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Senior Secondary School Term II Examination, 2022

Marking Scheme – MATHEMATICS (SUBJECT CODE – 041)

(PAPER CODE – 465)

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2. **"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC."**

3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**

4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

5. Evaluators will mark (√) wherever answer is correct. For wrong answer 'X' be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left-hand margin and encircled. This may be followed strictly.

7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

10. A full scale of marks ____0 to 40______ (example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

12. Ensure that you do not make the following common types of errors committed by the Examiner in the past :-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totalling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totalling on the title page.
    - Wrong totalling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
14. Any unassessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

MARKING SCHEME

Senior Secondary School Examination TERM–II, 2022

MATHEMATICS (Subject Code–241)

[Paper Code: 465]

Maximum Marks: 40

Section – A

(Questions number **1** to **6** carry **2** marks each)

1. (a) Evaluate:

$$\int_0^1 \frac{xe^x}{(x+1)^2}dx$$

Solution:

(a)

$$\int_0^1 \frac{xe^x}{(x+1)^2}dx = \int_0^1 \frac{(x+1)-1}{(x+1)^2}e^x dx = \int_0^1 \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right]dx$$ **[1]**

$$= \left[\frac{1}{x+1}e^x\right]_0^1 = \frac{e-2}{2}$$ $\left[\frac{1}{2}+\frac{1}{2}\right]$

OR

1. (b) Solve the following differential equation:

$$\frac{dy}{dx} = e^{x+y} + x^2e^y$$

Solution:

(b)

$$\frac{dy}{dx} = e^y(e^x + x^2)$$

$\Rightarrow$ $$\frac{dy}{e^y} = (e^x + x^2)dx$$ **[1]**

$\Rightarrow$ $$-e^{-y} = e^x + \frac{1}{3}x^3 + C$$ **[1]**

2. Find the present value of a perpetuity of ₹18,000 payable at the end of 6 months, if the money is worth 8% p.a. compounded semi-annually.

Solution:

Let P be the present value of the perpetuity.

Here, R = ₹18000 and $i = \frac{8}{100\times 2} = 0.04$ **[1]**

$\therefore\ P = \frac{R}{i} = \frac{18000}{0.4} = ₹4,50,000$ **[1]**

3. (a) Find the effective rate which is equivalent to nominal rate of 10% p.a. compounded monthly.
[Given that: $(1·00833)^{12} = 1·1047$]

Solution:

(a) Here, nominal rate = 10% and k =12

$\therefore$ Effective rate of interest = $\left(1+\frac{r}{100k}\right)^k - 1$

$$= \left(1+\frac{10}{1200}\right)^{12} - 1 = (1.00833)^{12} - 1 = 0.1047$$ $\left[1\frac{1}{2}\right]$

Hence, the effective rate of interest is 10.47% $\left[\frac{1}{2}\right]$

OR

3. (b) Abhay bought a mobile phone for ₹30,000. The mobile phone is estimated to have a scrap value of ₹3,000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years.

Solution:

(b) Here, original value of mobile phone (C) = ₹30,000

Scrap value of the phone (S) = ₹3,000

Useful life (n) = 3 years

Annual depreciation $\frac{\text{C}-\text{S}}{\text{n}} = \frac{30,000-3000}{3} = 9000$ **[1]**

$\therefore$ Book value of the mobile at the end of 2 years = ₹30000 − 2 × ₹9000 = ₹12000 **[1]**

4. Consider the following hypothesis:

$$\boldsymbol{H_0: \mu = 35}$$
$$\boldsymbol{H_1: \mu \neq 35}$$

A sample of 81 items is taken whose mean is 37.5 and the standard deviation is 5. Test the hypothesis at 5% level of significance.

[*Given*: Critical value of Z for a two-tailed test at 5% level of significance is 1·96]

Solution:

The question is not in conformity with the prescribed syllabus. Thus, 2 marks be given to each examinee.

