Class IX Session 2024-25
Subject - Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. π is **[1]**

a) a rational number b) an integer

c) an irrational number d) a whole number

2. The linear equation 3x - 5y = 15 has **[1]**

a) no solution b) infinitely many solutions

c) a unique solution d) two solutions

3. Two points having same abscissa but different ordinates lie on **[1]**

a) y-axis b) x-axis

c) a line parallel to y-axis d) a line parallel to x-axis

4. To draw a histogram to represent the following frequency distribution : **[1]**

Class interval	5-10	10-15	15-25	25-45	45-75
Frequency	6	12	10	8	15

The adjusted frequency for the class 25-45 is

a) 6 b) 5

c) 2 d) 3

5. The graph of the linear equation 2x + 3y = 6 is a line which meets the x-axis at the point **[1]**

a) (0,3) b) (3,0)

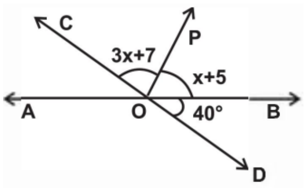
c) (2, 0)   d) (0 ,2)

6. Euclid stated that all right angles are equal to each other in the form of **[1]**

a) A postulate   b) A proof

c) An axiom   d) A definition

7. In the figure AB & CD are two straight lines intersecting at O, OP is a ray. What is the measure of $\angle AOD$. **[1]**



a) 128^o b) 40^o

c) 140^o d) 100^o

8. The diagonals AC and BD of a rectangle ABCD intersect each other at P. If $\angle$ABD = 50^o, then $\angle$DPC = **[1]**

a) 70^o b) 80^o

c) 90^o d) 100^o

9. Zero of the zero polynomial is - **[1]**

a) every real number   b) 1

c) not defined   d) 0

10. Express y in terms of x in the equation 5x - 2y = 7. **[1]**

a) $y = \frac{5x-7}{2}$ b) $y = \frac{7-5x}{2}$

c) $y = \frac{7x+5}{2}$ d) $y = \frac{5x+7}{2}$

11. ABCD is a Rhombus such that $\angle ACB$= 40°, then $\angle ADB$ is **[1]**

a) 100° b) 40°

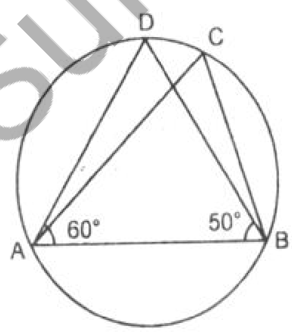
c) 60° d) 50°

12. Diagonals of a quadrilateral ABCD bisect each other. If $\angle$A = 45^o, then $\angle$B = **[1]**

a) 125^o b) 115^o

c) 120^o d) 135^o

13. In the figure, if $\angle DAB = 60^o$, $\angle ABD = 50^o$, then $\angle ACB$ is equal to : **[1]**



a) 80^o b) 60^o

c) 50^o d) 70^o

14. The simplest form of $0.5\bar{7}$ is **[1]**

a) $\frac{26}{45}$

b) $\frac{57}{99}$

c) $\frac{57}{100}$

d) $\frac{57}{90}$

15. Which of the following point does not lie on the line y = 2x + 3? **[1]**

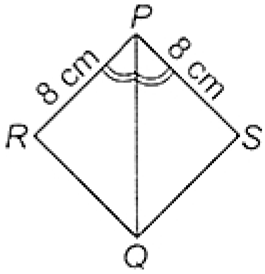
a) (-5, -7)

b) (-1, 1)

c) (3, 9)

d) (3, 7)

16. The congruence rule, by which the two triangles in the given figure are congruent is ________. **[1]**



a) ASA

b) SAS

c) SSS

d) RHS

17. In a histogram, which of the following is proportional to the frequency of the corresponding class? **[1]**

a) Width of the rectangle

b) Length of the rectangle

c) Perimeter of the rectangle

d) Area of the rectangle

18. The curved surface area of a cylinder and a cone is equal. If their base radius is same, then the ratio of the slant height of the cone to the height of the cylinder is **[1]**

a) 1 : 1

b) 2 : 3

c) 1 : 2

d) 2 : 1

19. **Assertion (A):** The sides of a triangle are 3 cm, 4 cm and 5 cm. Its area is 6 cm^2. **[1]**
Reason (R): If 2s = (a + b + c), where a, b, c are the sides of a triangle, then area = $\sqrt{(s-a)(s-b)(s-c)}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The point (1, 1) is the solution of x + y = 2. **[1]**
Reason (R): Every point which satisfy the linear equation is a solution of the equation.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

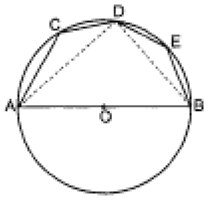
c) A is true but R is false.

d) A is false but R is true.

