

Handwritten Digit Recognition using various Classification Techniques

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Abstract

The purpose of this paper is to propose Classification models such as Logistic Regression, Support Vector Machine, Random Forest, etc. to recognize different Handwritten Digits. The feature vectors of the handwritten digits images are obtained from two different sources, MNIST dataset and USPS dataset. The results of various classifiers are observed using Confusion Matrices and are used to ensemble them. The results of individual classifiers are combined using Majority Voting.

1 Introduction

In this project, several classification methods to solve the handwritten digit recognition task is discussed. The feature vectors are obtained for each image from MNIST dataset by using pickle file. The feature vectors are obtained for each image from USPS dataset by re-sizing the image to 28x28 dimension and then flattening it. The target vector takes ten values, 0 to 9 corresponding to each digit. The MNIST dataset is used to train the classifiers and the program is tested on a subset of MNIST dataset and USPS dataset. The datasets contain approximately equal numbers of each class.

2 Experiment

The aim of this project is to train a Classification model to predict which digit does the handwritten digit image resemble. Various Models of the following Classification techniques have been implemented:

1. **Multi-class Logistic Regression**
2. **Neural Network**
3. **Support Vector Machine**
4. **Random Forest**

Various Models of the following Classification ensemble techniques have also been implemented using Scikit-Learn and Mlxtend Library:

1. **Bootstrap aggregating**
2. **Boosting**
3. **Stacking**

The results of all the individual classifiers are combined using Majority Voting.

3 Classification Models

Classification is a data mining function that assigns items in a collection to target categories or classes. The goal of classification is to accurately predict the target class for each case in the data [1].

3.1 Multi-class Logistic Regression (Mini-batch Stochastic Gradient Descent)

Softmax regression (or, Multi-class Logistic Regression) is used to describe data and to explain the relationship between one dependent binary variable and one or more nominal, ordinal, interval or ratio-level independent variables. Like all regression analyses, the softmax regression is a predictive analysis [2]. A softmax regression function $\mathbf{y}(\mathbf{x}, \mathbf{w})$ is defined as,

$$y(x, w) = \text{softmax}(w^\top X) \quad (1)$$

where,

$$\text{softmax}(z) = \frac{e^z}{\sum_{i=1}^k e^{z_i}} \quad (2)$$

where $\mathbf{w} = (\mathbf{w}_0, \mathbf{w}_1, \dots, \mathbf{w}_{m-1})$ is a weight vector to be learned from training samples and X is the data matrix. The Loss Function for Logistic Regression is given as,

$$E_D(w) = \frac{1}{m} (-\mathbf{t}^\top \log(y(x, w)) - (1 - \mathbf{t})^\top \log(1 - y(x, w))) \quad (3)$$

where $\mathbf{t} = (\mathbf{t}_0, \mathbf{t}_1, \dots, \mathbf{t}_n)$ is the vector of outputs. The derivative of the loss function with respect to weights is defined as,

$$\nabla E_D = \frac{1}{m} X^\top (y(x, w) - \mathbf{t}) \quad (4)$$

We modify the solution for each data sample using:

$$w^{\tau+1} = w^\tau + \Delta w^\tau \quad (5)$$

where,

$$\Delta w^\tau = -\eta^\tau \nabla \sum_{i=1}^m E_D(w) \quad (6)$$

is called mini-batch weight updates. η^τ is the Learning Rate.

3.2 Neural Network

Many experts define deep neural networks as networks that have an input layer, an output layer and at least one hidden layer in between. Each layer performs specific types of sorting and ordering in a process that some refer to as "feature hierarchy" [3]. A simple Neural Network architecture is shown in Figure 1.

