UNIT - I — Module

1.) Define the following with Examples:

a) Symmetric Matrice b) skew - symmetric matrix (9)
c) orthogonal Matrix (11) d) Hermitiation matrix (24)
e) skew - Hermitiation matrix (24) f) Unitary matrix. (25)

2.) Find the rank of the following matrices by reducing it into Echelon form and Normal form

a) $\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$ (40)

b) $\begin{bmatrix} 4 & 3 & 2 & 1 \\ 5 & 1 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 1 & -1 & 3 & 2 \end{bmatrix}$ (62)

3.) Find the value of K such that the rank of the following mathrix are 2.

a) $\begin{bmatrix} 0 & 1 & -3 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & K & 0 \end{bmatrix}$ (39)

b) $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & K & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ (40)

4.) Find the inverse of the matrix A using elementary operations (i.e., using Gauss - Jordan method)

a) $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & 3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ (82)

b) $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ 2 & -4 & -4 \end{bmatrix}$ (84)

5) Discuss for what values of λ, μ the simultaneous equations: $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have (i) No solution (ii) A unique solution iii) A infinite no. of solutions (96)

6.) S.T the eq^{n}s: $3x+4y+5z=a$, $4x+5y+6z=b$ and $5x+6y+7z=c$ do not have a solution unless $a+c=2b$. (115).

7.) Find for what values of λ the equations $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a solution and solve them completely in each case (117)

8.) S.T the only real number λ the for which the system $x+2y+3z=\lambda x$, $3x+y+2z=\lambda y$, $2x+3y+z=\lambda z$ has non-zero solution is 6 and solve them, when $\lambda=6$ (127)

9.) Examine whether the following vectors are linearly dependent or not (3,1,1), (2,0,-1), (4,2,1) (133)

10.) Determine the values of λ for which the following set of equations may possess non-trivial solution (133)
$3x_1+x_2-\lambda x_3=0$, $4x_1-2x_2-3x_3=0$, $2\lambda x_1+4x_2+\lambda x_3=0$.
For each permissible value of λ, determine the general solution.

11.) Solve the following system of equations by Gauss elimination method a) $3x+y+2z=3$, $2x-3y-z=-3$, $x+2y+z=4$
(144) b) $2x+y+2z+w=6$, $6x-6y+6z+12w=36$,
$4x+3y+3z-3w=-1$, $2x+2y-z+w=10$ (147)

12.) Solve the following by LU Decomposition method
a) $2x+10y+z=13$, $10x+y+z=12$, $2x+2y+10z=14$ (149)

b) $8x_1-3x_2+2x_3=20$, $4x_1+11x_2-x_3=33$, $6x_1+3x_2+12x_3=36$ (153)

UNIT-II - Module.

1.) Find Eigen values and corresponding Eigen vectors of the following matrices

a) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (186) b) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (189) c) $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 2 & 2 \\ -1 & -1 & 0 \end{bmatrix}$ (194)

2.) a) Find the sum and product of the eigen values of the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 4 & 2 \\ 1 & 0 & 2 \end{bmatrix}$ (213)

b) If 2, 3, 5 are the Eigen values of a matrix A, then find the eigen values of $2A^3 + 3A^2 + 5A + 3I$ (215)

3.) Find a matrix P which transform the following Matrices to diagonal form. Hence calculate A^4, A^8

a) $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ (236) b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ (238) c) $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (248)

d) $A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$ (259)

4.) verify cayley-Hamilton theorem. Find A^4 and A^{-1} using cayley-Hamilton theorem from the following

a) $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ (280) b) $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ (289)

5.) a) verify cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find A^{-1} and $B = A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$? (292)

b) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. find the value of the matrix
$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ (278)

6.) Define the following

i) Rank of a Quadratic form (Q.E) (320)

ii) canonical Form (or) Normal Form of a Q.E (320)

iii) Index of a Real Q.E (320)

iv) signature of Q.E (321)

v) Nature of Q.E (321)

7.) Find the rank, signature, index and Nature of the following Q.F by reducing it to canonical form or normal form or sum of square form. Also write the linear which brings about the normal reduction.

a) $2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 4x_1x_3 - 8x_2x_3$ (326)

b) $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$ (332)

8.) Reduce the following Q.F to C.F (or N.F or S.S.F) by orthogonal reduction and hence state, nature, rank idex and signature of the Q.F

a) $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ (350)

b) $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ (354)

9.) Prove the following properties of Eigen values and Eigen values.

i) Theorems: 1, 2 (205), 5 (206), 6 (207), 12 (209), 13 (209), 17 (211) and.

ii) Theorems: 1 (219), 2 (220), 3 (221).

UNIT - IV

1) State Rolle's, Lagrange's & cauchys mean value Theorems.

