Homework#1

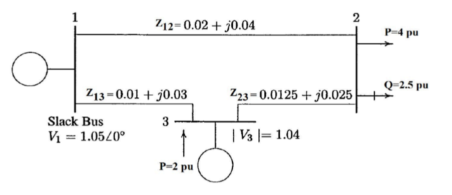
1) For the following non-linear equations:

$$2x_1^2+x_1x_2-x_1=2$$
$$x_1^2-x_2=0$$

a) Solve the above system of non-linear equations using Newton-Raphson method with $(x_1^{(0)}, x_2^{(0)})=(0.9,1.1)$. Perform the first two iterations (i.e. K=0 and K=1)
b) Write a computer code for solving above system of non-linear equations using Newton-Raphson method. The iterations should be stopped if $|g_1|<0.0001$ and $|g_2|<0.0001$

(2)

For the following network, perform power flow analysis. Only first two iterations are enough (i.e. for K=0 and K=1). Start with initial values of $|V_2^{(0)}|=1.0$, $\delta_2^{(0)}=0.0$, and $\delta_3^{(0)}=0.0$.



Solution #1

(a) $2x_1^2+x_2x_1-x_1=2$

$x_1^2-x_2=0$

$x_1^{(0)}=0.9;\ x_2^0=1.1$

Let, $g(x)=2x_1^2+x_1x_2-x_1-2=0$

$h(x)=x_1^2-x_2=0$

Now,

$$\frac{\partial g(x)}{\partial x_1}=g'_{x_1}=4x_1+x_2;\quad \frac{\partial g(x)}{\partial x_2}=g'_{x_2}=x_1.$$

Similarly,

$$\frac{\partial h(x)}{\partial x_1}=2x_1;\quad \frac{\partial h(x)}{\partial x_2}=-1$$

For any arbitrary function $\xi(x)$:

$$\frac{\partial \xi(x)}{\partial x}=\frac{\xi(x_0)-\xi(x)}{x_0-x}$$

$$\Rightarrow x_0-x=\left(\frac{1}{\frac{\partial \xi(x)}{\partial x}}\right)\xi(x_0)-\xi(x)$$

$$\Rightarrow x=x_0-\frac{1}{\xi'_x}\left(\xi(x_0)-\xi(x)\right)$$

Assuming $\xi(x)=0$ {At opt. sol^n for $\xi(x)=0$}

$$\Rightarrow x=x_0-\underbrace{\frac{1}{\xi'_x}\left(\xi(x_0)\right)}_{\Delta x}$$

without loss of generality : (For r^{th} iteration)

$$\Delta x=J^{-1}\left(\xi(x)\right)\ (\because J \text{ is Jacobian})$$

Aforementioned generalization can be further implemented in context of the given problem in 1(a):

$$\begin{bmatrix}\Delta x_1^{(r+1)}\\ \Delta x_2^{(r+1)}\end{bmatrix}=\begin{bmatrix}g'^{(r)}_{x_1} & g'^{(r)}_{x_2}\\ h'^{(r)}_{x_1} & h'^{(r)}_{x_2}\end{bmatrix}^{-1}\begin{bmatrix}g(x_1^{(r)},x_2^{(r)})\\ h(x_1^{(r)},x_2^{(r)})\end{bmatrix}$$

First iteration: $x_1^{(0)}=0.9$; $x_2^{(0)}=1.1$;

$g'^{(0)}_{x_1}=4x_1^{(0)}+x_2^{(0)}=3.6+1.1=4.7$;

$g'^{(0)}_{x_2}=x_1^{(0)}=0.9$;

$h'^{(0)}_{x_1}=2x_1^{(0)}=1.8$; $h'^{(0)}_{x_2}=-1$

$g(x_1^{(0)},x_2^{(0)})=2x_1^{(0)2}-x_1^{(0)}x_2^{(0)}-x_1^{(0)}-2=-2.27$

$h(x_1^{(0)},x_2^{(0)})=x_1^{(0)2}-x_2^{(0)}=-0.29$

$$\begin{bmatrix}\Delta x_1^{(1)}\\ \Delta x_2^{(1)}\end{bmatrix}=\begin{bmatrix}4.7 & 0.9\\ 1.8 & -1\end{bmatrix}^{-1}\begin{bmatrix}-2.27\\ -0.29\end{bmatrix}$$

