Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2024
APPLIED MATHEMATICS PAPER CODE 465

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (**✓**) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	• Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past :-

	● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

APPLIED MATHEMATICS (Subject Code–241)

(PAPER CODE: 465)

Section A

Q. No.	**EXPECTED OUTCOMES/VALUE POINTS**	**Marks**
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	In a 1 km race, player P beats player Q by 18 metres or 9 seconds. What is P's time to complete the race ? (A) 512 seconds (B) 502 seconds (C) 491 seconds (D) 481 seconds	
Sol.	(C) 491 seconds	**(1)**
2.	If $x > y$ and $z < 0$, then : (A) $xz > yz$ (B) $xz \geq yz$ (C) $\frac{x}{z} > \frac{y}{z}$ (D) $\frac{x}{z} < \frac{y}{z}$	
Sol.	(D) $\frac{x}{z} < \frac{y}{z}$	**(1)**
3.	If $AB = A$ and $BA = B$, then $(B^2 + B)$ equals : (A) 2A (B) O (C) 2I (D) 2B	
Sol.	(D) 2B	**(1)**
4.	The value of $\Delta = \begin{vmatrix} 42 & 2 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ is : (A) 0 (B) 1 (C) – 3 (D) –15	
Sol.	(A) 0	**(1)**

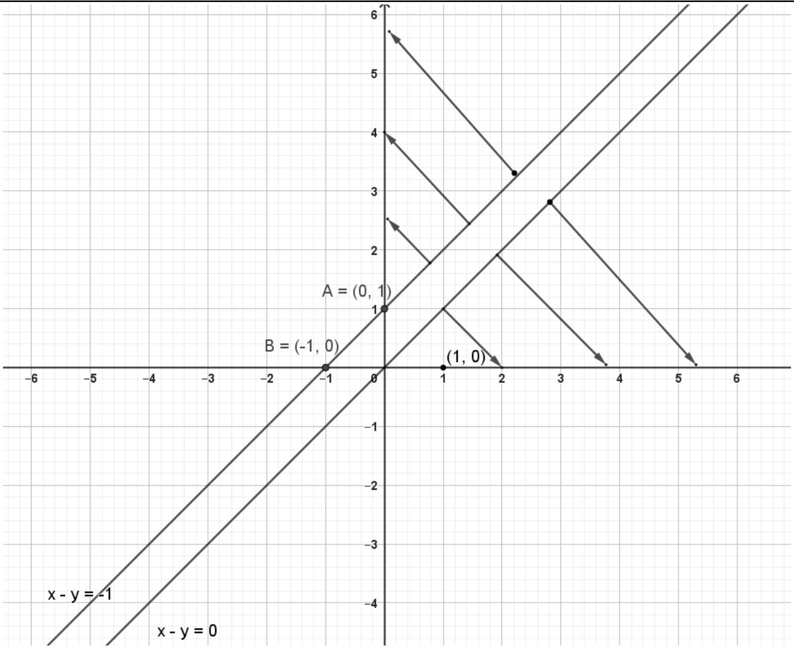
5.	If $y = e^{-2x}$, then $\frac{d^3y}{dx^3}$ is equal to : (A) $2e^{-2x}$ (B) e^{-4x} (C) $4e^{-4x}$ (D) $-8e^{-2x}$	
Sol.	(D) $-\mathbf{8e^{-2x}}$	**(1)**
6.	The function $f(x) = x^2 - x + 1$ is : (A) increasing in (0, 1) (B) decreasing in (0, 1) (C) increasing in $(0, \frac{1}{2})$ and decreasing in $(\frac{1}{2}, 1)$ (D) increasing in $(\frac{1}{2}, 1)$ and decreasing in $(0, \frac{1}{2})$	
Sol.	(D) increasing in $\left(\frac{1}{2}, 1\right)$ and decreasing in $\left(0, \frac{1}{2}\right)$	**(1)**
7.	The **order** and the **degree** of the differential equation $y\,dx + x \log\left(\frac{y}{x}\right) dy - 2x\,dy = 0$ are respectively : (A) 1, 1 (B) 1, 2 (C) 2, 1 (D) 1, not defined	
Sol.	(A) 1,1	**(1)**
8.	A fair coin is tossed twice and outcomes are noted. If the random variable X represents the number of heads that appeared in the experiment, then the mathematical expectation of X is : (A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $1\frac{1}{2}$	
Sol.	(A) 1	**(1)**
9.	What time will it be after 1275 hours, if the present time is 9:00 p.m. ? (A) 11 p.m. (B) 12 p.m. (C) 9 p.m. (D) 9 a.m.	
Sol.	Since correct answer is not in the options given So, it is suggested that 1 mark may be given to all who attempted this question	**(1)**

