

# Process Optimization Project Report

Module 3.1: Quantifying Environmental Load for Sustainable "Rendezvous"

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## Declaration of Tool Usage

I declare that in completing this assignment:

- I used an LLM-based tool (Gemini) for assistance in:
    - Brainstorming model formulations and exploring nonlinear crowding effects.
    - Formatting the mathematical expressions and generating LaTeX code.
    - Verifying the symbolic differentiation of the proposed objective function.
  - I understand the submitted solution fully.
  - I can explain and justify every part of my code and reasoning.
  - I have verified all results independently.
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## 1 Introduction

The "Rendezvous" cultural festival at IIT Delhi presents a significant logistical and environmental challenge. As part of the Sustainability Task Force, our objective in Module 3.1 is to develop a robust mathematical model to quantify the total environmental load ( $E$ ) of the festival. This model balances exploration of complex system dynamics (crowding, economies of scale) with the exploitation of rigorous optimization techniques to identify trade-offs and guide sustainable decision-making.

## 2 Nomenclature

The variables and parameters used in the mathematical model are defined in Table 1.

Table 1: Nomenclature Table

Symbol	Description	Units	Type
$N$	Number of attendees	persons	Decision Var
$S$	Number of food stalls/vendors	count	Decision Var
$A$	Total event activities	activity-hours	Decision Var
$E$	Total environmental load	kg CO <sub>2</sub> -eq	Objective Fn
$\alpha_1$	Base per-capita impact coefficient	kg CO <sub>2</sub> -eq/person	Parameter
$\alpha_2$	Base per-stall impact coefficient	kg CO <sub>2</sub> -eq/stall	Parameter
$\alpha_3$	Base per-activity impact coefficient	kg CO <sub>2</sub> -eq/activity-hr	Parameter
$\gamma_{NS}$	Congestion penalty coefficient	kg CO <sub>2</sub> -eq · stall/person <sup>2</sup>	Parameter
$\beta_N$	Crowding nonlinearity coefficient	kg CO <sub>2</sub> -eq/person <sup>1.3</sup>	Parameter
$\beta_S$	Stall scaling nonlinearity coefficient	kg CO <sub>2</sub> -eq/stall <sup>1.2</sup>	Parameter
$\beta_A$	Activity scaling nonlinearity coefficient	kg CO <sub>2</sub> -eq/activity-hr <sup>0.8</sup>	Parameter
$\epsilon$	Grid emission factor (India)	kg CO <sub>2</sub> /kWh	Parameter
$\tau$	Average event duration per attendee	hours	Parameter
$P_{light}$	Lighting power per attendee	kW/person	Sub-parameter
$P_{stall}$	Power per food stall	kW/stall	Sub-parameter
$P_{stage}$	Power per activity stage	kW/stage	Sub-parameter
$\phi_{waste}$	Base waste generation per person	kg/person/day	Sub-parameter
$k_{crowd}$	Crowding waste amplification factor	dimensionless	Sub-parameter
$\xi$	Sustainability Efficiency Metric	kg CO <sub>2</sub> -eq/(person·hr)	Derived Var

## 3 Assumptions and Justifications

To develop a tractable yet expressive model, the following assumptions are made based on Life Cycle Assessment (LCA) principles and chemical engineering process economics (Letterman, 1980):

1. **A1: Nonlinear Crowding Effects (Diseconomies of Scale).** *Justification:* Environmental impact does not scale linearly with attendee density. As crowd density increases, waste collection efficiency drops, littering increases (overflowing bins), and resource competition leads to higher per-capita waste generation. We model this with a superlinear term  $N^{\beta_1}$  where  $\beta_1 > 1$ .
2. **A2: Economie of Scale in Infrastructure.** *Justification:* High-fixed-cost infrastructure (generators, stages) becomes more efficient per unit of activity as scale increases.

However, the total load still grows. For activities  $A$ , we use a sublinear power term  $A^{0.8}$  for base energy load, reflecting standard equipment scaling laws (0.6-factor rule).

3. **A3: Congestion-Induced Waste Leakage.** *Justification:* When the ratio of attendees to service points (stalls  $S$ ) is high ( $N/S \gg \text{capacity}$ ), "leakage" occurs in the form of unsegregated waste and littering. This is modeled as a penalty term proportional to  $N^2/S$ .
4. **A4: Sub-component Decomposition.** *Justification:* The total load  $E$  is the sum of Energy ( $E_{\text{energy}}$ ), Waste ( $W_{\text{total}}$ ), and Emissions ( $E_{\text{emissions}}$ ), adhering to ISO 14040 LCA framework boundaries (Cradle-to-Grave for site operations).

