Mechanics

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Contents

Chapter 1

Introduction

Mechanics is the area of mathematics and physics concerned with the motions of physical objects, more specifically the relationships among **force**, **matter**, and **motion**.

1.1 Classical Mechanics

Classical mechanics is a physical theory describing the motion of macroscopic objects, from projectiles to parts of machinery, and astronomical objects, such as spacecraft, planets, stars, and galaxies. For objects governed by classical mechanics, if the present state is known, it is possible to predict how it will move in the future (**determinism**), and how it has moved in the past (**reversibility**).

1.1.1 Newtonian Mechanics

The earliest development of classical mechanics is often referred to as **Newtonian mechanics**. It consists of the physical concepts based on foundational works of **Sir Isaac Newton**, and the mathematical methods invented by **Gottfried Wilhelm Leibniz**, **Joseph-Louis Lagrange**, **Leonhard Euler**, and other contemporaries, in the 17th century to describe the motion of bodies under the influence of a system of **forces**.

1.1.2 Analytical Dynamics

Later, more abstract methods were developed, leading to the reformulations of classical mechanics known as **Lagrangian mechanics** and **Hamil-**

tonian mechanics. These advances, made predominantly in the 18th and 19th centuries, extend substantially beyond earlier works, particularly through their use of **analytical mechanics**.

1.2 Quantum Mechanics

Classical mechanics provides extremely accurate results when studying large objects that are not extremely massive and speeds not approaching the speed of light. When the objects being examined have about the size of an atom diameter, it becomes necessary to introduce the other major sub-field of mechanics: **quantum mechanics**.

1.3 Relativistic

To describe velocities that are not small compared to the **speed of light**, **Albert Einstein's** theory of **special relativity** is needed. In cases where objects become extremely massive, **Albert Einstein's** theory of **general relativity** becomes applicable.

1.4 Energy

In classical mechanics, **energy** is a conceptually and mathematically useful property, as it is a **conserved quantity**. Several formulations of mechanics have been developed using energy as a core concept.

Work is a function of energy.

$$W = \int_C \mathbf{F} \cdot d\mathbf{s}$$

This says that the work (W) is equal to the **line integral** of the force F along a path C. Work and thus energy is frame dependent.

The total energy of a system is sometimes called the **Hamiltonian**, after William Rowan Hamilton. The classical equations of motion can be written in terms of the Hamiltonian, even for highly complex or abstract systems. These classical equations have remarkably direct analogs in nonrelativistic quantum mechanics.

Another energy-related concept is called the **Lagrangian**, after Joseph-Louis Lagrange. This formalism is as fundamental as the Hamiltonian, and both can be used to derive the equations of motion or be derived from them. It was invented in the context of classical mechanics, but is generally useful in modern physics. Usually, the Lagrange formalism is mathematically more convenient than the Hamiltonian for non-conservative systems (such as systems with friction).

Noether's theorem (1918) states that any differentiable symmetry of the action of a physical system has a corresponding **conservation law**. Noether's theorem has become a fundamental tool of modern theoretical physics and the **calculus of variations**. A generalisation of the seminal formulations on constants of motion in Lagrangian and Hamiltonian mechanics (1788 and 1833, respectively), it does not apply to systems that cannot be modeled with a Lagrangian; for example, dissipative systems with continuous symmetries need not have a corresponding conservation law.

Part I

Classical Mechanics

Chapter 2

Introduction

Chapter 3

Frame of Reference

3.1 Coordinate System

A **coordinate system** is a system that uses one or more numbers, or **coordinates**, to uniquely determine the position of the **points** or other geometrci elements on a **manifold** (such as **Euclidean Space**).

3.2 Transformation

3.3 Inertial Frame of Reference

Within the realm of **Newtonian Mechanics**, an **inertial frame of reference**, or inertial reference frame, is one in which **Newton's first law of motion** is valid. However, the **principle of special relativity** generalizes the notion of inertial frame to include all physical laws, not simply Newton's first law.

Newton viewed the first law as valid in any reference frame that is in uniform motion relative to the fixed stars; this is, neither rotating nor accelerating relative to the starts. Today the notion of "absolute space" is abandonded, and an inertial frame in the field of **classical mechanics** is defined as:

Definition 3.3.1. An **inertial frame of reference** is one in which the motion of a particle not subjected to forces is in a straight line at constant speed.