10.	If for a Poisson variate X, $P(X = k) = P(X = k + 1)$, then the variance of X is : (A) $k - 1$ (B) k (C) $k + 1$ (D) $k + 2$	
Sol.	(C) $\mathbf{k + 1}$	**(1)**
11.	If the calculated value of $\lvert t \rvert < t_v(\alpha)$ (critical value of t), then the null hypothesis : (A) is rejected. (B) is accepted. (C) is neither accepted nor rejected. (D) cannot be determined.	
Sol.	(B) is accepted	**(1)**
12.	For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as : (A) $n_1 - n_2 + 2$ (B) $n_1 - n_2 - 2$ (C) $n_1 + n_2 - 2$ (D) $n_1 + n_2 + 2$	
Sol.	(C) $\boldsymbol{n_1 + n_2 - 2}$	**(1)**
13.	For the given values 23, 32, 40, 47, 58, 33, 42; the 5-yearly moving averages are : (A) 38, 40, 42 (B) 40, 42, 44 (C) 40, 42, 46 (D) 42, 44, 46	
Sol.	(B) 40, 42, 44	**(1)**
14.	Using flat rate method, the EMI to repay a loan of ₹ 20,000 in $2\frac{1}{2}$ years at an interest rate of 8% p.a. is : (A) ₹ 700 (B) ₹ 800 (C) ₹ 900 (D) ₹ 100	
Sol.	(B) ₹ 800	**(1)**

15.	A mobile phone costs ₹ 12,000 and its scrap value after a useful life of 3 years is ₹ 3,000. Then, the book value of the mobile phone at the end of 2 years is : (A) ₹ 3,000 (B) ₹ 6,000 (C) ₹ 5,000 (D) ₹ 7,000	
Sol.	(B) ₹ 6,000	**(1)**
16.	What sum of money should be deposited at the end of every 6 months to accumulate ₹ 50,000 in 8 years, if money is worth 6% p.a. compounded semi-annually ? [Given : $(1{\cdot}03)^{16} = 1{\cdot}6047$] (A) ₹ 3,432·53 (B) ₹ 2,783·08 (C) ₹ 2,480·57 (D) ₹ 2,149·93	
Sol.	(C) ₹ 2480.57	**(1)**
17.	The graph of the inequation $2x + 3y > 6$ is the : (A) entire XOY-plane (B) half-plane that contains the origin (C) half-plane that neither contains the origin nor the points on the line $2x + 3y = 6$ (D) whole XOY-plane excluding the points on the line $2x + 3y = 6$	
Sol.	(C) half-plane that neither contains the origin nor the points on the line $\mathbf{2x + 3y = 6}$	**(1)**
18.	In an LPP, if the objective function $Z = ax + by$ has same maximum value on two corner points of the feasible region, then the number of points at which maximum value of Z occurs is : (A) 0 (B) 2 (C) finite (D) infinite	
Sol.	(D) infinite	**(1)**

	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* The function $f(x) = x^2 - x + 1$ is strictly increasing on $(-1, 1)$. *Reason (R) :* If $f(x)$ is continuous on $[a, b]$ and derivable on (a, b), then $f(x)$ is strictly increasing on $[a, b]$ if $f'(x) > 0$ for all $x \in (a, b)$.	
Sol.	(D) Assertion (A) is false, but Reason (R) is true	**(1)**
20.	In a binomial distribution, $n = 200$ and $p = 0{\cdot}04$. Taking Poisson distribution as an approximation to the binomial distribution : *Assertion (A) :* Mean of Poisson distribution = 8. *Reason (R) :* $P(X = 4) = \frac{512}{3e^8}$.	
Sol.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is the not the correct explanation of the Assertion (A).	**(1)**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21(a).	If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find the value of k such that $A^2 - 8A + kI = 0$.	
Sol.	Given $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{7} \end{bmatrix}, \mathbf{A^2} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-8} & \mathbf{49} \end{bmatrix}$ $\mathbf{A^2 - 8A + kI = 0}$ gives $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-8} & \mathbf{49} \end{bmatrix} - \mathbf{8}\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{7} \end{bmatrix} + \mathbf{k}\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$ $\Rightarrow \begin{bmatrix} \mathbf{1 - 8 + k} & \mathbf{0 - 0 + 0} \\ \mathbf{-8 + 8 + 0} & \mathbf{49 - 56 + k} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$ $\Rightarrow \mathbf{k = 7}$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$

