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AI&DS II Experiment 06

AIM: To implement fuzzy set Properties

THEORY:

Fuzzy logic is a mathematical system for representing information that is uncertain, imprecise, or "fuzzy." Unlike classical logic, which only allows variables to be true (1) or false (0), fuzzy logic lets variables take *any value between 0 and 1*. This feature is very useful in real-world situations where boundaries are gradual rather than sharply defined. For example, how dirty your clothes are can't usually be classified as simply "clean" or "dirty"—there might be many intermediate degrees.

To practically use fuzzy logic, we define fuzzy sets like "Low Dirt," "Medium Dirt," "High Dirt," etc. Each set is described using a membership function, which assigns a *degree of membership* (from 0 to 1) for every input value.

What is a Membership Function?

A membership function $\mu(x)$ answers: *How much does this input x belong to a fuzzy set?*

If $\mu(x)=0$, then x doesn't belong at all.

If $\mu(x)=1$, then x fully belongs.

If $0<\mu(x)<1$, then x belongs partially.

Let's look at four popular types of membership functions commonly used in fuzzy logic:

1. Triangular Membership Function

Equation

$$\mu(x;a,b,c)=\begin{cases}0, & x\le a\\ \frac{(x-a)}{(b-a)}, & a<x\le b\\ \frac{(c-x)}{(c-b)}, & b<x<c\\ 0, & x\ge c\end{cases}$$

Simple Explanation

- The function looks like a triangle.
- Parameters a,b,c are three points that define the triangle's base (a and c) and its peak (b).
- Between a and b, the membership degree rises linearly to 1 at b.
- Between b and c, it falls linearly back to 0.

Used when there is one clear, central value (the peak) and membership fades smoothly to zero at the edges.

2. Trapezoidal Membership Function

Equation

$$\mu(x;a,b,c,d) = \begin{cases} 0, & x \le a \\ \frac{(x-a)}{(b-a)}, & a < x \le b \\ 1, & b < x < c \\ \frac{(d-x)}{(d-c)}, & c \le x < d \\ 0, & x \ge d \end{cases}$$

Simple Explanation

- Has a flat “top” region where membership is fully true (value 1).
- Points a,b determine where it starts rising, c,d where it starts falling.
- Membership rises from 0 to 1, stays at 1, then falls back to 0.

Useful for representing ranges that are definitely “true” for a while and then gradually become “false” at the edges.

3. Gaussian Membership Function

Equation

$$\mu(x;m,\sigma) = \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

Simple Explanation

- Looks like a bell curve (Gaussian curve).
- Parameter m sets the center (mean), σ sets the spread (standard deviation).
- Membership is highest in the center and drops off smoothly as you move away.

Great for representing concepts where “extreme” values are unlikely, and most membership is around a typical mean.

4. Sigmoidal Membership Function

Equation

$$\mu(x;a,c) = \frac{1}{1+\exp(-a(x-c))}$$

Simple Explanation

- Has an S-shape.
- Parameter a controls the steepness (how quickly it rises), c is the center value where the function transitions.
- Membership starts near 0, rises gradually, and approaches 1.

Ideal for situations where change happens progressively rather than suddenly.

