

AI&DS II Experiment 07

AIM: To implement Fuzzy Membership Functions.

THEORY:

Fuzzy set theory, introduced by Lotfi Zadeh in 1965, extends classical set theory by allowing elements to have degrees of membership between 0 and 1. This approach is useful for dealing with real-world scenarios where boundaries are not always clear, such as determining how dirty clothes are in a washing machine.

Key Concepts

- Fuzzy Set: A collection of elements, where each element has a membership value between 0 (not a member) and 1 (full member). For example, in a washing machine, the "Dirtiness" of clothes can be represented by a fuzzy set where each item's dirtiness level is given as a value between 0 and 1.
- Universe of Discourse: The complete set of all possible elements under consideration. In a washing machine context, this could be all the clothes in one load.
- Membership Function: Assigns to each element a membership value that reflects how well the element fits the concept described by the fuzzy set.

Basic Fuzzy Set Operations

1. Union ($A \cup B$):
 - Combines two fuzzy sets. The membership value for each element in the union is the higher value from the two sets.
 - $$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$
 - Example: If a shirt's dirtiness is 0.6 and its heaviness is 0.4, the union gives 0.6.
2. Intersection ($A \cap B$):
 - The membership value in the intersection is the lower value from the two sets for each element.
 - $$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$
 - Example: If dirtiness is 0.6 and heaviness is 0.4, intersection gives 0.4.
3. Complement (A'):
 - The complement of a fuzzy set measures how much an element does NOT belong to the set. Calculated as 1 minus the membership value.
 - $$\mu_{A'}(x) = 1 - \mu_A(x)$$
 - Example: If a shirt's dirtiness is 0.7, the complement is 0.3.
4. Scalar Multiplication:
 - Scales the membership values by a constant factor between 0 and 1.
 - $$\mu_{\alpha A}(x) = \alpha \cdot \mu_A(x)$$
 - Example: If dirtiness is scaled by 0.5, a value of 0.8 becomes 0.4.
5. Fuzzy Sum (Algebraic Sum):
 - Combines two fuzzy sets using the formula:
 - $$\mu_{A+B}(x) = \min(1, \mu_A(x) + \mu_B(x))$$
 - Useful for modeling the combined effect of two properties.

Applications in Washing Machine Control

Fuzzy sets can help washing machines make better decisions about wash cycles by considering imprecise information like "how dirty" or "how heavy" the clothes are. This enables more efficient, tailored cleaning.

CODE:

Import Libraries

```
import matplotlib.pyplot as plt
```

Define Fuzzy Sets

```
# Universe of discourse: individual clothing items
x = [1, 2, 3, 4, 5]
```

```
# Fuzzy set A ("Dirtiness")
dirtiness = [0.6, 0.1, 0.8, 0.3, 0.5]
```

```
# Fuzzy set B ("Load Size")
load_size = [0.2, 0.7, 0.4, 0.9, 0.5]
```

Compute Fuzzy Set Operations

```
# Union
union = [max(a, b) for a, b in zip(dirtiness, load_size)]

# Intersection
intersection = [min(a, b) for a, b in zip(dirtiness, load_size)]

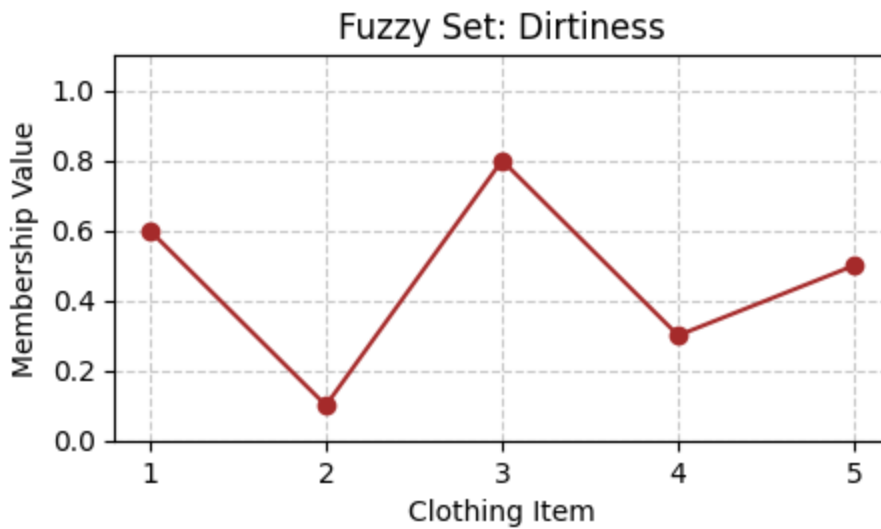
# Complement (Dirtiness')
complement_dirtiness = [1 - a for a in dirtiness]

# Scalar Multiplication (e.g., scale by 0.75)
scalar_mult_dirtiness = [0.75 * a for a in dirtiness]

# Fuzzy Sum (algebraic sum)
fuzzy_sum = [a + b - a * b for a, b in zip(dirtiness, load_size)]
```

