Process Optimization Project Report
Module 3.1: Quantifying Environmental Load for Sustainable "Rendezvous"

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Declaration of Tool Usage

I declare that in completing this assignment:

- I used an LLM-based tool (Gemini) for assistance in:

    - Brainstorming model formulations and exploring nonlinear crowding effects.
    - Formatting the mathematical expressions and generating LaTeX code.
    - Verifying the symbolic differentiation of the proposed objective function.

- I understand the submitted solution fully.

- I can explain and justify every part of my code and reasoning.

- I have verified all results independently.

Contents

1   Introduction

The "Rendezvous" cultural festival at IIT Delhi presents a significant logistical and environmental challenge. As part of the Sustainability Task Force, our objective in Module 3.1 is to develop a robust mathematical model to quantify the total environmental load (E) of the festival. This model balances exploration of complex system dynamics (crowding, economies of scale) with the exploitation of rigorous optimization techniques to identify trade-offs and guide sustainable decision-making.

2   Nomenclature

The variables and parameters used in the mathematical model are defined in Table **??**.

Table 1: Nomenclature Table

Symbol	Description	Units	Type
N	Number of attendees	persons	Decision Var
S	Number of food stalls/vendors	count	Decision Var
A	Total event activities	activity-hours	Decision Var
E	Total environmental load	kg CO_2-eq	Objective Fn
α_1	Base per-capita impact coefficient	kg CO_2-eq/person	Parameter
α_2	Base per-stall impact coefficient	kg CO_2-eq/stall	Parameter
γ_{NS}	Congestion penalty coefficient	kg CO_2-eq $\cdot$ stall/person^2	Parameter
β_1	Nonlinear crowding exponent (attendees)	dimensionless	Parameter
β_2	Nonlinear scaling exponent (stalls)	dimensionless	Parameter
ϵ	Grid emission factor	kg CO_2/kWh	Parameter
τ	Average event duration per attendee	hours	Parameter
ξ	Sustainability Efficiency Metric	kg CO_2-eq/(person·hr)	Derived Var

3   Assumptions and Justifications

To develop a tractable yet expressive model, the following assumptions are made based on Life Cycle Assessment (LCA) principles and chemical engineering process economics (Letterman, 1980):

1. **A1: Nonlinear Crowding Effects (Diseconomies of Scale)**. *Justification*: Environmental impact does not scale linearly with attendee density. As crowd density increases, waste collection efficiency drops, littering increases (overflowing bins), and resource competition leads to higher per-capita waste generation. We model this with a superlinear term N^{β_1} where $\beta_1 > 1$.

2. **A2: Economie of Scale in Infrastructure**. *Justification*: High-fixed-cost infrastructure (generators, stages) becomes more efficient per unit of activity as scale increases. However, the total load still grows. For activities A, we use a sublinear power term $A^{0.8}$ for base energy load, reflecting standard equipment scaling laws (0.6-factor rule).

3. **A3: Congestion-Induced Waste Leakage**. *Justification*: When the ratio of attendees to service points (stalls S) is high ($N/S \gg$ capacity), "leakage" occurs in the form of unsegregated waste and littering. This is modeled as a penalty term proportional to N^2/S.

4. **A4: Sub-component Decomposition**. *Justification*: The total load E is the sum of Energy (E_{energy}), Waste (W_{total}), and Emissions ($E_{emissions}$), adhering to ISO 14040 LCA framework boundaries (Cradle-to-Grave for site operations).

4 Mathematical Model Formulation

4.1 Objective Function Construction

We propose a multi-variable objective function $E(N, S, A)$ comprising three components: Linear Base Load (E_{base}), Nonlinear Scale Effects (E_{scale}), and Interaction/Congestion Penalties (E_{inter}).

$$E(N, S, A) = E_{base} + E_{scale} + E_{inter} \tag{1}$$

Substituting the terms:

$$E(N, S, A) = (\alpha_1 N + \alpha_2 S + \alpha_3 A) + (\beta_N N^{1.3} + \beta_S S^{1.2} + \beta_A A^{0.8}) + \left(\gamma_{NS} \frac{N^2}{S} \right) \tag{2}$$

Where:

- $\alpha_1 N$: Direct resource consumption per attendee.

- $N^{1.3}$: The superlinear crowding term.

- $\gamma_{NS} \frac{N^2}{S}$: The **Congestion Penalty**. As $S \to 0$ for fixed N, environmental load explodes due to waste management failure. As $S \to \infty$, this term vanishes (perfect service), but the direct cost $\alpha_2 S$ increases. This creates a trade-off.

