Mock Q1

Thursday, June 2, 2022 8:23 PM

1) Consider the following data set

x	y		$x_1 - 3x_2$
[1, 0]	+1	+1	1-3(0)
[2, 2]	+1	-1	2-3(2)
[3, 1]	+1	+1	3-3(1)
[4, 5]	−1	-1	4-3(5)
[5, 4]	−1	-1	5-4(3)
[6, 5]	−1	+1	6-3(5)

Sign'

[0.16 _ 0.17]

$= \frac{1}{6} = 0.166$

When a function that outputs the sign of $(x_1 - 3x_2)$ is used as a classification model to fit the data set, what is the resultant loss?

2) For the data set $(x^i, y^i) = [(1,2),(3,4),(4,3),(5,6)]$, i= 1 to 4, let the regression function be $\frac{x^2-2}{5}$. The mean squared error loss obtained while fitting this function on the data set is

[3, 4]

$y = mx + c$

$\frac{1}{n} \Sigma (y - y')^2$

x	y	y'	$(y-y')^2$
1	2	$-1/5$	4.84
3	4	$7/5$	6.76
4	3	$14/5$	0.04
5	6	$23/5$	1.96

$\frac{13.6}{4}$ $\boxed{3.4}$

$y = \frac{x^2 - 2}{5}$ $\left(\frac{1^2-2}{5}\right)$

$\frac{9-2}{5}$ $\frac{16-3}{5}$

$= \frac{25-6}{5}$

3) A probability model $P(X) = \begin{cases} \frac{1}{5}, & \text{if } x \in [0,5] \\ 0, & \text{otherwise} \end{cases}$ is obtained by the density estimation algorithm for the data points $x_1 = 2.5, x_2 = 1, x_3 = 3, x_4 = 4.5$ and $x_5 = 4.95$. Compute the negative log likelihood loss of the model.

$L = \frac{1}{n} \Sigma - \log (P_i(x))$

[$\log_{10} e$

$= \frac{1}{5} \left[-5 \log \left(\frac{1}{5}\right) \right)$

$= -1 \left[\log_e 5^{-1} \right] = \log_{10}(5) = \boxed{0.69897}$

4) The encoder and decoder functions expressed as $f(x) = x_1 - x_2 + 0.5x_3$ and $g(u) = [0.3u, -u, u]$ respectively are used to reduce the dimension of the data set given below.

-2+0.5

x	$f(x)$		$f(x)$	$g(f(x))$	$\lVert g(f(x_i)) - x_i \rVert^2$	
[−1, 1, −1]	-1-1+0.5	=	-1.5	[0.3(1.5), +1.5, -1.5]	[0.15, 1.5, -1.5]	[1.15, 0.5, 0.5]
[1, 2, 1]	1-2+0.5	=	-0.5	[0.3(-0.5), 0.5, -0.5]	[-0.15, 0.5, -0.5]	[1.15, -1.5, -1.5]
[1, 1, −1]	1+1+1		3	[0.3(3), -3, 3]	[0.9, -3, 3]	[-0.1, -2, 1]
[1, −1, 2]						
[0, 1, 3]	-1+1.5		0.5	[0.3(0.5), -0.5, 0.5]	[0.15, -0.5, 0.5]	[0.15, -1.5, -2.5]

What is the loss obtained for the model?

$\Rightarrow \frac{1.8225 + 5.8225 + 5.01 + 8.5225}{5} = 5.294$

[4.5, 5.5]

5) Let $g(x) = 2.5e^{-x^2+0.2x+2}$. Determine the equation of the tangent line at $x = 0.5$ and using it estimate the value of $g(1.5)$.

$g(0.5) = g(0.5) + g'(0.5)(x - 0.5)$

$g'(x) = 2.5\, e^{-x^2+0.2x+2} \quad (-2x+0.2)$

$g'(0.5) = 2.5\, e^{-0.5^2+0.2(0.5)+2} \quad (-2(0.5)+0.2)$

$= 2.5\,(-0.8)\, e^{2.35}$

$g'(0.5) = -12.71$

$-(0.5)^2 + 0.2(0.5)+2$

$g(0.5) = 2.5\, e$

$= 15.89$

$= 2.5\,($

$L[g(x)] = 15.89 + (-12.71)(x-0.5)$

$= 15.89 - 12.71x + 6.355$

$\boxed{L[g(x)] = 22.24 \quad L[g(x)(5)] = 3.175}$ $-12.71x$

6) The directional derivative of $f(x,y) = xy^2 + 3x^2$ at (1,1) in the direction of unit vector along $w = [-1, 1]$ is $[3, 4]$

$DD = (y^2 + 6x)\,\hat{i} + (2xy)\,\hat{j}$

$= 7\hat{i} + 2\hat{j}$

$\dfrac{-7+2}{\sqrt{2}} = \dfrac{-5}{\sqrt{2}} = -3.53$

Use the below information for Q7 & Q8

Compute the gradient of a function $g(x,y) = \sqrt{2x^2 - y^3}$ at $(2,1)$.

7) Enter the value of first element of the gradient.

