NUMERICAL ANALYSI & STATISTICAL METHODS (03191251)

Unit:2 Introduction to Probability

Prerequisites:

Fundamental Principle of Counting:

If one job can be done in m ways and another job can be done in n ways, then the number of ways of performing the combination of the above two jobs is m * n. It can be extended to any finite number of operations

Prerequisites:

Permutation

If n objects are given and we have to arrange r $(r \leq n)$ out of them and the order in which these objects are arranged is important, such an arrangement is called a permutation of n objects taken r at a time.

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Combination

If n objects are given and we have to choose $r(r \leq n)$ out of them and the order in which objects are arranged is not important, such a choice is called a combination of n objects taken at a time.

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

Basic Concept in Probability:

Experiment: The term experiment refers to describe an act which can be repeated under some given conditions.

Random Experiment or Trial: If in an experiment all possible outcomes are known in advance and none of the outcomes can be predicted with certainty, then such an experiment is called random experiment.
Eg: tossing a coin, throwing a dice.

Basic Concept in Probability:

Sample space: The set consisting of all outcomes of a random experiment is called sample space.
Eg: Sample space of tossing a coin once is {H, T}, Sample space of throwing a dice is {1, 2, 3, 4, 5, 6}.

Events: The outcomes of the random experiments are called the events.

Basic Concept in Probability:

Types of events

Certain event: An event whose occurrence is inevitable (or certain) is called a certain event.

Impossible event: An event whose occurrence is impossible is called impossible event.

Intersection of two events: Intersection of two events means the set of all the sample points belonging to both the events.

Basic Concept in Probability:

Mutually exclusive events: Two events are said to be mutually exclusive or incompatible when both cannot happen simultaneously in a single trial.

Or

The occurrence of any one of the events prevents the occurrence of the other.

Basic Concept in Probability:

Independent events: Two or more events are said to be independent when the outcome of one does not affect and is not affected by the other.

Eg. : If a coin is tossed twice the second throw is not affected by the outcome of the first and vice-versa. Also while throwing a dice the outcomes of none of the events are affected by the other.

Basic Concept in Probability:

Dependent events: Dependent events are those events in which the occurrence or non-occurrence of one event in any one trial affects the probability of other events in other trials.

Eg.: If a card is drawn from a pack of playing cards and is not replaced, this will alter the probability of next card drawn i. e. probability of drawing a queen from a pack of 52 cards is (4/52). But if the card drawn in first trial is queen and is not replaced then the probability of drawing queen in the next trial is (3/52).

Basic Concept in Probability:

Equally likely events: Events are said to be equally likely when one does not occur more often then the others.

Eg: If we throw a dice each face may be expected to be observed approximately the same number.

Exhaustive events: Events are said to be exhaustive when their totality includes all the possible outcomes of a random experiment.

Eg: In tossing a coin exhaustive number of cases are 2 i.e. head and tail.
In throwing a dice the exhaustive number of cases are 6 i.e. 1, 2, 3, 4, 5, 6.

Basic Concept in Probability:

Complementary events: For two events A and B, A is said to be complementary event of B, if A and B are mutually exclusive and exhaustive events.

Eg: When a dice is thrown, occurrence of even numbers (2, 4, 6) and odd numbers (1, 3, 5) are complementary events.

Favorable cases: The number of sample points favorable to the happening of an event A is known as favorable cases.

Eg: In drawing a card from a pack of cards, the favorable cases for getting a club is 13.

Probability:

$$p = probability\ of\ sucess$$
$$q = probability\ of\ failure$$

Mathematical or Classical approach: If an experiment results in ‘n’ equally likely ways, and out of which ‘m’ are favorable to the happening of the event A, then the probability of happening of event A is defined as the ration of m:n.