## 4 Mathematical Model Formulation

### 4.1 Objective Function Construction

We propose a multi-variable objective function  $E(N, S, A)$  comprising three components: Linear Base Load ( $E_{\text{base}}$ ), Nonlinear Scale Effects ( $E_{\text{scale}}$ ), and Interaction/Congestion Penalties ( $E_{\text{inter}}$ ).

$$E(N, S, A) = E_{\text{base}} + E_{\text{scale}} + E_{\text{inter}} \quad (1)$$

Substituting the terms:

$$E(N, S, A) = (\alpha_1 N + \alpha_2 S + \alpha_3 A) + (\beta_N N^{1.3} + \beta_S S^{1.2} + \beta_A A^{0.8}) + \left( \gamma_{NS} \frac{N^2}{S} \right) \quad (2)$$

Where:

- $\alpha_1 N$ : Direct resource consumption per attendee.
- $N^{1.3}$ : The superlinear crowding term.
- $\gamma_{NS} \frac{N^2}{S}$ : The **Congestion Penalty**. As  $S \rightarrow 0$  for fixed  $N$ , environmental load explodes due to waste management failure. As  $S \rightarrow \infty$ , this term vanishes (perfect service), but the direct cost  $\alpha_2 S$  increases. This creates a trade-off.

### 4.2 Sub-Component Integration

The aggregate function  $E$  is derived from specific engineering estimates:

$$E_{\text{energy}} = \epsilon \cdot \tau \cdot (P_{\text{light}} N + P_{\text{stall}} S + P_{\text{stage}} A) \quad (3)$$

$$W_{\text{total}} = \phi_{\text{waste}} \cdot N \cdot \left( 1 + k_{\text{crowd}} \frac{N}{S} \right) \quad (4)$$

$$CO_2e = E_{\text{transp}} + E_{\text{gen}} + E_{\text{mat}} \quad (5)$$

Equation (4) explicitly shows how waste generation rate depends on the crowding ratio  $N/S$ .

### 4.3 Parameter Estimation for Rendezvous, IIT Delhi

To ground the model in a realistic scenario, we estimate parameter values for the "Rendezvous" cultural festival at IIT Delhi, held on the institute's 325-acre (131 ha) green campus. Rendezvous attracts approximately 160,000 attendees over 4 days ( $\sim 40,000$  per day), features  $\sim 100$  food/vendor stalls and 15–20 parallel activity stages [1, 2, 3, 9]. The estimated values are presented in Table 2.

Table 2: Estimated Parameter Values for Rendezvous, IIT Delhi

Parameter	Value	Justification	Source
<i>Base Impact Coefficients (Linear Terms)</i>			
$\alpha_1$	2.5 kg CO <sub>2</sub> -eq/person	35% of total 7 kg/person/day on-site	[9]
$\alpha_2$	18 kg CO <sub>2</sub> -eq/stall	Energy (60 kWh $\times$ 0.727) + materials	[2]
$\alpha_3$	12 kg CO <sub>2</sub> -eq/act-hr	25 kW stage, adj. for 50% renewables	[2]
<i>Nonlinear Scaling Coefficients</i>			
$\beta_N$	0.002	Urban superlinear scaling ( $\sim 1.15$ – $1.3$ )	[1]
$\beta_S$	0.5	Supply-chain logistics fragmentation	[7]
$\beta_A$	5.0	Six-tenths rule, exponent 0.8	[10, 4]
<i>Congestion Penalty</i>			
$\gamma_{NS}$	0.0005	$S^* \approx 210$ stalls at $N=40,000$	[9]
<i>Physical / Engineering Constants</i>			
$\epsilon$	0.727 kg CO <sub>2</sub> /kWh	Indian grid FY 2023–24	[2]
$\tau$	6 hours	Avg. student attendance at campus fest	[8]
<i>Sub-Component Parameters</i>			
$P_{light}$	0.05 kW/person	LED area lighting, 5 W/m <sup>2</sup>	[2]
$P_{stall}$	7 kW/stall	Midpoint of 5–12 kW food truck range	[2]
$P_{stage}$	25 kW/stage	Sound (15) + LED (5) + AV (5) kW	[2]
$\phi_{waste}$	0.65 kg/person/day	Delhi per-capita MSW	[3]
$k_{crowd}$	0.002	Waste $\times 1.75$ at $N/S \approx 400$	[9]

**Key Observation:** With these values, the congestion term  $\gamma_{NS}N^2/S = 0.0005 \times 40000^2/100 = 8,000$  kg CO<sub>2</sub>-eq dominates the environmental load at full capacity, underscoring the critical importance of sufficient stall infrastructure. The optimal stall count  $S^* \approx N\sqrt{\gamma_{NS}/\alpha_2} = 40000\sqrt{0.0005/18} \approx 210$  stalls.

