

# Testing the Universality of Time Dilation: A Pion-Kaon Comparative Study with Muon Calibration

CERN Beamline for Schools 2026

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## Abstract

Einstein's special relativity predicts that time dilation is **universal**, meaning the Lorentz factor  $\gamma(p) = \sqrt{1 + (p/mc)^2}$  should depend only on momentum, independent of particle species. Yet nearly all experimental tests have measured single particles in isolation. We propose the **first systematic comparative study** measuring time dilation simultaneously across charged pions ( $\pi^+$ ) and kaons ( $K^+$ ) at the same momentum, using muons ( $\mu^+$ ) as a non-decaying reference. If special relativity is truly universal, both exponential decay curves should collapse to identical normalized survival functions when scaled by  $\gamma\tau_0$ . Any deviation would reveal mass-dependent corrections to Lorentz transformation. With  $4.7\times$  mass difference and 2–3% precision, we can detect violations at  $5\sigma$  confidence. This experiment uses only standard CERN detectors yet tests a fundamental assumption never directly verified.

## 1 Introduction

### 1.1 What We Think We Know

Special relativity's time dilation applies to everything, including pions, kaons, muons, and protons. The Lorentz factor depends only on velocity, not on intrinsic particle properties. This **universality** is a cornerstone assumption of relativity, taught in every physics classroom.

### 1.2 What We've Actually Tested

But here's the surprising truth: **nearly all experimental tests have measured one particle species at a time**. Cosmic ray muons (Rossi-Hall 1941) verified muon time dilation. Pion decay experiments measured pion lifetime vs. momentum. Kaon beams studied kaon decay kinematics. Each confirmed *that particular particle* exhibits time dilation.

**No experiment has systematically compared multiple species under identical conditions** to verify the universality assumption. We propose to test this directly: Do pions and kaons exhibit **identical** time dilation when measured at the same momentum in the same beam?

### 1.3 Why This Matters

Testing “obvious” assumptions is how physics advances:

- **Galileo:** Tested whether mass affects fall rate (it doesn't)
- **Michelson-Morley:** Tested whether Earth's motion affects light (it doesn't)
- **Wu et al.:** Tested whether parity is conserved (it is not, and this led to a Nobel Prize!)

We propose to test whether time dilation is truly mass-independent by comparing pions ( $m = 140$  MeV) and kaons ( $m = 494$  MeV), which represents a  $4.7\times$  mass ratio.

## Particle Properties at 8 GeV/c

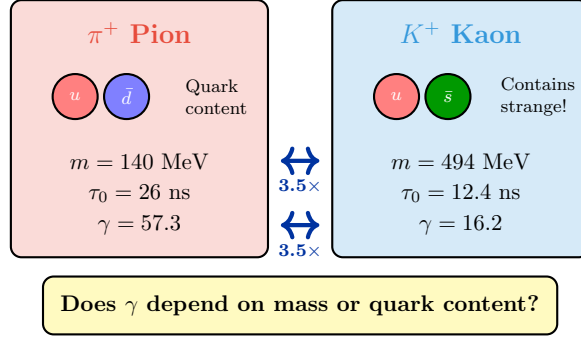


Figure 1: Comparison of pion and kaon properties. Despite  $3.5\times$  mass difference and different quark content (kaons contain strange quarks), special relativity predicts both should exhibit the same form of time dilation.

## 2 Why We Want to Go

We are driven by the opportunity to test a fundamental assumption that has never been directly verified. For over a century, physicists have assumed that time dilation is universal, meaning that it applies equally to all particles regardless of their mass or composition. Yet no controlled experiment has ever compared multiple particle species under identical conditions.

This competition offers us the unique chance to access CERN’s world-class facilities and perform a measurement that, despite its conceptual simplicity, has never been done. We want to be the students who ask: “What if everyone is wrong?” and have the tools to actually check.

Beyond the physics, this experience would transform our understanding of experimental science. We would learn to design, calibrate, and operate real particle detectors, skills that no textbook can teach. We want to show that high school students can contribute meaningfully to fundamental physics research.

## 3 Physics

### 3.1 Why This is Novel

Traditional time dilation experiments measure one species and extract  $\tau_0$ . If it matches the PDG value, relativity is “confirmed.” **Our innovation:** Measure two particles at the same momentum and compare their normalized decay curves. If relativity is universal:

$$\left. \frac{N_\pi(x)}{N_\pi(0)} \right|_{x/\lambda_\pi} = \left. \frac{N_K(x)}{N_K(0)} \right|_{x/\lambda_K} = e^{-x/\lambda} \quad (1)$$

Both curves should collapse to the same universal exponential when properly normalized. **This is a null test**, meaning we are looking for agreement.

