Process Optimization Project Report

Module 3.3: Waste Collection and Processing Logistics

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Declaration of Tool Usage

I declare that in completing this assignment:

- I used an LLM-based tool (Gemini) for assistance in:
- Formulating the Vehicle Routing Problem (VRP) variants.
- Structuring the optimization model for logistics.
- Formatting the report in LaTeX/Markdown.
- I understand the submitted solution fully.
- I can explain and justify every part of my code and reasoning.
- I have verified all results independently.

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1. Introduction

Efficient waste management for the "Rendezvous" festival requires a robust logistics network to transport waste from generation zones to processing facilities (recycling units, compost pits, landfills). Module 3.3 addresses this challenge by modeling a waste collection and processing system. The objective is to minimize total operational costs, including transportation and processing, while respecting vehicle capacities, facility throughput limits, and waste supply constraints.

2. Nomenclature

The variables and parameters used in the mathematical model are defined in Table 1.

Table 1: Nomenclature Table

Symbol	Description	Units	Type
i	Index for waste generation zones, i \in \{1, \dots, m\}	-	Index
j	Index for processing facilities, j \in \{1, \dots, p\}	-	Index
k	Index for vehicles, k \in \{1, \dots, K\}	-	Index
W_i	Total waste generated in zone i	kg	Parameter
Cap_j	Throughput capacity of facility j	kg	Parameter
V_{cap}	Capacity of a single waste collection vehicle	kg	Parameter
C_{proc,j}	Unit processing cost at facility j	INR/kg	Parameter
D_{ij}	Distance from zone i to facility j	km	Parameter
C_{dist}	Transport cost per km	INR/km	Parameter
C_{handling}	Handling cost per kg	INR/kg	Parameter
C_{fixed}	Fixed cost per vehicle used	INR	Parameter
x_{i,j}	Waste quantity transported from zone i to facility j	kg	Decision Var
v_k	Binary: 1 if vehicle k is deployed, 0 otherwise	-	Decision Var
Z_{total}	Total Cost (Transport + Processing + Fixed)	INR	Objective Fn

3. Assumptions and Justifications

1. **A1: Direct Transport Model.**

   - **Justification:** For simplicity in this module, we model direct transport from zones to facilities (Star network) rather than a complex multi-stop vehicle routing problem (VRP). The "route" variable mentioned in the problem statement is approximated by aggregate flow *x_{i,j}*.

- *Note:* A full VRP formulation would require variables *x_{i,j,k}* indicating if vehicle *k* travels from node *i* to *j*, which scales exponentially. The direct flow approximation provides a lower bound on cost and is tractable.

2. **A2: Homogeneous Fleet.**

- **Justification:** All vehicles are assumed to have identical capacity *V_{cap}* and costs.

3. **A3: Linear Costs.**

- **Justification:** Transportation and processing costs scale linearly with distance and mass, respectively.

4. Mathematical Model Formulation

The problem is modeled as a **Minimum Cost Flow Problem** with fixed costs for vehicle deployment.

4.1 Objective Function Construction

We minimize the Total Cost (*Z_{total}*), comprising: 1. **Transportation Cost:** Function of distance and weight.

$$C_{trans} = \sum_{i=1}^{m}\sum_{j=1}^{p} x_{i,j} \cdot (C_{dist} \cdot D_{ij} + C_{handling}) \tag{1}$$

2. **Processing Cost:** At facilities.

$$C_{proc} = \sum_{i=1}^{m}\sum_{j=1}^{p} x_{i,j} \cdot C_{proc,j} \tag{2}$$

3. **Vehicle Fixed Cost:** Based on the number of vehicles required.

$$C_{veh} = N_{vehicles} \cdot C_{fixed} \tag{3}$$

Where *N_{vehicles}* is determined by the total waste volume divided by vehicle capacity (approximate).

Combined Objective:

$$Z = \sum_{i,j} x_{i,j} (C_{dist} D_{ij} + C_{handling} + C_{proc,j}) + C_{fixed} \cdot \lceil \frac{\sum_{i,j} x_{i,j}}{V_{cap}} \rceil \tag{4}$$

To linearize the vehicle count for optimization:

$$Z = \sum_{i,j} C_{ij}^{total} \cdot x_{i,j} + C_{fixed} \cdot \sum_{k} v_k \tag{5}$$

4.2 Constraints Integration

1. Waste Clearance (Supply Constraint): All waste from zone *i* must be removed.

$$\sum_{j=1}^{p} x_{i,j} = W_i, \quad \forall i \tag{6}$$

2. Facility Capacity: Total waste sent to facility *j* cannot exceed its throughput.

$$\sum_{i=1}^{m} x_{i,j} \leq Cap_j, \quad \forall j \tag{7}$$

3. Vehicle Capacity (Fleet Sizing): The total waste transported must be covered by the deployed fleet.

$$\sum_{i=1}^{m} \sum_{j=1}^{p} x_{i,j} \leq \sum_{k=1}^{K} v_k \cdot V_{cap} \tag{8}$$

Or simply, defining *N* as integer variable for number of vehicles:

$$\sum_{i,j} x_{i,j} \leq N \cdot V_{cap} \tag{9}$$

4. Non-negativity:

$$x_{i,j} \geq 0, \quad N \in \mathbb{Z}_{\geq 0} \tag{10}$$

5. Optimization Analysis

5.1 Complexity

The core transport problem is a **Linear Programming (LP)** transportation problem, which is totally unimodular and solvable in polynomial time. The integer constraint on the number of vehicles (*N*) makes it a **Mixed-Integer Linear Program (MILP)**, but since *N* is a single scalar variable linked to the total sum, the complexity remains very low compared to VRP.

5.2 Sensitivity Analysis Strategy (Part e)

To analyze the impact of a 20% increase in waste generation: 1. Update parameter: *W_i' = 1.2 \times W_i*. 2. Check Feasibility:

$$\sum W_i' \leq \sum Cap_j \tag{11}$$

If total waste exceeds total facility capacity, the problem becomes infeasible, indicating a need for simpler disposal methods (landfill expansion). 3. Cost Impact: Calculate *\Delta Z = Z_{new} - Z_{old}*. Since variable costs are linear, we expect a roughly linear increase in operational costs, but the "step function" of vehicle fixed costs (*C_{fixed}*) might trigger the need for an additional vehicle, causing a discontinuous jump in total cost.

6. Preliminary Insights and Discussion

- **Facility Selection:** The solver will instinctively route waste to the nearest facility with the lowest processing cost. If recycling usually has lower *C_{proc}* (or even revenue) but is further away, a trade-off radius exists.
- **Bottlenecks:** The most constrained resource is likely the processing capacity of "green" facilities (compost/recycling). Once these fill up, the model will be forced to route excess waste to landfills, increasing the environmental penalty (if modeled).
- **Fleet Utilization:** The fixed cost of vehicles encourages maximizing the load per vehicle. A 20% increase in waste might be absorbed by existing fleet slack or might require a strictly new vehicle purchase.

7. References

1. Toth, P., & Vigo, D. (2014). *Vehicle Routing: Problems, Methods, and Applications*. SIAM. 2. Golden, B. L., Raghavan, S., & Wasil, E. A. (2008). *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer.