

# APMA 2822B Homework 3 Report

Yash Agrawal

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## Math Setup

Assume that  $\Delta x = \Delta y = h$ .

Using the second derivative approximations we were given, we have

$$f(x, y) = \frac{u[i-1, j] + u[i+1, j] + u[i, j-1] + u[i, j+1] - 4u[i, j]}{h^2}$$

Substituting in the given  $f(x, y)$ , we get

$$\begin{aligned} -8\pi^2 \sin(2\pi x) \cos(2\pi y) &= \frac{u[i-1, j] + u[i+1, j] + u[i, j-1] + u[i, j+1] - 4u[i, j]}{h^2} \\ -8\pi^2 h^2 \sin(2\pi x) \cos(2\pi y) &= u[i-1, j] + u[i+1, j] + u[i, j-1] + u[i, j+1] - 4u[i, j] \end{aligned}$$

Solving for  $u[i, j]$ , we have

$$\begin{aligned} 4u[i, j] &= u[i-1, j] + u[i+1, j] + u[i, j-1] + u[i, j+1] + 8\pi^2 h^2 \sin(2\pi x) \cos(2\pi y) \\ u[i, j] &= \frac{1}{4} (u[i-1, j] + u[i+1, j] + u[i, j-1] + u[i, j+1] + 8\pi^2 h^2 \sin(2\pi x) \cos(2\pi y)) \end{aligned}$$

We can use this equation to iteratively solve for  $u$  with the given  $f(x, y)$  using the Jacobi method.

We will consider our solution to have converged when the maximum residual in  $u$  across all grid points is less than a specified tolerance  $\epsilon$ .