Exercise 1.1

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Let

$$p = ax^2 + bx + c \tag{1}$$

Therefore

$$p'(x) = 2ax + b \tag{2}$$

Since p(x) interpolates u(x) at $u(\bar{x}), u(\bar{x} - h), u(\bar{x} - 2h)$ we have:

$$p(\bar{x}) = u(\bar{x}) = a\bar{x}^2 + b\bar{x} + c \tag{3}$$

$$p(\bar{x} - h) = u(\bar{x} - h) = a(\bar{x} - h)^2 + b(\bar{x} - h) + c \tag{4}$$

$$p(\bar{x} - 2h) = u(\bar{x} - 2h) = a(\bar{x} - 2h)^2 + b(\bar{x} - 2h) + c \tag{5}$$

Equation 4 and 5 can be rewritten as:

$$u(\bar{x} - h) = a\bar{x}^2 + b\bar{x} + c - h(2a\bar{x} + b) + ah^2 = u(\bar{x}) - hp'(\bar{x}) + ah^2 \tag{6}$$

$$u(\bar{x} - 2h) = a\bar{x}^2 + b\bar{x} + c - 2h(2a\bar{x} + b) + 4ah^2 = u(\bar{x}) - 2hp'(\bar{x}) + 4ah^2 \tag{7}$$

Subtracting 4 times Equation 6 from Equation 7 we get:

$$u(\bar{x} - 2h) - 4u(\bar{x} - h) = -3u(\bar{x}) + 2hp'(\bar{x}) \tag{8}$$

Rewriting for $p'(\bar{x})$ we get:

$$p'(\bar{x}) = \frac{1}{2h}[3u(\bar{x}) + u(\bar{x} - 2h) - 4u(\bar{x} - h)] \tag{9}$$