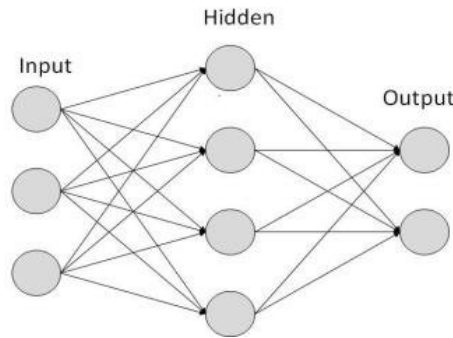


Figure 1: Neural Network architecture

3.3 Support Vector Machine

SVMs are supervised learning models with associated learning algorithms that analyze data used for classification and regression analysis. An SVM model is a representation of the examples as points in space, mapped so that the examples of the separate categories are divided by a clear gap that is as wide as possible. New examples are then mapped into that same space and predicted to belong to a category based on which side of the gap they fall. In addition to performing linear classification, SVMs can efficiently perform a non-linear classification using what is called the kernel trick, implicitly mapping their inputs into high-dimensional feature spaces [4].

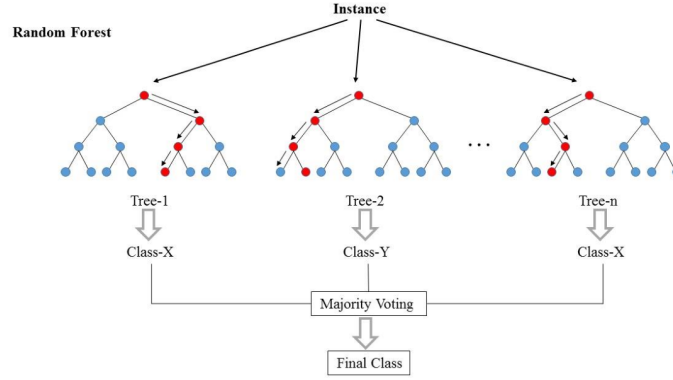


Figure 2: Random Forest Classifier

3.4 Random Forest Classifier

A random forest classifier is a meta estimator that fits a number of decision tree classifiers on various sub-samples of the dataset and uses averaging to improve the predictive accuracy and control over-fitting. Figure 2 shows simplified version of Random Forest Classifiers [5].

4 Ensemble Models

In statistics and machine learning, ensemble methods use multiple learning algorithms to obtain better predictive performance than could be obtained from any of the constituent learning algorithms alone [6].

4.1 Bootstrap Aggregating

Bootstrap aggregating, also called bagging, is a machine learning ensemble meta-algorithm designed to improve the stability and accuracy of machine learning algorithms used in statistical classification and regression. It also reduces variance and helps to avoid over-fitting [7]. Different subsets of data is fed to various classifier models and their results are combined using majority voting.

4.2 Boosting

The term ‘Boosting’ refers to a family of algorithms which converts weak learner to strong learners. Boosting is an ensemble method for improving the model predictions of any given learning algorithm. To correct the previous error, the observations that were incorrectly classified carry more weight than the observations that were correctly classified in the next iteration [8]. Figure 3 shows a simplified image of how bagging and boosting is implemented.

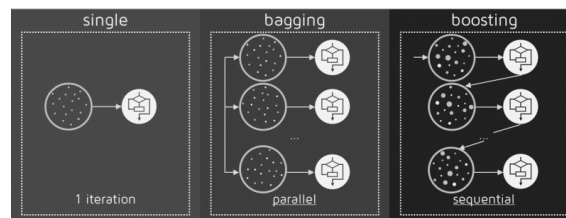


Figure 3: Comparison between Normal, Bagging, and Boosting Technique

4.3 Stacking

Stacking is an ensemble learning technique to combine multiple classification models via a meta-classifier. The individual classification models are trained based on the complete training set; then, the meta-classifier is fitted based on the outputs of the individual classification models in the

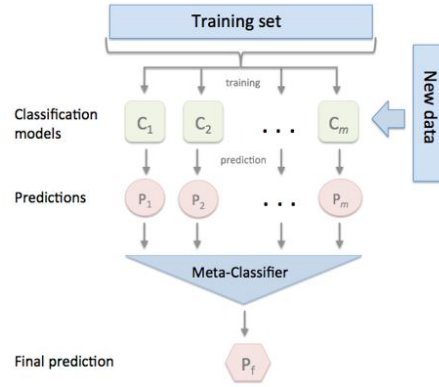


Figure 4: Stacking Technique

ensemble. The meta-classifier can either be trained on the predicted class labels or probabilities from the ensemble [9]. Figure 4 shows an image of how stacking is implemented.