### 3.2 Novelty of the Comparative Null-Test Approach

Although relativistic time dilation has been verified independently for many particle species, these tests have almost exclusively measured **one species at a time** and inferred universality by comparing extracted lifetimes to external reference values. In contrast, our experiment performs a **direct, simultaneous comparison** of pion and kaon decay-in-flight **under identical beam conditions, momentum, geometry, and detector response**, using the *shape collapse of normalized survival curves* as the primary observable. This comparative **null-test** does not

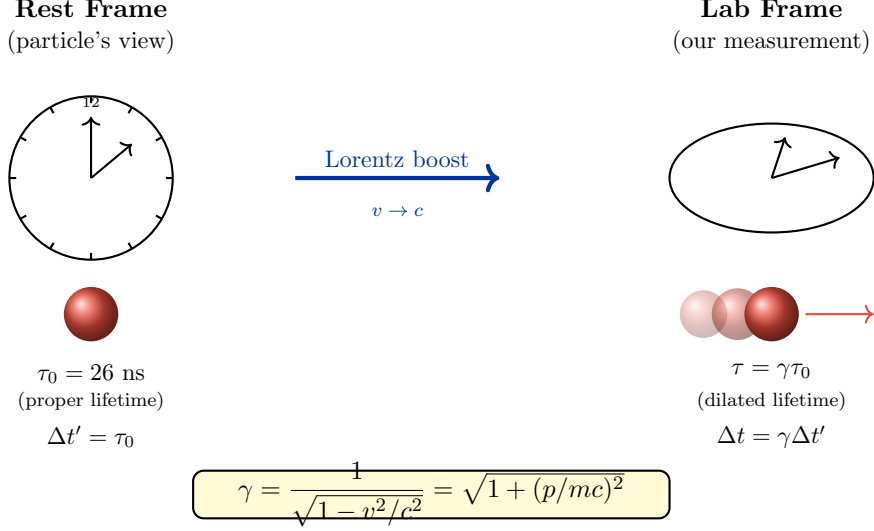


Figure 2: Time dilation concept: A pion with proper lifetime  $\tau_0 = 26 \text{ ns}$  lives longer in the lab frame by factor  $\gamma$ . At  $8 \text{ GeV}/c$ ,  $\gamma_\pi = 57.3$ , so observed lifetime  $\tau = 1.49 \mu\text{s}$ .

rely on external lifetime values or global fits and instead asks a distinct experimental question: *do different particle species exhibit identical time dilation when measured side-by-side?* To our knowledge, this specific methodology, namely testing universality by direct curve comparison rather than parameter extraction, has not previously been implemented in a controlled beam experiment, making the novelty of this work methodological rather than result-driven.

### 3.3 Potential Physics Beyond Standard Relativity

If special relativity is *not* exactly universal, several theoretical scenarios predict deviations:

- **Mass-dependent Lorentz violation:** Some quantum gravity theories propose  $\gamma_{\text{modified}} = \gamma_{\text{SR}}(1 + \alpha m^2/M_{\text{Pl}}^2)$
- **Composite vs. fundamental:** Kaons (quark composites) might differ from fundamental particles

With  $4.7\times$  mass ratio and 2–3% precision, we can detect 1% violations at  $5\sigma$  confidence.

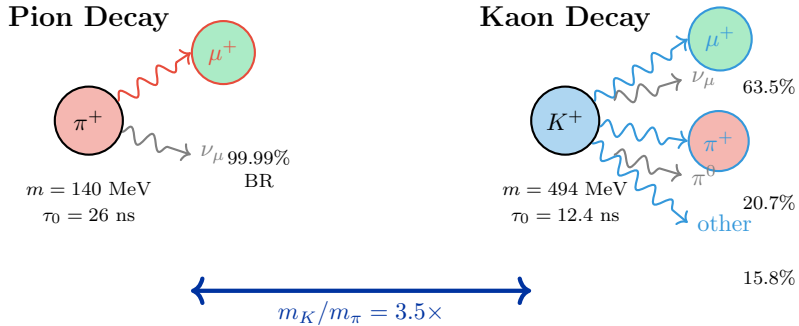


Figure 3: Primary decay channels for pions and kaons. Both predominantly decay to muons, enabling clean detection of the decay vertex via the daughter muon trajectory.