5. The following table shows the annual rainfall (in mm) recorded for Cherrapunji, Meghalaya:

Year	**Rainfall (in mm)**

2001	**1.2**
2002	**1.9**
2003	**2**
2004	**1.4**
2005	**2.1**
2006	**1.3**
2007	**1.8**
2008	**1.1**
2009	**1.3**

Solution:

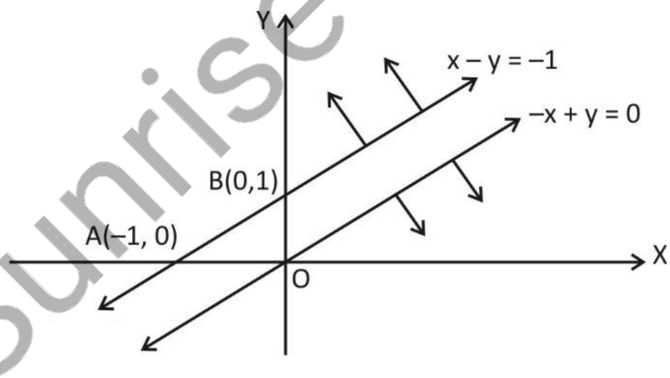
<table>
<tr><th>Year</th><th>Rain fall (in mm)</th><th>3 year moving total</th><th>3 year moving average</th><td rowspan="11">1 mark for 3-year moving totals
+
1 mark for 3-year moving averages</td></tr>
<tr><td>2001</td><td>1.2</td><td>---</td><td>---</td></tr>
<tr><td>2002</td><td>1.9</td><td>5.1</td><td>1.7</td></tr>
<tr><td>2003</td><td>2</td><td>5.3</td><td>1.77</td></tr>
<tr><td>2004</td><td>1.4</td><td>5.5</td><td>1.83</td></tr>
<tr><td>2005</td><td>2.1</td><td>4.8</td><td>1.6</td></tr>
<tr><td>2006</td><td>1.3</td><td>5.2</td><td>1.73</td></tr>
<tr><td>2007</td><td>1.8</td><td>4.2</td><td>1.4</td></tr>
<tr><td>2008</td><td>1.1</td><td>4.2</td><td>1.4</td></tr>
<tr><td>2009</td><td>1.3</td><td>---</td><td></td></tr>
</table>

6. **Maximize z = 3x + 4y, if possible,**
subject to the constraints:

$$x - y \leq -1$$
$$-x + y \leq 0$$
$$x, y \geq 0$$

Solution:

The graph of the given constraints is



[1]

Here, the feasible region is empty.

So, there exists no solution to the given LPP. **[1]**

Section – B

(Questions number **7** to **10** carry 3 marks each)

7. (a) The supply function of a commodity is 100p = (x + 20) ². Find the Producer's Surplus (PS), when the market price is ₹25.

Solution:

(a) Here, $p_0 = 25$

Putting $p = p_0$ and $x = x_0$ in the given supply function $100p = (x + 20)^2$.

We have $x_0 = 30$ **[1]**

Thus,

Producer's surplus $= p_0 x_0 - \int_0^{x_0} p\,dx = 750 - \frac{1}{100}\int_0^{30}(x+20)^2 dx$ **[1]**

$= 750 - \frac{1}{100}\left[\frac{1}{3}(x+20)^3\right]_0^{30}$

$= 750 - \frac{1}{300}[(50)^3 - (20)^3] = ₹\,360$ **[1]**

OR

(b) Find:

$$\int \frac{2x^2+1}{x^2-3x+2}dx$$

Solution:

(b) $I = \int \frac{2x^2+1}{x^2-3x+2}dx = \int (2 + \frac{6x-3}{x^2-3x+2})dx$ **[1]**

$= \int 2dx + \int \left[\frac{-3}{x-1} + \frac{9}{x-2}\right]dx$ {Using partial fractions} **[1]**

$= 2x - 3\,log|x-1| + 9\,log|x-2| + C$ **[1]**

8. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 for the following data:

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37

2006	40
2007	36

Solution:

Year	Production (Y)	Origin = 2004 (X)	X^2	XY
2001	30	– 3	9	–90
2002	35	– 2	4	–70
2003	36	– 1	1	–36
2004	32	0	0	0
2005	37	1	1	37
2006	40	2	4	80
2007	36	3	93	108
2009	$\sum Y = 246$	$\sum XY = 0$	$\sum X^2 = 28$	$\sum XY = 29$

[1]

Trend equation is $Y_C = a + bX$; and

Normal equations are:

$$\left.\begin{aligned} a &= \frac{\Sigma Y}{n} = \frac{246}{7} = 35.14\text{ ; and} \\ b &= \frac{\Sigma XY}{\Sigma X^2} = \frac{29}{28} = 1.03 \end{aligned}\right\}$$ **[1]**