Section B

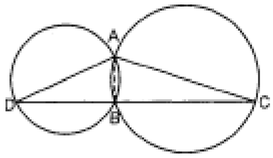
21. The base of an isosceles triangle measures 24 cm and its area is 192 cm^2. Find its perimeter. **[2]**

22. In given figure, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of $\angle$ACD + $\angle$ BED. **[2]**



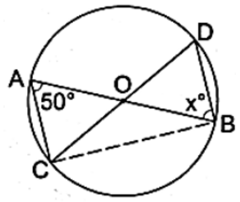
23. The outer diameter of a spherical shell is 10 cm and the inner diameter is 9 cm. Find the volume of the metal contained in the shell. **[2]**

24. In the given figure, two circles intersect at two points A and B. AD and AC are diameters to the two circles. Prove that B lies on the line segment DC. **[2]**



OR

If O is the centre of the circle, find the value of x in given figure:



25. Find whether the given equation have x = 2, y = 1 as a solution:x + y + 4 = 0. **[2]**

OR

Find whether $(\sqrt{2}, 4\sqrt{2})$ is the solution of the equation x – 2y = 4 or not?

Section C

26. Give three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$. **[3]**

27. Find the value of k, if x - 1 is a factor of p(x) in case: $p(x) = 2x^2 + kx + \sqrt{2}$ **[3]**

28. From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle. **[3]**

OR

The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs2000 per m^2 a year. A company hired one of its walls for 6 months. How much rent did it pay?

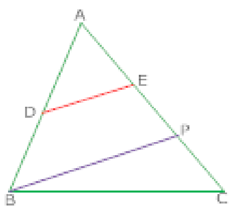
29. Find solutions of the form x = a, y = 0 and x = 0, y = b for the following pairs of equations. Do they have any common such solution? **[3]**

3x + 2y = 6 and 5x + 2y = 10

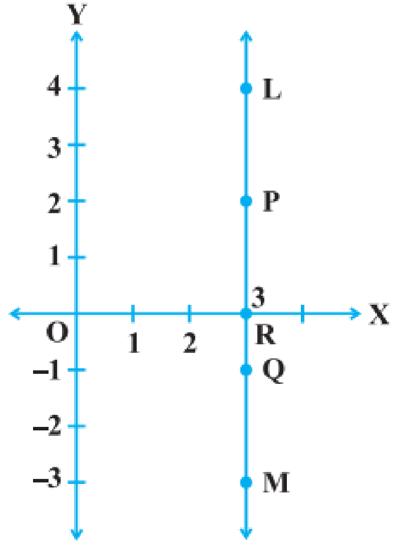
30. Show that the quadrilateral formed by joining the mid-points the sides of a rhombus, taken in order, form a rectangle. **[3]**

OR

In figure D is mid-points of AB. P is on AC such that PC = $\frac{1}{2}$AP and DE || BP, then show that AE = $\frac{1}{3}$AC.



31. In Figure, LM is a line parallel to the y-axis at a distance of 3 units. **[3]**



i. What are the coordinates of the points P, R and Q?

ii. What is the difference between the abscissa of the points L and M?

Section D

32. Find the values of a and b if $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}=a+b\sqrt{5}$. **[5]**

OR

If $p=\frac{3-\sqrt{5}}{3+\sqrt{5}}$ and $q=\frac{3+\sqrt{5}}{3-\sqrt{5}}$, find the value of $p^2 + q^2$.

33. In the adjoining figure, name: **[5]**

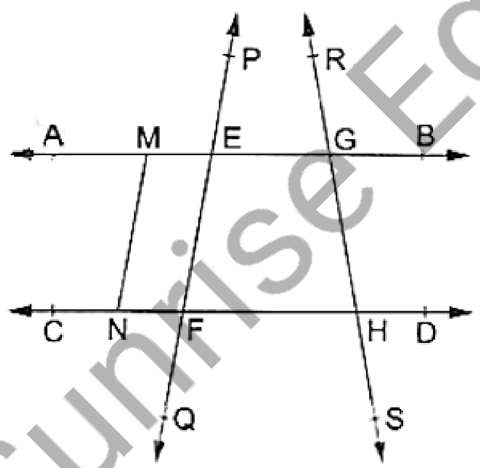
i. Six points

ii. Five line segments

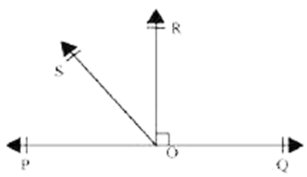
iii. Four rays

iv. Four lines

v. Four collinear points

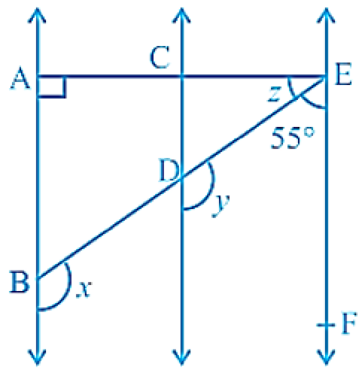


34. In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS=\frac{1}{2}(\angle QOS-\angle POS)$. **[5]**



OR

Fig., AB || CD and CD || EF. Also, EA ⊥ AB. If ∠BEF = 55°, find the values of x, y and z.