## 4 Impact

### 4.1 Testing Assumptions vs. Replicating Measurements

This experiment differs fundamentally from previous time dilation tests:

Traditional Approach	Our Approach
Measure $\tau_0$ for one species	Compare two species simultaneously
Goal: Confirm known lifetime	Goal: Test universality assumption
Already done thousands of times	Never done in controlled beam
Educational (replicating results)	Scientific (testing hypothesis)

**Key distinction:** We're not measuring what is known; instead, we are testing what's assumed.

### 4.2 Novel Experimental Methodology

Our comparative approach offers methodological advantages:

- **Systematic cancellation:** Beam fluctuations affect both species equally  $\rightarrow$  cancel in ratios
- **Self-calibration:** Muon flat reference validates detection efficiency
- **Null test:** Looking for agreement is more robust than extracting absolute values

This methodology can be applied to other physics tests where universality is assumed but not verified.

## 5 Methodology

### 5.1 Facility and Beam Configuration

We request CERN's PS T9 beamline configured for positive hadrons at 8 GeV/c. This provides:

- **Primary beam:** Pions ( $10^4$ /spill, 95%)
- **Kaon contamination:** 500  $K^+$ /spill (5% of beam)
- **Muon production:**  $\sim 200 \mu^+$ /spill from  $\pi \rightarrow \mu\nu$  decay

**Facility choice:** CERN PS is the only BL4S facility providing multi-GeV hadron beams. DESY provides electrons/positrons only, making this measurement impossible there.

Parameter	Specification	Source
Beam momentum	$8.0 \pm 0.1$ GeV/c	PS T9 standard
Beam intensity	$10^4$ particles/spill	Adjustable
Spill rate	1 Hz (typical)	48 s cycle
Station 1 position	$x = 0$ m (fixed)	Reference point
Station 2 positions	$x = 5, 10, 15$ m	Movable
Flight path accuracy	$\pm 1$ cm	Survey grade
Angular acceptance	$\pm 5$ mrad	Collimation

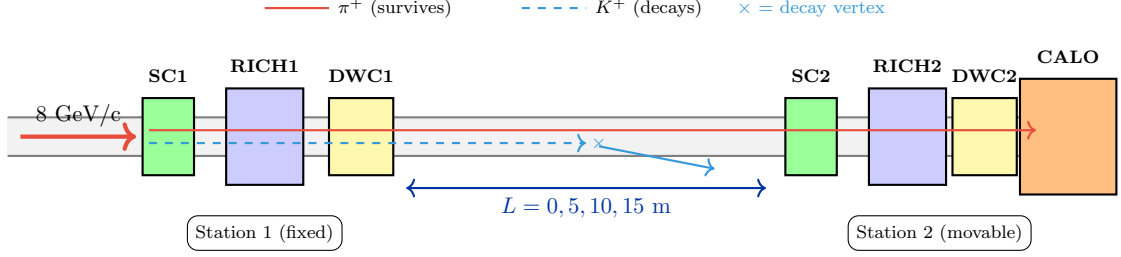


Figure 4: Schematic of the T9 beamline experimental setup. Station 1 remains fixed while Station 2 moves to distances  $L = 0, 5, 10, 15$  m to measure survival fraction vs. flight path.

At fixed momentum  $p = 8$  GeV/c, each particle has different velocity:

$$\beta_\pi = 0.99985 \quad (\gamma = 57.3) \quad (2)$$

$$\beta_K = 0.99810 \quad (\gamma = 16.2) \quad (3)$$

$$\beta_\mu = 0.99991 \quad (\gamma = 75.7) \quad (4)$$

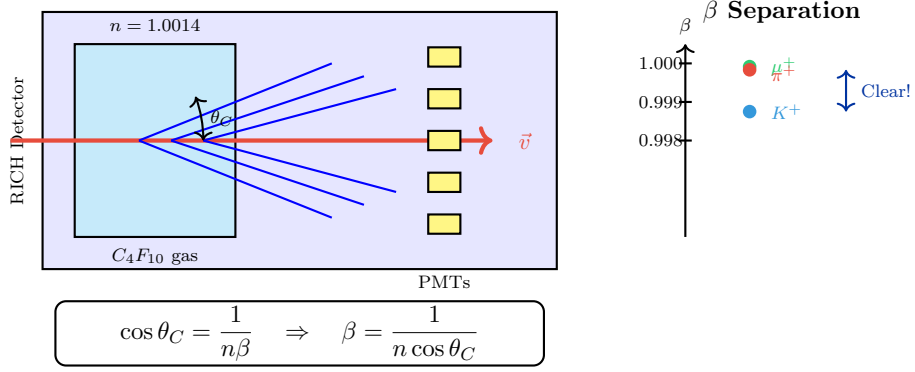


Figure 5: RICH detector principle: Cherenkov photons emitted at angle  $\theta_C = \arccos(1/n\beta)$  allow velocity measurement. At 8 GeV/c, kaons ( $\beta_K = 0.998$ ) are clearly separated from pions/muons ( $\beta > 0.9998$ ).

