Birla Institute of Technology & Science, Pilani
Pilani Campus – Rajasthan

Mid-Semester Exam – ECON-F354/ FIN F311
Derivative & Risk Management (DRM)
Session – 2022-23 (I)
Closed Book

Dated: 02/Nov./2022

Maximum Marks: 105
Time Duration: 90 Minutes (Max)

Instructions:

- Do not forget to write your Name and ID number on the answer sheet
- Read question specific instructions before giving your answers
- **To get the full score, you need to show all the steps required to arrive at the final answer with proper interpretation**
- **Calculator is allowed**

[15 Marks]

Q1.
There are two stocks in the market i.e., stock A and stock B. The current value of stock A is Rs.75. The price of stock A will be Rs.64 next year if the economy is in a recession, Rs.87 if the economy is normal, and Rs.97 if the economy is expanding. The probabilities of recession, normal and expansion are 0.2, 0.6 and 0.2 respectively. The correlation coefficient of stock A with market portfolio is 0.7. Stock B has an expected return of 14% and standard deviation of 34% with correlation coefficient with market portfolio 0.24. In addition, the stock B has correlation with stock A of 0.36, and the standard deviation of market portfolio is given as 18%. Based on your analysis, answers the following questions:

a. If you are a risk averse investor, which stock would you prefer and why?
b. What are the expected return and standard deviation of a portfolio consisting of 70 percent of stock A and 30 percent of stock B? check calculation
c. What is the beta of the portfolio is part (b)?

[30 Marks]

Q2.
There are three securities in the market. The following chart shows their possible payoffs:

State of Economy	Probability	Return on 1		Return on 2	Return on 3
1	0.15	0.20	0.075	0.20	0.05
2	0.35	0.15	0.025	0.10	0.10
3	0.35	0.10	0.025	0.15	0.15
4	0.15	0.05	0.075	0.05	0.20

a. What are the expected returns and standard deviation of each of the security?
b. What are the covariances and correlations between the pairs of securities?
c. What are the expected return and standard deviation of a portfolio with half of its funds invested in security 1 and half in security 2?
d. What are the expected return and standard deviation of a portfolio with half of its funds invested in security 1 and half in security 3?
e. What are the expected return and standard deviation of a portfolio with half of its funds invested in security 2 and half in security 3?
f. What do you answer in parts (a), (c), (d), and (e) imply about diversification?

[25 Marks]

Q3.
After deciding to buy a new car, you can either lease the car or purchase it with a three-year loan plan. The car you wish to buy is Rs.28000. The dealer has a leasing arrangement where you pay Rs.2400 today and Rs.380 per month for the next three years. If you purchase the car, you will pay it off in monthly payments over the three years at 6% interest rate per annum. You believe that you will be able to sell the car for Rs.17000 in three years. Should you buy or lease the car? What break-even resale price would make you indifferent between buying and leasing?

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Q4: **[30+5=35 Marks]**

(A) The current price of a stock is 200, and the continuously compounded risk-free interest rate is 4%. A dividend will be paid every quarter for the next 3 years, with the first dividend occurring 3 months from now. The amount of the first dividend is 1.50, but each subsequent dividend will be 1% higher than the one previously paid. Calculate the fair price of a 3-year forward contract on this stock. [Hint: $D1=D0(1+g)$, where D1: Dividend at t=1, D0: Dividend at t=0, g=growth rate of dividend]

(B) A market maker in stock index forward contracts observe a 6-month forward price of 112 on the index. The index spot price is 110 and the continuously compounded dividend yield on the index is 2%. The continuously compounded risk-free interest rate is 5%. Describe actions the market maker could take to exploit an arbitrage opportunity and calculate the resulting profit.

Mid-Sem DRM Solution

Q1:

a. A typical, risk-averse investor seeks high returns and low risks. For a risk-averse investor holding a well-diversified portfolio, beta is the appropriate measure of the risk of an individual security. To assess the two stocks, we need to find the expected return and beta of each of the two securities.