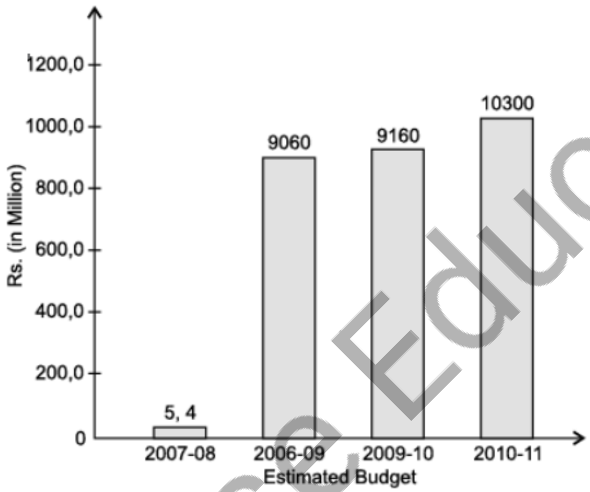
35. Find the values of a and b so that the polynomial $(x^4 + ax^3 - 7x^2 - 8x + b)$ is exactly divisible by $(x + 2)$ as well as $(x + 3)$. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Ladli Scheme was launched by the Delhi Government in the year 2008. This scheme helps to make women strong and will empower a girl child. This scheme was started in 2008.

The expenses for the scheme are plotted in the following bar chart.



i. What are the total expenses from 2009 to 2011? (1)

ii. What is the percentage of no of expenses in 2009-10 over the expenses in 2010-11? (1)

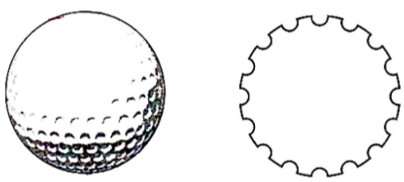
iii. What is the percentage of minimum expenses over the maximum expenses in the period 2007-2011? (2)

OR

What is the difference of expenses in 2010-11 and the expenses in 2006-09? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

A golf ball is spherical with about 300 - 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.



i. Find the surface area of one such dimple. (1)

ii. Find the volume of the material dug out to make one dimple. (1)

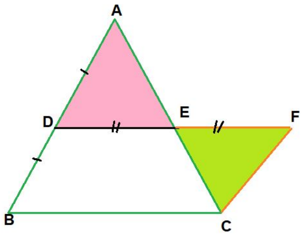
iii. Find the total surface area exposed to the surroundings. (2)

OR

Find the volume of the golf ball. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Haresh and Deep were trying to prove a theorem. For this they did the following



i. Draw a triangle ABC

ii. D and E are found as the mid points of AB and AC

iii. DE was joined and DE was extended to F so DE = EF

iv. FC was joined.

Questions:

i. $\triangle$ADE and $\triangle$EFC are congruent by which criteria? (1)

ii. Show that CF$\parallel$ AB. (1)

iii. Show that CF = BD. (2)

OR

Show that DF = BC and DF$\parallel$ BC. (2)

Solution

Section A

1.

(c) an irrational number

Explanation: π = 3.14159265359…….., which is non-terminating non-recurring.

Hence, it is an irrational number.

2.

(b) infinitely many solutions

Explanation:

Given linear equation: 3x - 5y = 15 Or, x = $\frac{5y+15}{3}$

When y = 0, x = $\frac{15}{3}$ = 5

When y = 3, x = $\frac{30}{3}$ = 10

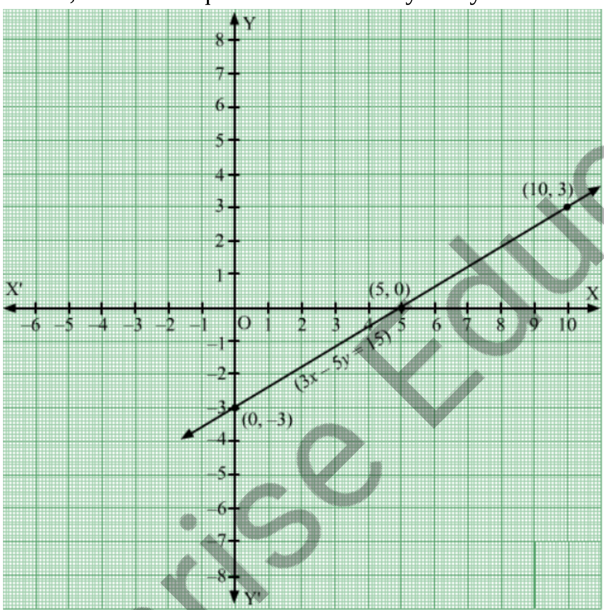
When y = -3, x = $\frac{0}{3}$ = 0

xx	5	10	0
yy	0	3	-3

Plot the points A(5,0) , B(10,3) and C(0,−3). Join the points and extend them in both the directions.

We get infinite points that satisfy the given equation.

Hence, the linear equation has infinitely many solutions.



3.

(c) a line parallel to y-axis

Explanation: Two points having same abscissa but different ordinate always make a line which is parallel to the y-axis as abscissa is fixed and the only ordinate keeps changing.