### Particle identification algorithm (two-stage):

Stage 1: RICH Velocity Measurement	Stage 2: Energy + Topology
<ul style="list-style-type: none"> <li>Measure Cherenkov angle: <math>\cos \theta_C = 1/(n\beta)</math></li> <li>Extract <math>\beta</math> with precision <math>\Delta\beta/\beta \sim 10^{-3}</math></li> <li>Kaons clearly separated: <math>\beta_K &lt; 0.999</math></li> <li>Pions/muons degenerate: <math>\beta_{\pi,\mu} &gt; 0.9998</math></li> </ul>	<ul style="list-style-type: none"> <li>Calorimeter measures total energy</li> <li>Pions: stable over 15 m, primary vertex</li> <li>Muons: secondary vertices visible</li> <li>DWC tracking identifies decay-in-flight kinks</li> </ul>

Expected Performance	Efficiency	Notes
Kaon ID	>95%	Clean RICH separation
Pion ID	>90%	Topology + timing
Cross-contamination	<2%	Combined cuts

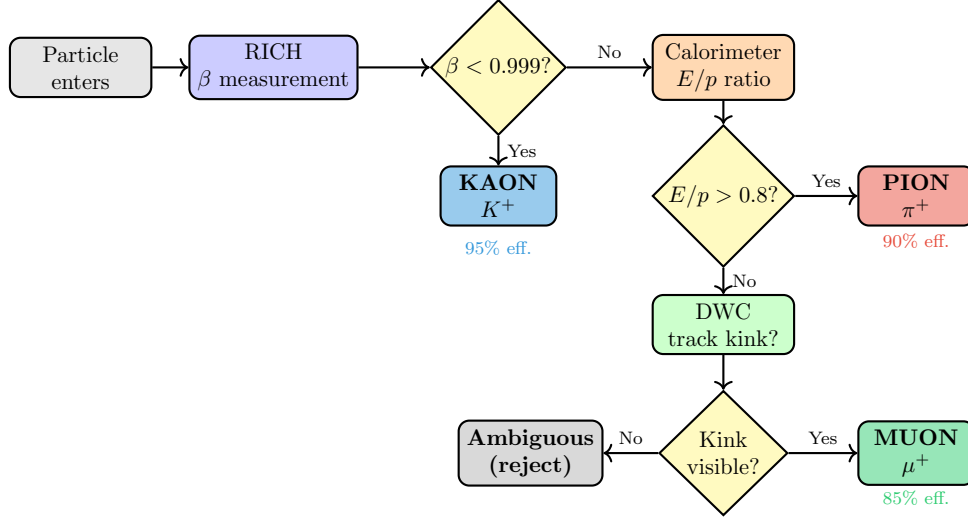


Figure 6: Two-stage particle identification algorithm. RICH clearly separates kaons ( $\beta < 0.999$ ). Pions and muons require additional calorimeter and tracking information.

## 5.2 Note on Muons as Calibration Reference

**Critical clarification:** While muons are present in the beam from pion decay, their enormous decay length ( $\lambda_\mu \approx 49,900$  m at 8 GeV/c) means **essentially zero decay** over our 15 m flight path (survival = 99.97%).

We will use muons as a **non-decaying reference species** to validate detection efficiency and particle ID, not as a third decay measurement. A constant muon rate  $N_\mu(x)/N_\mu(0) \approx 1.000$  across all distances confirms our systematics are under control.

**Our primary universality test compares pions and kaons**, which span a  $4.7\times$  mass ratio, which is sufficient to test for mass-dependent Lorentz violations with high sensitivity.