<u>CODE:</u>

Import Required Libraries

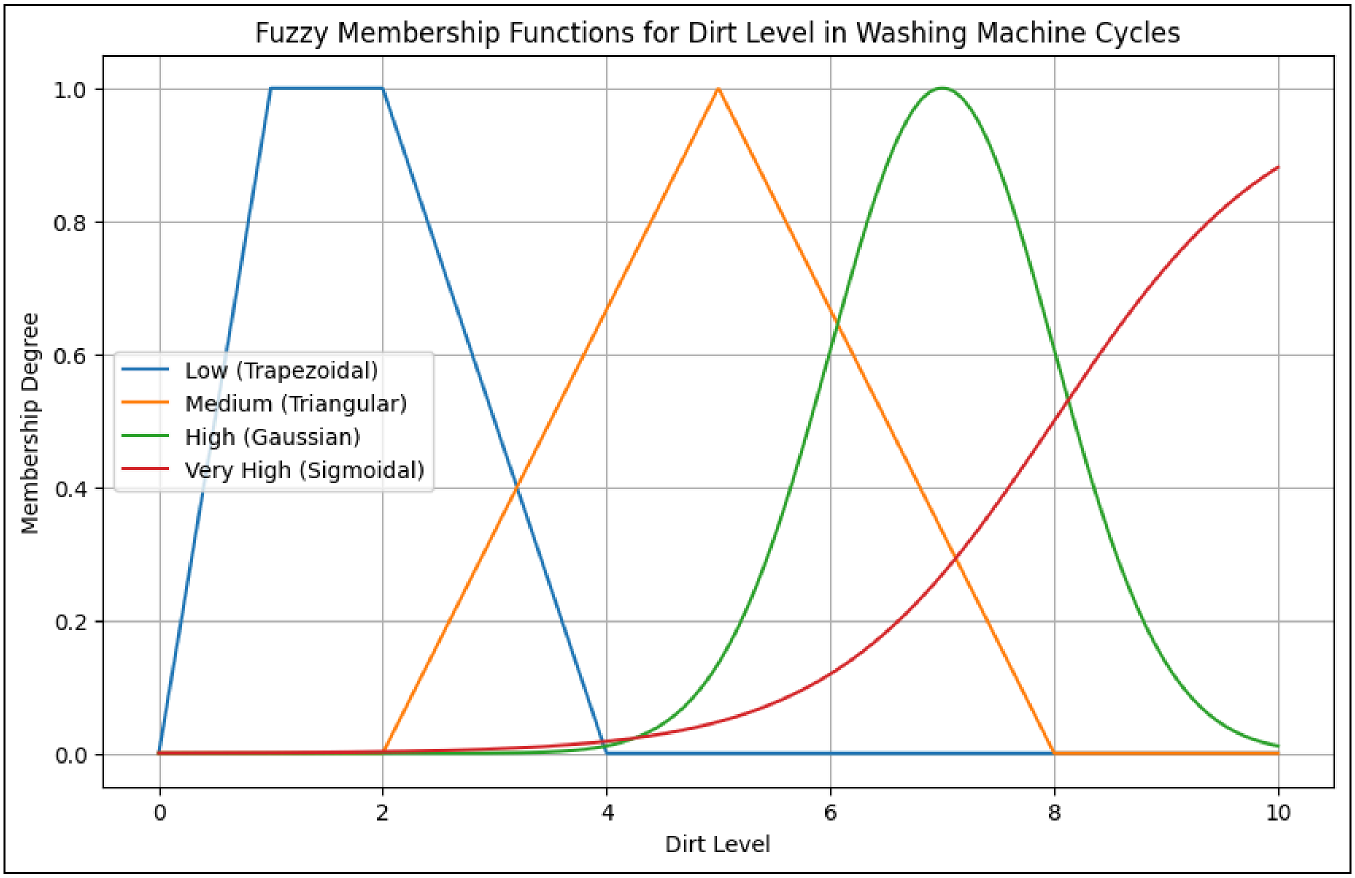
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
```

Define Membership Functions

```
def triangular(x, a, b, c):
    return np.maximum(np.minimum((x - a) / (b - a), (c - x) / (c - b)), 0)

def trapezoidal(x, a, b, c, d):
    return np.maximum(np.minimum(np.minimum((x-a)/(b-a), 1), (d-x)/(d-c)), 0)

def gaussian(x, mean, sigma):
    return np.exp(-0.5 * np.square((x - mean) / sigma))

