Exercise 2.2

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A.

$$G_0''(x) = 0 \quad G_0'(0) = 1 \quad G_0(1) = 0$$

$$G_0(x) = x - 1$$

$$G_1''(x) = 0 \quad G_1'(0) = 0 \quad G_1(1) = 1$$

$$G_0(x) = 1$$

$$G''(x, \bar{x})(x) = 0 \quad G'(x, \bar{x})(0) = 0 \quad G(x, \bar{x})(1) = 0$$

$$G(x, \bar{x}) = \begin{cases} ax + b, & \text{if } 0 \le x < \bar{x} \\ cx + d, & \text{if } \bar{x} \le x \le 1 \end{cases}$$

$$\begin{cases} G'(x, \bar{x})(0) = 0 \Rightarrow a = 0 \\ G(x, \bar{x})(1) = 0 \Rightarrow c = -d \\ \text{Continuity: } a\bar{x} + b = c\bar{x} + d \\ \text{Derivative: } c - a = 1 \end{cases}$$

$$G(x, \bar{x}) = \begin{cases} \bar{x} - 1, & \text{if } 0 \le x < \bar{x} \\ x - 1, & \text{if } \bar{x} \le x \le 1 \end{cases}$$

B.

Matrix A:

$$\begin{bmatrix} -4 & 4 & 0 & 0 & 0 \\ 16 & -32 & 16 & 0 & 0 \\ 0 & 16 & -32 & 16 & 0 \\ 0 & 0 & 16 & -32 & 16 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Let $B = A^{-1}$:

$$B_{i0} = G_0 x_i$$

$$B_{i4} = G_1 x_i$$

$$B_{ij} = hG(x_i; x_j)$$

$$\begin{bmatrix} -1 & -3/16 & -1/8 & -1/16 & 1 \\ -3/4 & -3/16 & -1/8 & -1/16 & 1 \\ -1/2 & -1/8 & -1/8 & -1/16 & 1 \\ -1/4 & -1/16 & -1/16 & --1/16 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$