4.

(c) 2

Explanation: Adjusted frequency = $\left(\frac{\text{frequency of the class}}{\text{width of the class}}\right) \times 5$

Therefore, Adjusted frequency of 25 - 45 = $\frac{8}{20} \times 5 = 2$

5.

(b) (3,0)

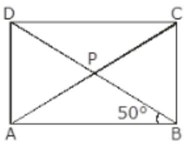
Explanation: 2x + 3y = 6 meets the X-axis.

Put y = 0,
2x + 3(0) = 6
x = 3
Therefore, graph of the given line meets X-axis at (3, 0).

6. **(a)** A postulate
Explanation: Eucid's fourth postulate states that all right angles are equal to one another.

7.
(c) 140^o
Explanation: 140^o
From the figure it follows that
(3x + 7) + (x + 5) + 40 = 180
⇒ 4x + 52 = 180
⇒ 4x = 180 - 52 = 128
⇒ x = 32
Now,
$\angle AOD = \angle COP + \angle POB$
$\Rightarrow \angle AOD = (3x + 7) + (x + 5)$
$\Rightarrow \angle AOD = 4x + 12$
$\Rightarrow \angle AOD = 4 \times 32 + 12$
$\Rightarrow \angle AOD = 128 + 12$
$\Rightarrow \angle AOD = 140$

8.
(b) 80^o
Explanation:
Given,
ABCD is a rectangle



Diagonals AC & BD intersect each other at P
∠ABD = 50^o
∵ diagonals of rectangle bisect each other and are equal in length
⇒ ∠ABD = ∠PDC [alternate angles]
⇒ ∠PDC= ∠PCD = 50°
In △DPC
⇒ ∠DPC + ∠PCD + ∠PDC = 180^o
⇒ ∠DPC + 50^o + 50^o = 180^o
⇒ ∠DPC = 180^o - 100^o = 80^o

9. **(a)** every real number
Explanation: Zero of the zero polynomial is any real number.
e.g., Let us consider zero polynomial be 0(x - k), where k is a real number.
For determining the zero, put x - k = 0 ⇒ x = k Hence, zero of the zero polynomial be any real number.

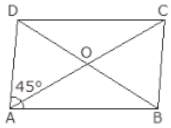
10. **(a)** $y = \frac{5x-7}{2}$
Explanation: 5x - 2y = 7
- 2y = 7 - 5x
2y = 5x - 7
y = $\frac{5x-7}{2}$

11.
(d) 50°

Explanation: In Rhombus, digonals bisect each other right angle. By using angle sum property in any of the four triangles formed by intersection of diagonals, we get ∠CBD = 50 and ∠CBD = ∠ADC (alternate angles).
So, ∠ADC = 50

12.

(d) 135^{o}
Explanation:
Given,



ABCD is a quadrilateral

$\angle A = 45^{o}$,

∵ diagonals of quadrilateral bisects each other hence ABCD is a parallelogram,

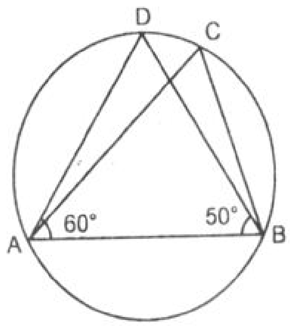
$\Rightarrow \angle A + \angle B = 180^{o}$

$\Rightarrow 45^{o} + \angle B = 180^{o}$

$\Rightarrow \angle B = 180^{o} - 45^{o} = 135^{o}$

13.

(d) 70^{o}
Explanation:



In, △ABD

$\angle D = 180^{o} - \angle A - \angle B$

$=180^{o} - 110^{o} = 70^{o}$

Since angles made by same chord at any point of circumference are equal so, $\angle ACB = \angle ADB = 70^{0}$

14. **(a)** $\frac{26}{45}$
Explanation: $0.5\overline{7} = \frac{57-5}{90}$
$= \frac{52}{90} = \frac{26}{45}$

15.

(d) (3, 7)
Explanation: Let us put x = 3 in the give equation,
Then, y = 2(3) + 3
y = 6 + 3 = 9
So, the point will be (3, 9)
For x = 3, y = 9. But in the given option, y = 7
So, the given point (3, 7) will not lie on the line y = 2x + 3.

16.

(b) SAS
Explanation: In △PQR and △PQS
PR = PS = 8 cm
∠RPQ = ∠SPQ (Given)

PQ = PQ (Common)

$\therefore \triangle PQR \cong \triangle PQS$ (By SAS congruency)

17.

(b) Length of the rectangle

Explanation: In, Histogram each rectangle is drawn, where width equivalent to class interval and height equivalent to the frequency of the class.

18.

(d) 2 : 1

Explanation: CSA of cone = CSA of cylinder

πrl = 2πrh

l = 2h

l : h = 2 : 1

19.

(c) A is true but R is false.