Hence, with respect to an inertial frame, an object or body **accelerates** only when a physics force is applied, and (following Newton's first law of motion), in the absence of a net force, a body at **rest** will remain at rest and a body in motion will continue to move uniformly - that is, in a straigth line and at constant speed. Newtonian inertial frames transform among each other according to the **Galiliean group of symmetries**.

If this rule is interpreted as saying that straight-line motion is an indication of zero net force, the rules does not identify inertial reference frames because straight-line motion can be observed in a vareity of frames. If the rule is interpreted as defining an inertial frame, then we have to be able to determine when zero net force is applied.

3.4 Galiliean Transformation

3.4.1 Galiliean Invariance

3.5 Euler Angles

3.6 Generalized Coordinate

3.7 Constraints and Degrees of Freedom

3.7.1 Holonomic Constraint

3.7.2 Non-holonomic Constraint

Chapter 4

Kinematics

*"**Kinematics** is often described as **applied geometry**, where the movement of a mechanical system is described using the **rigid transformation** of **Euclidean geometry**."*

4.1 Introduction

Kinematics is a subfield of **physics**, developed in **classical mechanics**, that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering the forces that cause them to move. Kinematics, as a field of study, is often referred to as the "**geometry of motion**" and is occasionally seen as a branch of mathematics. A kinematics problem begins by describing the geometry of the system and declaring the initial conditions of any known values of **position**, **velocity** and/or **acceleration** of points within the system.

4.2 Position

The **position** of a **particle** is defined as the coordinate vector from the origin of a coordinate frame to the particle. It expresses both the distance of the point from the origin and its direction from the origin. In three dimensions, the position vector $\mathbf{r}$ can be expressed as

$$\mathbf{r} = (x, y, z) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

where, x, y and z are the **Cartesian coordinates** and $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are the unit vectors along the x, y and z coordinate axes, respectively. The **magnitude** of the positon vector $|\mathbf{r}|$ gives the distance between the point $\mathbf{r}$ and the origin.

$$|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}.$$

the **direction cosines** of the position vector provide a quantitative measure of direction.

The **trajectory** of a particle is a vector function of time, $\mathbf{r}(t)$, which defines the curve traced by the moving particle, given by

$$\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}},$$

where $x(t)$, $y(t)$ and $z(t)$ describe each coordinate of the particle's position as a function of time.

4.3 Velocity

The **velocity** of a **particle** is a vector quantity that describes the magnitude as well as direction of motion of the particle. Consider the ratio formed by dividing the difference of two positions of a particle by the time interval. This ratio is called the **average velocity** over that time interval and is defined as

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{r}}{\Delta t},$$

where $\Delta\mathbf{r}$ is the change in the position vector during the time interval Δt. In the limit that the time interval Δt approaches zero, the average velocity approaches the instantaneous velocity, defined as the time derivative of the position vector,

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \dot{x}\hat{\mathbf{i}} + \dot{y}\hat{\mathbf{j}} + \dot{z}\hat{\mathbf{k}},$$

where the dot denotes a derivative with respect to time. Furthermore, this velocity is **tangent** to the particle's trajectory at every position along its path. Note that in a non-rotating frame of reference, the derivatives of the coordinate directions are not considered as their directions and magnitudes are constants.

The **speed** of an object is the magnitude of its velocity. It is a **scalar** quantity:

$$v = |\mathbf{v}| = \frac{ds}{dt},$$

where s is the arc-length measured along the trajectory of the particle. This arc-length must always increase as the particle moves. Hence, ds/dt is non-negative, which implies that speed is also non-negative.

4.4 Acceleration

The **acceleration** of a particle is the vector defined by the rate of change of the velocity vector. The **average acceleration** of a particle over a time interval is defined as the ratio.

$$\mathbf{a}_{\text{avg}} = \frac{\Delta \mathbf{v}}{\Delta t} ,$$

where $\Delta \mathbf{v}$ is the difference in the velocity vector and Δt is the time interval.