$\Rightarrow$ Trend equation is $Y_C = 35.14 + 1.03$ X $\left[\frac{1}{2}\right]$

Thus, trend value for 2008 is [35.14 + 1.03(4)] = 39.26 $\left[\frac{1}{2}\right]$

9. Ten cartons are taken at random from an automatic packing machine. The mean net weight of the ten cartons is 11.8 kg and standard deviation is 0.15 kg. Does the sample mean differ significantly from the intended mean of 12 kg?
[Given that for d.f. = 9, $t_{0 \cdot 05} = 2 \cdot 26$]

Solution:

We are given $n = 10$, $\bar{x} = 11.8$ kg and s = 0.15 kg

Let Null hypothesis be $H_0 = \mu = 12$ kg, and

Alternate hypothesis be H_1 ; $\mu \neq 12$ kg **[1]**

Under H_0, the test statistic is

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \frac{11.8-12}{\frac{0.15}{3}} = -4 \qquad \textbf{[1]}$$

Since the tabulated value of t for *d.f.* = 9 is $t_{0.05}$ = 2.26 and the calculated |t| is much greater than the tabulated value, null hypothesis is rejected. Thus, we conclude that the sample mean differs significantly from the intended mean of 12 kg. **[1]**

10. Madhu exchanged her old car valued at ₹1,50,000 with a new one priced at ₹6,50,000. She paid ₹x as down payment and the balance in 20 monthly equal instalments of ₹21,000 each. The rate of interest offered to her is 9% p.a. Find the value of x.
[Given that: $(1\cdot0075)^{-20} = 0\cdot86118985$]

Solution:

Here, $i = \frac{9}{1200} = 0.0075$, n = 20 and E = ₹ 21,000

P = ₹ (650000 – 150000 – x) = $(500000 - x)$ **[1]**

By the reducing balance method, we have

$$E = \frac{Pi}{1-(1+i)^{-n}} \qquad \textbf{[1]}$$

$$\Rightarrow \quad 21000 = \frac{(500000-x)(0.0075)}{1-(1.0075)^{-20}} = \frac{3750-0.0075x}{1-0.86118985} = \frac{3750-0.0075x}{0.1381015} \qquad \left[\frac{1}{2}\right]$$

$\Rightarrow \quad x$ = ₹ 1,11,332 $\left[\frac{1}{2}\right]$

Section – C

(Questions number **11** to **14** carry **4** marks each)

11. In a certain culture of bacteria, the rate of increase of bacteria is proportional to the number present. It is found that there are 10,000 bacteria at the end of 3 hours and 40,000 bacteria at the end of 5 hours. Determine the number of bacteria present in the beginning.

Solution:

Let P be the number of bacteria present in the culture after t hours. Then,

$$\frac{dP}{dt} \alpha P \qquad \textbf{[1]}$$

$$\Rightarrow \quad \frac{dP}{dt} = kP$$

$$\Rightarrow \quad \int \frac{dP}{P} = \int k dt$$

$$\Rightarrow \quad log\, P = kt + C$$

$$\Rightarrow \quad P(t) = e^{kt+C} + \lambda e^{kt} \qquad \ldots. \text{(i)} \qquad \textbf{[1]}$$

We are given that P(3) = 10000 and P(5) = 40000 we get

$$\lambda e^{k(3)} = 10000 \text{ and } \lambda e^{k(5)} = 40000 \quad \ldots \text{(ii)}$$

$\Rightarrow \quad e^{2k} = 4$, or $e^{k} = 2$ **[1]**

From (ii), λ = 1250

Thus, from (i), we have

$P(t) = 1250(2)^t$

$\Rightarrow$ $P(t) = 1250(2)^0 = 1250$

Hence, there were 1250 bacteria present in the beginning. **[1]**

12. (a) Calculate the EMI under 'Flat Rate System' for a loan of ₹5,00,000 with 10% annual interest rate for 5 years.

Solution:

(a) Here, P = ₹500000

$I = 500000 \times \frac{10}{100} \times 5 =$ ₹ 250000 **[1]**

n = 5 years = 5 × 12 months = 60 months **[1]**

$\text{EMI} = \frac{P+I}{n} = \frac{500000+250000}{60}$ **[1]**

= ₹12500 **[1]**

OR

12. (b) A machine costing ₹2,00,000 has effective life of 7 years and its scrap value is ₹30,000. What amount should the company put into a sinking fund earning 5% p.a., so that it can replace the machine after its usual life? Assume that a new machine will cost ₹ 3,00,000 after 7 years.