5 Framework

For analysis of the effect of various Classifiers listed below, the models were fitted on the testing data of MNIST dataset and whole USPS dataset to predict the digits and compare it with original labels of testing data to determine the accuracy.

- **Logistic Regression:** Three Models of Logistic Regression have been implemented. The first model implements the mini batch stochastic gradient descent and runs 200 epochs on the data with the batch size of 512. The other two models have been implemented by using the Scikit-Learn library *Logistic Regression* with solver set as *lbfgs* and multi_class is set as *ovr* and *multinomial* respectively.
- **Neural Network:** Four Models of Neural Network have been implemented. The first model is a Deep Neural Network architecture which uses the keras framework and has been implemented from scratch. The next two models have been implemented by using the Scikit-Learn library *MLPClassifier* with solver set as *sgd* and *adam* respectively. The last Model is a Convolutional Neural Network architecture.
- **Support Vector Machine:** Three Models of Support Vector Machine have been implemented using the Scikit-Learn library *SVC* with kernel set as *linear* and *rbf* and gamma parameter set as *0.001* respectively.
- **Random Forest** Three Models of Random Forest have been implemented using the Scikit-Learn library *RandomForestClassifier* with number of trees set as *10*, *100*, and *200* respectively.
- **Bootstrap Aggregating:** Four Models of Bagging have been implemented using the Scikit-Learn library *BaggingClassifier* with base_estimator set as *Logistic Regression*, *Neural Network*, *Support Vector Machine*, and *Random Forest* respectively. The bagging algorithm takes at maximum 80% of features and dataset as input.
- **Boosting:** Two Models of Boosting have been implemented using the Scikit-Learn library *AdaBoostClassifier* with base_estimator set as *Logistic Regression* and *Random Forest* respectively.
- **Stacking:** Three Models of Stacking have been implemented using the library *Mlxtend* with classifier set as *Neural Network*, *Support Vector Machine*, and *Random Forest*. Meta_Classifier is set as *Logistic Regression*. The models don't use probability, uses average probability, and uses stacked probabilities respectively. Sample code of the three models of stacking are shown below.

```
sclf1 = StackingClassifier(classifiers=[clf_SVM, clf_NN, clf_RF],
meta_classifier=clf_LR)

sclf2 = StackingClassifier(classifiers=[clf_SVM, clf_NN, clf_RF],
use_probab=True, average_probab=True, meta_classifier=clf_LR)

sclf3 = StackingClassifier(classifiers=[clf_SVM, clf_NN, clf_RF],
use_probab=True, average_probab=False, meta_classifier=clf_LR)
```

The results of different classifiers were combined using majority voting. Accuracy on both the datasets and confusion matrix were observed for each classifier.

6 Observations

The accuracy of all models on the MNIST testing dataset and USPS dataset is analyzed with the help of gr tables. The observation is listed below model wise for both the datasets.

6.1 Logistic Regression

The accuracy obtained by performing Logistic (Softmax) Regression on MNIST and USPS dataset were observed and compared to other models. The Accuracy obtained from different models of Logistic Regression are shown in the table given below.

Models	Accuracy (MNIST)	Accuracy (USPS)
Mini-batch SGD Logistic Regression	87.07	34.10
Scikit-Learn (multi_class='ovr')	92.06	34.99
Scikit-Learn (multi_class='multinomial')	92.54	34.51

As it can be seen in the table, the best performance on MNIST dataset was obtained by the model constructed using the Scikit-Learn library with the multi_class parameter set as *multinomial*. However, the best performance on USPS dataset was obtained by the model with the multi_class parameter set as 'one versus rest'. The confusion matrices for these models are shown in Figure 5. From the figure, it can be observed that the model mostly performs quite accurately for each digit of MNIST dataset. However, it fails in some cases where it wrongly classifies some digits which look similar. An example can be seen for digit 5 getting misclassified as 3 and digit 4 and 9 getting misclassified. In USPS dataset, we mostly observe incorrect classification except for digit 5 where the model performs satisfactorily. This can be explained using the "No Free Lunch Theorem".