Explanation: s = $\frac{a+b+c}{2}$

s = $\frac{3+4+5}{2}$ = 6 cm

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{(6)(6-3)(6-4)(6-5)}$

= $\sqrt{(6)(3)(2)(1)}$ = 6 cm^2

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Putting (1, 1) in the given equation, we have

L.H.S = 1 + 1 = 2 = R.H.S

L.H.S = R.H.S

Hence (1, 1) satisfy the x + y = 2. So it is the solution of x + y = 2.

Section B

21. Let $\triangle$ABC be an isosceles triangle and let AL $\perp$ BC

$\therefore \frac{1}{2} \times BC \times AL = 192\text{cm}^2$

$\Rightarrow \frac{1}{2} \times 24\text{cm} \times h = 192\text{cm}^2$

$\Rightarrow h = \left(\frac{192}{12}\right)\text{cm} = 16\text{cm}$

Now, $BL = \frac{1}{2}(BC) = \left(\frac{1}{2} \times 24\right)$ cm = 12 cm and AL = 16 cm.

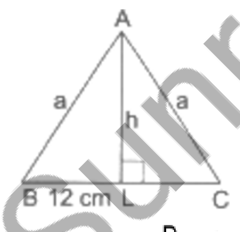
In $\triangle$ABL AB^2 = BL^2 + AL^2

$\Rightarrow$ a^2 = BL^2 + AL^2

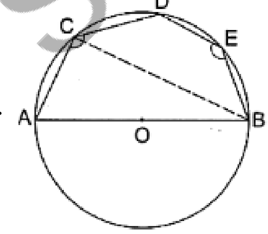
$\therefore a = \sqrt{BL^2 + AL^2} = \sqrt{(12)^2 + (16)^2}\text{cm} = \sqrt{144 + 256}\text{cm}$

$\Rightarrow a = \sqrt{400}\text{cm} = 20\text{cm}$

Hence, perimeter = (20 + 20 + 24) cm = 64 cm.



22.



Join BC,

Then, $\angle$ACB = 90° (Angle in the semicircle)

Since DCBE is a cyclic quadrilateral.

$\angle$BCD + $\angle$BED = 180°

Adding ∠ACB both the sides, we get

∠BCD + ∠BED + ∠ACB = ∠ACB +180°

(∠BCD + ∠ACB) + ∠BED = 90° + 180°

∠ACD + ∠BED = 270°

23. Outer diameter = 10 cm

$\therefore$ Outer radius (R) = $\frac{10}{2}$ cm = 5 cm

As Inner diameter = 9 cm

$\therefore$ Inner radius (r) = $\frac{9}{2}$ cm

Volume of the metal contained in the shell = $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3} \times \frac{22}{7} \times [(5)^3 - (\frac{9}{3})]$

$= \frac{4}{3} \times \frac{22}{7} \times (125 - \frac{729}{8})$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{271}{8} = \frac{2981}{21}\ cm^3$

24. In the given diagram join AB. Also ∠ABD = 90° (because angle in a semicircle is always 90°)

Similarly, we have ∠ABC = 90°

So, ∠ABD + ∠ABC = 90° + 90° = 180°

Therefore, DBC is a line i.e., B lies on the line segment DC.

OR

$\angle ODB = \angle OAC = 50^\circ$ [angles in the same segment]

OB = OD ⇒ $\angle OBD = \angle ODB = 50^\circ = x^\circ$

=> $x^o = 50^o$

25. For x = 2, y = 1

x + y + 4 = 0

L.H.S. = x + y + 4

= 2 + 1 + 4 = 7

$\neq$ R.H.S

$\therefore$ x = 2, y = 1 is not a solution of x + y + 4 = 0.

OR

x-2y=4

Put x = $\sqrt{2}$, y = $4\sqrt{2}$ in given equation, we get

$\sqrt{2} - 2(4\sqrt{2}) = \sqrt{2} - 8\sqrt{2} = -7\sqrt{2}$

which is not 4.

$\therefore (\sqrt{2}, 4\sqrt{2})$ is not a solution of given equation.

Section C

26. Here a = $\frac{1}{3}$, b = $\frac{1}{2}$, n = 3

$\therefore \frac{b-a}{n+1} = \frac{\frac{1}{2}-\frac{1}{3}}{3+1} = \frac{\frac{3-2}{6}}{4} = \frac{\frac{1}{6}}{4} = \frac{1}{24}$

$\therefore$ Three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$ are

$\frac{1}{3} + \frac{1}{24}, \frac{1}{3} + 2\left(\frac{1}{24}\right), \frac{1}{3} + 3\left(\frac{1}{24}\right)$

$\frac{1}{3} + \frac{1}{24}, \frac{1}{3} + \frac{1}{12}, \frac{1}{3} + \frac{1}{8}$

$\frac{3}{8}, \frac{5}{12}, \frac{11}{24}$

27. $p(x) = 2x^2 + kx + \sqrt{2}$

We know that according to the factor theorem

$p(a) = 0,$ if $x - a$ is a factor of $p(x)$

We conclude that if $(x - 1) isafactorofp(x) = 2x^2 + kx + \sqrt{2}$ then $p(1) = 0$

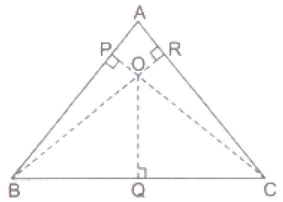
$p(1) = 2(1)^2 + k(1) + \sqrt{2} = 0,$ or

$2 + k + \sqrt{2} = 0$

$k = -\left(2 + \sqrt{2}\right).$

Therefore, we can conclude that the value of k is $-\left(2 + \sqrt{2}\right)$

28. Let ABC be an equilateral triangle, O be the interior point and OQ, OR and OC are the perpendicular drawn from points O. Let the sides of an equilateral triangle be a m.



