MOCK QUIZ 1

MLF

30 May 2022 12:48

1. (5 points) A sugar patient records his daily Blood Glucose levels in the morning before taking a breakfast. He continued to do that for an year. The table below shows a few samples of the dataset with food items he had on the previous nights. Being a data scientist, he developed and trained a simple linear model.Eventually, he settled with the following model $f(\mathbf{x}) = 5.8x_1 + 3.4x_2 + 20.9x_3 + 1.2x_4 + 79.8$ to predict the glucose level.

Idli (x_1)	Dosa(x_2)	Ice Cream(x_3)	Slept by 11 p.m (x_4)	Level (mg/dL)
⋮	⋮	⋮	⋮	⋮
1	1	1	1 (Yes)	113.4
0	4	0.5	1 (Yes)	106.1
1	2	0	-1 (No)	92.2
⋮	⋮	⋮	⋮	⋮

One fine day, he had three Idli, two Dosa, one Ice cream and continued working beyond 11 p.m. to submit the MLF assignment.On the next day he found the actual glucose level was at 118.2 mg/dL. Then, he used the trained model to predict the glucose level. The squared error between the model's prediction and the actual level would be?

$= w_1x_1 + w_2x_2 \quad + b$

$f(x) = 5.8x_1 + 3.4x_2 + 20.9x_3 + 1.2x_4 + 79.8$

$f_p(x) = 5.8(3) + 3.4(2) + 20.9(1) + 1.2(-1) + 79.8$

$f_p(x) = 123.7$

[30.25]

$x_1 = 3$

$x_2 = 2$

$x_3 = 1$

$x_4 = -1$

$f_a(x) = 118.2$ mg/dL

$\text{loss} = (\;\cdot\; - f_a(x))^2$

$= (123.7 - 118.2)^2$

$\text{loss} = 30.25$

2. (5 points) The ML team in a movie production company wanted to predict whether a movie will be successful or not by using some classification models.Therefore, the team collected presence/absence of various factors $\mathbf{x} = [x_1, x_2, x_3]$ from both successful and unsuccessful movies in the past.The data and the corresponding label is shown in the table below.

[0.2]

$\mathbf{x}$	y	z
[0,1,1]	0	0
[1,0,1]	1	1
[1,0,0]	1	1
[1,1,1]	1	1
[1,1,0]	1	0

$z = 0.5x_1 - 0.8x_2 + 0.4x_3$

$\frac{1}{5} = 0.2$

$0.5 + 0.4 \to 0.9$

$0.5(0) - 0.8 + 0.4$

Compute the loss of the model if they use

$$u(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

and $z = 0.5x_1 - 0.8x_2 + 0.4x_3$

$0.5 - 0.8 \quad I \quad x \quad y \quad z$

3. (5 points) The dimensionality of the data points shown in the table is to be reduced from $\mathbb{R}^3$ to $\mathbb{R}$. To achieve it the following encoder function $f(x_1, x_2, x_3) = \frac{2x_1 - x_2}{2}$ and the corresponding decoder function $g(u) = [u, 2u, 3u]$ is proposed .Compute the loss (or reconstruction error) of the proposed model.

$2(1) - 2$

$2(2) - 3$

$\frac{2(-1)+0}{2}$

x
[1,2,3]
[2,3,4]
[-1,0,1]
[0,1,1]

[14.25]

x	$f(x) = \frac{2x_1 - x_2}{2}$	$g(f(x))$	$\lVert g(f(x_i)) - x_i \rVert^2$
[1 2 3]	0	[0, 0, 0]	$1^2 + 2^2 + 3^2 = 15$
[2 3 4]	$\frac{1}{2}$	$[\frac{1}{2}, 1, \frac{3}{2}]$	$[(2-\frac{1}{2})^2 + (2)^2 + (4-\frac{3}{2})^2] = 12.5$
[-1 0 1]	-1	[-1, -2, -3]	$(2)^2 + 4^2 = 20$
[0 1 1]	$-\frac{1}{2}$	$[-\frac{1}{2}, -1, -\frac{3}{2}]$	$\frac{1}{4} + 4 + (\frac{5}{2})^2 = 10.5$

$= 14.25$

4. (5 points) Consider the following table that shows the data points (x) and two possible distributions ($P_1(x)$, $P_2(x)$) which might have generated those data points.

$L_1 \quad L_2$ [-0.36]

Density est. model

x	$P_1(x)$	$P_2(x)$
0.2	$\frac{1}{6}$	0.5
0.4	$\frac{1}{6}$	0.2
0	$\frac{1}{6}$	0.1
-0.2	$\frac{1}{6}$	0.1
-0.4	$\frac{1}{6}$	0.05
0.1	$\frac{1}{6}$	0.05

Rohan argues that the data points are most likely generated by $P_1(x)$. Let L_1 be the loss for $P_1(x)$ and L_2 be the loss for $P_2(x)$. Verify his claim by computing the average loss for both distributions and enter the difference $L_1 - L_2$.