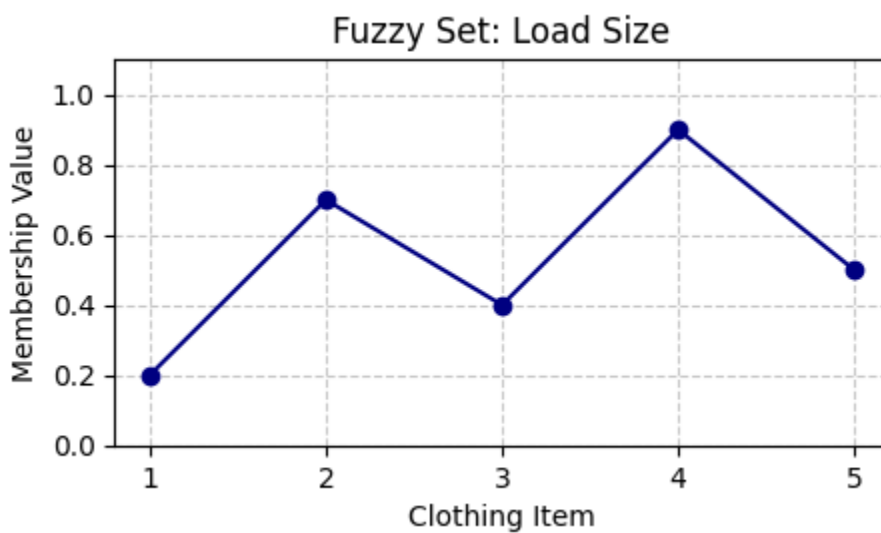
Plot - Dirtiness (Fuzzy Set A)

```
plt.figure(figsize=(5,2.5))
plt.plot(x, dirtiness, 'o-', color='brown')
plt.xlabel('Clothing Item')
plt.ylabel('Membership Value')
plt.title('Fuzzy Set: Dirtiness')
plt.ylim(0, 1.1)
plt.xticks(x)
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()
```



Plot - Load Size (Fuzzy Set B)

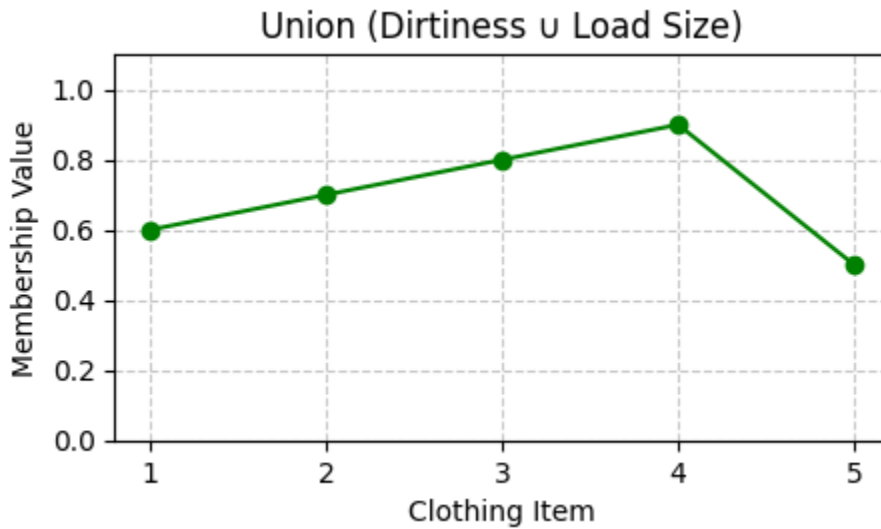
```
plt.figure(figsize=(5,2.5))
plt.plot(x, load_size, 'o-', color='navy')
plt.xlabel('Clothing Item')
plt.ylabel('Membership Value')
plt.title('Fuzzy Set: Load Size')
plt.ylim(0, 1.1)
plt.xticks(x)
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()
```