1.51

No, the answer is incorrect.
Score: 0

Accepted Answers:

(Type: Range) 1,2

8) Enter the value of second element of the gradient. $[\cancel{1.5}, -0.5]$

0.56

$\nabla g(x,y) = \dfrac{1}{2\sqrt{2x^2 - y^3}}\,(4x)\,\hat{i}$

$= \dfrac{1}{\cancel{2}\sqrt{7}} \times 4(\cancel{x})$

$\nabla g(\,g(1,1)) = \dfrac{1}{2\sqrt{2x^2 - y^3}} \; -3y^2$

$= \dfrac{1}{2}\,\dfrac{1}{\sqrt{7}}\,(-3)$

$= -0.56$

9) A function $f(x,y) = x^2 + 2xy^3$ is approximated linearly in the neighbourhood of (2,-2). Use the approximation to approximate $f(2.3, -2.2)$.

$\dfrac{\partial f}{\partial x} = 2x + 2y^3 = 2(2) + 2(-2)^3 = 4 - 16 = -12$

$\dfrac{\partial f}{\partial y} = 2x(3y^2) = 6(2)(-2)^2 = 12 \times 4 = 48$

$f(2,-2) = 4 + 4(-2)^3$

$= 4 - 32 = -28$

$f(x,y) = (-28) + (-12)(x-2) + 48(y+2)$

$= -28 - 12x + 24 + 48y + 96$

$= -12x + 48y - 4 + 96$

$f(x,y) = -12x + 48y + 92$

$f(2.3, -2.2) = -12(2.3) + 48(-2.2) + 92$
$= -41.2$

10) The dominant eigen value of the matrix $\begin{bmatrix} 2 & 0 & 4 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{vmatrix} 2-\lambda & 0 & 4 \\ 0 & 3-\lambda & 0 \\ 0 & 1 & 2-\lambda \end{vmatrix} = 0$

$(2-\lambda)(3-\lambda)(2-\lambda) = 0$

$\lambda = \boxed{3}, 2, 2$

11) The length of error vector obtained by projecting $u = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$ onto $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is

$9+4+1$

$\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{(-1+8+6)}{14}(1,2,3) = \frac{13}{14}(1,2,3)$

$\text{error} = u - p = [-1, 4, 2] - [\frac{13}{14}, \frac{26}{14}, \frac{39}{14}]$

$= [-1-\frac{13}{14}, 4-\frac{26}{14}, 2-\frac{39}{14}]$

$= [\frac{-27}{14}, \frac{30}{14}, \frac{-11}{14}]$

$= \frac{(27)^2 + (30)^2 + (11)^2}{(14)^2}$

$= 8.92$

A vector $x = [a, b]$ has dot products $x \cdot r = 1$ and $x \cdot s = 0$ with two given vectors $r = [2, -1]$ and $s = [-1, 2]$.

$2a - b = 1$
$-a + 2b = 0 \rightarrow a = 2b$

$2a - b = 1$
$2(2b) - b = 1$

$3b = 1 \Rightarrow \boxed{b = \frac{1}{3}}$

$\boxed{a = \frac{2}{3}}$

14) Consider a line with zero slope and y-intercept of 2. The sum of squared residuals obtained when this line is used to fit the following data set is

x	y	
1	2	2
2	4	2
3	1	2
4	3	2

$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 3 \end{bmatrix}$ $y = 2$

$A^T b = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \\ 3 \end{bmatrix} = [2$

$A^T b = [$

$A^T A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1+4+9+16 & 10 \\ 10 & 5 \end{bmatrix}$ (1)

$(2)^2 + 1^2 + 1^2 = 6$

$A^T A x = \begin{bmatrix} 30 & 10 \\ 10 & 5 \end{bmatrix}$

$\begin{bmatrix} 30 & 10 \\ 10 & 5 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 25 \\ 10 \end{bmatrix}$

$30\theta_1 + 10\theta_0 = 25$

$- \quad 20\theta_1 + 10\theta_0 = 20$

$10\theta_1 = 5$

$\boxed{\theta_1 = 0.5}$

$15 + 10\theta_0 = 25$

$\boxed{\theta_0 = 1}$

Find the value of a such that $f(x)$ is continuous at $x = 0$:

$$f(x) = \begin{cases} \frac{\sqrt{1+ax} - \sqrt{1-ax}}{x}, & \text{if } -1 \le x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$

NOTE: Enter your answer to the nearest integer.

Response Type : Numeric

$\lim_{x \to 0^-} \frac{\sqrt{1+ax} - \sqrt{1-ax}}{x} = \lim_{x \to 0^+} \frac{2x}{}$

$\lim_{x \to 0^-} \frac{1(a)}{2\sqrt{1+ax}} - \frac{(-a)}{2\sqrt{1-ax}} = \frac{1}{-1}$

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

$\frac{a}{2} + \frac{a}{2}$ $\boxed{a = -1}$

Text Areas : PlainText

Possible Answers :

-1

15) If the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$ is reduced to its triangular echelon form, the number of free variables is

$$\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \Rightarrow$$

$x_1 \to x_3$ depends

free x_2, x_4, x_5