## 5 Optimization Analysis

### 5.1 Unconstrained Optimization of $E$

We seek to minimize  $E(N, S, A)$  over the domain  $N, S, A \geq 0$ .

**Gradient Check:**

$$\frac{\partial E}{\partial N} = \alpha_1 + 1.3\beta_N N^{0.3} + \frac{2\gamma_{NS}N}{S} > 0 \quad (\text{always positive for } N, S > 0) \quad (6)$$

$$\frac{\partial E}{\partial A} = \alpha_3 + 0.8\beta_A A^{-0.2} > 0 \quad (7)$$

Since the gradient components for productivity variables  $(N, A)$  are strictly positive, the unconstrained global minimum lies at the trivial solution:

$$(N^*, S^*, A^*) = (0, 0, 0) \quad (8)$$

**Insight:** Minimizing total environmental footprint to zero implies having no festival. This formulation, while chemically accurate, does not capture the *value* of the event.

### 5.2 Sub-Optimization: Optimal Stall Count

However, for a **fixed number of attendees**  $N$ , there exists an optimal infrastructure level  $S^*$ . Differentiating  $E$  with respect to  $S$ :

$$\frac{\partial E}{\partial S} = \alpha_2 + 1.2\beta_S S^{0.2} - \gamma_{NS} \frac{N^2}{S^2} \quad (9)$$

Setting  $\frac{\partial E}{\partial S} = 0$ :

$$\alpha_2 + 1.2\beta_S S^{0.2} = \gamma_{NS} \frac{N^2}{S^2} \quad (10)$$

The LHS represents the *Marginal Environmental Cost* of adding a stall (embodied carbon, power). The RHS represents the *Marginal Environmental Benefit* of reducing congestion (less litter). The unique intersection  $S^*(N)$  defines the optimal resource allocation for any crowd size. Approximating for small nonlinearities ( $\beta_S \approx 0$ ):

$$S^* \approx N \sqrt{\frac{\gamma_{NS}}{\alpha_2}} \quad (11)$$

This square-root law suggests that infrastructure should scale linearly with crowd size to maintain constant congestion levels ( $N/S$ ), but considering base costs, an optimal ratio emerges.

### 5.3 Sustainability Efficiency (Part e)

To address the trivial minimum problem, we refine the objective to minimize **Impact Intensity** or maximize **Sustainability Efficiency** ( $\xi$ ):

$$\text{minimize } \xi(N, S, A) = \frac{E(N, S, A)}{N \cdot \tau} \quad (12)$$

This function represents the environmental cost per "attendee-hour".

$$\xi = \frac{\alpha_1}{\tau} + \frac{\alpha_2 S}{N\tau} + \frac{\alpha_3 A}{N\tau} + \frac{\gamma_{NS} N}{S\tau} + \dots \quad (13)$$

Minimizing  $\xi$  with respect to  $N$  now involves trade-offs between fixed environmental costs (stalls, stages) which benefit from economies of scale (larger  $N$ ), and crowding costs ( $\gamma_{NS} N/S$ ) which penalize scale. This formulation yields a non-trivial measure of the "greenest" event scale.

## 6 Preliminary Insights and Discussion

- **Crowding is Environmentally Expensive:** The term  $\gamma_{NS} N^2/S$  is the dominant driver of inefficiency at large scales. Investing in sufficient infrastructure ( $S$ ) is not just operationally necessary but environmentally critical to prevent waste leakage.
- **Economies of Scale:** The sublinear energy term for activities ( $A^{0.8}$ ) suggests that consolidating entertainment into fewer, high-quality events is greener than many small, fragmented setups.
- **Critical Ratio:** The analysis reveals a critical Attendee-to-Stall ratio  $(N/S)_{crit} = \sqrt{\alpha_2/\gamma_{NS}}$ . Operating above this ratio causes environmental load to spike due to congestion.

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