[947	0	3	2	0	3	16	1	8	0]	[627	2	151	137	217	264	70	187	80	265]
[0	1096	5	3	1	2	4	0	24	0]	[62	550	41	209	501	135	25	355	96	26]
[16	22	845	26	19	0	28	22	47	7]	[93	50	1198	146	77	211	73	81	41	29]
[5	3	22	881	1	29	8	20	27	14]	[43	10	301	897	19	581	9	58	51	31]
[3	9	5	0	857	1	17	2	11	77]	[46	42	47	77	880	146	21	476	179	86]
[26	15	6	82	24	643	29	9	41	17]	[74	15	98	199	48	1375	70	75	32	14]
[19	5	13	2	13	18	882	0	6	0]	[90	8	513	117	102	531	601	14	14	10]
[4	39	26	1	13	0	4	889	10	42]	[100	83	191	545	119	214	17	573	114	44]
[9	17	14	39	12	22	18	14	809	20]	[192	20	107	508	135	580	116	111	181	50]
[14	13	11	12	52	10	1	27	11	858]	[36	40	95	526	133	109	11	642	292	116]

Figure 5: Confusion Matrices for LR of MNIST and USPS dataset respectively

6.2 Neural Network

The accuracy obtained by performing classification using Neural Network on MNIST dataset were not great when compared to other models. The Accuracy obtained from different models of Ne are shown in the table given below.

Models	Accuracy (MNIST)	Accuracy (USPS)
Deep Neural Network	94.06	39.42
Scikit-Learn (solver='adam')	93.4	37.51
Scikit-Learn (solver='sgd')	84.33	31.81
Convolutional Neural Network	98.91	70.8

[975	0	0	0	0	0	2	1	2	0]	[769	5	28	19	181	32	41	5	16	904]
[0	1119	0	4	0	1	2	3	5	1]	[5	945	11	23	543	60	25	326	19	43]
[1	0	1015	3	3	0	1	5	4	0]	[25	10	1608	81	26	142	27	30	41	9]
[0	0	0	1002	0	3	0	0	4	1]	[2	2	22	1756	3	194	0	4	10	7]
[0	0	0	0	974	0	1	0	0	7]	[2	31	6	3	1534	102	5	225	38	54]
[2	0	0	5	0	884	1	0	0	0]	[8	5	8	26	1	1898	4	13	7	30]
[1	2	0	0	1	4	948	0	2	0]	[71	30	25	8	34	51	1741	1	30	9]
[0	2	1	1	0	0	0	1019	2	3]	[3	150	193	417	14	54	6	1111	32	20]
[2	0	1	2	0	4	0	1	960	4]	[18	7	24	253	40	592	29	68	841	128]
[0	0	0	0	2	8	1	2	1	995]	[3	40	35	63	82	39	0	602	147	989]

Figure 6: Confusion Matrices for CNN of MNIST and USPS dataset respectively

As it can be seen in the table, the best performance on MNIST dataset was obtained by the model constructed using the Convolutional Neural Network Architecture. Similarly, the best performance on USPS dataset was obtained by the same model. We observe that Convolutional Neural network gives the highest accuracy for both datasets among all the classification models implemented. The confusion matrices for these models are shown in Figure 6. From the figure, it can be observed that the model performs very accurately for each digit of MNIST dataset. In USPS dataset, we observe satisfactory results except for digit 1,8, and 9 which gets misclassified as 4,5, and 7.