Probability:

$$P(A) = \frac{Number\ of\ favorable\ cases}{Total\ number\ of\ equally\ likely\ cases} = \frac{m}{n}$$

The number of favorable cases is always less than or equal to the total number of equally likely cases.

$$0 \leq m \leq n$$

$$0 \leq \frac{m}{n} \leq 1$$

$$0 \leq P(A) \leq 1$$

Thus the probability of an event is always between 0 and 1.

Probability:

The probability of a certain event is always 1 and that of impossible event is 0.
If ‘m’ cases are favorable to the happening of event A, then ‘n-m’ are the cases not favorable to the happening of A i.e. favorable to the happening of complement of event A.

$$P(A') = \frac{n-m}{n}$$

$$P(A') = \frac{n}{n} - \frac{m}{n}$$

$$P(A') = 1 - \frac{m}{n}$$

$$P(A') = 1 - P(A)$$

$$P(A) + P(A') = 1$$

Probability:

Thus the probability of happening and non-happening of an event is 1.

Shortcomings of the classical approach:

It cannot be applied whenever it is not possible to find the no. of equally likely cases.

It cannot be applied if exhaustive cases are infinite.

It cannot be applied if the events are not equally likely.

Probability:

Statistical or Empirical or A posteriori definition of Probability: If an experiment is repeated under same conditions for a large number of times then the limit of the ratio of number of times the event happens to the total number of trials is Statistical Probability.

Probability:

Modern or Axiomatic definition of Probability: The axiomatic approach to probability was introduced by the Russian mathematician A. N. Kolmogorov in the year 1993 in the book *Foundations of Probability*.

If A is any event from the sample space S, then $P(A)$is called the probability of A if it satisfies the following axioms:

$$0 \leq P(A) \leq 1$$

$$P(S) = 1$$

If A and B are mutually exclusive events then the probability of occurrence of either A or B is denoted by $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B)$$

Probability:

Example: Find the probability of getting an odd number when a cubical die is thrown.

Example: Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that both are kings.

Probability:

Addition Theorem: If A and B are two events, the probability of occurrence of either A or B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events then $A \cap B = \varphi$ hence $P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B)$$

For 3 events A, B and C, the probability that at least one will occur is

$$P(A \cup B \cup C)$$
$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$P(A \cup B) = 1 - P(A' \cap B')$$

Probability:

Multiplication Theorem: If two events A and B are independent, the probability of occurrence of both is equal to the product of the individual probabilities of both the events.

$$P(A \cap B) = P(A) * P(B)$$

Probability:

Example: If $P(A) = \frac{1}{3}, P(B') = \frac{1}{4}$, and $P(A \cap B) = \frac{1}{6}$, find $P(A \cup B), P(A' \cap B')$ and $P(\frac{A'}{B'})$.

Example: There are 5 red ad 7 black balls in an urn. Two balls are drawn at random one after the other. If they are drawn (i) with replacement (ii) without replacement, find the probability that both the balls are red.

Example: If an unbiased dice is rolled. Find the probability of getting: (i) Even Number (ii) A perfect square (iii) A number divisible by 3.

Probability:

Example: A card is drawn at random from a pack of well shuffled 52 cards. Find the probability that the card drawn is (i) a king (ii) not a diamond (iii) An ace of red hearts or diamonds

Example: The Probability that a contractor will get a contract is $\frac{2}{3}$ and the probability that he will get on other contract is $\frac{5}{9}$ .If the probability of getting at least one contract is $\frac{4}{5}$, what is the probability that he will get both the contracts?

Example: Two students X and Y work independently on a problem. The probability that X will solve it is $\frac{3}{4}$ and probability that Y will solve it is $\frac{2}{3}$. What is the probability that the problem will be solved?