Area of $\Delta AOB = \frac{1}{2} \times AB \times OP$

[$\because$ Area of a triangle $= \frac{1}{2} \times (base \times height)$]

$= \frac{1}{2} \times a \times 14 = 7a\,cm^2$ …(1)

Area of $\Delta OBC = \frac{1}{2} \times BC \times OQ = \frac{1}{2} \times a \times 10$

= 5a cm^2 …(2)

Area of $\Delta OAC = \frac{1}{2} \times AC \times OR = \frac{1}{2} \times a \times 6$

= 3a cm^2…(3)

$\therefore$ Area of an equilateral ΔABC

= Area of $(\Delta OAB + \Delta OBC + \Delta OAC)$

= (7a + 5a + 3a) cm^2

= 15a cm^2…(4)

We have, semi-perimeter $s = \frac{a+a+a}{2}$

$\Rightarrow\ s = \frac{3a}{2} cm$

$\therefore$ Area of an equilateral $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$

$= \frac{\sqrt{3}}{4}a^2$ …(5)

From equations (4) and (5), we get

$\frac{\sqrt{3}}{4}a^2 = 15a$

$\Rightarrow\ a = \frac{15\times4}{\sqrt{3}} = \frac{60}{\sqrt{3}}$

$\Rightarrow\ a = \frac{60}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}cm$

On putting $a = 20\sqrt{3}$ in equation (5), we get

Area of $\Delta ABC = \frac{\sqrt{3}}{4}(20\sqrt{3})^2 = \frac{\sqrt{3}}{4} \times 400 \times 3 = 300\sqrt{3}cm^2$

Hence, the area of an equilateral triangle is $300\sqrt{3}cm^2$.

OR

The sides of triangular side walls of flyover which have been used for advertisements are 13 m, 14 m, 15 m.

$s = \frac{13+14+15}{2} = \frac{42}{2} = 21m$

$= \sqrt{21(21-13)(21-14)(21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6}$

$= \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 7 \times 3 \times 2}$

$= 7 \times 3 \times 2 \times 2 = 84m^2$

It is given that the advertisement yield an earning of Rs. 2,000 per m^2 a year.

$\therefore$ Rent for 1 m^2 for 1 year = Rs. 2000

So, rent for 1 m^2 for 6 months or $\frac{1}{2}year = Rs(\frac{1}{2} \times 2000)$ = Rs. 1,000.

$\therefore$ Rent for 84 m^2 for 6 months = Rs. (1000 × 84) = Rs. 84,000.

29. 3x + 2y = 6

Put y = 0, we get

3x + 2(0) = 6

⇒ 3x = 6

⇒ $x = \frac{6}{3} = 2$

$\therefore$ (2, 0) is a solution.

3x + 2y = 6

put x = 0, we get

3(0) + 2y = 6

⇒ 2y = 6

$\Rightarrow y=\frac{6}{2}=3$

∴ (0, 3) is a solution.

5x+ 2y = 10

Put y = 0, we get

5x + 2(0) = 10

⇒ 5x = 10

$\Rightarrow x=\frac{10}{5}=2$

∴ (2, 0) is a solution.

5x + 2y = 10

Put x = 0, we get

5(0) + 2y = 10

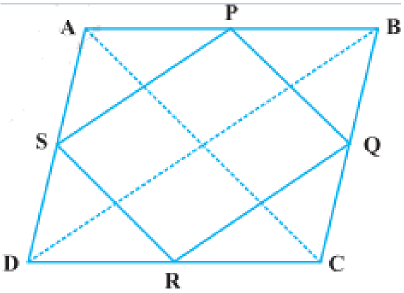
⇒ 2y = 10

$\Rightarrow y=\frac{10}{2}=5$

∴ (0, 5) is a solution.

The given equations have a common solution (2, 0).

30. Let ABCD be a rhombus and P, Q, R and S be the mid-points of sides AB, BC, CD and DA, respectively (Fig.). Join AC and BD.