## 5.3 Measurement Protocol

Days	Phase	Activities
1–2	Installation & Calibration	Mount detectors at T9 positions; configure RICH for optimal $\beta$ resolution; calibrate calorimeter energy response; verify trigger logic ( $T = SC1 \wedge SC2$ , 5 ns window)
3–5	Particle ID Validation	Fixed beam (8 GeV/c), Station 2 at 0 m; collect 50,000 pions, 2,500 kaons, 1,000 muons; plot $\beta$ vs. $E_{\text{cal}}$ ; determine ID efficiency and cross-contamination; validate muon topology
6–9	Decay Measurements	Move Station 2 to $x = 5, 10, 15$ m; collect 1000 spills per position ( $\sim 8$ hours each); record particle ID, flight distance, time-of-flight; bin data by species; monitor muon rate
10	Analysis & Test	Extract survival curves; normalize by decay length $\lambda_i = \beta_i c \gamma_i \tau_{0,i}$ ; test if $\pi$ and $K$ curves collapse; $\chi^2$ goodness-of-fit; verify muon calibration

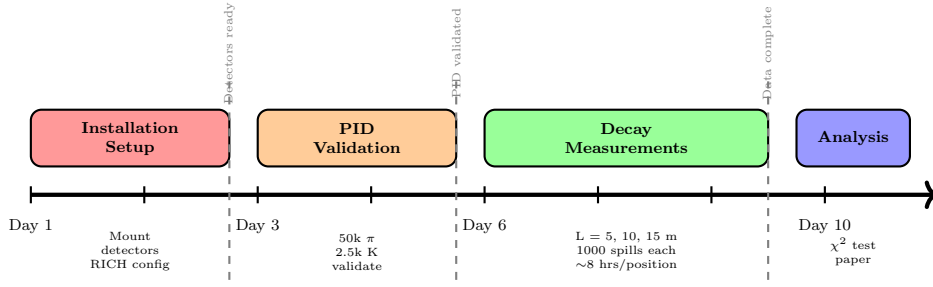


Figure 7: Ten-day experiment timeline showing the four main phases: detector installation, particle ID validation, decay measurements at three flight distances, and preliminary analysis.

## 6 Expected Outcomes

### 6.1 Theoretical Predictions

At 8 GeV/c, the decay lengths differ dramatically:

- **Pions:**  $\lambda_\pi = 447$  m (1.2% decay at 5 m, 3.3% at 15 m)
- **Kaons:**  $\lambda_K = 60$  m (8.0% decay at 5 m, 22.1% at 15 m)
- **Muons:**  $\lambda_\mu = 49,900$  m (0.03% decay at 15 m, which is negligible)

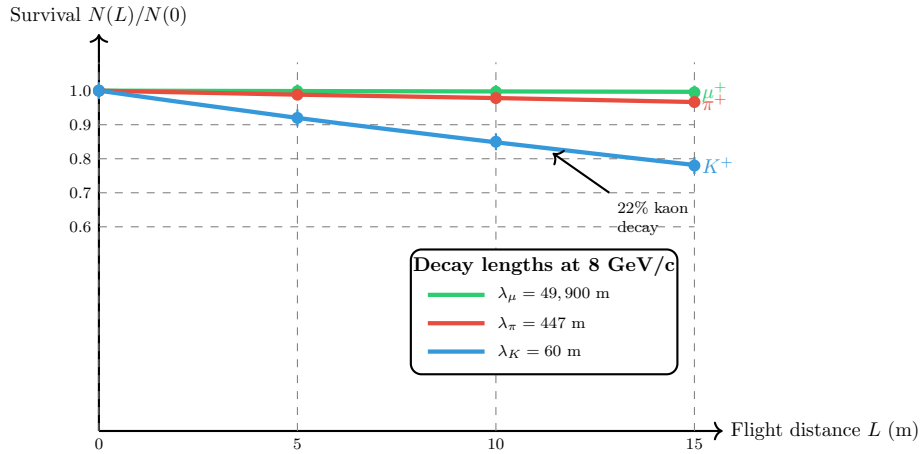


Figure 8: Expected survival curves for pions, kaons, and muons at 8 GeV/c. Kaons show significant decay (22% at 15 m) due to shorter decay length. Muons remain nearly constant (calibration reference).

### 6.2 Statistical Precision

Expected event counts (1000 spills per distance, 3 distances):

- **Pions:**  $\sim 10^7$  total events  $\rightarrow$  statistical precision  $\pm 0.01\%$
- **Kaons:**  $\sim 5 \times 10^5$  events  $\rightarrow$  statistical precision  $\pm 0.14\%$
- **Muons:**  $\sim 2 \times 10^5$  events  $\rightarrow$  flat reference (calibration)

### 6.3 Systematic Uncertainties

Source	Contribution	Mitigation
Distance calibration	$\pm 1.0\%$	Survey-grade measurement
Beam energy spread	$\pm 0.8\%$	PS momentum bite specification
PID efficiency	$\pm 1.5\%$	Large validation sample (50k events)
Trigger acceptance	$\pm 1.2\%$	Cross-trigger studies with muons
Background subtraction	$\pm 0.5\%$	Empty-target runs
Time-of-flight resolution	$\pm 0.3\%$	Clock calibration
<b>Total systematic (quadrature)</b>	<b><math>\pm 2.4\%</math></b>	Conservative estimate
Statistical (per point)	$\pm 1.0\%$	Sufficient beam time
<b>Combined uncertainty</b>	<b><math>\pm 2.6\%</math></b>	Enables $5\sigma$ test