The acceleration of the particle is the limit of the average acceleration as the time interval approaches zero, which is the time derivative,

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \dot{v}_x \hat{\mathbf{i}} + \dot{v}_y \hat{\mathbf{j}} + \dot{v}_z \hat{\mathbf{k}}$$

or

$$\mathbf{a} = \ddot{\mathbf{r}} = \ddot{x}\hat{\mathbf{i}} + \ddot{y}\hat{\mathbf{j}} + \ddot{z}\hat{\mathbf{k}}$$

Note 4.4.1. *In a non-rotating frame of reference, the derivatives of the coordinate directions are not considered as their directions and magnitudes are constants.*

The magnitude of the acceleration of an object is the magnitude $|\mathbf{a}|$ of its acceleration vector. It is a **scalar** quantity:

$$|\mathbf{a}| = |\dot{\mathbf{v}}| = \frac{dv}{dt},$$

4.4.1 Particle trajectories in cylindrical-polar coordinates

Consider a particle P that moves only on the surface of a circular cylinder $\mathbf{r}(t) =$ constant, it is possible to align the Z axis of the fixed frame F with the axis of the cylinder. Then, the angle θ around this axis in the $X\check{\ }Y$ plane can be used to define the trajectory as,

$$\mathbf{r}(t) = R\cos(\theta(t))\hat{\mathbf{i}} + R\sin(\theta(t))\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$$

where the constant distance from the center is denoted as R, and $\theta = \theta(t)$ is a function of time.
The cylindrical coordinates for $\mathbf{r}(t)$ can be simplified by introducing the radial and tangential unit vectors,

$$\mathbf{e}_r = \cos(\theta(t))\hat{\mathbf{i}} + \sin(\theta(t))\hat{\mathbf{j}}, \quad \mathbf{e}_\theta = -\sin(\theta(t))\hat{\mathbf{i}} + \cos(\theta(t))\hat{\mathbf{j}}.$$

and their time derivatives from elementary calculus:

$$\frac{d}{dt}\mathbf{e}_r = \dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta \frac{d}{dt}\dot{\mathbf{e}}_r = \ddot{\mathbf{e}}_r = \ddot{\theta}\mathbf{e}_\theta - \dot{\theta}\mathbf{e}_r \frac{d}{dt}\mathbf{e}_\theta = \dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r \frac{d}{dt}\dot{\mathbf{e}}_\theta = \ddot{\mathbf{e}}_\theta = -\ddot{\theta}\mathbf{e}_r - \dot{\theta}^2\mathbf{e}_\theta.$$

Using this notation, $\mathbf{r}(t)$ takes the form,

$$\mathbf{r}(t) = R\mathbf{e}_r + z(t)\hat{\mathbf{k}}.$$

In general, the trajectory $\mathbf{r}(t)$ is not constrained to lie on a circular cylinder, so the radius R varies with time and the trajectory of the particle in cylindrical-polar coordinates becomes:

$$\mathbf{r}(t) = R(t)\mathbf{e}_r + z(t)\hat{\mathbf{k}}.$$

Where R, θ, and z might be continuously differentiable functions of time and the function notation is dropped for simplicity. The velocity vector $\mathbf{v}_P$ is the time derivative of the trajectory $\mathbf{r}(t)$, which yields:

$$\mathbf{v}_P = \frac{d}{dt}\left(R\mathbf{e}_r + z\hat{\mathbf{k}}\right) = \dot{R}\mathbf{e}_r + R\dot{\mathbf{e}}_r + \dot{z}\hat{\mathbf{k}} = \dot{R}\mathbf{e}_r + R\dot{\theta}\mathbf{e}_\theta + \dot{z}\hat{\mathbf{k}}.$$

Similarly, the acceleration $\mathbf{a}_P$, which is the time derivative of the velocity $\mathbf{v}_P$, is given by:

$$\mathbf{a}_P = \frac{d}{dt}\left(\dot{R}\mathbf{e}_r + R\dot{\theta}\mathbf{e}_\theta + \dot{z}\hat{\mathbf{k}}\right) = \left(\ddot{R} - R\dot{\theta}^2\right)\mathbf{e}_r + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\mathbf{e}_\theta + \ddot{z}\hat{\mathbf{k}}.$$

The term $-R\dot{\theta}^2\mathbf{e}_r$ acts toward the center of curvature of the path at that point on the path, is commonly called the centripetal acceleration. The term $2\dot{R}\dot{\theta}\mathbf{e}_\theta$ is called the Coriolis acceleration.