$|J^0|=-4.7-1.62=-6.32$

$$\begin{bmatrix}\Delta x_1^{(1)}\\ \Delta x_2^{(1)}\end{bmatrix}=\frac{1}{-6.32}\begin{bmatrix}-1 & -0.9\\ 1.8 & -4.7\end{bmatrix}\begin{bmatrix}-2.27\\ -0.29\end{bmatrix}$$

$$\left.\begin{aligned}\Delta x_1^{(1)}&=-0.509\\ \Delta x_2^{(1)}&=-1.68\end{aligned}\right\}\Rightarrow \begin{aligned}x_1^{(1)}&=x_1^{(0)}-\Delta x_1^{(1)}=1.409\\ x_2^{(1)}&=x_2^{(0)}-\Delta x_2^{(1)}=2.78\end{aligned}$$

Second iteration:

$$\begin{bmatrix}\Delta x_1^{(2)}\\ \Delta x_2^{(2)}\end{bmatrix}=\begin{bmatrix}8.416 & 1.409\\ 2.818 & -1\end{bmatrix}^{-1}\begin{bmatrix}-3.36\\ 1.371\end{bmatrix}$$

$$\left.\begin{aligned}\Delta x_1^{(2)}&=0.04\\ \Delta x_2&=-1.31\end{aligned}\right\}\Rightarrow \begin{aligned}x_1^{(2)}&=1.369\\ x_2^{(2)}&=4.09\end{aligned}$$

Solution #2

Step 1 (Y_{Bus} Formulation)

$$Y_{Bus}=\begin{bmatrix}y_{12}+y_{13} & -y_{12} & -y_{13}\\ -y_{12} & y_{12}+y_{13} & -y_{32}\\ -y_{13} & -y_{32} & y_{13}+y_{23}\end{bmatrix}$$

$y_{12}=\frac{1}{z_{12}}=10-j20$ pu (12)

$y_{23}=\frac{1}{z_{23}}=16-j32$ pu (23)

$y_{31}=\frac{1}{z_{31}}=10-j30$ pu (31)

$$Y_{Bus}=\begin{bmatrix}20-j50 & -10+j20 & -10+j30\\ -10+j20 & 26-j52 & -16+j32\\ -10+j30 & -16+j32 & 26-j62\end{bmatrix}$$

Step 2 Bus Classification (Variable formalization)

Bus #	Type	P_{inj} (net)	Q_{inj} (net)
1	Slack	?	?
2	PQ	-4	-2.5
3	PV	2	?

$\delta_1=0°$; $\delta_2=$?; $\delta_3=$?; $|V_2|=$?

$V_1=1$

$|V_3|=1.04$

Step 3 Jacobian Derivation

$$f(\theta)=f(\theta_0)+\Delta\theta f'(\theta_0)+\frac{(\Delta\theta)^2}{2!}f''(\theta_0)+\dots$$

Assuming insensitivity towards higher order terms:

$$f(\theta)=f(\theta_0)+\Delta\theta f'(\theta_0)$$

$$\Rightarrow \Delta\theta=-\frac{\left(f(\theta_0)-f(\theta)\right)}{f'(\theta_0)}$$

Here we can write the same equations for power systems.

In power systems applications, $f(\theta)$ is often known.

Hence, $S_i=V_iI_i^*=V_i\left(\sum_N V_jY_{ij}\right)^*$

Here, N is set of all buses in the system

$$P_{inj}(net)=\sum_{j\in N}|V_i||V_j||Y_{ij}|\cos(\delta_i-\delta_j-\theta_{ij})$$

$$Q_{inj}(net)=\sum_{j\in N}|V_i||V_j||Y_{ij}|\sin(\delta_i-\delta_j-\theta_{ij})$$

Now,

$$\frac{\partial P_i}{\partial \delta_i}=-\sum_{j\in N}|V_i||V_j||Y_{ij}|\sin(\delta_i-\delta_j-\theta_{ij})$$

$$\frac{\partial P_i}{\partial \delta_j}=|V_i||V_j||Y_{ij}|\sin(\delta_i-\delta_j-\theta_{ij})$$

$$|V_i|\frac{\partial P_i}{\partial |V_i|}=\sum_{\substack{j\in N\\ j\neq i}}|V_i||V_j||Y_{ij}|\cos(\delta_i-\delta_j-\theta_{ij})+2|V_i||V_i||Y_{ij}|\cos(\theta_{ij})$$

$$|V_j|\frac{\partial P_i}{\partial |V_j|}=|V_i||V_j||Y_{ij}|\cos(\delta_i-\delta_j-\theta_{ij})$$

$$\frac{\partial Q_i}{\partial \delta_i}=\sum_{j\in N}|V_i||V_j||Y_{ij}|\cos(\delta_i-\delta_j-\theta_{ij})$$