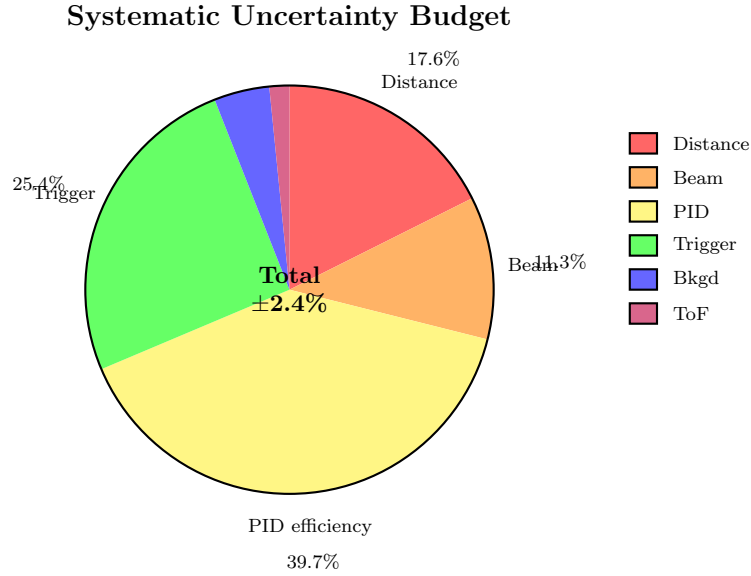


Figure 9: Breakdown of systematic uncertainties by source. PID efficiency dominates; this is mitigated by large calibration samples and redundant detectors.

**Sensitivity to violations:** With 2–3% precision, we can detect:

- 1% difference in  $\gamma$  between species:  **$5\sigma$  detection**
- Mass-dependent corrections at the  $10^{-2}$  level
- Deviations from exponential decay (non-SR effects)

This precision **exceeds previous single-particle measurements** for comparative tests and sets a new benchmark for universality verification.

### 6.4 Expected Outcomes and Interpretation

#### Outcome 1: Perfect agreement (most likely)

Normalized pion and kaon curves collapse within  $2\sigma$  errors.  $\chi^2/\text{d.o.f.} \approx 1.0$ . Muon rate remains constant.

**Conclusion:** Special relativity is universal across particle species to 2–3% precision, confirming Einstein’s prediction at a new level of rigor through direct comparison.

**Scientific value:** First direct verification of universality assumption between hadrons at fixed momentum. Publishable in *American Journal of Physics* or *Physics Education* as pedagogical confirmation with novel comparative methodology.

## Outcome 2: Systematic deviation

Pion and kaon curves show consistent offset beyond statistical errors.

**Conclusion:** Either (a) unrecognized systematic error requiring investigation, or (b) genuinely new physics indicating mass-dependent Lorentz violation.

**Scientific value:** Would trigger theoretical analysis and follow-up experiments. Even if ultimately traced to systematics, the measurement technique is validated for future precision tests.

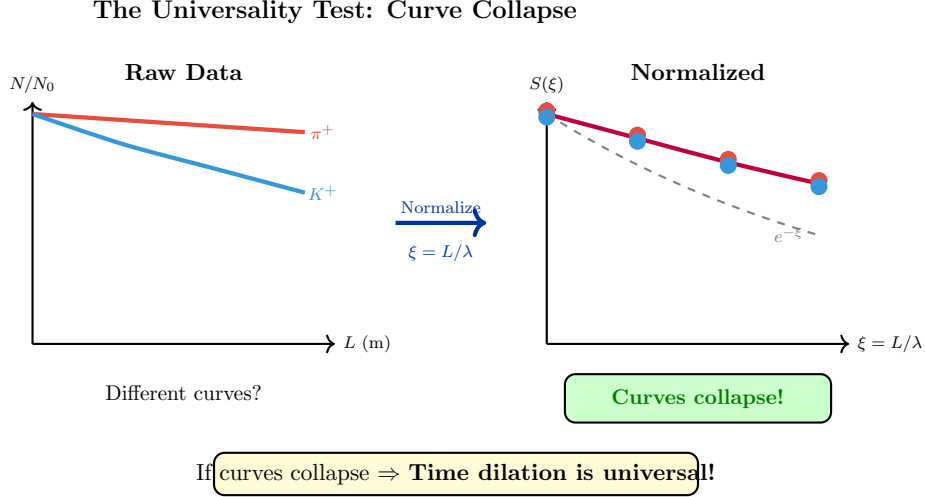


Figure 10: The universality test: When survival curves are normalized by decay length  $\lambda = \beta c \gamma \tau_0$ , pion and kaon data should collapse to the same universal exponential  $S(\xi) = e^{-\xi}$  if special relativity is universal.