	OR	
21(b).	If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the values of x, y, z and w.	
Sol.	Here, $\mathbf{x - y = -1, 2x - y = 0, 2x + z = 5, 3z + w = 13}$ Solving these equations, we get $\mathbf{x = 1, y = 2, z = 3, w = 4}$	**(1)** **(1)**
22.	Using Cramer's rule, solve the following system of equations : $2x_1 + 3x_2 = 5$ $11x_1 - 5x_2 = 6$	
Sol.	Here, $\mathbf{D} = \begin{vmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{11} & \mathbf{-5} \end{vmatrix} = \mathbf{-43 \neq 0}$ $\mathbf{D_1} = \begin{vmatrix} \mathbf{5} & \mathbf{3} \\ \mathbf{6} & \mathbf{-5} \end{vmatrix} = \mathbf{-43}$ $\mathbf{D_2} = \begin{vmatrix} \mathbf{2} & \mathbf{5} \\ \mathbf{11} & \mathbf{6} \end{vmatrix} = \mathbf{-43}$ So, $\mathbf{x_1} = \frac{\mathbf{D_1}}{\mathbf{D}} = \frac{\mathbf{-43}}{\mathbf{-43}} = \mathbf{1}$, $\mathbf{x_2} = \frac{\mathbf{D_2}}{\mathbf{D}} = \frac{\mathbf{-43}}{\mathbf{-43}} = \mathbf{1}$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
23.	Find the solution to the following linear programming problem (if it exists) graphically : Maximize $Z = x + y$ subject to the constraints $x - y \le -1$ $-x + y \le 0$ $x, y \ge 0$.	

<table>
<tr><td>Sol.</td><td>

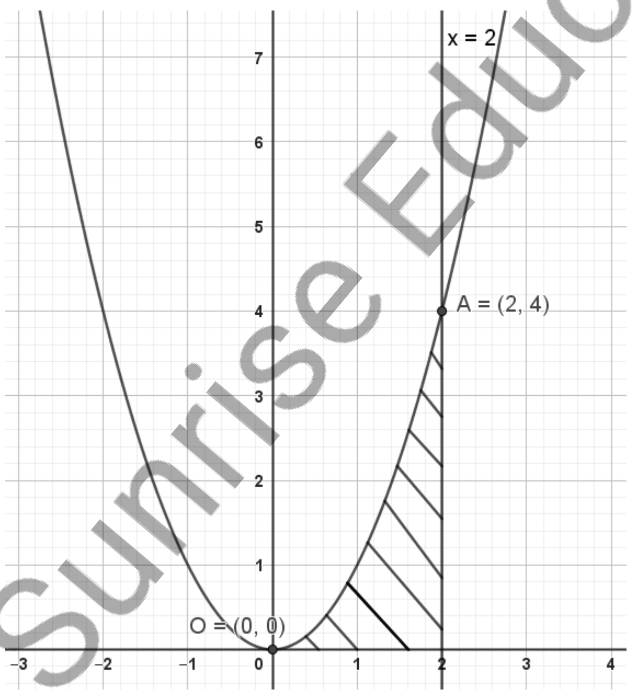
Since feasible region is empty, there is no solution to the problem.</td><td>$(1\frac{1}{2})$ for correct graph

$(\frac{1}{2})$</td></tr>
<tr><td>24.</td><td>At 6% p.a., compounded quarterly, find the present value of a perpetuity of ₹ 600 payable at the end of each quarter.</td><td></td></tr>
<tr><td>Sol.</td><td>Here, $\mathbf{R = 600, i = \frac{0.06}{4} = 0.015}$

So, $\mathbf{PV = \frac{R}{i} = \frac{600}{0.015} =}$ ₹ $\mathbf{40,000}$</td><td>(1)

(1)</td></tr>
<tr><td>25 (a).</td><td>Assume an investment's starting value is ₹ 20,000 and it grows to ₹ 50,000 in 3 years. Calculate CAGR (Compounded Annual Growth Rate) [Use : $(2\cdot5)^{1/3} = 1\cdot355$]</td><td></td></tr>
<tr><td>Sol.</td><td>CAGR $= \left[\left(\frac{50000}{20000}\right)^{1/3} - 1\right] \times 100$
$= [(2.5)^{1/3} - 1] \times 100 = (1.355 - 1) \times 100$
$= 0.355 \times 100 = 35.5\%$</td><td>(1)

(1)</td></tr>
<tr><td></td><td>OR</td><td></td></tr>
<tr><td>25 (b).</td><td>A man bought an item for ₹ 12,000. At the end of the year, he decided to sell it for ₹ 15,000. If the inflation rate was 6%, find the nominal and real rate of return.</td><td></td></tr>
<tr><td>Sol.</td><td>Nominal rate of return $= \frac{15000-12000}{12000} \times 100$
$= \frac{3000}{12000} \times 100 = 25\%$</td><td>$(\frac{1}{2})$
$(\frac{1}{2})$</td></tr>
</table>

	Real rate of return =Nominal rate of return – Inflation rate $= \mathbf{25\% - 6\% = 19\%}$	**(1)**
	SECTION C **This section comprises short answer (SA) type questions of 3 marks each.**	
26.	A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated 4 more times. Determine the quantity of juice in the container after final replacement. [Use $(0{\cdot}9)^5 = 0{\cdot}59049$]	
Sol.	Juice contained in the container after final replacement $= \mathbf{50\left(1-\frac{5}{50}\right)^5 = 50\left(\frac{9}{10}\right)^5}$ $= \mathbf{50 \times 0.59049 = 29.5}$ litres	**(2)** **(1)**
27(a).	Evaluate : $\int_0^2 x^2\,dx$ and hence show the region on the graph whose area it represents.	
Sol.	Required Area $= \int_0^2 \mathbf{x^2 dx} = \left\| \frac{x^3}{3} \right\|_0^2 = \frac{8}{3}$  	**($1\frac{1}{2}$)** **($1\frac{1}{2}$)** **for correct graph**
	OR	

27(b).	Evaluate : $\int_0^1 \frac{e^{-x}}{1+e^x}\,dx$	
Sol.	$\mathbf{I = \int_0^1 \frac{dx}{e^x(1+e^x)}}$ $\mathbf{= \int_1^e \frac{dt}{t^2(1+t)}}$ **(Put $\mathbf{e^x = t \Rightarrow e^x dx = dt}$)** $\mathbf{= \int_1^e \left(-\frac{1}{t}+\frac{1}{t^2}+\frac{1}{1+t}\right) dt}$ $\mathbf{= \left[-\log(t) - \frac{1}{t} + \log(1+t)\right]_1^e}$ $\mathbf{= \left[\log\left(\frac{1+t}{t}\right) - \frac{1}{t}\right]_1^e}$ $\mathbf{= \left[\log\left(\frac{1+e}{e}\right) - \frac{1}{e}\right] - [\log 2 - 1]}$ $\mathbf{= \log\left(\frac{1+e}{2e}\right) - \frac{1}{e} + 1}$	$(\frac{1}{2})$ **(1)** **(1)** $(\frac{1}{2})$
28.	Find the differential equation of all circles in the first quadrant which touches both the coordinate axes.	
Sol.	Here, the equation of the circle is $\mathbf{(x-a)^2 + (y-a)^2 = a^2}$ … (1) i.e., $\mathbf{x^2 + y^2 - 2ax - 2ay + a^2 = 0}$ $\mathbf{\Rightarrow 2x + 2y\frac{dy}{dx} - 2a - 2a\frac{dy}{dx} = 0}$ $\mathbf{\Rightarrow a = \frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}}$ From (1), we have $\mathbf{\left(x - \frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)^2 + \left(y - \frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)^2 = \left(\frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)^2}$ or $\mathbf{\left(\frac{dy}{dx}\right)^2 (x^2 - 2xy) - 2xy\frac{dy}{dx} + y^2 - 2xy = 0}$	**(1)** **(1)** **(1)**
29.	Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, determine the probability that a candidate who takes the test will score between 90 and 110. [Given P (Z < 1) = 0·8413 and P (Z < – 1) = 0·1587]	