$\text{Loss} = +\frac{1}{n} \sum (-\log_e(P_i(x))$

$L_2 = \frac{-1}{6}\left[-\log\left(\frac{1}{2}\right) - \log(0.2) - 2\log(0.1) - 2\log(0.05)\right)$

$= 2.14$

$L_1 - L_2 = 1.79 - 2.14$

$= -0.35$

$L_1 = \frac{1}{6}\left[-\log\left(\frac{1}{6}\right) + (-\log\left(\frac{1}{6}\right) + \ldots + (-\log\left(\frac{1}{6}\right)\right)$

$= \frac{1}{6}\left[-6\log\left(\frac{1}{6}\right)\right)$ $\qquad \log\left(\frac{1}{x}\right) = \log x^{-1}$

$= \frac{1}{6}(-6)\log(6^{-1}) = \frac{1}{\cancel{6}} \times (\cancel{-6})(-1)\log 6$

$L = \log 6 = \ln 6 = 1.79$

5. (2 points) Linear approximation of $f(x) = 5x^2 + 3x - 9$ around x=0.1 is

$f(x) = 5x^2 + 3x - 9$

$f'(x) = 10x + 3$

$a = 0.1$

$= 10(0.1) + 3$

$f'(x) = 4$

$f(x) = 5(0.1)^2 + 0.3 - 9$

$= -8.65$

$L[f(x)] = f(x) + f'(x)(x-a)$

$= f(a) + f'(a)(x-a)$

$= -8.65 + 4(x - 0.1)$

$= -8.65 + 4x - 0.4$

$\boxed{Lf(x) = 4x - 9.05}$

6. (2 points) First order derivative of $f(x) = \frac{e^x}{\sqrt{2+x}}$ at $x = 0$ is $[0.53]$

$u = e^x \quad v = \frac{1}{\sqrt{2+x}} \qquad = \frac{u}{v} \qquad \overline{}\, v^2$

$v = (2+x)^{-1/2}$

$f(x) = \frac{e^x}{\sqrt{2+x}} \rightarrow u.$

$f'(x) = e^x\left(-\frac{1}{2}\frac{1}{(2+x)^{3/2}}\right) + \frac{1}{\sqrt{2+x}}e^x \qquad x = 0$

$= 1\left(-\frac{1}{2}\frac{1}{(2)^{3/2}} + \frac{1}{\sqrt{2}}\right.$

$= \frac{1}{\sqrt{2}}\left[-\frac{1}{2}\times\frac{1}{2} + 1\right] = \frac{1}{\sqrt{2}}\left[1 - \frac{1}{4}\right] = \frac{3}{4}\frac{1}{\sqrt{2}} = \frac{3}{4\times 1.41} = 0.53$

7. (2 points) Linear approximation of $f(x_1, x_2) = 2x_1^2 + 2x_2^2$ around (1,1) is

$L[f(x_1,x_2)] = f(x_1,x_2)_{(1,1)} + \left(\frac{\partial f}{\partial x_1}\right)_{(a_1)}(x_1 - a_1) + \left(\frac{\partial f}{\partial x_2}\right)_{(a_2)}(x_2 - a_2)$

$\left(\frac{\partial f}{\partial x_1}\right)_{(1,1)} = 2(2x_1) = 4x_1 = 4$

$= 4 + 4(1)(x_1 - 1) + 4(1)(x_2 - 1)$

$\left(\frac{\partial f}{\partial x_2}\right)_{(1,1)} = 4x_2 = 4$

$= 4 + 4(x_1 - 1) + 4(x_2 - 1)$

$f(x_1,x_2)_{(1,1)} = 2(1)^2 + 2(1)^2$

$= 4$

$= \cancel{4} + 4x_1 - \cancel{4} + 4x_2 - 4$

$= 4ey + 4e^{z} \, 4$

8. (2 points) Let $f(x, y, z) = -3x + 4ye^{6z}$. Find directional derivative of f at (1,-1,0) in the direction of unit vector along (1,0,-1)

(1, 0, -1)

$DD_{(1,-1,0)} =$ (1, 0, -1)

$DD = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$DD_{(1,-1,0)} = -3\hat{i} + (4e^{6z})\hat{j} + 4y(6)e^{6z}$

$= -3\hat{i} + 4\hat{j} + 4(-1)(6)$

$DD_{(1,-1,0)} = (-3\hat{i}\ 4\hat{j} - 24\hat{k})$

$= \frac{1 \times -3 + 4(0) - 1(-24)}{\sqrt{2}}$

$= \frac{-3 + 24}{\sqrt{2}}$

$= \frac{21}{\sqrt{2}} = 14.84924$

9. (2 points) $f(x) = c \times cos|x| + d \times e^{|x|}$ is differentiable at $x = 0$, Then which of the following options are correct.