6.3 Support Vector Machine

The accuracy obtained by performing classification using Support vector Machine on MNIST dataset were observed and compared to other models. The Accuracy obtained from different models of Support Vector Machine are shown in the table given below.

Models	Accuracy (MNIST)	Accuracy (USPS)
Scikit-Learn (kernel='linear')	93.9	33.97
Scikit-Learn (kernel='rbf')	94.35	40.85
Scikit-Learn (gamma='0.001')	94.01	40.79

As it can be seen in the table, the best performance on MNIST dataset was obtained by the model constructed using the Scikit-Learn library with kernel parameter set as 'radial basis function'. Similarly, the best performance on USPS dataset was obtained by the same model. The confusion matrix for these models are shown in Figure 7. From the figure, it can be observed that the model performs quite accurately for each digit of MNIST dataset. In USPS dataset, we mostly observe wrong classification except for digit 2 and 5 where the model performs satisfactorily. This can be explained using the "No Free Lunch Theorem".

[967	0	1	0	0	5	4	1	2	0]	[594	3	351	18	295	207	72	32	5	423]
[0	1120	2	3	0	1	3	1	5	0]	[63	611	76	135	332	217	42	493	15	16]
[9	1	962	7	10	1	13	11	16	2]	[132	27	1341	66	58	198	69	71	25	12]
[1	1	14	950	1	17	1	10	11	4]	[68	5	151	1149	13	492	6	65	27	24]
[1	1	7	0	937	0	7	2	2	25]	[16	73	59	11	1231	231	21	196	61	101]
[7	4	5	33	7	808	11	2	10	5]	[84	20	136	110	25	1484	50	52	27	12]
[10	3	4	1	5	10	924	0	1	0]	[192	9	417	21	130	411	797	4	9	10]
[2	13	22	5	7	1	0	954	4	20]	[42	236	444	253	54	420	17	461	48	25]
[4	6	6	14	8	24	10	8	891	3]	[70	25	182	182	90	1014	94	36	279	28]
[10	6	0	12	33	5	1	14	6	922]	[24	197	192	236	232	158	13	505	219	224]

Figure 7: Confusion Matrices for SVM of MNIST and USPS dataset respectively

6.4 Random Forest

The accuracy obtained by performing classification using Random Forest on MNIST dataset were observed and compared to other models. The Accuracy obtained from different models of Random Forest are shown in the table given below.

Models	Accuracy (MNIST)	Accuracy (USPS)
Scikit-Learn (n_estimator='10')	94.45	37.18
Scikit-Learn (n_estimator='100')	96.86	44.84
Scikit-Learn (n_estimator='200')	96.87	45.34

[969	0	0	0	0	1	4	1	4	1]	[671	6	258	67	440	90	56	40	4	368]
[0	1121	3	4	1	2	2	0	1	1]	[9	676	54	125	38	123	36	932	6	1]
[5	0	1000	6	3	0	2	9	7	0]	[100	21	1413	63	53	143	15	182	6	3]
[0	0	9	976	0	6	0	8	8	3]	[42	3	78	1418	46	261	2	126	5	19]
[1	0	0	0	955	0	5	0	3	18]	[7	154	46	26	1209	142	15	312	57	32]
[4	0	1	16	3	852	6	3	6	1]	[106	16	85	70	11	1625	12	61	5	9]
[6	3	0	1	3	5	937	0	3	0]	[311	28	216	30	101	260	991	44	10	9]
[1	4	19	1	1	0	0	989	1	12]	[48	230	528	275	30	233	37	604	4	11]
[5	0	5	7	7	5	3	4	928	10]	[66	22	182	226	97	965	68	61	274	39]
[5	5	1	15	10	2	1	4	6	960]	[23	186	310	344	235	87	14	473	141	187]

Figure 8: Confusion Matrices for RF of MNIST and USPS dataset respectively

As it can be seen in the table, the best performance on MNIST dataset was obtained by the model constructed using the Scikit-Learn library with the number of trees set as '200'. Similarly, the best performance on USPS dataset was obtained by the same model. The confusion matrices for these models are shown in Figure 8. From the figure, it can be observed that the model performs quite accurately for each digit of MNIST dataset except 9 where it misclassifies some data as 4. In USPS dataset, we mostly observe incorrect classification except for digit 2,4, and 5 where the model performs satisfactorily.