Stock A:

Since Stock A pays no dividends, the return on Stock A is: $(P_1 – P_0) / P_0$. So, the return for each state of the economy is:

$R_{\text{Recession}}$ = (\$64 – 75) / \$75 = –.147, or –14.70%

R_{Normal} = (\$87 – 75) / \$75 = .160, or 16.00%

$R_{\text{Expanding}}$ = (\$97 – 75) / \$75 = .293, or 29.30%

The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

$$E(R_A) = .20(–.147) + .60(.160) + .20(.293) = .1253, \text{ or } 12.53\%$$

And the variance of the stock is:

$$\sigma_A^2 = .20(–.147 – 0.1253)^2 + .60(.160 – .1253)^2 + .20(.293 – .1253)^2$$

$$\sigma_A^2 = .0212$$

Which means the standard deviation is:

$$\sigma_A = .0212^{1/2}$$

$$\sigma_A = .1455, \text{ or } 14.55\%$$

Now we can calculate the stock's beta, which is:

$$\beta_A = (\rho_{A,M})(\sigma_A) / \sigma_M$$

$$\beta_A = (.70)(.1455) / .18$$

$$\beta_A = .566$$

For Stock B, we can directly calculate the beta from the information provided. So, the beta for Stock B is:

$$\beta_B = (\rho_{B,M})(\sigma_B) / \sigma_M$$

$$\beta_B = (.24)(.34) / .18$$

$$\beta_B = .453$$

The expected return on Stock B is higher than the expected return on Stock A. The risk of Stock B, as measured by its beta, is lower than the risk of Stock A. Thus, a typical risk-averse investor holding a well-diversified portfolio will prefer Stock B. Note, this situation implies that at least one of the stocks is mispriced since the higher risk (beta) stock has a lower return than the lower risk (beta) stock.

b. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$E(R_P) = X_A E(R_A) + X_B E(R_B)$

$E(R_P) = .70(.1253) + .30(.14)$

$E(R_P) = .1297$, or 12.97%

To find the standard deviation of the portfolio, we first need to calculate the variance. The variance of the portfolio is:

$\sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \sigma_A \sigma_B \rho_{A,B}$

$\sigma_P^2 = (.70)^2(.1455)^2 + (.30)^2(.34)^2 + 2(.70)(.30)(.1455)(.34)(.36)$

$\sigma_P^2 = .02825$

And the standard deviation of the portfolio is:

$\sigma_P = .02825^{1/2}$

$\sigma_P = .1681$, or 16.81%

c. The beta of a portfolio is the weighted average of the betas of its individual securities. So the beta of the portfolio is:

$\beta_P = .70(.566) + .30(.453)$

$\beta_P = .532$

Q2:

a. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the expected return and standard deviation of each stock are:

Asset 1:

$E(R_1) = .15(.20) + .35(.15) + .35(.10) + .15(.05) = .1250$, or 12.50%

$\sigma_1^2 = .15(.20 – .1250)^2 + .35(.15 – .1250)^2 + .35(.10 – .1250)^2 + .15(.05 – .1250)^2 = .00213$

$\sigma_1 = (.00213)^{1/2} = .0461$, or 4.61%

Asset 2:

$E(R_2) = .15(.20) + .35(.10) + .35(.15) + .15(.05) = .1250$, or 12.50%

$\sigma_2^2 = .15(.20 – .1250)^2 + .35(.10 – .1250)^2 + .35(.15 – .1250)^2 + .15(.05 – .1250)^2 = .00213$

$\sigma_2 = (.00213)^{1/2} = .0461$, or 4.61%

Asset 3:

$E(R_3) = .15(.05) + .35(.10) + .35(.15) + .15(.20) = .1250$, or 12.50%

$\sigma_3^2 = .15(.05 – .1250)^2 + .35(.10 – .1250)^2 + .35(.15 – .1250)^2 + .15(.20 – .1250)^2 = .00213$

$\sigma_3 = (.00213)^{1/2} = .0461$, or 4.61%

b. To find the covariance, we multiply each possible state times the product of each asset's deviation from the mean in that state. The sum of these products is the covariance. The correlation is the covariance divided by the product of the two standard deviations. So, the covariance and correlation between each possible set of assets are:

Asset 1 and Asset 2:

$\text{Cov}(1,2) = .15(.20 – .1250)(.20 – .1250) + .35(.15 – .1250)(.10 – .1250)$
$+ .35(.10 – .1250)(.15 – .1250) + .15(.05 – .1250)(.05 – .1250)$

$Cov(1,2) = .00125$

$\rho_{1,2} = Cov(1,2) / \sigma_1 \sigma_2$
$\rho_{1,2} = .00125 / (.0461)(.0461)$
$\rho_{1,2} = .5882$

Asset 1 and Asset 3:
$Cov(1,3) = .15(.20 – .1250)(.05 – .1250) + .35(.15 – .1250)(.10 – .1250)$
$+ .35(.10 – .1250)(.15 – .1250) + .15(.05 – .1250)(.20 – .1250)$
$Cov(1,3) = –.002125$

