MA 374 LAB 01

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Ques1: The initial option value for European call and put option is as follows:

Number of steps:	CALL Option Value	PUT Option Value
1	2.405371888328072	1.8598554962991374
5	2.169396202324439	1.543635235302061
10	2.1177567446693577	1.4813095477175118
20	2.1260189044358127	1.484167308526397
50	2.1257832976133915	1.4806690053130285
100	2.1229497842829432	1.4767445457201847
200	2.119850759749658	1.473099410559442

Binomial Pricing Algorithm:

1. At time $t = t_i$ ($= i.\delta t$), there are $i + 1$ possible asset prices, i.e,

$S_n^i = d^{i-n} u^n S_0$, $0 <= n <= i$

2. Since continuous compounding convention is used, gross return is $R = e^{r.\delta t}$.

3. The probability (p) of an upward return in price is $\frac{R-d}{u-d}$.

4. At expiry, i.e, t = T, we calculate the price of the option using the respective payoff function for both the call and put option, i.e,

C_n^M = max(S_n^M - K, 0) , $0 <= n <= M$

P_n^M = max(K - S_n^M, 0) , $0 <= n <= M$

where, C_n^M is the nth possible price of the call option for the Mth interval, and

P_n^M is the nth possible price of the put option for the Mth interval

5. Now, we continuously apply **Backward Induction** to find out the option price at t = 0 by using following relation:

$C_n^i = (1 – p) . C_{n+1}^{i+1} + p . C_n^{i+1}$, $0 <= n <= i$ & $0 <= i <= M - 1$

$P_n^i = (1 – p) . C_{n+1}^{i+1} + p . P_n^{i+1}$, $0 <= n <= i$ & $0 <= i <= M - 1$

C_0^0 and P_0^0 are the required values, i.e, initial option prices

<u>No-arbitrage Condition:</u>

In order for no arbitrage opportunity to exist, following relation must exist:

$d < R < u$

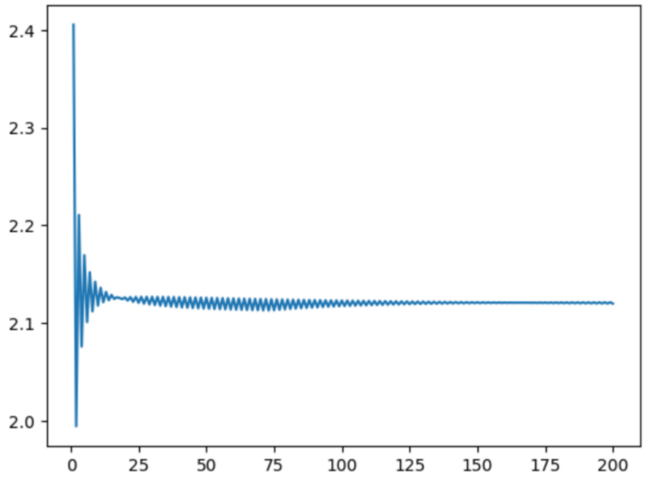
where,

$R = e^{r.\delta t}$

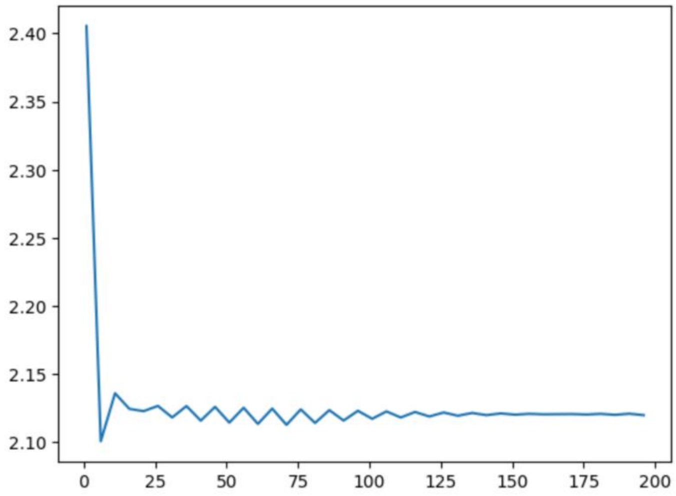
$d = e^{-\sigma\sqrt{\delta t} + \left(r - \frac{1}{2}\sigma^2\right)\delta t}$ $u = e^{\sigma\sqrt{\delta t} + \left(r - \frac{1}{2}\sigma^2\right)\delta t}$, where

$$\delta t = \frac{T}{M}$$

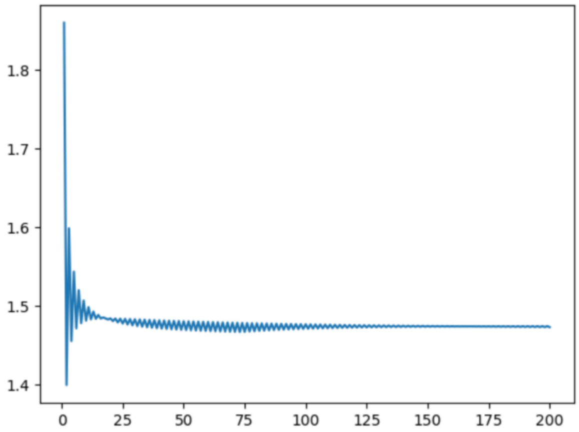
QUESTION 2:



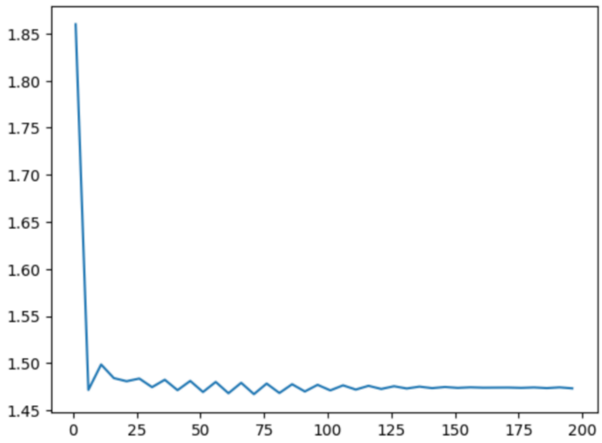
Call option value with 1 time step



Call option value with 5 time step



Put option value with 1 time step



Put option value with 5 time step

Observation:

1. From all the plots, we observe that the initial Call Option price converges to 2.12 (approx.) while the initial Put Option price converges to 1.47 (approx.).
2. The convergence of the plot is faster when step value is 5. The deviations from the convergence value is higher when the value of M is less.
3. The convergence is not perfect in the sense that the values tend to oscillate around the specified values (*in point 1*) even if the sub-intervals number is increased beyond 200.
4. But this oscillation is normal for such numerical algorithms and gives a correct approximation of the required value since fluctuations tend to happen at $3^{rd}/4^{th}$ decimal place onwards.

We can say that the convergence of the values is fast enough as not too many iterations are required to approximately attain the converged value

QUESTION 3:

VALUES IN IPYNB FILE.