6.5 Bootstrap Aggregating

The accuracy was obtained by performing bagging on different classifiers and then testing on MNIST and USPS dataset. The results were observed and compared to other models. The Accuracy obtained from different bagging models are shown in the table given below.

Models	Accuracy (MNIST)	Accuracy (USPS)
Scikit-Learn (base_estimator='LR')	92.45	34.69
Scikit-Learn (base_estimator='NN')	96.72	46.23
Scikit-Learn (base_estimator='SVM')	96.11	42.27
Scikit-Learn (base_estimator='RF')	94.22	35.87

As it can be seen in the table, the best performance on MNIST dataset was obtained by the model constructed using the Scikit-Learn library with base classifier set as 'Neural Network'. Similarly, the best performance on USPS dataset was obtained by the same model. The confusion matrices for these models are shown in Figure 9. From the figure, it can be observed that the bagging model performs very accurately for each digit of MNIST dataset. In USPS dataset, we mostly observe incorrect classification except for digit 2 and 3 where the model performs satisfactorily.

[967	0	0	1	3	1	3	2	2	1]	[563	17	102	56	197	27	48	27	367	596]
[0	1121	2	0	1	3	1	1	6	0]	[28	541	303	123	204	25	18	233	232	293]
[5	4	995	3	8	0	0	12	2	3]	[33	56	1405	176	59	19	1	40	157	53]
[0	0	4	974	0	1	0	6	24	1]	[21	13	174	1446	3	64	2	33	185	59]
[1	0	1	0	957	0	1	0	7	15]	[28	69	32	23	1026	22	26	140	284	350]
[1	2	0	3	1	846	5	0	26	8]	[48	22	34	94	20	1282	23	17	323	137]
[6	8	0	1	6	1	930	0	6	0]	[169	117	61	81	222	68	1023	7	199	53]
[0	4	11	5	0	0	0	984	6	18]	[19	85	528	433	35	6	2	546	241	105]
[3	5	2	10	5	5	0	2	932	10]	[188	39	80	274	107	110	51	41	963	147]
[2	2	0	5	7	0	0	3	24	966]	[16	77	156	260	77	13	6	463	481	451]

Figure 9: Confusion Matrices for Bagging of MNIST and USPS dataset respectively

6.6 Boosting

The accuracy was obtained by performing boosting on different classifiers and then testing on MNIST and USPS dataset. The results were observed and compared to other models. The Accuracy obtained from different boosting are shown in the table given below.

Models	Accuracy (MNIST)	Accuracy (USPS)
Scikit-Learn (base_estimator='LR')	85.31	32.06
Scikit-Learn (base_estimator='RF')	95.59	39.66

[964	0	1	1	1	3	5	2	3	0]	[519	7	326	50	447	126	93	58	53	321]
[0	1119	3	2	1	1	3	0	6	0]	[22	606	153	111	321	120	35	568	47	17]
[7	2	981	6	3	2	2	8	18	3]	[95	37	1262	92	71	159	39	155	73	16]
[0	0	15	961	0	10	2	8	10	4]	[50	8	154	1158	39	365	18	85	79	44]
[1	2	4	0	936	0	9	0	6	24]	[11	128	77	35	1150	144	25	231	130	69]
[3	1	3	27	2	833	10	1	9	3]	[87	30	123	151	37	1345	22	100	81	24]
[5	3	1	1	3	6	933	0	5	1]	[242	19	260	70	174	297	814	38	67	19]
[2	8	18	3	4	1	0	967	5	20]	[74	284	419	231	131	211	32	472	125	21]
[2	1	9	10	3	10	5	5	919	10]	[64	28	202	250	144	716	68	73	407	48]
[5	4	3	12	17	2	2	7	11	946]	[24	224	186	212	354	119	9	446	226	200]

