35.4.2 ▪ Rock-Paper-Scissors Model

The Rock-Paper-Scissors model is a model for multiple species where species U preys on species V, species V preys on species W, and species W preys on species U, [29]. The system of equations is

$$
\begin{aligned}
\frac{\partial U}{\partial t} &= \nabla^2 U + U(1 - U - \alpha W), && (35.45)\\
\frac{\partial V}{\partial t} &= \nabla^2 V + V(1 - V - \alpha U), && (35.46)\\
\frac{\partial W}{\partial t} &= \nabla^2 W + W(1 - W - \alpha V), && (35.47)\\
U(\mathbf{x}, 0) &= \frac{1}{1+\alpha}\sigma, && (35.48)\\
V(\mathbf{x}, 0) &= \frac{1}{1+\alpha}\sigma, && (35.49)\\
W(\mathbf{x}, 0) &= \frac{1}{1+\alpha}\sigma, && (35.50)\\
\left.\frac{\partial U}{\partial n}\right|_{\Gamma} &= 0, && (35.51)\\
\left.\frac{\partial V}{\partial n}\right|_{\Gamma} &= 0, && (35.52)\\
\left.\frac{\partial W}{\partial n}\right|_{\Gamma} &= 0, && (35.53)
\end{aligned}
$$

where α is the depredation rate. The term σ in the initial conditions should be a random value at each grid point drawn from a uniform distribution on the interval $0 \le \sigma \le 1$.

Assignment. Use the ADI method by modifying Equations (35.32)–(35.35) to solve the Rock-Paper-Scissors model on a box of dimensions $[-L, L] \times [-L, L]$ for $L = 60$ and with parameter value $\alpha = 9$. Solve until the terminal time $T = 200$.

Your program should take three arguments with a fourth optional seed value so that your program named `rps` can be executed like this:

```
$ rps 128 9.0 20000
```

where arguments in order are the number of grid points in each dimension, $N = 128$, the double precision depredation rate $\alpha = 9.0$, and the number of time steps, $M = 20{,}000$. If a fourth argument is specified, then it is a long integer seed value, s, for generating the initial random values. You will need to check the value of `argc` to determine if s was given. Your program must print the arguments given including the value of the seed, and you should verify that your seed argument is implemented correctly by taking the value printed by your program from the above test run and adding it to the argument list. For example, if your program reports using a seed of 12345, then execute your program again like this:

```
$ rps 128 9.0 20000 12345
```

and verify that you get the exact same results.

Your grid will be $N \times N$ so that $x_0 = -L$ and $x_{N-1} = L$. Your program should output the grid values for U, V and W into three files called "`RPSU.out`", "`RPSV.out`" and "`RPSW.out`" that contain the data at the time points $t = 200k$ for $k = 0, 1, \ldots, 10$. The files should contain *only* the values of U, V, and W respectively on the grid and nothing else.

Your program must also print to the screen the total time required to complete the calculation. Vary the values of N to generate a plot that illustrates the computational cost as a function of N.

Plot the isocontours of U, V, and W and over time and a pattern of waves and rotating spirals should emerge as shown in Figure 35.3.

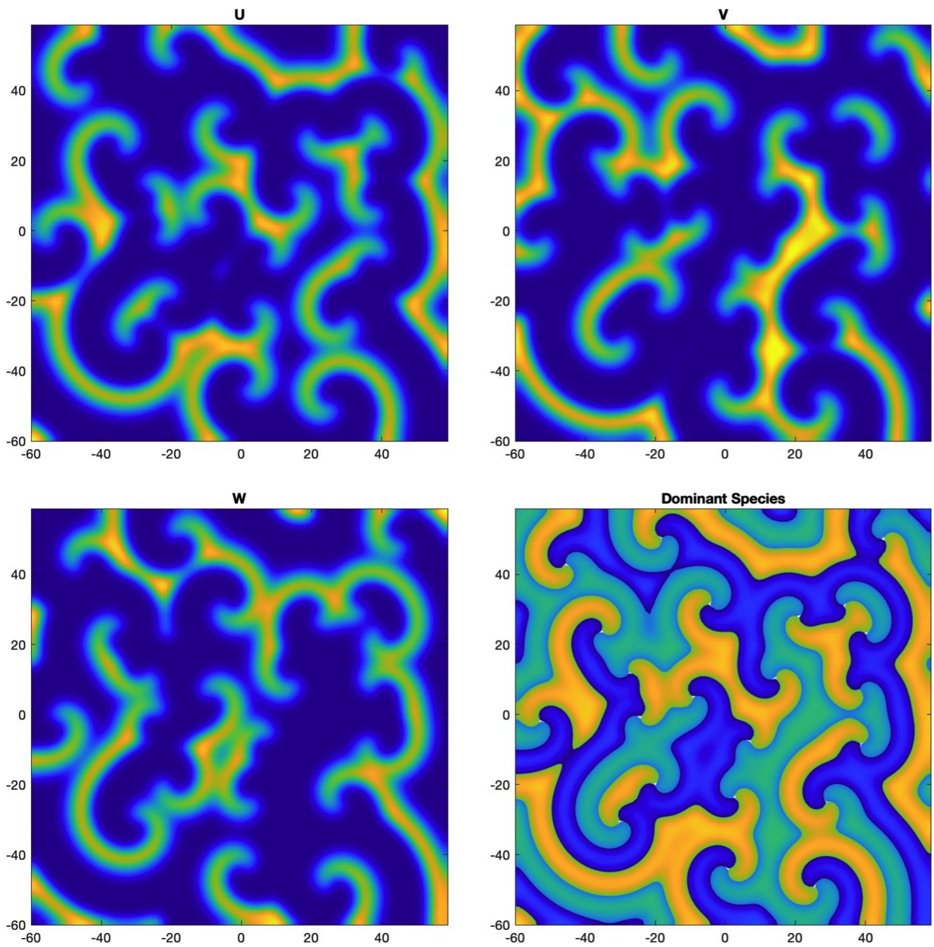

Figure 35.3. *Sample contour plots for the Rock-Paper-Scissors model. The contour plots of U, V, W at the terminal time $T = 2000$. The last plot is a color coded representation of $\max(U - \max(V, W), V - \max(U, W), W - \max(U, V))$ according to which species is largest.*