A. $c - d = 0$

B. $c = 0$

C. $d = 0$

D. Any values of c and d

$f(x) = c\cos|x| + d \times e^{|x|}$

$f'(x) = c\cdot(-\sin(x))(-1) + d \times e^{|x|}(-1)$ $x<0$

$c(-\sin(x))(+1) + d \times e^{|x|}(1)$

$\lim_{x \to 0^-} f'(x) = -[g(x)]$ $x<0$

$\lim_{x \to 0^+} f'(x) = g(x)$ $x>0$

10. (2 points) Find the element $P_{2,2}$ of the projection matrix P of vector $v = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$. (Assume matrix index starts from 1)

0.904

$P = \frac{vv^T}{v^Tv}$

$vv^T = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix} [-1\ 7\ 2] = \begin{bmatrix} +1 & -7 & -2 \\ -7 & 49 & 14 \\ -2 & 14 & 4 \end{bmatrix}$

$v^Tv = [-1\ 7\ 2] \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$

$= 1 + 49 + 4$

$= 54$

$P = \frac{1}{54} \begin{bmatrix} 1 & -7 & -2 \\ -7 & \boxed{49} & 14 \\ -2 & 14 & 4 \end{bmatrix}$

$P_{22} = \frac{49}{54} = 0.904$

11. (3 points) Find the projection of vector $b = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 2 \end{bmatrix}$ onto vector $a = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}$

$\|a\|^2 = 1 + 4 + 1 + 9$

$$\vec{p} = \frac{(\vec{b}^T \cdot \vec{a})}{\|a\|^2}\vec{a} \qquad = 15$$

$$= \frac{(2+6+1+6)}{15} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}$$

12. (3 points) The best fit line using least squares method for the data set

x	y
1	2
2	3
3	7
4	10
5	13

is

$$A^T A \hat{\theta} = A^T b$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 7 \\ 10 \\ 13 \end{bmatrix} = \begin{bmatrix} 134 \\ 35 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 3 \\ 7 \\ 10 \\ 13 \end{bmatrix}$$

$$A^T A \hat{\theta} = A^T b$$

$$A^T A = \underbrace{\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}}_{2\times 5} \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \end{bmatrix}}_{5\times 2} = \begin{bmatrix} 55 & 5 \\ 15 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 55 & 5 \\ 15 & 5 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 134 \\ 35 \end{bmatrix}$$

θ_1 and $\theta_0 = \begin{bmatrix} 2.9 \\ -1.7 \end{bmatrix}$

13. (2 points) The second degree polynomial which best fits the following data

x	y
1	2
2	3
3	7

is

$$y = \theta_0 + \theta_1 x + \theta_2 x^2$$

(x_1, y_1) (x_2, y_2) (x_3, y_3)

$$2 = \theta_0 + \theta_1 + \theta_2$$
$$3 = \theta_0 + 2\theta_1 + 4\theta_2$$
$$7 = \theta_0 + 3\theta_1 + 4\theta_3$$

$\theta_0 = 4$

$\theta_1 = -3.5$

$\theta_2 = 1.5$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$$

14. (5 points) The two eigenvalues of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & p \end{bmatrix}$ have a ratio 3:1 for p=2. What is another value of p for which eigenvalues have the same ratio 3:1?

$$\begin{bmatrix} 2 & 1 \\ 1 & p \end{bmatrix} \qquad \frac{\lambda_1'}{\lambda_2'} = \frac{3}{1} \qquad \lambda_1 = 3\lambda_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{3}{1}$$

$$\boxed{\frac{3(4+4p+p^2)}{4\times 4} = 2p-1}$$

$$\lambda_1 + \lambda_2 = \mathrm{Tr}(A)$$

$$\lambda_1 + \lambda_2 = 2 + p \Rightarrow 4\lambda_2 = 2+p$$

$$3\lambda_2^2 = 2p-1$$

$$\lambda_1\lambda_2 = 2p-1$$

$$3\left(\frac{2+p}{4}\right)^2 = 2p-1$$

$$p = 2, \boxed{\frac{14}{3}}$$

15. (5 points) The eigenvalues and corresponding eigenvectors of a 2 × 2 matrix are given by

$$\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$

Eigenvalue	Eigenvector
8	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
4	$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is

$$A\underline{x} = \lambda \underline{x}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 8\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a+b \\ c+d \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \end{bmatrix}$$

$$a+b=8$$
$$a-b=4$$
$$a=6$$
$$b=2$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} a-b \\ c-d \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$