Figure 10: Confusion Matrices for Boosting of MNIST and USPS dataset respectively

As it can be seen in the table, the best performance on MNIST dataset was obtained by the model constructed using the Scikit-Learn library with base classifier set as 'Random Forest'. Similarly, the best performance on USPS dataset was obtained by the same model. The confusion matrices for these models are shown in Figure 10. From the figure, it can be observed that the boosting model performs very accurately for each digit of MNIST dataset. In USPS dataset, we mostly observe incorrect classification. This can be explained from the fact that over-fitting can occur while carrying out boosting of a classifier and the classifier performance might decrease on another dataset. This can be the reason that such low accuracy on USPS data was seen compared to other ensemble techniques even though the classifier performs well on MNIST dataset.

6.7 Stacking (Combining Classifiers)

The accuracy was obtained by performing Stacking different classifiers and then testing on MNIST and USPS dataset. The results were observed and compared to other models. The Accuracy obtained from different Stacking models are shown in the table given below.

Models	Accuracy (MNIST)	Accuracy (USPS)
Mlxtend (probability='False')	90.85	26.69
Mlxtend (probability='Stacked')	96.47	42.95
Mlxtend (probability='Average')	96.46	41.25

As it can be seen in the table, the best performance on MNIST dataset was obtained by the model constructed using the Mlxtend library in which the probability of each classifier is stacked and not averaged. Similarly, the best performance on USPS dataset was obtained by the same model. The confusion matrices for these models are shown in Figure 11. From the figure, it can be observed that the stacking model performs very accurately for each digit of MNIST dataset. In USPS dataset, we mostly observe incorrect classification.

[971	0	1	0	0	2	1	2	3	0]	[528	4	144	307	89	34	67	266	132	429]
[0	1124	2	1	0	1	2	0	5	0]	[20	478	55	205	13	55	29	717	200	228]
[7	1	991	6	1	5	2	6	13	0]	[145	38	1246	158	7	99	26	80	189	11]
[3	0	7	980	0	2	0	8	9	1]	[133	11	239	1306	1	139	4	52	103	12]
[0	0	1	1	947	0	7	3	3	20]	[10	56	21	79	829	20	215	244	131	395]
[2	3	22	10	0	833	5	2	13	2]	[62	37	262	66	5	1451	17	28	65	7]
[6	3	1	1	4	7	929	0	7	0]	[114	115	196	69	53	157	995	27	234	40]
[0	2	7	7	0	0	0	993	5	14]	[113	33	314	580	3	51	10	589	237	70]
[4	0	5	9	3	7	11	5	923	7]	[114	31	273	295	11	288	140	89	698	61]
[3	5	1	15	10	0	1	13	5	956]	[15	47	68	324	104	15	38	573	346	470]

Figure 11: Confusion Matrices for Stacking of MNIST and USPS dataset respectively

6.8 Majority Voting (Combining Classifiers)

Once the analysis of each classifier was done, the predicted values of each model are put in an algorithm to get the most voted label for each data. The accuracy of **96.51%** was obtained for MNIST dataset and **44.69%** for USPS dataset. The confusion matrix obtained after carrying out the majority voting analysis is shown in Figure 12. From the figure, it can be observed that after majority voting the predictions are very accurate for each digit of MNIST dataset. However, some misclassifications are observed such as for digit 4 and 5 as most of the classifier classified 4 as 9,