$$\frac{\partial Q_i}{\partial \delta_j}=-|V_i||V_j||Y_{ij}|\cos(\delta_i-\delta_j-\theta_{ij})$$

$$|V_i|\frac{\partial Q_i}{\partial |V_i|}=\sum_{\substack{j\in N\\ j\neq i}}|V_i||V_j||Y_{ij}|\sin(\delta_i-\delta_j-\theta_{ij})+2|V_i||V_i||Y_{ij}|\sin(-\theta_{ij})$$

$$|V_j|\frac{\partial Q_j}{\partial |V_j|}=|V_i||V_j||Y_{ij}|\sin(\delta_i-\delta_j-\theta_{ij})$$

$$J=\begin{bmatrix}\frac{\partial P_2}{\partial\delta_2} & \frac{\partial P_2}{\partial\delta_3} & |V_2|\frac{\partial P_2}{\partial|V_2|}\\ \frac{\partial P_3}{\partial\delta_2} & \frac{\partial P_3}{\partial\delta_3} & |V_2|\frac{\partial P_2}{\partial|V_2|}\\ \frac{\partial Q_2}{\partial\delta_2} & \frac{\partial Q_2}{\partial\delta_3} & |V_2|\frac{\partial Q_2}{\partial|V_2|}\end{bmatrix};\ \Delta x=\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \frac{\Delta V_2}{|V_2|}\end{bmatrix}$$

we know that,

$$\Delta x^{(r)}=\frac{1}{J^{-1}}\left(P_{cal}^{(r)}-P_{meas}\right)$$

& $x^{(r+1)}=x^{(r)}-\Delta x^{(r)}$

Iteration #1 : $P_2^{(0)}=|V_2^{(0)}||V_1^{(0)}||Y_{12}|\cos(\delta_2-\delta_1-\theta_{12})+|V_2^{(0)}||V_2^{(0)}||Y_{22}|\cos(\theta_{22})+|V_2^{(0)}||V_3^{(0)}||Y_{23}|\cos(\delta_2-\delta_3-\theta_{23})$

$$\Rightarrow \Delta P_2^{(0)}=-2.86$$

$Q_2^{(0)}=|V_2^0||V_1^{(0)}||Y_{12}|\sin(\delta_2^0-\delta_1^0-\theta_{12})-|V_2^0||V_2^0||Y_{22}|\sin(\theta_{22})+|V_2^0||V_3^0||Y_{33}|\sin(\delta_2^0-\delta_3^0-\theta_{23})$

$$\Rightarrow \Delta Q_2^{(0)}=-0.22$$

$P_3^{(0)}=1.04\left[|Y_{13}|\cos(\theta_{13})+|Y_{33}|\cos(\theta_{33})+|Y_{32}|\cos(\theta_{32})\right]$

$$\Rightarrow \Delta P_3^{(0)}=1.44$$

Also,

$$J^{(0)}=\begin{bmatrix}54.5 & -33.28 & 22\\ -33.28 & 67.0592 & -16.64\\ -30 & 16.64 & 49.5\end{bmatrix}$$

$$\Delta X^{(0)}=J^{-1}\begin{pmatrix}\Delta P\\ \Delta Q\end{pmatrix}=\begin{bmatrix}-0.038\\ -0.01\\ -0.05\end{bmatrix}$$

$$\Rightarrow X^{(0)}=\begin{bmatrix}0\\ -0.038\\ -0.01\end{bmatrix}$$

Iteration #2:

$$J^{(1)}=\begin{bmatrix}53.46 & -32.46 & 21.48\\ -33.39 & 67.05 & -15.53\\ -29.48 & 17.39 & 48.46\end{bmatrix}$$

$$\Delta X^{(1)}=\begin{bmatrix}-0.003\\ 0.0006\\ -0.03\end{bmatrix}$$

$$X^{(1)}=\begin{bmatrix}0\\ -0.041\\ -0.009\end{bmatrix}$$

$$\overline{V}=\begin{bmatrix}1.05\\ 0.99\\ 1.04\end{bmatrix};\ \overline{I}=\begin{bmatrix}0 & -1.42 & -0.41\\ 1.44 & 0 & 1.83\\ 0.41 & -1.88 & 0\end{bmatrix}$$

$$\overline{S}_{ij}=\begin{bmatrix}0 & -1.493 & -0.413\\ 1.441 & 0 & 1.83\\ 0.41 & -1.88 & 0\end{bmatrix}$$

$$\overline{S}_{loss}=\begin{bmatrix}0 & -2.937 & -6.825\\ 2.93 & 0 & 3.719\\ 0.82 & -3.719 & 0\end{bmatrix}$$

(Line Loss)