Sol.	Here, $\mathbf{Z = \frac{X-100}{10}}$ $\mathbf{P(90 < X < 110) = P(X < 110) - P(X < 90)}$ $\Rightarrow \mathbf{P(90 < X < 110) = P(-1 < Z < 1) = P(Z < 1) - P(Z < -1)}$ $\mathbf{= 0.8413 - 0.1587}$ $\mathbf{= 0.6826}$	$(\frac{1}{2})$ **(2)** $(\frac{1}{2})$
30.	The mean weekly sales of a 4-wheeler was 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful. [Given $\sqrt{5} = 2 \cdot 24$, $t_{19}(0 \cdot 05) = 1 \cdot 729$]	
Sol.	Here, $\mathbf{\mu_0 = 50, \bar{x} = 55, n = 20}$ and $\mathbf{S = 10}$ $\mathbf{H_0: \mu = 50}$ (The advertisement campaign was not successful) $\mathbf{H_\alpha: \mu > 50}$ (The advertisement campaign was successful) The test statistic t is given by $\mathbf{t = \frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}} = \frac{55-50}{\frac{10}{\sqrt{20}}} = \frac{2\sqrt{5}}{2} = \sqrt{5} = 2.24}$ Degree of freedom $= \mathbf{20 - 1 = 19}$ Here, $\mathbf{t > t_{19}(0.05)}$ as $\mathbf{2.24 > 1.729}$ $\Rightarrow$ null hypothesis is rejected i.e., Advertising campaign was successful	$(\frac{1}{2})$ $(1\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
31(a).	A recent accounting graduate opened a new business and installed a computer system that costs ₹ 45,200. The computer system will be depreciated linearly over 3 years and will have a scrap value of ₹ 0. (i) What is the rate of depreciation ? (ii) Give a linear equation that describes the computer system's book value at the end of t^{th} year, where $0 \le t \le 3$. (iii) What will be the computer system's book value at the end of the first year and a half ?	
Sol.	(i) Annual amount of depreciation $= \mathbf{\frac{45200-0}{3}} =$ ₹ $\mathbf{\frac{45200}{3}}$ Rate of depreciation $= \mathbf{\frac{\frac{45200}{3}}{45200} \times 100 = 33.3}\%$	**(1)**

	(ii) $\mathbf{v(t) = mt + C = -\frac{45200}{3}t + 45200}$ (iii) $\mathbf{v\left(1\frac{1}{2}\right) = -\frac{45200}{3} \times \frac{3}{2} + 45200 = ₹\,22600}$	**(1)** **(1)**
	OR	
31(b).	Find the effective rate which is equivalent to normal rate of 10% p.a. compounded : (i) semi-annually. (ii) quarterly. [Given $(1{\cdot}05)^2 = 1{\cdot}1025$, $(1{\cdot}025)^4 = 1{\cdot}1038$]	
Sol.	(i) $\mathbf{r_e = \left(1 + \frac{r}{200}\right)^2 - 1 = \left(1 + \frac{10}{200}\right)^2 - 1}$ $\mathbf{= (1.05)^2 - 1 = 1.1025 - 1 = 10.25\%}$ (ii) $\mathbf{r_e = \left(1 + \frac{r}{400}\right)^4 - 1 = \left(1 + \frac{10}{400}\right)^4 - 1}$ $\mathbf{= (1.025)^4 - 1 = 1.1038 - 1 = 10.38\%}$	**(1)** $\mathbf{(\frac{1}{2})}$ **(1)** $\mathbf{(\frac{1}{2})}$
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32.	A cistern has three pipes A, B and C. Pipes A and B are inlet pipes whereas C is an outlet pipe. Pipes A and B can fill the cistern separately in 3 hours and 4 hours respectively; while pipe C can empty the completely filled cistern in 1 hour. If the pipes A, B and C are opened in order at 5, 6 and 7 a.m. respectively, at what time will the cistern be empty ?	
Sol.	Let the cistern be emptied in $\boldsymbol{n}$ hours after 5 a.m. Clearly pipes A and B fill the cistern for $\boldsymbol{n}$ and $\boldsymbol{n-1}$ hours respectively, while pipe C empties the tank for $\boldsymbol{n-2}$ hours $\therefore \boldsymbol{\frac{n}{3} + \frac{n-1}{4} - \frac{n-2}{1} = 0}$ Solving, we get $\boldsymbol{n = \frac{21}{5}}$ i.e., 4 hours 12 minutes past 5 am i.e., at 9:12 am	$\mathbf{(1\frac{1}{2})}$ $\mathbf{(1\frac{1}{2})}$ **(1)** **(1)**