$\rho_{1,3} = Cov(1,3) / \sigma_1 \sigma_3$
$\rho_{1,3} = –.002125 / (.0461)(.0461)$
$\rho_{1,3} = –1$

Asset 2 and Asset 3:
$Cov(2,3) = .15(.20 – .1250)(.05 – .1250) + .35(.10 – .1250)(.10 – .1250)$
$+ .35(.15 – .1250)(.15 – .1250) + .15(.05 – .1250)(.20 – .1250)$
$Cov(2,3) = –.00125$

$\rho_{2,3} = Cov(2,3) / \sigma_2 \sigma_3$
$\rho_{2,3} = –.00125 / (.0461)(.0461)$
$\rho_{2,3} = –.5882$

c. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 2:

$E(R_P) = X_1E(R_1) + X_2E(R_2)$
$E(R_P) = .50(.1250) + .50(.1250)$
$E(R_P) = .1250$, or 12.50%

The variance of a portfolio of two assets can be expressed as:

$\sigma^2_P = X^2_1 \sigma^2_1 + X^2_2 \sigma^2_2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2}$
$\sigma^2_P = .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(.5882)$
$\sigma^2_P = .001688$

And the standard deviation of the portfolio is:

$\sigma_P = (.001688)^{1/2}$
$\sigma_P = .0411$ or 4.11%

d. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 3:

$E(R_P) = X_1E(R_1) + X_3E(R_3)$
$E(R_P) = .50(.1250) + .50(.1250)$
$E(R_P) = .1250$, or 12.50%

The variance of a portfolio of two assets can be expressed as:

$$\sigma_P^2 = X_1^2\sigma_1^2 + X_3^2\sigma_3^2 + 2X_1X_3\sigma_1\sigma_3\rho_{1,3}$$
$$\sigma_P^2 = .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(–1)$$
$$\sigma_P^2 = .000000$$

Since the variance is zero, the standard deviation is also zero.

e. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 2 and Asset 3:

$$E(R_P) = X_2E(R_2) + X_3E(R_3)$$
$$E(R_P) = .50(.1250) + .50(.1250)$$
$$E(R_P) = .1250\text{, or } 12.50\%$$

The variance of a portfolio of two assets can be expressed as:

$$\sigma_P^2 = X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2X_2X_3\sigma_2\sigma_3\rho_{2,3}$$
$$\sigma_P^2 = .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(–.5882)$$
$$\sigma_P^2 = .000438$$

And the standard deviation of the portfolio is:

$$\sigma_P = (.000438)^{1/2}$$
$$\sigma_P = .0209\text{, or } 2.09\%$$

f. As long as the correlation between the returns on two securities is below 1, there is a benefit to diversification. A portfolio with negatively correlated securities can achieve greater risk reduction than a portfolio with positively correlated securities, holding the expected return on each stock constant. Applying proper weights on perfectly negatively correlated securities can reduce portfolio variance to 0.

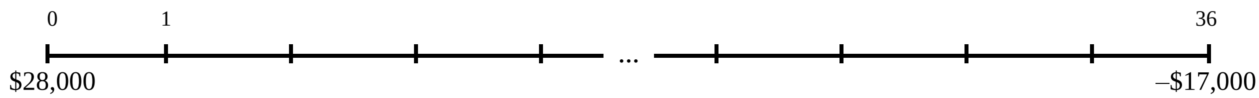
Q3:

To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing option is the PV of the lease payments, plus the $2,400. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:

0	1									36
$2,400	$380	$380	$380	$380	...	$380	$380	$380	$380	$380

$$PV = \$2,400 + \$380\{1 – [1 / (1 + .06 / 12)^{12(3)}]\} / (.06 / 12)$$
$$PV = \$14,890.99$$

The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:



PV = $17,000 / [1 + (.06 / 12)]$^{12(3)}$
PV = $14,205.96

The PV of the decision to purchase is:

$28,000 – 14,205.96 = $13,794.04

In this case, it is cheaper to buy the car than lease it since the PV of the leasing cash flows is lower. To find the break-even resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