Plot - Union

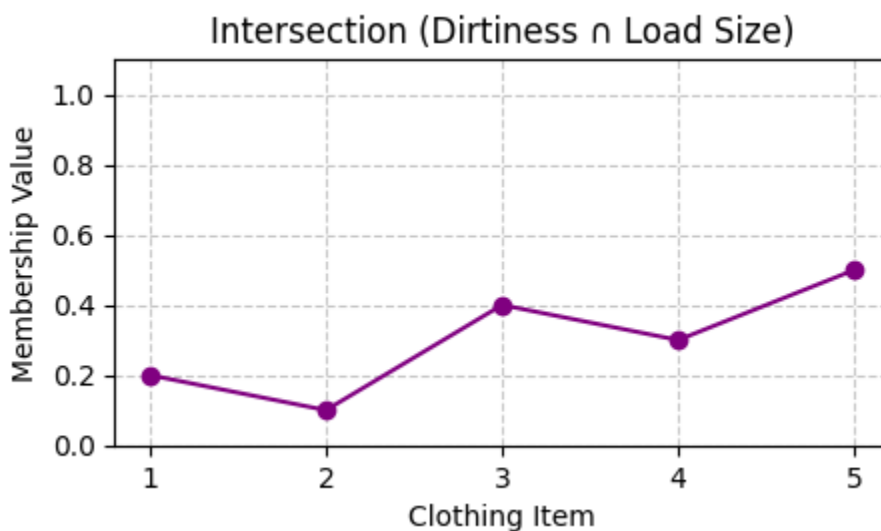
```
plt.figure(figsize=(5,2.5))
plt.plot(x, union, 'o-', color='green')
plt.xlabel('Clothing Item')
plt.ylabel('Membership Value')
plt.title('Union (Dirtiness  $\cup$  Load Size)')
plt.ylim(0, 1.1)
plt.xticks(x)
```

```
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()
```



Plot - Intersection

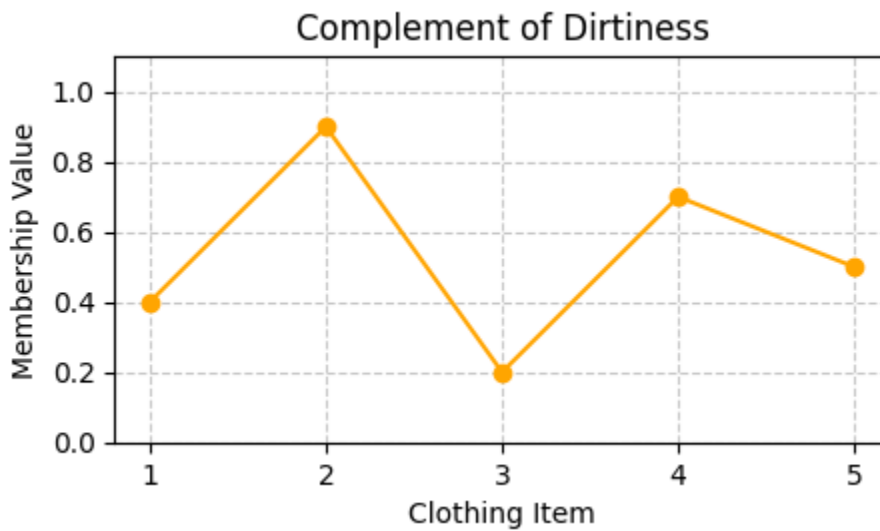
```
plt.figure(figsize=(5,2.5))
plt.plot(x, intersection, 'o-', color='purple')
plt.xlabel('Clothing Item')
plt.ylabel('Membership Value')
plt.title('Intersection (Dirtiness ∩ Load Size)')
plt.ylim(0, 1.1)
plt.xticks(x)
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()
```