[970	0	1	1	0	2	4	1	1	0]	[628	1	289	48	306	154	40	97	22	415]
[0	1123	2	2	0	1	3	1	3	0]	[28	622	53	179	169	172	25	703	42	7]
[6	2	994	4	3	1	7	8	6	1]	[104	30	1453	73	46	164	29	75	17	8]
[0	0	12	971	0	9	0	11	7	0]	[45	4	154	1326	8	372	2	56	26	7]
[1	1	4	0	955	0	6	0	2	13]	[11	91	42	19	1210	136	11	305	101	74]
[9	3	1	24	3	829	7	1	12	3]	[66	14	121	133	18	1540	20	58	25	5]
[8	3	3	1	5	6	930	0	2	0]	[193	18	429	34	104	335	870	8	4	5]
[2	7	20	3	3	0	0	982	1	10]	[44	165	318	454	24	288	10	591	91	15]
[5	3	5	15	8	18	8	6	902	4]	[93	15	167	281	83	893	67	69	309	23]
[8	6	1	14	15	3	1	13	4	944]	[9	142	153	366	146	101	6	625	262	190]

Figure 12: Confusion Matrices for Majority voting of MNIST and USPS dataset respectively

etc. In USPS dataset, we mostly observe incorrect classification except for digit 2,3, and 5 where the results obtained are satisfactory because many classifiers performed well for these digits.

7 Conclusion

In this paper, various Machine Learning Classification methods are proposed to solve the hand-written digit classification problem. Hyper-parameters are tuned for each model and the results obtained was observed. The best results for each model is shown in the Figure 13.

- **Which classifier has the overall best performance?:**

The best accuracy of 98.91% and 70.8% was achieved by using Convolutional Neural Network for classification on both MNIST and USPS dataset respectively.

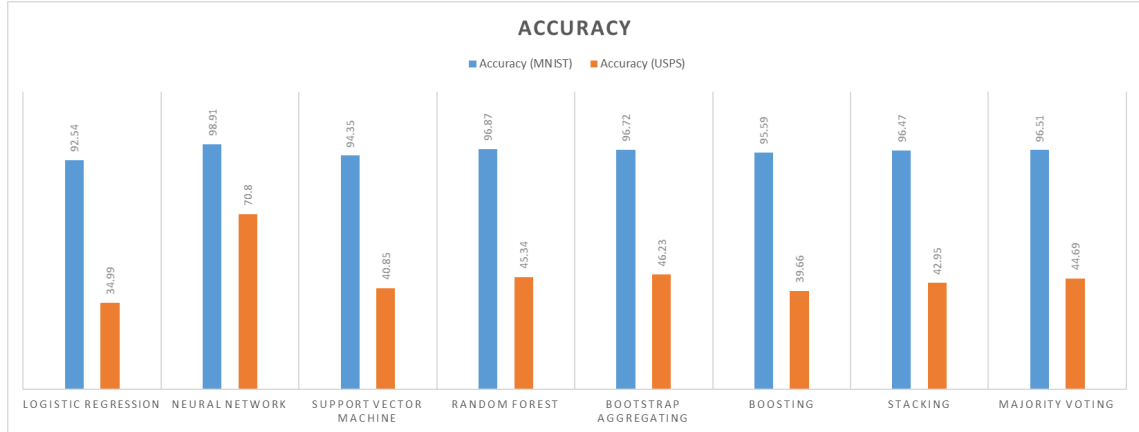


Figure 13: Comparison between different Classification Techniques

- **Does the results obtained support the “No Free Lunch” theorem?:**

Hume (1739–1740) pointed out that ‘even after the observation of the frequent or constant conjunction of objects, we have no reason to draw any inference concerning any object beyond those of which we have had experience’. This can be explained as following in the context of this project: No model is universal. Though every classification model after training on MNIST dataset gives good performance on its testing data, it can’t be expected to perform similarly for a different dataset even when the dataset is similar for example, in this case, handwritten digits. If a model is trained on one dataset then it should only be expected to run properly on that dataset and not any other dataset. This is why we see that even though both datasets contain handwritten digit images, the models perform properly only on one it’s trained on or ‘had experience with’.

- **Is the overall combined performance better than that of any individual classifier?:**

The Accuracy obtained by implementing Majority Voting on MNIST dataset is higher than all models except Convolutional Neural Network, Random Forest with 100 and 200 decision trees, and Bagging model with Neural Network as the base classifier. Similar results were obtained for USPS dataset.

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