From triangle ABD, we have $\mathrm{SP}=\frac{1}{2}\mathrm{BD}$ and

SP || BD (Because S and P are mid-points)

Similarly $\mathrm{RQ}=\frac{1}{2}\mathrm{BD}$ and RQ || BD

Therefore, SP = RQ and SP || RQ

So, PQRS is a parallelogram ...(1)

Also, AC ⊥ BD (Diagonals of a rhombus are perpendicular)

Further PQ || AC (From △BAC)

As SP || BD, PQ || AC and AC ⊥ BD,

therefore, we have BD ⊥ PQ, i.e. ∠SPQ = 90º. ..(2)

Therefore, PQRS is a rectangle. [From (1) and (2)]

OR

In △ABP

D is mid points of AB and DE||BP

∴ E is midpoint of AP

∴ AE = EP

Therefore,AP=2AE

Also PC = $\frac{1}{2}$ AP

2PC = AP

2PC = 2AE

⇒ PC = AE

∴ AE = PE = PC

∴ AC = AE + EP + PC

AC = AE + AE + AE

⇒ AE = $\frac{1}{3}$ AC

Hence Proved.

31. Given LM is a line parallel to the Y-axis and its perpendicular distance from Y-axis is 3 units.

i. Coordinate of point P = (3,2)
Coordinate of point Q = (3,-1)
Coordinate of point R = (3, 0) [since its lies on X-axis, so its y coordinate is zero].

ii. Abscissa of point L = 3, abscissa of point M=3
$\therefore$ Difference between the abscissa of the points L and M = 3 – 3 = 0

Section D

32. LHS

$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{7\times 3-7\sqrt{5}+3\sqrt{5}\times 3-3\sqrt{5}\times\sqrt{5}}{3^2-\sqrt{5}^2} - \frac{7\times 3+7\sqrt{5}-3\sqrt{5}\times 3-3\sqrt{5}\times\sqrt{5}}{3^2-\sqrt{5}^2}$

$= \frac{21-7\sqrt{5}+9\sqrt{5}-15}{9-5} - \frac{21+7\sqrt{5}-9\sqrt{5}-15}{9-5}$

$= \frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4}$

$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$

$= \frac{0+4\sqrt{5}}{4}$

$= 0 + \sqrt{5}$

We know that,

$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$

$0 + \sqrt{5} = a + b\sqrt{5}$

a = 0 and b = 1

OR

$p = \frac{3-\sqrt{5}}{3+\sqrt{5}}$

$= \frac{3-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$

$= \frac{(3-\sqrt{5})^2}{3^2-\sqrt{5}^2}$

$= \frac{9+5-6\sqrt{5}}{9-5}$

$= \frac{14-6\sqrt{5}}{4}$

$= \frac{7-3\sqrt{5}}{2}$

$q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$

$= \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{(3+\sqrt{5})^2}{3^2-\sqrt{5}^2}$

$= \frac{9+5+6\sqrt{5}}{9-5}$

$= \frac{14+6\sqrt{5}}{4}$

$= \frac{7+3\sqrt{5}}{2}$

$p^2 + q^2$

$= \left(\frac{7-3\sqrt{5}}{2}\right)^2 + \left(\frac{7+3\sqrt{5}}{2}\right)^2$

$= \frac{49+45-42\sqrt{5}}{4} + \frac{49+45+42\sqrt{5}}{4}$

$= \frac{94-42\sqrt{5}}{4} + \frac{94+42\sqrt{5}}{4}$

$= \frac{47-21\sqrt{5}}{2} + \frac{47+21\sqrt{5}}{2}$

$= \frac{47-21\sqrt{5}+47+21\sqrt{5}}{2}$

$= \frac{94}{2}$

= 47

33.
- Six points: A,B,C,D,E,F
- Five line segments: $\overline{EG}, \overline{FH}, \overline{EF}, \overline{GH}, \overline{MN}$
- Four rays: $\overrightarrow{EP}, \overrightarrow{GR}, \overrightarrow{GB}, \overrightarrow{HD}$

- Four lines: = $\overleftrightarrow{AB}, \overleftrightarrow{CD}$, $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$
- Four collinear points: M,E,G,B

34. To Prove: $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Given: OR is perpendicular to PQ, or $\angle$QOR = 90°

From the given figure, we can conclude that $\angle$POR and $\angle$QOR form a linear pair.

We know that sum of the angles of a linear pair is 180°.

$\therefore$ $\angle$POR + $\angle$QOR = 180°

or $\angle$POR = 90°

From the figure, we can conclude that

$\angle$POR = $\angle$POS + $\angle$ROS

$\Rightarrow$ $\angle$POS + $\angle$ROS = 90°

$\Rightarrow$ $\angle$ROS = 90° - $\angle$POS...(i)

Again,

$\angle$QOS + $\angle$POS = 180°

$\Rightarrow \frac{1}{2}(\angle QOS + \angle POS) = 90^\circ$.(ii)

Substitute (ii) in (i), to get

$\angle ROS = \frac{1}{2}(\angle QOS + \angle POS) - \angle POS$

$= \frac{1}{2}(\angle QOS - \angle POS)$.

Therefore, the desired result is proved.

OR

Since corresponding angles are equal.

$\therefore$ x = y ... (i)

We know that the interior angles on the same side of the transversal are supplementary.

$\therefore$ y + 55^o = 180^o

$\Rightarrow$ y = 180^o - 55^o = 125^o

So, x = y = 125^o

Since AB || CD and CD || EF.

