate Geometry:** The branch of mathematics in which geometric problems are solved through algebra by using the coordinate system is known as coordinate geometry.
- **Coordinate System:** Coordinate axes: The position of a point in a plane is determined with reference to two fixed mutually perpendicular lines, called the coordinate axes.



In this system, position of a point is described by ordered pair of two numbers.

- **Ordered pair:** A pair of numbers  $a$  and  $b$  listed in a specific order with ' $a$ ' at the first place and ' $b$ ' at the second place is called an ordered pair  $(a,b)$

Note that  $(a,b) \neq (b,a)$

Thus  $(2,3)$  is one ordered pair and  $(3,2)$  is another ordered pair.

In given figure  $O$  is called origin.

The horizontal line  $XO$

is called the X-axis.

The vertical line  $YOY'$  is called the Y-axis.



$P(a,b)$  be any point in the plane. 'a' the first number denotes the distance of point from Y-axis and 'b' the second number denotes the distance of point from X-axis.

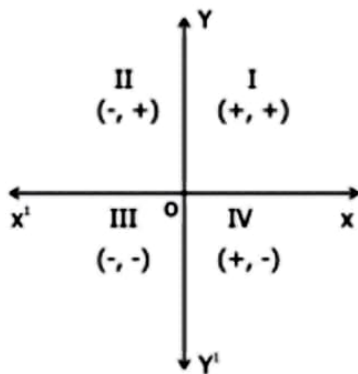
a - X - coordinate | abscissa of P.

b - Y - coordinate | ordinate of P.

The coordinates of origin are  $(0,0)$

Every point on the x-axis is at a distance 0 unit from the Y-axis. So its ordinate is 0.

Every point on the y-axis is at a distance 0 unit from the X-axis. So, its abscissa is 0.



Note: Any point lying on X-axis or Y-axis does not lie in any quadrant.

## Linear Equations in Two Variables

1. Linear Equations
2. Solution of a Linear Equation
3. Graph of a Linear Equation in Two Variables
4. Equations of Lines Parallel to x-axis and y-axis

- An equation of the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are real numbers such that  $a$  and  $b$  are not both zero is called a linear equation in two variables.
- A pair of values of  $x$  and  $y$  which satisfy the equation  $ax + by + c = 0$  is called a solution of the equation.
- **Graph:** The graph of every linear equation in two variables is a straight line. Every point on the graph of a linear equation in two variables is a solution of the linear equation. Conversely, every solution of the linear equation is a point on the graph of the linear equation.
- A linear equation in two variables has infinitely many solutions.
- The graph of every linear equation in two variables is a straight line.
- $y = 0$  is the equation of x-axis and  $x = 0$  is equation of y-axis.
- The graph of  $x = a$  is a straight line parallel to the y-axis.
- The graph of  $y = a$  is a straight line parallel to the x-axis.
- An equation of the type  $y = mx$  represent a line passing through the origin.

## Introduction to Euclid's Geometry

### 1. Euclid's Definitions, Axioms and Postulates

### 2. Equivalent Versions of Euclid's Fifth Postulate

The Greeks developed geometry in a systematic manner. Euclid (300 B.C.) a Greek mathematician, father of geometry introduced the method of proving mathematical results by using deductive logical reasoning and the previously proved result. The Geometry of plane figure is known as "Euclidean Geometry".

**Axioms:** The basic facts which are taken for granted without proof are called axioms. Some of Euclid's axioms are:

- (i) Things which are equal to the same thing are equal to one another. i.e.  $a = b, b = c \Rightarrow a = c$
- (ii) If equals are added to equals, the wholes are equal i.e.  $a = b \Rightarrow a + c = b + c$
- (iii) If equals are subtracted from equals, the remainders are equal i.e.  $a = b \Rightarrow a - c = b - c$
- (iv) Things which coincide with one another are equal to one another.
- (v) The whole is greater than the part.