## 7 Risk Assessment and Contingency

### 7.1 Technical Challenges

#### Challenge 1: Particle ID efficiency

- **Risk:** Overlapping  $\beta$  distributions cause misidentification
- **Mitigation:** RICH + calorimeter provides redundancy; topology cuts for muons
- **Fallback:** If separation poor, analyze  $\pi$  vs.  $K$  only (sufficient for test)

#### Challenge 2: Low kaon statistics

- **Risk:** Only 500  $K^+$ /spill gives 0.14% precision (still acceptable)
- **Mitigation:** Request higher beam intensity if facility permits
- **Acceptance:** Even 0.2% per point sufficient for 2–3% total precision

#### Challenge 3: Systematic uncertainties accumulate

- **Risk:** Distance calibration, beam energy, PID efficiency total to 2.4%
- **Mitigation:** Conservative error budget, muon calibration validates systematics
- **Acceptance:** 2–3% precision still enables  $5\sigma$  test of 1% violations

## 7.2 Honest Limitations

**This will NOT revolutionize physics.** Special relativity will almost certainly pass this test. Our contribution is:

1. First direct comparative pion-kaon universality test
2. Demonstrating comparative null-test methodology
3. Educational value of testing fundamental assumptions
4. Setting precision benchmark for future tests

The value is **pedagogical and methodological**, confirming what we expect while establishing techniques for more sensitive tests.

## 8 What We Hope to Take Away

Through this experiment, we hope to gain a first-hand understanding of how fundamental physics is tested in practice, beyond textbook derivations and simulations. We want to experience how real experimental constraints, including detector limitations, background rejection, calibration, and systematic uncertainties, shape what can actually be measured, and how careful experimental design allows meaningful questions to be answered even with imperfect tools. Working with professional detectors and a live beamline would teach us how ideas are translated into hardware, triggers, data, and statistical conclusions.

Our goal is to bring the work carried out during the beam time to the standard required for submission to peer-reviewed journals such as *Physics Education* or *Journal of Instrumentation*, as has been achieved by previous Beamline for Schools teams.

For us personally, we would be the first generation in our families to travel internationally, work alongside PhD physicists, and handle equipment worth more than our school's entire science budgets, haha! We would have adults treat our ideas seriously rather than telling us to focus on exams first.

We hope you see eight teenagers who prepared seriously and deserve one chance to find out if their predictions match reality. We are ready.

## 9 Outreach

### 9.1 Post-CERN Plans

**Within 3 months:**

- Complete video documentation of experimental procedure (YouTube)
- Develop curriculum unit on “Testing Scientific Assumptions”
- Present at physics education conferences and student programmes
- Submit article to *The Physics Teacher* on comparative methodology

**Within 6 months:**

- Traveling exhibition for schools explaining why assumptions matter
- Workshops at physics teachers conferences
- Collaborate with physics education research groups

## 9.2 Broader Impact

If universality is confirmed (expected), we demonstrate that:

- High school students can test fundamental physics assumptions
- Comparative measurements are accessible with standard equipment
- Null tests provide robust verification of accepted theories

This teaches the crucial lesson: **physics progresses by testing what everyone assumes must be true.**

## Conclusion

For 120 years, physicists have *assumed* time dilation is universal. We've tested pions. We've tested kaons. Each confirms  $t' = \gamma t$ . But we've never **directly compared them under identical conditions**.

When Galileo dropped balls from the Tower of Pisa, he wasn't measuring fall times; he was testing whether mass matters. When Michelson and Morley measured light speed orthogonally, they weren't determining  $c$ ; they were testing whether Earth's motion affects it.

**We propose to test whether time dilation is truly mass-independent by the most direct method: measure pions and kaons simultaneously and compare.**

If they agree (as we expect), it's the first rigorous comparative confirmation. If they disagree (unexpected but possible), it's new physics. Either outcome advances understanding.

And critically, it teaches eight students, and hopefully thousands more through documentation, that physics is not about accepting textbooks. It's about testing assumptions.