def sigmoidal(x, a, c):
    return 1 / (1 + np.exp(-a * (x - c)))
```

Plot All Membership Functions in a Single Plot

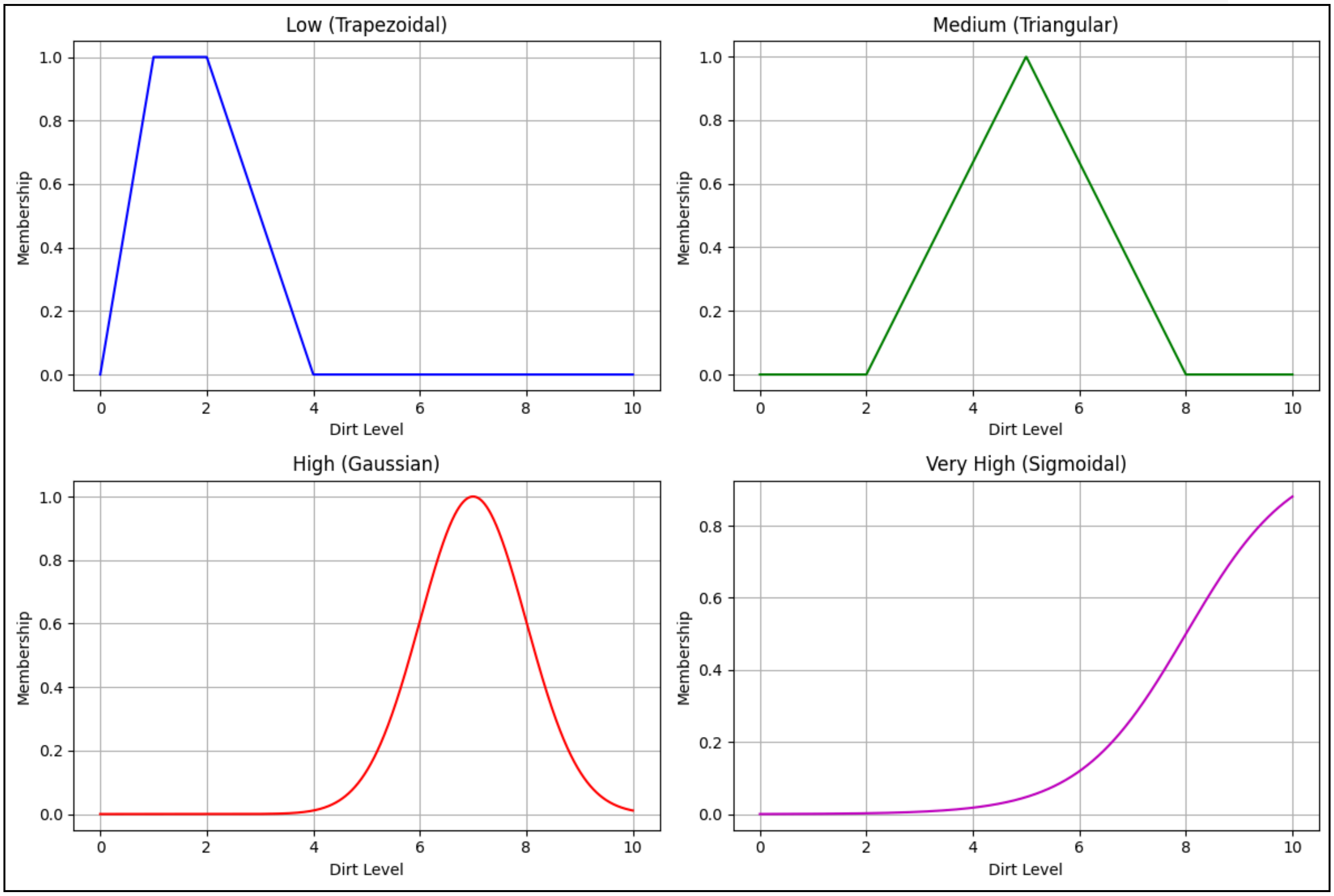
```
x = np.linspace(0, 10, 1000)

# Fuzzy sets for Dirt Level
low = trapezoidal(x, 0, 1, 2, 4)
medium = triangular(x, 2, 5, 8)
high = gaussian(x, 7, 1)
very_high = sigmoidal(x, 1, 8)

plt.figure(figsize=(10,6))
plt.plot(x, low, label='Low (Trapezoidal)')
plt.plot(x, medium, label='Medium (Triangular)')
plt.plot(x, high, label='High (Gaussian)')
plt.plot(x, very_high, label='Very High (Sigmoidal)')
plt.title('Fuzzy Membership Functions for Dirt Level in Washing Machine Cycles')
plt.xlabel('Dirt Level')
plt.ylabel('Membership Degree')
plt.legend()
plt.grid(True)
plt.show()
```



Plot Each Membership Function Individually

```
fig, axs = plt.subplots(2, 2, figsize=(12, 8))


axs[0,0].plot(x, low, color='b')

axs[0,0].set_title('Low (Trapezoidal)')

axs[0,0].set_xlabel('Dirt Level')

axs[0,0].set_ylabel('Membership')

axs[0,0].grid(True)


axs[0,1].plot(x, medium, color='g')

axs[0,1].set_title('Medium (Triangular)')

axs[0,1].set_xlabel('Dirt Level')

axs[0,1].set_ylabel('Membership')

axs[0,1].grid(True)


axs[1,0].plot(x, high, color='r')

axs[1,0].set_title('High (Gaussian)')

axs[1,0].set_xlabel('Dirt Level')

axs[1,0].set_ylabel('Membership')

axs[1,0].grid(True)
```

```
axs[1,1].plot(x, very_high, color='m')
axs[1,1].set_title('Very High (Sigmoidal)')
axs[1,1].set_xlabel('Dirt Level')
axs[1,1].set_ylabel('Membership')
axs[1,1].grid(True)

plt.tight_layout()
plt.show()
```



CONCLUSION:

Fuzzy membership functions help us represent real-world situations where values are not just true or false, but can be partly true to different degrees. By using functions like triangular, trapezoidal, Gaussian, and sigmoidal, we can model these gradual changes and make smarter decisions, such as selecting the right washing cycle based on dirt level. This approach makes systems work more like human thinking in handling uncertainty.

[39_colab_07]