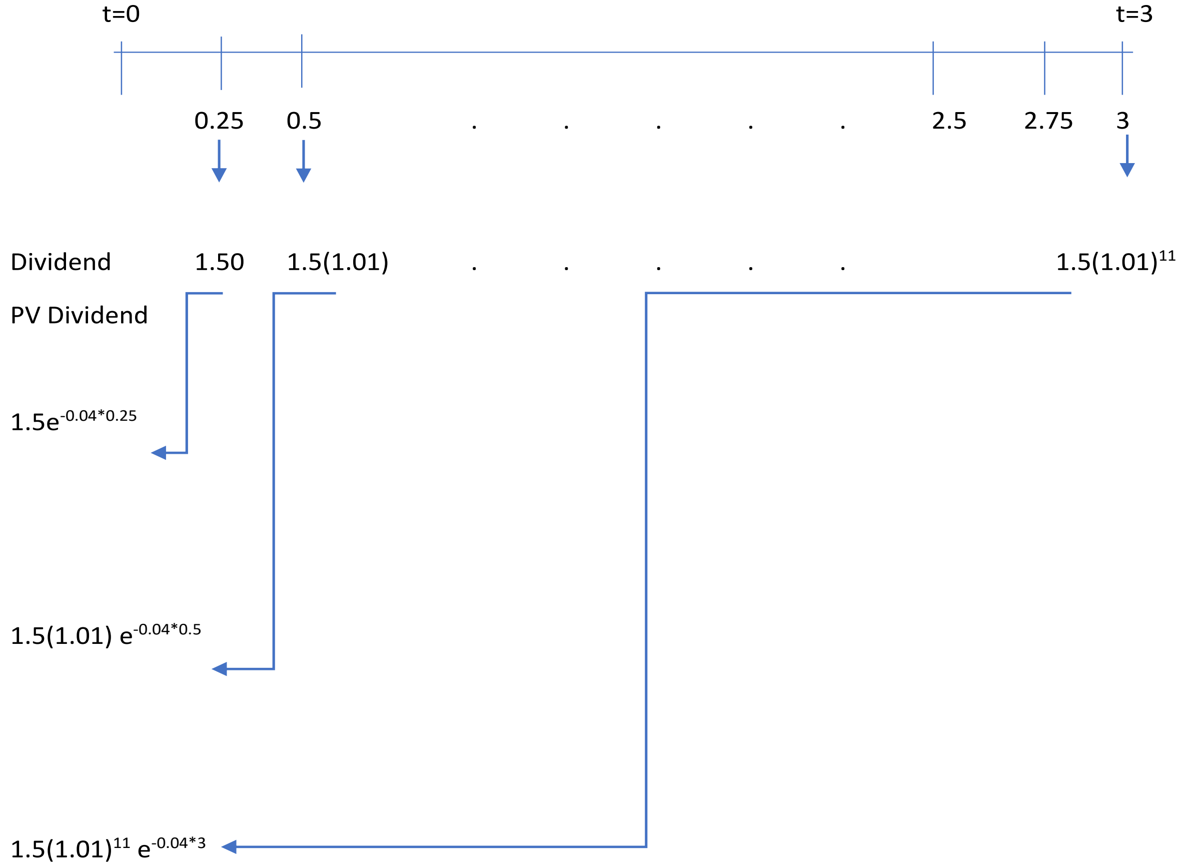
$28,000 – PV of resale price = $14,890.99
PV of resale price = $13,109.01

The resale price that would make the PV of the lease versus buy decision equal is the FV of this value, so:

Break-even resale price = $13,109.01[1 + (.06 / 12)]$^{12(3)}$
Break-even resale price = $15,687.30

Q4. (A)

The current value of the stock = Rs. 200
R= 4% with continuous compounding
T= 3 years
D= Rs. 1.50
Present dividend will be 1% higher than the previous one. Thus,
$D_1 = D_0(1+0.01)$
$D_2 = D_1\,(1+0.01)$ and so on…



Thus, the PV of all the dividends = Rs. 17.81
$F_0=(S_0-I)\,e^{rt} = (200-17.81)\,e^{0.04*3}$ =Rs. 205.41

(B) **Q:** A market maker in stock index forward contracts observe a 6-month forward price of 112 on the index. The index spot price is 110 and the continuously compounded dividend yield on the index is 2%. The continuously compounded risk-free interest rate is 5%. Describe actions the market maker could take to exploit an arbitrage opportunity and calculate the resulting profit.

Answer:

The fair value of the forward contract is given by $S\,e^{(r-d)T} =$ Rs. $110e^{(0.05-0.02)0.5} =$ Rs.111.66.

This is Rs. 0.34 less than the observed price. Thus, one could exploit this arbitrage opportunity by selling the observed forward at Rs.112 and buying a synthetic forward at

111.66, making 112 – 111.66 = Rs. 0.34 profit.