Probability:

Example: From well shuffled pack of 52 playing cards, one card is drawn. What is the probability that (i) It is either red card or black card (ii) it is either a jack or an ace card (iii) it is either red card or spade card

Example: If the probability of getting a contract A is 0.25, probability of getting contract B is 0.15 and the probability of getting at least one contract is 0.30, Calculate: (i) the probability of getting both the contracts (ii) the probability of getting contract A only (iii) the probability of getting contract B only (iv) the probability of not getting both the contracts (v) the probability of not getting at least one contract

Conditional Probability:

If A and B are two events such that the probability of one event is influence or effected by whether a related event already occur or not.
The Condition probability A given B is denoted By (A/B).
If A and B are dependent events, the probability related to dependent events is called conditional probability.
Conditional probability of B given A has already occurred is given by

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

or

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Note: If events $A\ and\ B$ are Independent then $P(A \cap B) = P(A)P(B)$

Conditional Probability:

Example: The personnel department of a company has records which show the following analysis of its 200 engineers.

Age(Year)	Bachelor's Degree only	master's Degree	Total
Under 30	90	10	100
30 to 40	20	30	50
Over 40	40	10	50
	150	50	200

Conditional Probability:

If one engineer is selected at random from thee company, find (i) The probability that he has only a batcher's degree; (ii) The probability that he has a master's degree given that he is over 40; (iii) The probability that he is under 30 given that he has only a bachelor's degree.

Example: In a certain town, males and females form 50 percent of the population.it is known that 20 percent of the males and 5 percent of the females are unemployed. A research student studying the employment situation select unemployed person selected is (a) male (b) female?

Conditional Probability:

Example: A department store has been the target of many shoplifters during the past month, but owing to increased security precautions, 250 shoplifters have been caught. Each shoplifter's sex is noted, also noted is whether he/she was a first-time or repeat offender. The data are summarized in the table below.

Sex	First-Time Offender	Repeat Offender
Males	60	70
Females	44	76

Conditional Probability:

Assuming that an apprehended shoplifter is chosen at random, find:

a) The Probability that the shoplifter is male.

b) The Probability that the shoplifter is a first-time offender, given that the shoplifter is male.

c) The Probability that the shoplifter is female, given that the shoplifter is a repeat offender.

d) The Probability that the shoplifter if female, given that the shoplifter is a first-time offender.

Conditional Probability:

Example: In a certain college, 25% students failed in mathematics, 15% students failed statistics and 30% students failed in at least one of the subject selected at random. Find the probability that (i) he failed in both mathematics and statistics (ii) he failed in mathematics if he also failed in statistics (iii) he failed in statistics if he also failed in mathematics (iv)he failed in mathematics given that he passed in statistics.

Example: A box has 3 black, 4 white and 5 red balls. Balls are drawn one after one from the box (without replacement). Find the probability that the both balls drawn are black.

Bayes' Theorem:

- A formula for determining conditional probability named after 18th-century British mathematician Thomas Bayes. The theorem provides a way to revise existing predictions or theories given new or additional evidence.
- The formula is as follows:

$$P(Ai|B) = \frac{P(Ai)P(B|Ai)}{P(A1)P(B|A1) + P(A2)P(B|A2) + \ldots\ldots\ldots\ldots + P(An)P(B|An)}$$

Bayes' Theorem:

Example: The prior for event A1, A2, A3 are $p(A1)=.20$ $p(A2)=.50$ $p(A3)= .30$. the conditional probability of B given A1, A2, A3 are $p(B|A1)=.50$ and $p(B|A2)= .40$, and $P(B|A3)= .30$ Use Bayes' theorem to compute $P(A1|B)$, $P(A2|B)$, $P(A3|B)$.

Example: An entomologist spots what might be a rare subspecies of beetle, due to the pattern on its back. In the rare subspecies, 98% have the pattern. In the common subspecies, 5% have the pattern. The rare subspecies accounts for only 0.1% of the population. Using Bayes' theorem, find the p(Rare | Pattern).

Bayes' Theorem:

Example: There are 1000 people in a locality. Three newspapers A, B and C are available to them. 500 people read A, 400 people read B and 400 read C. 100 people read both A and B, 150 read both B and C and 200 read both A and c. 40 people read all the three newspapers. Find the probability that a person selected at random from that locality reads at least one of the three papers.

Thank You