Constant radius

If the trajectory of the particle is constrained to lie on a cylinder, then the radius R is constant and the velocity and acceleration vectors simplify. The velocity of vP is the time derivative of the trajectory r(t),

$$\mathbf{v}_P = \frac{d}{dt}\left(R\mathbf{e}_r + z\hat{\mathbf{k}}\right) = R\dot{\theta}\mathbf{e}_\theta + \dot{z}\hat{\mathbf{k}}.$$

Planar circular trajectories

A special case of a particle trajectory on a circular cylinder occurs when there is no movement along the Z axis:

$$\mathbf{v}_P = \frac{d}{dt}\left(R\mathbf{e}_r + z_0\hat{\mathbf{k}}\right) = R\dot{\theta}\mathbf{e}_\theta = R\omega\mathbf{e}_\theta,$$

where $\omega = \dot{\theta}$ is the angular velocity of the unit vector $\mathbf{e}_\theta$ around the z axis of the cylinder. The acceleration $\mathbf{a}_P$ of the particle P is now given by:

$$\mathbf{a}_P = \frac{d}{dt}\left(R\dot{\theta}\mathbf{e}_\theta\right) = -R\dot{\theta}^2\mathbf{e}_r + R\ddot{\theta}\mathbf{e}_\theta.$$

The components

$$a_r = -R\dot{\theta}^2, \quad a_\theta = R\ddot{\theta},$$

are called, respectively, the radial and tangential components of acceleration. The notation for angular velocity and angular acceleration is often defined as

$$\omega = \dot{\theta}, \quad \alpha = \ddot{\theta},$$

so the radial and tangential acceleration components for circular trajectories are also written as

$$a_r = -R\omega^2, \quad a_\theta = R\alpha.$$

4.5 Moving Reference Frame

4.6 Rotation

4.7 Kinematics Constraints

Chapter 5

Force

5.1 Funcamental Forces

5.2 Non-Funcamental Forces

5.2.1 Normal Force

5.2.2 Friction

5.2.3 Tension

5.2.4 Elastic Force

5.2.5 Continuum Mechanics

Chapter 6

Newtonian Mechanics

6.1 Introduction

6.1.1

6.2 Momentum

In **Newtonian mechanics**, **linear momentum**, **translational momentum**, or simply **momentum** is the product of the **mass** and **velocity** of an object. It is a **vector quantity**, possessing a magnitude and a direction. If m is an object's mass and $\mathbf{v}$ is its velocity, then the object's momentum $\mathbf{p}$ is

$$\mathbf{p} = m\mathbf{v}.$$

Momentum depends on the frame of reference, but in any inertial frame it is a conserved quantity, meaning that if a closed system is not affected by external forces, its total linear momentum does not change.

It is an expression of one of the fundamental symmetries of space and time: translational symmetry.

In continuous systems such as electromagnetic fields, fluid dynamics and deformable bodies, a momentum density can be defined, and a continuum version of the conservation of momentum leads to equations such as the Navier–Stokes equations for fluids or the Cauchy momentum equation for deformable solids or fluids.

6.2.1 Dependence on reference frame

6.2.2 conservation

6.2.3 Angular Momentum

6.3 Dynamics

6.3.1 Newton's Laws of Motion

Newton's First Law

Newton's Second Law

Newton's Third Law

6.3.2 Euler's Laws of Motion

Euler's First Law

Euler's Second Law

6.3.3 Roational Dynamics

Torque

6.3.4 Statics

6.4 Oscillation

6.4.1 Harmonic Oscillator

Chapter 7

Energy

7.1 Work

Work is the energy transferred to or from an object via the application of force along a displacement. In its simplest form, it is often represented as the product of force and displacement. A force is said to do positive work if (when applied) it has a component in the direction of the displacement of the point of application. A force does negative work if it has a component opposite to the direction of the displacement at the point of application of the force.

Work is a scalar quantity, so it has only magnitude and no direction. Work transfers energy from one place to another, or one form to another. The SI unit of work is the joule (J), the same unit as for energy.