4.2 Sub-Component Integration

The aggregate function E is derived from specific engineering estimates:

$$E_{energy} = \epsilon \cdot \tau \cdot (P_{light} N + P_{stall} S + P_{stage} A) \tag{3}$$

$$W_{total} = \phi_{waste} \cdot N \cdot (1 + k_{crowd} \frac{N}{S}) \tag{4}$$

$$CO2e = E_{transp} + E_{gen} + E_{mat} \tag{5}$$

Equation (**??**) explicitly shows how waste generation rate depends on the crowding ratio N/S.

5 Optimization Analysis

5.1 Unconstrained Optimization of E

We seek to minimize $E(N, S, A)$ over the domain $N, S, A \geq 0$.
    Gradient Check:

$$\frac{\partial E}{\partial N} = \alpha_1 + 1.3 \beta_N N^{0.3} + \frac{2 \gamma_{NS} N}{S} > 0 \quad \text{(always positive for } N, S > 0) \tag{6}$$

$$\frac{\partial E}{\partial A} = \alpha_3 + 0.8 \beta_A A^{-0.2} > 0 \tag{7}$$

Since the gradient components for productivity variables (N, A) are strictly positive, the unconstrained global minimum lies at the trivial solution:

$$(N^*, S^*, A^*) = (0, 0, 0) \tag{8}$$

Insight: Minimizing total environmental footprint to zero implies having no festival. This formulation, while chemically accurate, does not capture the *value* of the event.

5.2 Sub-Optimization: Optimal Stall Count

However, for a **fixed number of attendees** N, there exists an optimal infrastructure level S^*. Differentiating E with respect to S:

$$\frac{\partial E}{\partial S} = \alpha_2 + 1.2\beta_S S^{0.2} - \gamma_{NS}\frac{N^2}{S^2} \tag{9}$$

Setting $\frac{\partial E}{\partial S} = 0$:

$$\alpha_2 + 1.2\beta_S S^{0.2} = \gamma_{NS}\frac{N^2}{S^2} \tag{10}$$

The LHS represents the *Marginal Environmental Cost* of adding a stall (embodied carbon, power). The RHS represents the *Marginal Environmental Benefit* of reducing congestion (less litter). The unique intersection $S^*(N)$ defines the optimal resource allocation for any crowd size. Approximating for small nonlinearities ($\beta_S \approx 0$):

$$S^* \approx N\sqrt{\frac{\gamma_{NS}}{\alpha_2}} \tag{11}$$

This square-root law suggests that infrastructure should scale linearly with crowd size to maintain constant congestion levels (N/S), but considering base costs, an optimal ratio emerges.

5.3 Sustainability Efficiency (Part e)

To address the trivial minimum problem, we refine the objective to minimize **Impact Intensity** or maximize **Sustainability Efficiency** (ξ):

$$\text{minimize } \xi(N, S, A) = \frac{E(N, S, A)}{N \cdot \tau} \tag{12}$$

This function represents the environmental cost per "attendee-hour".

$$\xi = \frac{\alpha_1}{\tau} + \frac{\alpha_2 S}{N\tau} + \frac{\alpha_3 A}{N\tau} + \frac{\gamma_{NS}N}{S\tau} + \dots \tag{13}$$

Minimizing ξ with respect to N now involves trade-offs between fixed environmental costs (stalls, stages) which benefit from economies of scale (larger N), and crowding costs ($\gamma_{NS}N/S$) which penalize scale. This formulation yields a non-trivial measure of the "greenest" event scale.

6 Preliminary Insights and Discussion

- **Crowding is Environmentally Expensive**: The term $\gamma_{NS}N^2/S$ is the dominant driver of inefficiency at large scales. Investing in sufficient infrastructure (S) is not just operationally necessary but environmentally critical to prevent waste leakage.

- **Economies of Scale**: The sublinear energy term for activities ($A^{0.8}$) suggests that consolidating entertainment into fewer, high-quality events is greener than many small, fragmented setups.

- **Critical Ratio**: The analysis reveals a critical Attendee-to-Stall ratio $(N/S)_{crit} = \sqrt{\alpha_2/\gamma_{NS}}$. Operating above this ratio causes environmental load to spike due to congestion.

7 References

1. Edgar, T.F., Himmelblau, D.M., & Lasdon, L.S. (2001). *Optimization of Chemical Processes*, 2nd ed. McGraw-Hill.

2. Letterman, R.D. (1980). Economic analysis of granular-bed filtration. *Journal of the Environmental Engineering Division*, 106, 279–291.

3. ISO 14040:2006. Environmental management — Life cycle assessment — Principles and framework.

4. Prakash, O. (2025). *Course Lecture Notes: Project Details*, CLL782/CHL7204, IIT Delhi.