

Exercise 2.3

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A^T :

$$\begin{bmatrix} -h & 1 & 0 & . & . & 0 & 0 \\ h & -2 & 1 & . & . & 0 & 0 \\ 0 & 1 & -2 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & -2 & h \\ 0 & 0 & 0 & . & . & 1 & -h \end{bmatrix}$$

Null(A):

$$\begin{bmatrix} 1 \\ h \\ h \\ h \\ . \\ . \\ . \\ h \\ 1 \end{bmatrix}$$

For the system $AU = F$ to be solvable, $v^T F = 0 \quad \forall v \in \text{null}(A^T)$. Therefore:

$$\begin{bmatrix} 1 & h & h & h & . & . & . & h & 1 \end{bmatrix} \begin{bmatrix} \sigma_0 + \frac{h}{2}f(x_0) \\ f(x_1) \\ f(x_2) \\ f(x_3) \\ . \\ . \\ . \\ f(x_m) \\ -\sigma_1 + \frac{h}{2}f(x_{m+1}) \end{bmatrix} = 0$$

Therefore

$$\frac{h}{2}f(x_0) + h \sum_{i=1}^m f(x_i) + \frac{h}{2}f(x_{m+1}) = \sigma_1 - \sigma_0$$