7.1.1 Computation

Work is the result of a **force** on a point that follows a curve X, with a velocity $\mathbf{v}$, at each instant. The small amount of work δW that occurs over an instant of time dt is calculated as

$$\delta W = \mathbf{F} \cdot d\mathbf{s} = \mathbf{F} \cdot \mathbf{v} dt$$

where the $\mathbf{F} \cdot \mathbf{v}$ is the power over the instant dt. The sum of these small amounts of work over the trajectory of the point yields the work,

$$W = \int_{t_1}^{t_2} \mathbf{F} \cdot d\mathbf{s} = \int_{t_1}^{t_2} \mathbf{F} \cdot \frac{d\mathbf{s}}{dt} dt = \int_C \mathbf{F} \cdot d\mathbf{s}$$

where C is the trajectory from $\mathbf{x}(t_1)$ to $\mathbf{x}(t_2)$. This integral is computed along the trajectory of the particle, and is therefore said to be path dependent.

If the force is always directed along this line, and the magnitude of the force is $\mathbf{F}$, then this integral simplifies to

$$W = \int_C F\,ds$$

where s is the displacement along the line. If F is constant, in addition to being directed along the line, then the integral simplifies further to

$$W = \int_C F\,ds = F\int_C ds = Fs$$

where s is the displacement of the point along the line.

This calulation can be generalized for a constant force that is not directed along the line, followed by the particle. In this case the dot product $\mathbf{F} \cdot ds = F\cos\theta ds$, where θ is the angle between the force vector and the direction of movement, that is

$$W = \int_C \mathbf{F} \cdot d\mathbf{s} = Fs\cos\theta$$

7.1.2 Work-energy Principle

The **principle of work and kinetic energy** (also known as the **work–energy principle**) states that the work done by all forces acting on a particle (the work of the resultant force) equals the change in the kinetic energy of the particle. That is, the work W done by the resultant force on a particle equals the change in the particle's kinetic energy E_k,

$$W = \Delta E_k = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2,$$

where v_1 and v_2 are the speeds of the particle before and after the work is done, and m is its mass.

The derivation of the work–energy principle begins with Newton's second law of motion and the resultant force on a particle. Computation of the scalar product of the forces with the velocity of the particle evaluates the instantaneous **power** added to the system.

Constraints define the direction of movement of the particle by ensuring there is no component of velocity in the direction of the constraint force. This also means the constraint forces do not add to the instantaneous power. The time integral of this scalar equation yields work from the instantaneous power, and kinetic energy from the scalar product of velocity and acceleration. The fact that the work–energy principle eliminates the constraint forces underlies **Lagrangian mechanics**.

7.2 Kinetic Energy

Kinetic energy of an object is the energy that it possesses due to its motion. It is defined as the work needed to accelerate a body of a given mass from rest to its stated velocity. Having gained this energy during its acceleration, the body maintains this kinetic energy unless its speed changes. The same amount of work is done by the body when decelerating from its current speed to a state of rest. Formally, a kinetic energy is any term in a system's Lagrangian which includes a derivative with respect to time.

In classical mechanics, the kinetic energy of a non-rotating object of mass m traveling at a speed $\mathbf{v}$ is $\frac{1}{2}m\mathbf{v}^2$. In relativistic mechanics, this is a good approximation only when **v** is much less than the speed of light.

7.3 Potential Energy

Potential energy is the energy held by an object because of its position relative to other objects, stresses within itself, its electric charge, or other factors.

Potential energy is associated with forces that act on a body in a way that the total work done by these forces on the body depends only on the initial and final positions of the body in space. These forces, that are called **conservative forces**, can be represented at every point in space by vectors expressed as gradients of a certain scalar function called **potential**.

7.3.1 Conservative Force

A **conservative force** is a force with the property that the total work done in moving a particle between two points is independent of the path taken.

A conservative force depends only on the position of the object. If a force is conservative, it is possible to assign a numerical value for the potential at any point and conversely, when an object moves from one location to another, the force changes the potential energy of the object by an amount that does not depend on the path taken, contributing to the mechanical energy and the overall conservation of energy. If the force is not conservative, then defining a scalar potential is not possible, because taking different paths would lead to conflicting potential differences between the start and end points.

Definition 7.3.1. A force field $\mathbf{F}$, defined everywhere in space (or within a simply-connected volume of space), is called a **conservative force** or **conservative vector field** if it meets any of these three equivalent conditions:

1. The **curl** of $\mathbf{F}$ is the zero vector:
   - $\nabla \times \mathbf{F} = \mathbf{0}$.
2. There is zero net **work**(W) done by the force when moving a particle through a trajectory that starts and ends in the same place:
   - $W = \oint_C \mathbf{F} \cdot d\mathbf{r} = \mathbf{0}$.
3. The force can be written as the negative **gradient** of a **potential**, Φ:
   - $\mathbf{F} = -\nabla\Phi$.