[Given that: $(1{\cdot}05)^7 = 1{\cdot}407$]

Solution:

(b) Cost of new machine = ₹300000; scrap value = ₹30000

$\Rightarrow$ Money required to buy new machine after 7 years is

₹ (300000 – 30000) = ₹ 270000 **[1]**

So, A = ₹ 270000, i = 0.05 and n = 7

Using the formula: $A = R\left[\frac{(1+i)^n-1}{i}\right]$ **[1]**

$\Rightarrow 270000 = R\left[\frac{(1.05)^7-1}{0.05}\right]$

$\Rightarrow R = \frac{270000\times0.05}{0.407} = ₹33169.33$ **[1+1]**

Hence, the company should put ₹ 33169.33 into sinking fund.

13. A start-up company invested ₹3,00,000 in shares for 5 years. The value of this investment was ₹3,50,000 at the end of second year, ₹3,80,000 at the end of third year and on maturity, the final value stood at ₹4,50,000. Calculate the Compound Annual Growth Rate (CAGR) on the investment.
[Given that: $(1{\cdot}5)^{1/5} = 1{\cdot}084$]

Solution:

Here, initial value of investment (V_i) = ₹ 300000

Final value of investment (V_{fi}) = ₹ 450000 **[1]**

and n = 5 years.

Now, $i = \left(\frac{V_f}{V_i}\right)^{1/n} - 1$

$\Rightarrow \quad i = \left(\frac{450000}{300000}\right)^{1/5} - 1 = (1.5)^{1/5} - 1 = 1.084 - 1 = 0.084$ **[1+1]**

$\Rightarrow$ CAGR (%) = 8.4%

Thus, the compound annual growth rate is 8.4%. **[1]**

14. A dietician wishes to mix two types of foods F_1 and F_2 in such a way that the vitamin content of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food F_1 contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C, while Food F_2 contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5 per kg to purchase Food F1 and ₹7 per kg to purchase Food F_2.

Based on the above information, answer the following questions:

(a) To find out the minimum cost of such a mixture, formulate the above problem as a LPP.

(b) Determine the minimum cost of the mixture.

Solution:

Let the mixture contain x kg of food F_1 and y kg of food F_2. Then, LPP becomes

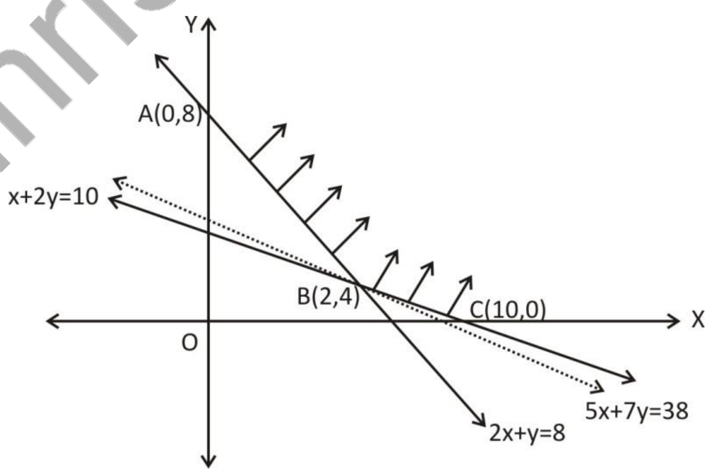
(a) Minimize Z = 5x + 7y $\left[\frac{1}{2}\right]$

Subject to the constraints

$2x + y \geq 8$

$x + 2y \geq 10$

$x \geq 0, y \geq 0$ **[1]**



(b) The corner points are A(0, 8), B(2,4) and C (10, 0). **[1]**

The value of Z at these corner points are

$Z_A = 56$; $Z_B = 38$; $Z_C = 50$ $\left[\frac{1}{2}\right]$

Since the feasible region is unbounded, we draw the graph of

$5x + 7y < 38$.

As the graph of $5x + 7y < 38$ does not have any point common with the Feasible region, so the minimum cost of the mixture is ₹ 38. **[1]**