33(a).	Find all the points of local maxima and local minima of the function : $$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105.$$	
Sol.	$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$ $f'(x) = -3x^3 - 24x^2 - 45x$ and $f''(x) = -9x^2 - 48x - 45$ $f'(x) = 0 \Rightarrow x = 0$ or $x^2 + 8x + 15 = 0$ i.e., $x = 0, -3, -5$ At $x = 0, f''(0) < 0 \Rightarrow 0$ is a point of local maxima At $x = -3, f''(-3) > 0 \Rightarrow -3$ is a point of local minima At $x = -5, f''(-5) < 0 \Rightarrow -5$ is a point of local maxima	**(1)** **(1)** **(1)** **(1)** **(1)**
	OR	
33(b).	Find the intervals in which the following function f is strictly increasing or strictly decreasing : $$f(x) = 20 - 9x + 6x^2 - x^3.$$	
Sol.	$f(x) = 20 - 9x + 6x^2 - x^3$ $f'(x) = -9 + 12x - 3x^2 = -3(x^2 - 4x + 3)$ $= -3(x-1)(x-3)$ $f'(x) = 0 \Rightarrow x = 1,3$ Now intervals are $(-\infty, 1), (1,3)$ and $(3, \infty)$	$(\frac{1}{2})$ **(1)** $(\frac{1}{2})$ $(\frac{1}{2})$

Intervals	Sign of $f'(x)$
$(-\infty, 1)$	$-$ve
$(1,3)$	$+$ve
$(3, \infty)$	$-$ve

$(1\frac{1}{2})$

$\Rightarrow f(x)$ is strictly increasing in $(1,3)$ or $[1,3]$ $(\frac{1}{2})$

And $f(x)$ is strictly decreasing in $(-\infty, 1) \cup (3, \infty)$ or $(-\infty, 1] \cup [3, \infty)$ $(\frac{1}{2})$

34(a).	Let X denote the number of hours a Class 12 student studies during a randomly selected school day. The probability that X can take the values x_i, for an unknown constant 'k' : $$P(X = k) = \begin{cases} 0.1 & \text{if} \quad x_i = 0 \\ kx_i & \text{if} \quad x_i = 1 \text{ or } 2 \\ k(5 - x_i) & \text{if} \quad x_i = 3 \text{ or } 4 \end{cases}$$ (i) Find the value of k. (ii) Determine the probability that the student studied for at least 2 hours. (iii) Determine the probability that the student studied for at most 2 hours.	
Sol.	(i) $\mathbf{0.1 + k + 2k + 2k + k = 1}$ $\Rightarrow \mathbf{0.1 + 6k = 1}$ $\Rightarrow \mathbf{k = \frac{3}{20}}$ (ii) $\mathbf{P(X \geq 2) = P(2) + P(3) + P(4)}$ $\mathbf{= 2k + 2k + k}$ $\mathbf{= 5k = \frac{3}{4}}$ (iii) $\mathbf{P(X \leq 2) = P(0) + P(1) + P(2)}$ $\mathbf{= 0.1 + k + 2k}$ $\mathbf{= \frac{1}{10} + \frac{9}{20} = \frac{11}{20}}$	**(1)** **(1)** **($\frac{1}{2}$)** **(1)** **($\frac{1}{2}$)** **(1)**
	OR	
34(b).	A river near a small town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation ? Also, calculate the probability of 3 or less overflows and floods in a 10-year interval. [Given $e^{-2} = 0.13534$]	
Sol.	Here, mean expectation $= \boldsymbol{\lambda = 2}$ $\mathbf{P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}}$ $\mathbf{P(X \leq 3) = P(0) + P(1) + P(2) + P(3)}$ $\mathbf{= e^{-2}\left(1 + 2 + 2 + \frac{4}{3}\right)}$ $\mathbf{= 0.13534 \times \frac{19}{3} = 0.8571 \text{ or } 0.86}$	**(1)** **(1)** **(2)** **(1)**