2) Verify Rolle's Mean value theorem from the following a) $f(x) = (x-a)^m (x-b)^n$ where m, n are positive integers in $[a,b]$ [Page No. 491]

b) $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a,b]$, $a>0$, $b>0$ [Pg 492]

c) $f(x) = |x|$ in $[-1,1]$ [Pg 492]

d) $f(x) = e^x \sin x$ in $[0,\pi]$ [Pg 495]

3) Verify Lagranges mean value Theorem for the following. a) $f(x) = x(x-2)(x-3)$ in $(0,4)$ (Pg 502)

4) a) show that $h < \sin^{-1} h < \frac{h}{\sqrt{1-h^2}}$ for $0 < h < 1$ (Pg 505)

b) Show that $|\sin b - \sin a| \leq |b-a|$ (Pg No. 509)

c) show that if $x>0$, $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$ (Pg 510)

d) P.T $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$ (Pg 507)

5) If $a<b$ P.T $\frac{b-a}{(1+b^2)} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{(1+a^2)}$ using Lagranges mean value theorem. Deduce the following (Pg 504)

(i) $\pi/4 + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$

(ii) $\frac{5\pi+4}{20} < \tan^{-1} 2 < \frac{\pi+2}{4}$

6) Verify cauchy's mean value theorem from the following.

a) $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ in $[a,b]$; $0 < a < b$ (513)

b) $f(x) = e^x$ and $g(x) = e^{-x}$ in $[a,b]$; $(a, b > 0)$ (514)

c) $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x}$ on $[a,b]$ (515)

d) If $f(x) = \log x$ and $g'x = x^2$ in $[a,b]$ with $b > a > 1$ using cauchy's theorem S.T $\frac{\log b - \log a}{b-a} = \frac{a+b}{2c^2}$ (516)

7) Expand (a) $e^{x \sin x}$ in powers of x (528)

(b) $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$ in powers of x (526)

8) a) Expand $\log_e(1+x)$ using Maclaurin's series (524)

b) Obtain the Taylor's series expansion of $\sin x$ in powers of $x - \pi/4$ (520)

9) a) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(0 < b < a)$ about the major axis (539)

b) Find the volume formed by the revolution of the loop of the curve $y^2(a+x) = x^2(3a-x)$ about the x-axis (543)

10) a) Find the surface area of the solid generated by the revolution of the parabola $y^2 = 4ax$ about its axis, by the arc form the vertex to one end of the latus-rectum (557)

13) Evaluate the following by using β-$\sqrt{\ }$ functions.

a) $\int_0^2 (8-x^3)^{-1/3} \cdot dx$ (623)

c) $\int_0^\infty \frac{x\,dx}{(1+x^6)}$ (614)

b) $4\int_0^\infty \frac{x^2}{1+x^4} \cdot dx$ (622)

d) $\int_0^\infty x^4 e^{-2x^2} dx$ (612)

e) $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}} \cdot dx$ (610)

14) Evaluate the following

a) $\int_0^2 x(8-x^3)^{\frac{1}{3}} \cdot dx$ (590)

b) $\int_0^\infty e^{-4x} x^{3/2} dx$ (592)

c) $\int_0^\infty e^{-x^2} dx$ (593)

d) $\int_0^{\pi/2} \sqrt{\cot\theta} \cdot d\theta$ (595)

e) $\int_0^\infty 3^{-4x^2} \cdot dx$ (598)

f) $\int_0^1 x^4 \left(\log \frac{1}{x}\right)^3 dx$ (598)

g) $\int_0^{\pi/2} \sin^{7/2}\theta \cos^{3/2}\theta \, d\theta$ (604)

b) Find the surface area of the solid generated by revolution of the ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ (a > b) \text{ about the x-axis } (556)$$

11) Prove the following.

a) $B(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \cdot d\theta$ (569)

b) $B(m,n) = \beta(m+1,n) + \beta(m,n+1)$ (570)

c) $\beta(m,n) = \dfrac{(m-1)b\,(n-1)b}{(m+n-1)b}$ (5 70)

d) $\beta(m,n) = \int_0^\infty \dfrac{x^{m-1}}{(1+x)^{m+n}} \cdot dx = \int_0^\infty \dfrac{x^{n-1}}{(1+x)^{m+n}} dx$ (571)

e) $\beta(m,n) = \int_0^1 \dfrac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} \cdot dx$ (572)

f) $\int_b^a (x-b)^{m-1} (a-x)^{n-1} \cdot dx = (a-b)^{m+n-1} \cdot \beta(m,n)$; $m > 0, n > 0$ (574)

12) Prove the following.

a) $B(m,n) = \dfrac{\sqrt{m} \cdot \sqrt{n}}{\sqrt{(m+n)}}$ where $m > 0$, $n > 0$ (583)

b) $\sqrt{(n)}\sqrt{(1-n)} = \dfrac{\pi}{\sin n\pi}$ [584)

c) $\sqrt{\left(\frac{1}{2}\right)} = \sqrt{\pi}$ (585)

(1)

Moduel - IV

UNIT - IV I^{st} Order I^{st} Degree Differential Equations

* Tutorial Questions *

1Q:- Solve $2xy\,dy - (x^2 - y^2 + 1)dx = 0$.