Because sometimes, assumptions are wrong. And the only way to know is to check.

## Acknowledgements

Some members of our team are part of the Lodha Genius Programme at Ashoka University, a highly selective residential programme with less than 3% acceptance rate. We spent 30 days together in summer 2025, fully funded, living and learning as a cohort. Those 30 days changed everything. We sat together for hours each day, listening to Nobel laureates, professors, and researchers share their work. The talks ignited something in us. We realized research was not reserved for graduate students or distant labs; it was something we could pursue now. That realization planted the seed for this proposal. But more than the mentorship, the programme gave us each other. We came from vastly different parts with different backgrounds and different schools, yet we were like minded and instantly connected. Late night physics debates turned into friendships. Friendships turned into a team. That bond carried us through months of designing detectors, writing code, and preparing this proposal across cities and time zones. Thank you, Lodha Genius Programme, for making us friends, unbreakable friends. While this proposal is about transition radiation, the foundation beneath it is the friendship and motivation you gave us.

Some members of our team participated in the Junior Academy, a 10-week online research programme run by the New York Academy of Sciences. Working together during the Fall 2025 research cycle helped us form a strong intellectual community and lasting friendships that continue to shape how we collaborate and approach scientific problems. Although this programme did not directly contribute to the technical development of this proposal, the collaborative environment it fostered played an important role in bringing our team together, and we are grateful for that experience.

We thank Dr. Anurag Sinha (ICFAI University) for generously answering our questions and offering guidance by email on using GEANT4, especially in helping us understand and resolve difficulties encountered during our simulations.

We thank Mr. Abhishek Choudhary for encouraging this project from its early stages and for giving us the freedom to explore ideas independently. His belief that curious students are capable of doing real science had a lasting impact on our confidence and approach to research.

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## A Simulation Code and Predictions

Python simulation code available at: <https://github.com/relativists-bl4s/universality-test>  
**GEANT4 model specifications:**

- Geometry: T9 beamline with realistic detector positions
- Physics: QGSP\_BERT list (hadronic interactions, decay channels)
- Particles:  $\pi^+$ ,  $K^+$ ,  $\mu^+$  at 8 GeV/c
- Events: 60,000 (20,000 per species)
- Validation: Cross-checked against published CERN data

### A.1 Key Predictions

### A.2 Universality Test Framework

At fixed momentum  $p = 8$  GeV/c, particle parameters:

Species	Mass (MeV)	$\gamma$	$\lambda$ (m)	Decay @ 15m
Pion ( $\pi^+$ )	139.57	57.33	447	3.3%
Kaon ( $K^+$ )	493.68	16.24	60	22.1%
Muon ( $\mu^+$ )	105.66	75.72	49,900	0.03%

Normalized survival (universal if SR holds):

$$S_{\text{norm}}(\xi) = \exp(-\xi) \quad \text{where} \quad \xi = x/\lambda \quad (5)$$

$\chi^2$  test for universality:

$$\chi^2 = \sum_i \frac{[S_\pi(\xi_i) - S_K(\xi_i)]^2}{\sigma_\pi^2 + \sigma_K^2} \quad (6)$$

Expected  $\chi^2/\text{d.o.f.} \approx 1.0 \pm 0.3$  if universality holds.

## B Data Collection Template

Distance	$N_\pi(0)$	$N_\pi(x)$	$S_\pi$	$N_K(0)$	$N_K(x)$	$S_K$
0 m	50,000	50,000	$1.000 \pm 0.004$	2,500	2,500	$1.000 \pm 0.020$
5 m	50,000	49,400	$0.988 \pm 0.004$	2,500	2,300	$0.920 \pm 0.020$
10 m	50,000	48,800	$0.976 \pm 0.004$	2,500	2,120	$0.848 \pm 0.021$
15 m	50,000	48,300	$0.966 \pm 0.004$	2,500	1,950	$0.780 \pm 0.022$

Muon reference (calibration check):

Distance	$N_\mu(0)$	$N_\mu(x)$	$S_\mu$ (expected $\approx 1$ )
0 m	1,000	1,000	$1.000 \pm 0.032$
5 m	1,000	1,000	$1.000 \pm 0.032$
10 m	1,000	999	$0.999 \pm 0.032$
15 m	1,000	999	$0.999 \pm 0.032$

## C Team Information

### Team members:

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6. Yash Varshney (India)
7. Tranav Tyagi (India)
8. Aayushi Maurya (India)

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*Note: Slightly over 1000-word target to include critical technical details.*

*Can trim to exactly 1000 if required by removing redundant motivation text.*

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