35.	Amrita buys a car for which she makes a down payment of ₹ 2,50,000 and the balance is to be paid in 2 years by monthly instalments of ₹ 25,448 each. If the financer charges interest at the rate of 20% p.a, find the actual price of the car. [Given $\left(\frac{61}{60}\right)^{-24} = 0{\cdot}67253$]	
Sol.	$\mathbf{n = 2 \times 12 = 24,}$	$(\frac{1}{2})$
	$\mathbf{i = \frac{20}{1200} = \frac{1}{60}}$	$(\frac{1}{2})$
	EMI $= \frac{\mathbf{Pi}}{\mathbf{1-(1+i)^{-n}}}$	
	$\mathbf{25448 = \frac{P \times \frac{1}{60}}{1-(1+\frac{1}{60})^{-24}}}$	**(1)**
	$\mathbf{P = 25448 \times 60\left[1-\left(1+\frac{1}{60}\right)^{-24}\right]}$	**(1)**
	$\mathbf{P = 25448 \times 60(1-0.67253)}$	$(\frac{1}{2})$
	$=$ ₹ $\mathbf{5,00,000}$ approx.	**(1)**
	Hence the actual price of the car is ₹ 7,50,000 approx.	$(\frac{1}{2})$
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36. On her birthday, Prema decides to donate some money to children of an orphanage home.

If there are 8 children less, everyone gets ₹ 10 more. However, if there are 16 children more, everyone gets ₹ 10 less.

Let the number of children in the orphanage home be x and the amount to be donated to each child be ₹ y.

Based on the above information, answer the following questions :

(i) Write the system of linear equations in x and y formed of the given situation. *1*

(ii) Write the system of linear equations, obtained in (i) above, in matrix form AX = B. *1*

(iii) (a) Find the inverse of matrix A. *2*

OR

(b) Determine the values of x and y. *2*

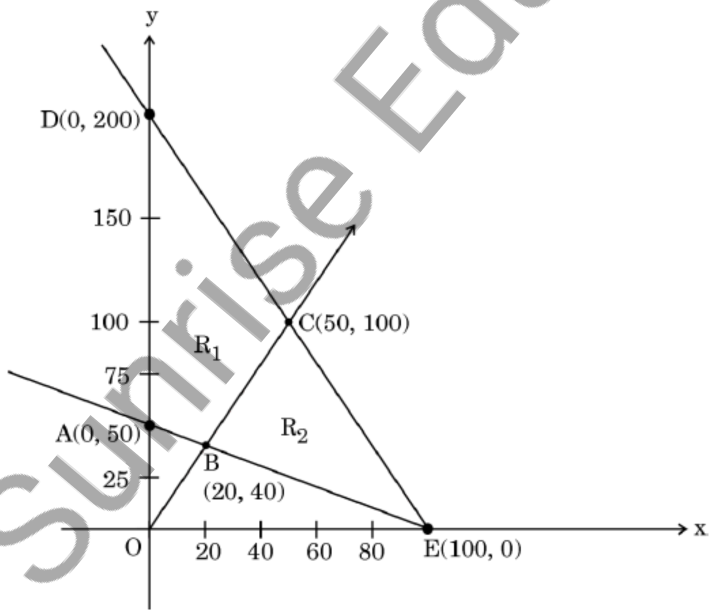
Sol.

(i) Let number of children be x and the amount donated to each child be ₹ y

$\therefore (x-8)(y+10) = xy$ and $(x+16)(y-10) = xy$ **$(\frac{1}{2})$**

i.e., $5x - 4y = 40$ and $5x - 8y = -80$ **$(\frac{1}{2})$**

(ii) $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$ **(1)**

	(iii) (a) $\lvert A \rvert = -40 + 20 = -20 \neq 0$ adj (A) $= \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$ $\Rightarrow A^{-1} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 8 & -4 \\ 5 & -5 \end{bmatrix}$ **OR** (iii) (b) $X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$ $= \frac{1}{20}\begin{bmatrix} 8 & -4 \\ 5 & -5 \end{bmatrix}\begin{bmatrix} 40 \\ -80 \end{bmatrix}$ $= \frac{1}{20}\begin{bmatrix} 640 \\ 600 \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$ $\Rightarrow x = 32, y = 30$	$(\frac{1}{2})$ **(1)** $(\frac{1}{2})$ $(\frac{1}{2})$ **(1)** $(\frac{1}{2})$
37.	**Case Study – 2** In number theory, it is often important to find factors of an integer N. The number N has two trivial factors, namely 1 and N. Any other factor, if exists, is called non-trivial factor of N. Naresh has plotted a graph of some constraints (linear inequations) with points A(0, 50), B(20, 40), C(50, 100), D(0, 200) and E(100, 0). This graph is constructed using three non-trivial constraints and two trivial constraints. One of the non-trivial constraints is $x + 2y \geq 100$. 	

