COMBINATORICS :

Agenda :

- Counting
- Permutation
- Combination

Pen and paper for today's session.

Q1) You are given 3 questions answer of which is either True or False. Total ways to answer them all.

Ans $\underset{T/F}{2}$ and $\underset{T/F}{2}$ and $\underset{T/F}{2}$

T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

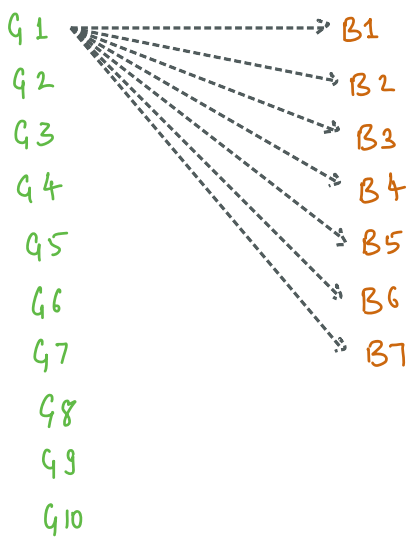
2 x 2 x 2 = 8

Q1 x Q2 x Q3

When it's and we multiply

Q.2) There are 10 girls and 7 boys. Count no. of ways to pick a pair of boy & girl.

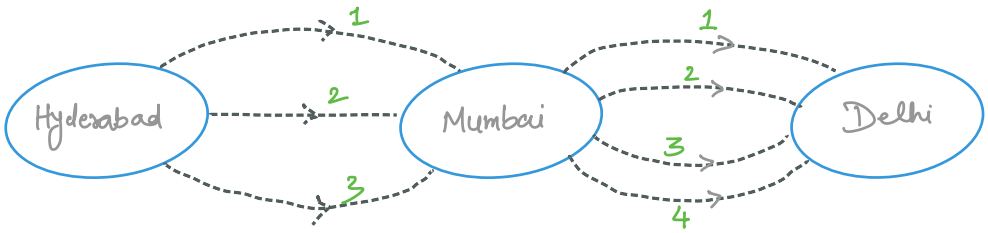
Ans:



10 / Girl and 7 / Boy

10 x 7 => 70 pairs

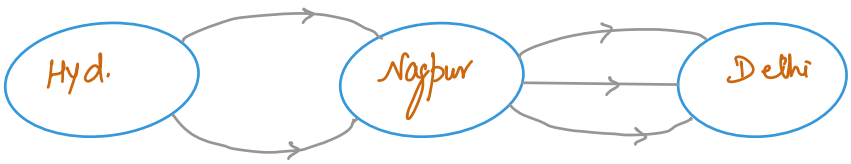
Q.3)



How many possible routes

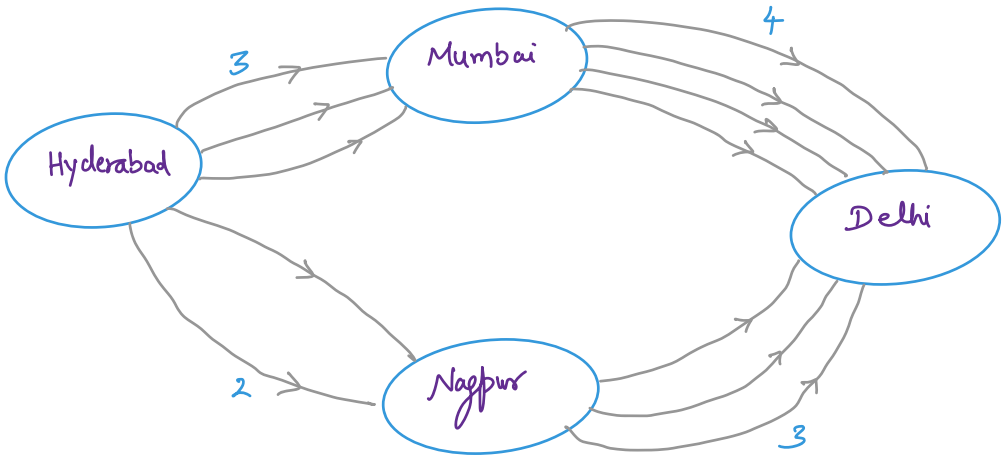
3 and 4 $\Rightarrow$ $3 \times 4 = 12$

Q.4)



Ans $2 \times 3 = 6$

Q.5) Considering yourself as a normal human being. How many way to move from Hyderabad to Delhi



Ans: Hyd. $\xrightarrow{\text{Mumbai}}$ Delhi = 3×4 = 12 ways

Hyd $\xrightarrow{\text{Nagpur}}$ Delhi = 2×3 = 6 ways

Total routes = 12 + 6 = 18 routes

Q.6) You can gift one of the following combos. Count Total number of ways to form a gift hamper.

Ans) Given:

i) 1 Pen and 1 book
ii) 1 flower and 1 chocolate
iii) 1 ring

∪

pen = 3
books = 5
Choco = 3
Flowers = 7
Rings = 3

Pen & book

$\frac{3}{\text{pen}}$ and $\frac{5}{\text{book}}$ = 15

Flower & chocolate

$\frac{7}{\text{flower}}$ and $\frac{3}{\text{chocolate}}$ = 21

$\frac{3}{\text{ring}}$ = 3

Total = 15 + 21 + 3 = 39 Ans

★ **Permutation :**

- The total number of arrangements of an object in defined order. Order matters here.