Plot - Complement of Dirtiness

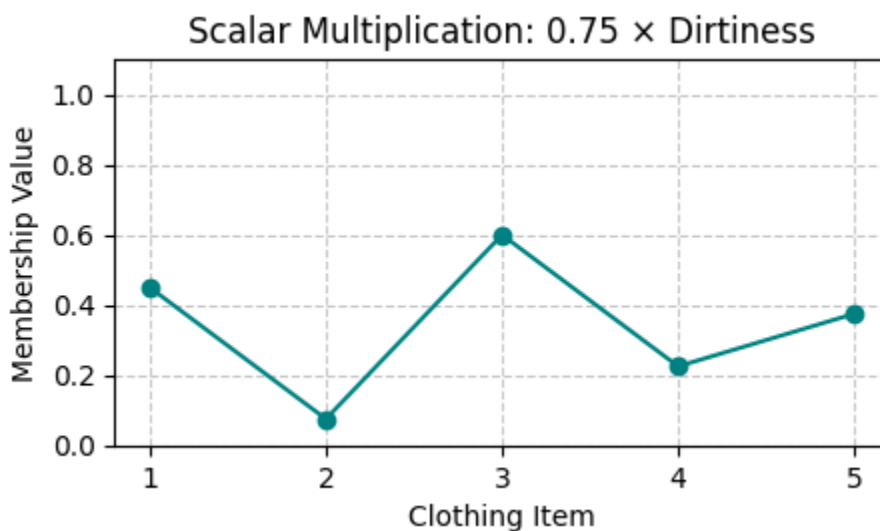
```
plt.figure(figsize=(5,2.5))
plt.plot(x, complement_dirtiness, 'o-', color='orange')
plt.xlabel('Clothing Item')
plt.ylabel('Membership Value')
plt.title('Complement of Dirtiness')
```

```
plt.ylim(0, 1.1)
plt.xticks(x)
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()
```



Plot - Scalar Multiplication ($0.75 \times \text{Dirtiness}$)

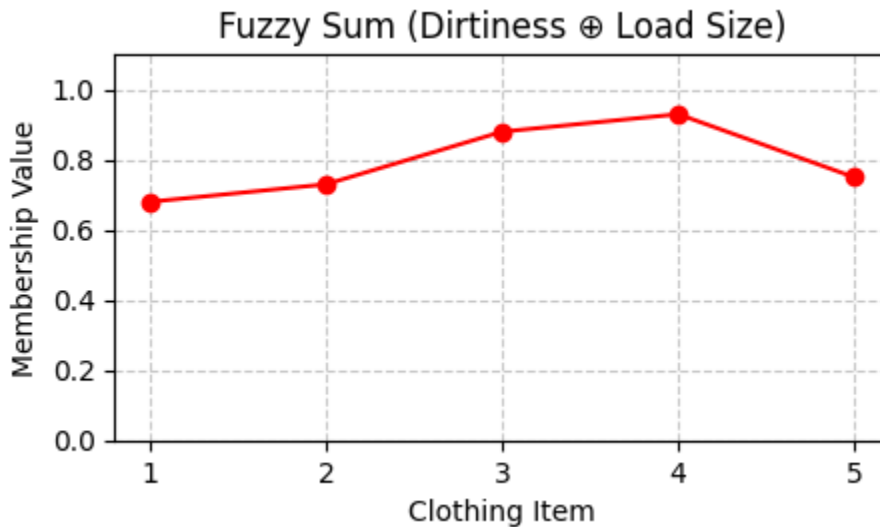
```
plt.figure(figsize=(5,2.5))
plt.plot(x, scalar_mult_dirtiness, 'o-', color='teal')
plt.xlabel('Clothing Item')
plt.ylabel('Membership Value')
plt.title('Scalar Multiplication: 0.75 × Dirtiness')
plt.ylim(0, 1.1)
plt.xticks(x)
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()
```



Plot - Fuzzy Sum

```
plt.figure(figsize=(5,2.5))
plt.plot(x, fuzzy_sum, 'o-', color='red')
plt.xlabel('Clothing Item')
```

```
plt.ylabel('Membership Value')
plt.title('Fuzzy Sum (Dirtiness  $\oplus$  Load Size)')
plt.ylim(0, 1.1)
plt.xticks(x)
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()
```



CONCLUSION:

Fuzzy membership functions help us represent real-world situations where values are not just true or false, but can be partly true to different degrees. By using functions like triangular, trapezoidal, Gaussian, and sigmoidal, we can model these gradual changes and make smarter decisions, such as selecting the right washing cycle based on dirt level. This approach makes systems work more like human thinking in handling uncertainty

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