	Based on the above information, answer the following questions : (i) What are the two trivial constraints ? *1* (ii) (a) If R_1 is the feasible region, then what are the other two non-trivial constraints ? *2* **OR** (b) If R_2 is the feasible region, then what are the other two non-trivial constraints ? *2* (iii) If R_1 is the feasible region, then find the maximum value of the objective function $z = 5x + 2y$. *1*	
Sol.	(i) $x \geq 0, y \geq 0$ (ii) (a) $2x - y \leq 0$, $2x + y \leq 200$ **OR** (ii) (b) $2x + y \leq 200$, $2x - y \geq 0$ (iii) Corner points of R_1 are $A(0,50), B(20,40), C(50,100)$ and $D(0,200)$ $\mathbf{Z_A = 100;\ Z_B = 180; Z_C = 450;\ Z_D = 400}$ So, Z is maximum at C and maximum value of $Z = 450$	**(1)** **(1)** **(1)** **(1)** **(1)** $\mathbf{(\frac{1}{2})}$ $\mathbf{(\frac{1}{2})}$

38.

When observed over a long period of time, a time series data can predict trends that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

The table below shows the sale of an item in a district during 1996 – 2001 :

Year :	1996	1997	1998	1999	2000	2001
Sales (in lakh ₹) :	6·5	5·3	4·3	6·1	5·6	7·8

Based on the above information, answer the following questions :

(i) Determine the equation of the straight-line trend. *2*

(a) Tabulate the trend values of the years and also compute expected sales trend for the year 2002. *2*

OR

(b) Fit a straight-line trend by the method of least squares for the following data : *2*

Year :	2004	2005	2006	2007	2008	2009	2010
Profit (₹ '000)	114	130	126	144	138	156	164

Sol.	(i)	

Year (x_i)	Index Number (y)	$x = \frac{x_i - A}{0.5}$	x^2	xy
1996	6.5	-5	25	-32.5
1997	5.3	-3	9	-15.9
1998	4.3	-1	1	-4.3
1999	6.1	1	1	6.1
2000	5.6	3	9	16.8
2001	7.8	5	25	39
n = 6	$\sum y = 35.6$	$\sum x = 0$	$\sum x^2 = 70$	$\sum xy = 9.2$

(1) for correct table

$a = \frac{\sum y}{n} = \frac{35.6}{6}, b = \frac{\sum xy}{\sum x^2} = \frac{9.2}{70} = 0.13$ **$(\frac{1}{2})$**

∴ Equation of straight-line trend is given by

$y = a + bx = 5.9 + 0.13x$ **$(\frac{1}{2})$**

(ii) (a) Trend Values

1996	$5 \cdot 9 + (-5) \times 0 \cdot 13 = 5 \cdot 25$	
1997	$5 \cdot 9 + (-3) \times 0 \cdot 13 = 5 \cdot 51$	
1998	$5 \cdot 9 + (-1) \times 0 \cdot 13 = 5 \cdot 77$	
1999	$5 \cdot 9 + (1) \times 0 \cdot 13 = 6.03$	
2000	$5 \cdot 9 + (3) \times 0 \cdot 13 = 6.29$	
2001	$5 \cdot 9 + (5) \times 0 \cdot 13 = 6.55$	

(1) mark for correct trend values

Expected sales trend for 2002

$= 5.9 + 0.13\left(\frac{2002-1998.5}{0.5}\right)$ **$(\frac{1}{2})$**

$=$ ₹ 6.81 lakhs **$(\frac{1}{2})$**

OR

(ii) (b)

Year (x_i)	Profit (y)	$x = x_i - A$	x^2	xy
2004	114	-3	9	-342
2005	130	-2	4	-260
2006	126	-1	1	-126
2007	144	0	0	0
2008	138	1	1	138
2009	156	2	4	312
2010	164	3	9	492
n = 7	$\sum y = 972$	$\sum x = 0$	$\sum x^2 = 28$	$\sum xy = 214$

(1) for correct table

$$a = \frac{\sum y}{n} = \frac{972}{7} = 138.86, b = \frac{\sum xy}{\sum x^2} = \frac{214}{28} = 7.64$$ $\left(\frac{1}{2}\right)$

So, required equation of straight- line trend is

$$y = a + bx = 138.86 + 7.64x$$ $\left(\frac{1}{2}\right)$