**Postulates:** Axioms are the general statements, postulates are the axioms relating to a particular field.

Euclid's five postulates are.

- (i) A straight line may be drawn from any one point to any other point.
- (ii) A terminated line can be produced indefinitely.
- (iii) A circle can be drawn with any centre and any radius.
- (iv) All right angles are equal to one another.
- (v) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely meet on that side on which the angles are less than two right angles.

**Statements:** A sentence which is either true or false but not both, is called a statement.

eg. (i)  $4+9=6$  is a false sentence, so it is a statement.

(ii) Sajay is tall. This is not a statement because he may be tall for certain persons and may not be taller for others.

**Theorems:** A statement that requires a proof is called a theorem.

eg. (i) The sum of the angles of a triangle is  $180^\circ$ .

(ii) The angles opposite to equal sides of a triangle are equal.

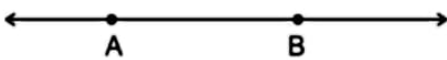
**Corollary** - Result deduced from a theorem is called its corollary.

## Lines and Angles

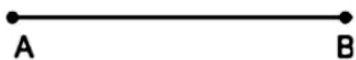
1. Basic Terms and Definitions
2. Intersecting Lines and Non-Intersecting Lines
3. Pairs of Angles
4. Parallel Lines and a Transversal
5. Lines Parallel to the same Line
6. Angle Sum Property of a Triangle

(1) **Point**- We often represent a point by a fine dot made with a fine sharpened pencil on a piece of paper.

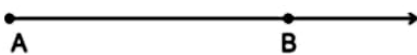
(2) **Line**- A line is completely known if we are given any two distinct points. Line AB is represented by as  $\overleftrightarrow{AB}$ . A line or a straight line extends indefinitely in both the directions.



(3) **Line segment**- A part (or portion) of a line with two end points is called a line segment.



(4) **Ray**- A part of line with one end point is called a ray.



(5) **Collinear points**- If three or more points lie on the same line, they are called collinear points otherwise they are called non-collinear points.

### Types of Angles-

- (1) **Acute angle**- An acute angle measure between  $0^\circ$  and  $90^\circ$
- (2) **Right angle**- A right angle is exactly equal to  $90^\circ$
- (3) **Obtuse angle**- An angle greater than  $90^\circ$  but less than  $180^\circ$
- (4) **Straight angle**- A straight angle is equal to  $180^\circ$
- (5) **Reflex angle**- An angle which is greater than  $180^\circ$  but less than  $360^\circ$  is called a reflex angle.
- (6) **Complementary angles**- Two angles whose sum is  $90^\circ$  are called complementary angles.
- (7) **Supplementary angle**- Two angles whose sum is  $180^\circ$  are called supplementary angles.
- (8) **Adjacent angles**- Two angles are adjacent, if they have a common vertex, a common arm and their non-common arms are on different sides of common arm.



(9) **Linear pair**- Two angles form a linear pair, if their non-common arms form a line.

(10) **Vertically opposite angles**- Vertically opposite angles are formed when two lines intersect each other at a point.

## **TRANSVERSAL:**

(a) Corresponding angles

(b) Alternate interior angles

(c) Alternate exterior angles

(d) Interior angles on the same side of the transversal.

- If a transversal intersects two parallel lines, then
  - (i) each pair of corresponding angles is equal.
  - (ii) each pair of alternate interior angles is equal.
  - (iii) each pair of interior angle on the same side of the transversal is supplementary.
- If a transversal interacts two lines such that, either
  - (i) any one pair of corresponding angles is equal, or
  - (ii) any one pair of alternate interior angles is equal or
  - (iii) any one pair of interior angles on the same side of the transversal is supplementary then the lines are parallel.
- Lines which are parallel to a given line are parallel to each other.
- The sum of the three angles of a triangle is  $180^\circ$
- If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