2Q:-(a) Solve $(y - x^2)dx + (x^2 \cot y - x)dy = 0$

b) Solve $(1 + xy)x\,dy + (1 - yx)y\,dx = 0$.

3Q:- Solve $(3xy^2 - y^3)dx - (2x^2 y - xy^2)dy = 0$.

4Q:- Solve $y(1 - xy)dx - x(1 + xy)dy = 0$.

5Q:- Solve the differential equation $y(xy + e^x)dx - e^x dy = 0$.

6Q:- $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

7Q:- Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$.

(a) Define newton's law of cooling.

8Q:- A body kept in air with temperature 25°c cools from 140°c to 80°c in 20 min. Find when the body cools down to 35°c.

(a) Define natural growth or decay.

9Q:- In a culture of yeast, the active ferment doubles itself in 3 hours. Determine the number of times it multiplies it self in 15 hrs.

10Q:- If 30% of a radio active substance disappears in 10 days, how long will it take for 90% of it to disappear?

11Q:a) Solve $x^2p^2 + xyp - 6y^2 = 0$

b) $p^2 + 2py\cot x = x^2$

12Q:- Solve (a) $y = p\tan p + \log\cos p$

b) $y + px = x^4 p^2$.

13Q:- Solve a) $y = 2px + p^3y^2$

b) $x = y + a\log p$

14Q:- (a) $(y - px)(p - 1) = p$

b) $p = \tan(px - y)$; $y = px + p^2$

15Q:- Solve $x^2(y - px) = yp^2$

16Q:- Find the Orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$ where 'a' is the parameter.

17) Find the Orthogonal Trajectories of the family of Cardioids $r = a(1 - \cos\theta)$

18) Find the O.T of the family of curves $r^n \sin n\theta = a^n$ and $r = 2a(\cos\theta + \sin\theta)$

19) If a population is increasing exponentially at the rate of 2% per year, what will be the % increase over a period of 10 years.

20) A generator having emf 100 volts is connected in series with a 10 ohm resistor and an inductor of 2 henries. If the switch is closed at a time $t = 0$, find the current at time $t > 0$.

(2)

Module - V
UNIT - IV Higher Order Differential Equations.

1 a) Solve $y'' + y' - 2y = 0$, $y(0) = 4$; $y'(0) = 1$

b) Solve $\frac{d^3x}{dt^3} - x = 0$.

2) Solve $(D^2+4)y = \tan 2x$.

3) a) Solve $(4D^2 - 4D + 1)y = 100$.

b) $(D^2 - 3D + 2)y = \cosh x$.

4) Solve $(D^2 - D - 2)y = 3e^{2x}$, $y(0) = 0$, $y'(0) = -2$.

b) $(D^3 - 1)y = (e^x + 1)^2$

5) a) $(D^2 - 4)y = 2\cos^2 x$.

b) Solve $(D^2 + 1)y = \sin x \sin 2x$.

6) (a) $(D^2+9)\cancel{x} = \cos 3\cancel{t} + \sin 2\cancel{t}$. $(D^2+9)x = \cos 3t + \sin 2t$.

(b) $(D^2 + D + 1)y = x^3$

(c) $(D^3 + 2D^2 + D)y = x^3$

7) Solve $(D^2 - 2D + 1)y = x^2 e^{3x} - \sin 2x + 3$.

8) a) $(D^4 + 2D^2 + 1)y = x^2 \cos^2 x$.

b) $(D^2 - 2D + 1)y = xe^x \sin x$.

9) Solve $x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 12y = x^3 \log x$.

10) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin(\log(1+x))$.

11) Solve $(x^2D^2 - 3xD + 1)y = \log x \left(\frac{\sin(\log x) + 1}{x}\right)$

12) Find Wronskian of $e^x \sin x$ & $e^x \cos x$. for what values of x, W zero

(b) $(D^2+1)y = \operatorname{cosec} x$ Method of Variation of Parameters.

13) An Electrical Circuit consists of an inductance of 0.1 henries, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads. Find the charge q and the current i at any time t, given that at $t=0$, $q=0.05$ coulomb. $i=\frac{dq}{dt}=0$, when $t=0$.

14) The Charge $q(t)$ on the capacitor is given by D.E.

$$10\frac{d^2q}{dt^2}+120\frac{dq}{dt}+1000q=17\sin(2t).$$

At time zero the current is zero and the charge on the capacitor is $\frac{1}{2000}$ coulomb. Find the charge on the capacitor for $t>0$.