$\therefore$ AB || EF

$\Rightarrow$ $\angle$EAB + $\angle$FEA = 180^o [$\because$ Interior angles on the same side of the transversal EA are supplementary]

$\Rightarrow$ 90^o + z + 55^o = 180^o

$\Rightarrow$ z = 35^o

35. The given polynomial is,

f(x) = $x^4 + ax^3 - 7x^2 - 8x + b$

Now, x + 2 = 0 $\Rightarrow$ x = -2

By the factor theorem, we can say: f(x) will be exactly divisible by (x + 2) if f(-2) = 0

Therefore, we have:

f(-2) = [$(-2)^4$ + a $\times$ $(-2)^3$ - 7 $\times$ $(-2)^2$ - 8 $\times$ (-2) + b]

= (16 - 8a - 28 + 16 + b)

= (4 - 8a + b)

$\therefore$ f(-2) = 0 $\Rightarrow$ 8a - b = 4 ...(i)

Also, x + 3 = 0 $\Rightarrow$ x = -3

By the factor theorem, we can say: f(x) will be exactly divisible by (x + 3) if f(-3) = 0

Therefore, we have:

f(-3) = [$(-3)^4$ + a $\times$ $(-3)^3$ - 7 $\times$ $(-3)^2$ - 8 $\times$ (-3) + b]

= (81 - 27a - 63 + 24 + b)

= (42 - 27a + b)

$\therefore$ f(-3) = 0 $\Rightarrow$ 27a - b = 42 ...(ii)

Subtracting (i) from (ii), we have:

$\Rightarrow$ 19a = 38

$\Rightarrow$ a = 2

Putting the value of a, we get the value of b, i.e., 12

$\therefore$ a = 2 and b = 12

Section E

36. i. Expenses in 2009-10 = 9160 Million

Expenses in 2010-11 = 10300 Million

Total expenses from 2009 to 2011

= 9160 + 10300

= 19460 Million

ii. Expenses in 2009-10 = 9160 Million

Expenses in 2010-11 = 10300 Million

Thus percentage of no of expenses in 2009-10 over the expenses in 2010-11

= $\frac{9160}{10300} \times 100$

= 88.93%

iii. The minimum expenses (in 2007-08) = 5.4 Million

The maximum expenses (in 2010-11) = 10300 Million

Thus percentage of no of minimum expenses over the maximum expenses

= $\frac{5.4}{10300} \times 100$

= 0.052%

OR

The expenses in 2010-11 = 10300 Million

The expenses in 2006-09 = 9060 Million

The difference = 10300 - 9060 Million

= 1240 Million

37. i. Diameter of golf ball = 4.2 cm

Radius of golf ball, R = 2.1 cm

Radius of dimple, r = 2mm = 0.2 cm

Surface area of each dimple = $2\pi r^2$

$2 \times \frac{22}{7} \times (0.2)^2 = 0.08$ cm^2

ii. Diameter of golf ball = 4.2 cm

Radius of golf ball, R = 2.1 cm

Radius of dimple ,r = 2mm = 0.2cm

Volume of the material dug out to make one dimple

= Volume of 1 dimple

$= \frac{2}{3}\pi r^3$

$= \frac{0.016\pi}{3}$ cm^3

iii. Diameter of golf ball = 4.2 cm

Radius of golf ball, R = 2.1 cm

Radius of dimple, r = 2mm = 0.2cm

The total surface area exposed to the surroundings

= surface area of golf ball − surface area of 315 dimples

= $4\pi R^2 - 315 \times 0.08\pi$

= $70.56\pi - 25.2\pi$ cm^2

= 45.36π cm^2

OR

Diameter of golf ball = 4.2 cm

Radius of golf ball, R = 2.1 cm

Radius of dimple, r = 2mm = 0.2cm

volume of the golf ball = volume of sphere − volume of 315 dimples

= $\frac{4}{3}\pi R^3 - 315 \times \frac{2}{3}\pi r^3$

= $\frac{4}{3}\pi(74.088 - 10.08)$

= $97.344\ \pi$cm^3

38. i. $\triangle ADE$ and $\triangle CFE$

DE = EF (By construction)

$\angle AED = \angle CEF$ (Vertically opposite angles)

AE = EC(By construction)

By SAS criteria $\triangle ADE \cong \triangle CFE$

ii. $\triangle ADE \cong \triangle CFE$

Corresponding part of congruent triangle are equal

$\angle EFC = \angle EDA$

alternate interior angles are equal

$\Rightarrow AD \parallel FC$

$\Rightarrow CF \parallel AB$

iii. $\triangle ADE \cong \triangle CFE$

Corresponding part of congruent triangle are equal.

CF = AD

We know that D is mid point AB

$\Rightarrow AD = BD$

$\Rightarrow CF = BD$

OR

$DE = \frac{BC}{2}$ {line drawn from mid points of 2 sides of $\triangle$ is parallel and half of third side}

$DE \parallel BC$ and $DF \parallel BC$

DF = DE + EF

$\Rightarrow DF = 2DE(BE = EF)$

$\Rightarrow DF = BC$