## Triangles

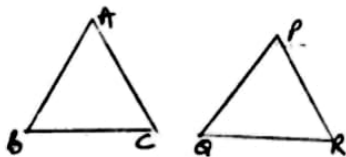
### 1. Congruence of Triangles

### 2. Criteria for Congruence of Triangles

### 3. Some Properties of a Triangle

### 4. Inequalities in a Triangle

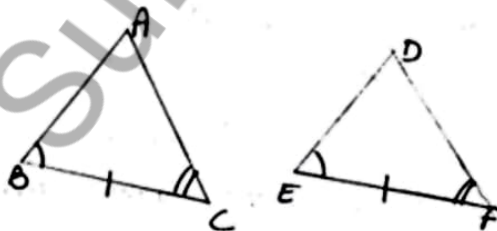
- **Triangle**- A closed figure formed by three intersecting lines is called a triangle. A triangle has three sides, three angles and three vertices.
- **Congruent figures**- Congruent means equal in all respects or figures whose shapes and sizes are both the same for example, two circles of the same radii are congruent. Also two squares of the same sides are congruent.
- **Congruent Triangles**- two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.
- If two triangles ABC and PQR are congruent under the correspondence  $A \leftrightarrow P, B \leftrightarrow Q$  and  $C \leftrightarrow R$  then symbolically, it is expressed as  $\triangle ABC \cong \triangle PQR$



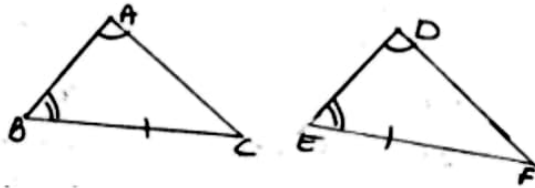
- In congruent triangles corresponding parts are equal and we write 'CPCT' for corresponding parts of congruent triangles.
- **SAS congruency rule** - Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle. For example:  $\triangle ABC$  and  $\triangle PQR$  as shown in the figure satisfy SAS congruent criterion.



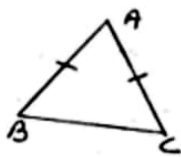
- **ASA Congruence Rule**- Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle. For examples  $\triangle ABC$  and  $\triangle DEF$  shown below satisfy ASA congruence criterion.



- **AAS Congruence Rule-** Two triangle are congruent if any two pairs of angles and one pair of corresponding sides are equal for example  $\triangle ABC$  and  $\triangle DEF$  shown below satisfy AAS congruence criterion.



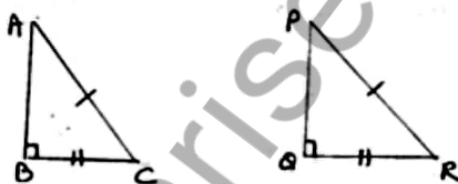
- AAS criterion for congruence of triangles is a particular case of ASA criterion.
- **Isosceles Triangle-** A triangle in which two sides are equal is called an isosceles triangle. For example:  $\triangle ABC$  shown below is an isosceles triangle with  $AB=AC$ .



- Angle opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- Each angle of an equilateral triangle is  $60^\circ$ .
- **SSS congruence Rule -** If three sides of one triangle are equal to the three sides of another triangle then the two triangles are congruent for example  $\triangle ABC$  and  $\triangle DEF$  as shown in the figure satisfy SSS congruence criterion.



- **RHS Congruence Rule-** If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle then the two triangle are congruent. For example:  $\triangle ABC$  and  $\triangle PQR$  shown below satisfy RHS congruence criterion.



RHS stands for right angle - Hypotenuse side.

- A point equidistant from two given points lies on the perpendicular bisector of the line segment joining the two points and its converse.
- A point equidistant from two intersecting lines lies on the bisectors of the angles formed by the two lines.
- In a triangle, angle opposite to the longer side is larger (greater)
- In a triangle, side opposite to the large (greater) angle is longer.
- Sum of any two sides of a triangle is greater than the third side.

