APMA 2822B Homework 3 Report

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Math Setup

Assume that $\Delta x = \Delta y = h$.

Using the second derivative approximations we were given, we have

$$f(x,y) = \frac{u[i-1,j] + u[i+1,j] + u[i,j-1] + u[i,j+1] - 4u[i,j]}{h^2}$$

Substituting in the given $f(x,y)$, we get

$$\begin{aligned} -8\pi^2 \sin(2\pi x)\cos(2\pi y) &= \frac{u[i-1,j] + u[i+1,j] + u[i,j-1] + u[i,j+1] - 4u[i,j]}{h^2} \\ -8\pi^2 h^2 \sin(2\pi x)\cos(2\pi y) &= u[i-1,j] + u[i+1,j] + u[i,j-1] + u[i,j+1] - 4u[i,j] \end{aligned}$$

Solving for $u[i,j]$, we have

$$\begin{aligned} 4u[i,j] &= u[i-1,j] + u[i+1,j] + u[i,j-1] + u[i,j+1] + 8\pi^2 h^2 \sin(2\pi x)\cos(2\pi y) \\ u[i,j] &= \frac{1}{4}\left(u[i-1,j] + u[i+1,j] + u[i,j-1] + u[i,j+1] + 8\pi^2 h^2 \sin(2\pi x)\cos(2\pi y)\right) \end{aligned}$$

We can use this equation to iteratively solve for u with the given $f(x,y)$ using the Jacobi method.

We will consider our solution to have converged when the maximum residual in u across all grid points is less than a specified tolerance ϵ.