Example : You have 2 balls of different colors.

⇒ These are different combinations

Example →

1	2	3
2	1	3
1	3	2
3	2	1

Q.1) Count no. of ways to ~~arrange~~ "a", "b", "c". Without repitition.

Ans :

a	b	c
a	c	b
b	a	c
b	c	a
c	a	b
c	b	a

Here we are fixing one element & we'll see how many ways can we arrange rest of the elements.

3 x 2 x 1 = 6

Q.8) "a", "b", "c", "d"

Ans

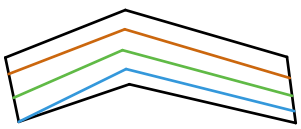
4 x 3 x 2 x 1 = 24

n!

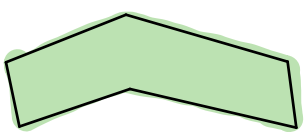
No. of ways to arrange n distinct elements at n positions.

Q.10 : Number of ways to arrange 0 element.

Ans ! 0! = 1



=> 3 x 2 x 1 = 6 ways



1 : empty wallet

Q.11) Given 5 characters, how many ways to arrange them in 2 places?

Ans: $\underline{5} \times \underline{4} = 20$ Ans

a b	b a	c a	d a	e a
a c	b c	c b	d b	e b
a d	b d	c d	d c	e c
a e	b e	c e	d e	e d

★ Number of ways to arrange n-distinct object at r positions.

Example : $n = 5$, $r = 3$

$$\underline{5} \times \underline{4} \times \underline{3} = 60$$
$$n \times (n-1) \times (n-2)$$

→ n = No. of elements

→ r = positions.

if r = 3 : $n \times (n-1) \times (n-2)$

if r = 4 : $n \times (n-1) \times (n-2) \times (n-3)$

if $r = k$ $\quad n \times (n-1) \times (n-2) \times \ldots \times (n-(k-1))$

$$\Rightarrow \frac{n \times (n-1) \times (n-2) \ldots (n-k+1) \times \boxed{(n-k) \times (n-k-1) \times \ldots 1}}{(n-k) \times (n-k-1) \times \ldots \times 1}$$

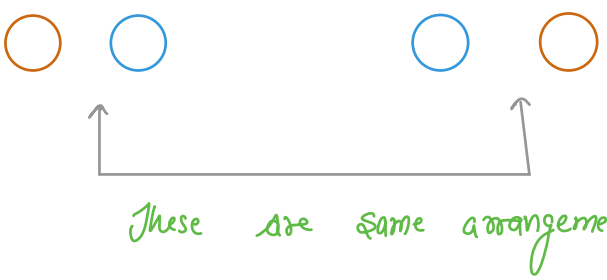
$$\Rightarrow \frac{n!}{(n-k)!}$$

$$\Rightarrow \boxed{\frac{n!}{(n-r)!}}$$

{ n distinct elements at r positions }

$$\Rightarrow {}^{n}P_{r} = \frac{n!}{(n-r)!}$$

★ **Combination :** It's about selection of object.
Order doesn't matter



These are same arrangements

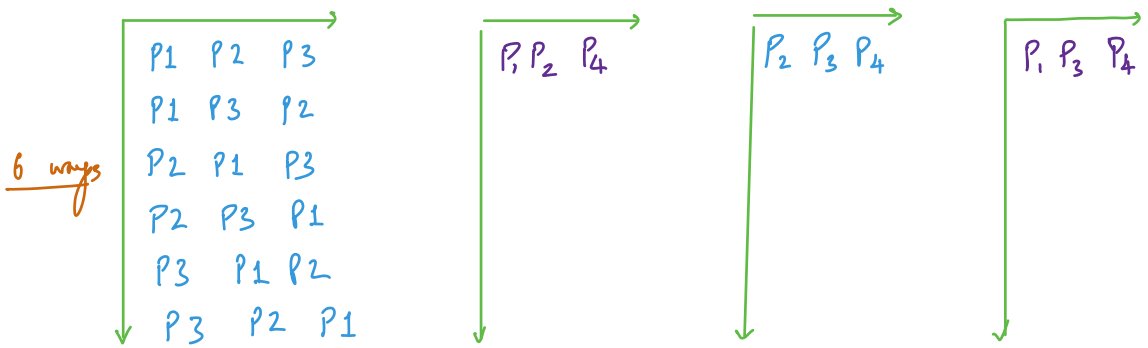
Example :

S1 : Dhoni , Kohli , Bumrah , Rohit , Chahal

S2 : Chahal, Bumrah , Kohli , Dhoni, Rohit

Q.12) In how many ways can we select 3 batsmen from a pool of 4 cricketers.

Any : P1 , P2 , P3 , P4



Permutations : $4 \times 3 \times 2 = 24$

Combinations : 4 Any

Note : $\frac{n!}{(n-r)! \times r!} = \frac{4 \times 3 \times 2 \times 1}{1! \times 3!} = 4$

Combination :

⇒ no. of ways to arrange n objects at r slots

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

⇒ r object at r position. Arrangements

⇒ r!

Combination : $\frac{n!}{(n-r)!\ r!}$

⇒ $^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$

★ How many combinat to select 1 player out of 5 players

Ans 5

⇒ $^{5}C_{1} = \frac{5!}{(5-1)! \times 1!}$ ⇒ $\frac{5 \times 4!}{4! \times 1!}$

$= 5$