## Quadrilaterals

- **Angle Sum Property of a Quadrilateral**
- **Types of Quadrilaterals**
- **Properties of a Parallelogram**
- **The Mid-Point Theorem**

- (1) Sum of the angles of a quadrilateral is  $360^\circ$
- (2) A diagonal of a parallelogram divides it into two congruent triangles.
- (3) In a parallelogram
  - (a) diagonals bisect each other.
  - (b) opposite angles are equal.
  - (c) opposite sides are equal
- (4) Diagonals of a square bisect each other at right angles and are equal, and vice-versa.
- (5) A line through the mid-point of a side of a triangle parallel to another side bisects the third side. (mid-point theorem)
- (6) The line through the mid points of sides of a  $\Delta$   $\parallel$  to third side and half of it.
- (7) A quadrilateral is a parallelogram, if
  - (a) its opposite angles are equal.
  - (b) its opposite sides are equal.
  - (c) its diagonals bisect each other.
  - (d) a pair of opposite sides is equal and parallel.
- (8) Diagonals of a rectangle bisect each other and are equal and vice-versa.
- (9) Diagonals of a rhombus bisect each other at right angles and vice-versa.
- (10) A line through the mid-point of a side of a triangle parallel to another side bisects the third side.
- (11) The line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.



## **Circles**

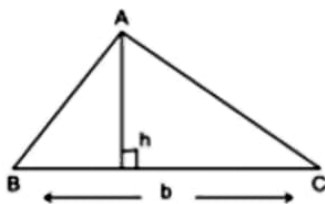
- **Circles and its Related Terms : A Review**
  - **Angle Subtended by a Chord at a Point**
  - **Perpendicular from the Centre to a Chord**
  - **Circle through Three Points**
  - **Equal Chords and their Distances from the Centre**
  - **Angle Subtended by an Arc of a Circle**
  - **Cyclic Quadrilaterals**
- 
- **Circle-** circle is locus of such points which are at equidistant from a fixed point in a plane.
  - **Concentric circle-** Circle having same centre called concentric circle.
  - Two arc of a circle called congruent if they have the same degree measure.
  - If two arc equal then their corresponding chords are equal.
  - The perpendicular from centre to chord of circle, it bisects the chord and converse.
  - There is one and only one circle passing through three non-collinear points.
  - Equal chords of circle are equidistant from centre.
  - The angle subtend by an arc at the centre of circle is twice the angle which subtend at remaining part of circumference.
  - Any two angles in the same segment of the circle are equal.
  - Angle of semicircle is right angle.
  - Equal chords of circle subtend equals angle at the centre of circle.
  - If the all vertices of a quadrilateral lie on the circumference of circle, then quadrilateral called cyclic.
  - In a cycle quadrilateral the sum of opposite angles is  $180^\circ$  and converse.
  - The exterior angle of a cycle quadrilateral is equal to the opposite interior angle.

## Heron's Formula

1. Area of a Triangle - by Heron's Formula
2. Application of Heron's Formula in finding Areas of Quadrilaterals

- Triangle with base 'b' and altitude 'h' is

$$\text{Area} = \frac{1}{2} \times (b \times h)$$



- Triangle with sides a, b and c

(i) Semi perimeter of triangle  $s = \frac{a+b+c}{2}$

(ii) Area  $= \sqrt{s(s-a)(s-b)(s-c)}$  square units.



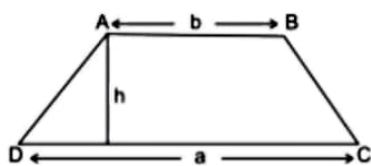
- Equilateral triangle with side 'a'

$$\text{Area} = \frac{\sqrt{3}}{4} a^2 \text{ square units}$$



- Trapezium with parallel sides 'a' & 'b' and the distance between two parallel sides as 'h'.

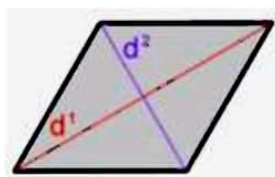
$$\text{Area} = \frac{1}{2} (a+b)h \text{ square units}$$



- Rhombus with diagonals  $d_1$  and  $d_2$

$$\text{Area} = \frac{1}{2} d_1 \times d_2;$$

$$\text{Perimeter} = 2\sqrt{d_1^2 + d_2^2}$$



## Surface Areas and Volumes

1. Surface Area of a Cuboid and a Cube
2. Surface Area of a Right Circular Cylinder
3. Surface Area of a Right Circular Cone
4. Surface Area of a Sphere
5. Volume of a Cuboid
6. Volume of a Cylinder
7. Volume of a Right Circular Cone
8. Volume of a Sphere

- **Polyhedrons Shapes:**

- (i) **Cube:**

Cube whose edge =  $a$

Diagonal of Cube =  $\sqrt{3}a$

Lateral Surface Area of Cube =  $4a^2$

Total Surface Area of Cube =  $6a^2$

Volume of Cube =  $a^3$

- (ii) **Cuboid:**

Cuboid whose length =  $l$ , breadth =  $b$  and height =  $h$

Diagonal of Cuboid =  $\sqrt{l^2 + b^2 + h^2}$

Lateral Surface Area of Cuboid =  $2(l + b)h$

Total Surface Area of Cuboid =  $2(lb + bh + hl)$

Volume of Cuboid =  $lbh$

- **Non-polyhedrons:**

- (i) **Cylinder:**

Cylinder whose radius =  $r$ , height =  $h$

Curved Surface Area of Cylinder =  $2\pi rh$

Total Surface Area of Cylinder =  $2\pi rh(r + h)$



Volume of Cylinder =  $\pi r^2 h$

(ii) **Cone:**

Cone having height = h, radius = r and slant height = l

Slant height of Cone (l) =  $\sqrt{r^2 + h^2}$

Curved Surface Area of Cone =  $\pi r l$

Total Surface Area of Cone =  $\pi r(r + l)$

Volume of Cone =  $\frac{1}{3} \pi r^2 h$

(iii) **Sphere:**

Sphere whose radius = r

Surface Area of a Sphere =  $4\pi r^2$

Volume of Sphere =  $\frac{4}{3} \pi r^3$

(iv) **Hemisphere:**

Hemisphere whose radius = r

Curved Surface Area of Hemisphere =  $2\pi r^2$

Total Surface Area of Hemisphere =  $3\pi r^2$

Volume of Hemisphere =  $\frac{2}{3} \pi r^3$

## Statistics

### 1. Collection of Data

### 2. Presentation of Data

### 3. Graphical Representation of Data

### 4. Measures of Central Tendency

- Statistics is the area of study that deals with the collection presentation, analysis and interpretation of data.
- Data:** Facts or figures, collected with a definite purpose, are called data.
- There are two types of data (i) Primary (ii) Secondary
- We can represent the data by (i) ungrouped and grouped frequency distribution.
- Data can also represent by (i) bar graph (ii) Histogram (iii) Frequency polygons
- Class mark of grouped data is  $\frac{\text{lower limit} + \text{upper limit}}{2}$
- Measure of central tendencies by mean, median, mode.
- Mean:**  $(\bar{x}) = \frac{\text{sum of all observations}}{\text{Total no. of observations}}$
- If observations denoted by and their occurrence i.e. frequency is denoted by then mean is  $(\bar{x}) = \frac{\sum}{\sum f_i} (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$
- Median:** Arrange the observations in ascending or descending order then if numbers of observations (n) are odd then then median is  $\frac{n+1}{2}$  term.

If no. of observations (n) are even, then median is average of  $\frac{n}{2}$ th and  $\frac{n}{2} + 1$ th terms.

- Mode:** The observation whose frequency is greatest.
- Mode = 3 median - 2 mean.