Many forces (particularly those that depend on velocity) are not force fields. In these cases, the above three conditions are not mathematically equivalent.

7.3.2 Non-conservative Force

7.4 Conservation of Energy

Chapter 8

Newton's Law of Universal Gravitation

Every **point mass** attracts every single other point mass by a **force** acting along the line intersecting both points. The force is proportionla to the product of the two masses and iversely proportional to the square of the distance between them:

$$F = G\frac{m_1 m_2}{r^2}$$

where:

- F is the force between the masses
- G is the **gravitation constant** ($6.674 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$)
- m_1 is the first mass
- m_2 is the second mass
- r is the distance between the centers of the masses

8.1 Vector Form

Newton's law of universal gravitation can be written as a **vector equation** to account for the direction of the gravitational force as well as its magnitude:

$$\mathbf{F}_{21} = -G\frac{m_1 m_2}{|\mathbf{r}_{21}|^2}\hat{\mathbf{r}_{21}}$$

where:

- $\mathbf{F}_{21}$ is the force applied on object 2 excerted by object 1
- G is the **gravitation constant**
- m_1 and m_2 are respectively the masses of objects 1 and 2
- $|\mathbf{r}_{21}| = |\mathbf{r}_2 - \mathbf{r}_1|$ is the distance between objects 1 and 2
- $\hat{\mathbf{r}}_{21} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r})_2 - \mathbf{r}_1|}$ is the **unit vector** from object 1 to object 2

8.2 Gravitational Field

The **gravitational field** $\mathbf{g}$ around a single particle of mass M is a **vector field** consisting at every point of a vector pointing directly towards the particle. The magnitude of the field at every point is calculated by applying the universal law, and represents the force per unit mass on any object at that point in space. Because the force field is conservative, there is a scalar potential evergy per unit mass, Φ, at each point in space associated with the force fields; this is called **gravitational potential**. The gravitational field equation is

$$\mathbf{g} = \frac{\mathbf{F}}{m} = \frac{d^2\mathbf{R}}{dt^2} = -GM\frac{\hat{\mathbf{R}}}{|\mathbf{R}|^2} = -\nabla\Phi$$

where:

- $\mathbf{F}$ is the gravitational force
- m is the mass of the test particle
- $\mathbf{R}$ is the position of the **test particle**
- $\hat{\mathbf{R}}$ is a unit vector in the radial direction of $\mathbf{R}$
- t is time
- G is the gravitational constant
- ∇ is the **del operator**.

The equivalent field equation in terms of mass density ρ of the attracting mass is:

$$\nabla \cdot \mathbf{g} = -\nabla^2\Phi = -4\pi G\rho$$

which contains **Gauss's law for gravity**, and **Poisson's equation for gravity**. Newton's law implies Gauss's law, but not vice-versa.

Chapter 9

Lagrangian Mechanics

9.1 Introduction

9.1.1

Chapter 10

Hamiltonian Mechanics

10.1 Introduction

10.1.1

Part II

Quantum Mechanics

Part III

Relativistic

Chapter 11

Special Relativity

11.1 Lorentz Transformation

Define an event to have spacetime coordinates (t, x, y, z) in system S and (t', x', y', z') in a reference frame moving at a velocity $\mathbf{v}$ with respect to that frame, S'. Then the Lorentz transformation specifies that these coordinates are related in the following way:

$$\begin{aligned}
t' &= \gamma\left(t - vx/c^2\right) \\
x' &= \gamma\left(x - vt\right) \\
y' &= y \\
z' &= z,
\end{aligned}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is the **Lorentz factor** and c is the **speed of light** in vacuum, and the velocity $\mathbf{v}$ of S', relative to S, is parallel to the x-axis. For simplicity, the y and z coordinates are unaffected; only the x and t coordinates are transformed. These Lorentz transformations form a one-parameter group of linear mappings, that parameter being called $rapidity$.

Solving the four transformation equations above for the unprimed coordinates yields the inverse Lorentz transformation:

$$\begin{aligned}
t &= \gamma(t' + vx'/c^2) \\
x &= \gamma(x' + vt') \\
y &= y' \\
z &= z'.
\end{aligned}$$