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Tutorial - 1

Name - Yash Mehar

Section - CST Core

Semester - IV

Class Roll no. - 37

University Roll no. - 2017584

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Yash

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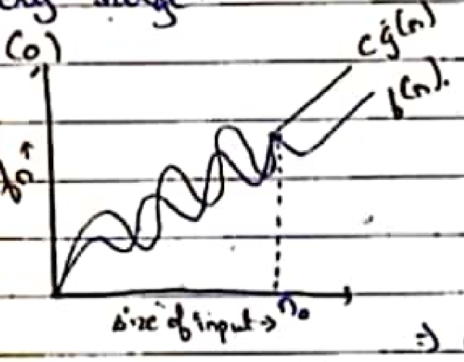
Que:1 Asymptotic Notations
↳ Tending to infinity

They help you find the complexity an algorith when input is very large.

1) Big O(o)



$f(n) = O(g(n))$
iff $f(n) \leq c \cdot g(n)$.
$\forall\, n \geq n_0$
for some constant $c > 0$

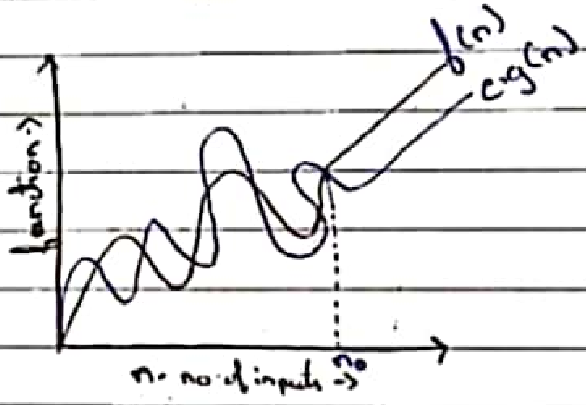
⇒ g(n) is 'tight' upper bound of f(n)

ii) Big Omega (Ω)

$f(n) = \Omega(g(n))$

g(n) is 'tight' lower bound of f(n)

$f(n) = \Omega(g(n))$.
iff $f(n) \geq c\,g(n)$
$\forall\, n \geq n_0$ for some constant $c > 0$



iii) Theta (θ)

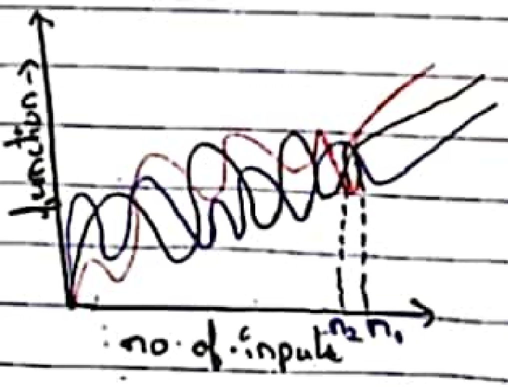
$f(n) = \theta(g(n))$.

g(n) is both 'tight' upper & lower bound of fn f(n)

$f(n) = \theta(g(n))$

iff

$c_1 g(n) \leq f(n) \leq c_2 \cdot g(n)$
$\forall\, n \geq \max(n_1, n_2)$
for some constant $c_1 > 0$ & $c_2 > 0$



4) Small o(O)

$f(n) = o(g(n))$

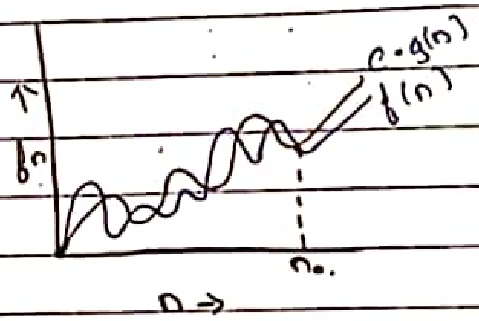
g(n) is upper bound of fn f(n).

$f(n) = o(g(n))$

when $f(n) < c \cdot g(n)$

$\forall\ n > n_0$

& $\forall\ c > 0$



5) small omega (ω)

$f(n) = \omega(g(n))$

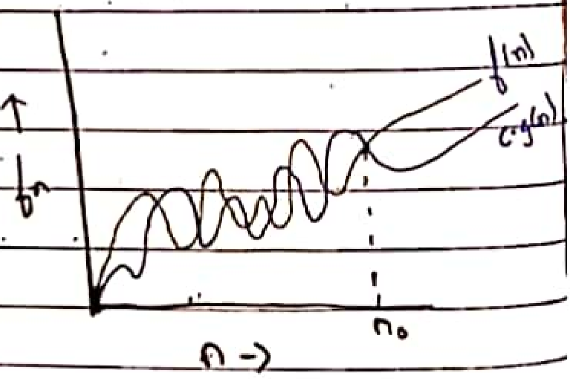
g(n) is lower bound of fn f(n)

$f(n) = \omega(g(n))$

when $f(n) > c \cdot g(n)$

$\forall\ n > n_0$

& $\forall\ c > 0$



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Ques-2 What should be time complexity of
for(i=1 to n) {i=i*2}

for(i=1 to n) // $i = 1, 2, 4, 8 \ldots n$
{ i=i*2} // O(1)

$\Rightarrow \sum_{i=1} 1+2+4+8+\ldots n$

GP kth value $\Rightarrow T_k = ar^{k-1}$
$\Rightarrow 1 \times 2^{k-1}$
$\Rightarrow n = 2^{k-1}$

$\Rightarrow 2n = 2^k$
$\Rightarrow \log 2n = k \log 2$
$\Rightarrow \log_2 + \log n = k \log 2$
$\Rightarrow \log n + 1 = k$

$\Rightarrow O(k) = O(1+\log n)$
$= O(\log n)$

Q-3 $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

$T(n) = 3T(n-1)$ — ①

put $n = n-1$
$T(n-1) = 3\{T(n-2)\}$ — ②
from 1 & 2

putting $n = n-2$ in ①

$T(n) = 3(T(n-3))$ — ④

$\Rightarrow T(n) = 27(T(n-3))$

$\Rightarrow T(n) = 3^n (T(n-k))$

putting $n-k=0$

$\Rightarrow n = k$.

$\Rightarrow T(n) = 3^n [T(n-n)]$

$\Rightarrow T(n) = 3^n T(0)$

$\Rightarrow T(n) = 3^n \times 1$ [$T(0) =$

$\Rightarrow T(n) = O(3^n)$

4) $T(n) = \{ 2T(n-1) - 1$ if $n > 0$, otherwise

$T(n) = 2T(n-1) - 1$ — ①

let $n = n-1$

$\Rightarrow T(n-1) = 2T(n-2) - 1$ — ②

$\Rightarrow$ from ① & ②

$\Rightarrow T(n) = 2[2T(n-2) - 1] - 1$

$\Rightarrow T(n) = 4T(n-2) - 2 - 1$ — ③

let $n = n-2$

$\Rightarrow T(n-2) = 2T(n-3) - 1$ — ④.

from ③ & ④

$\Rightarrow T(n) = 4[2T(n-3) - 1] - 2 - 1$

$\Rightarrow T(n) = 8T(n-3) - 4 - 2 - 1$

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$\Rightarrow T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \cdots 1$

$\Rightarrow GP = 2^{k-1} + 2^{k-2} + 2^{k-3} + \cdots 1$

$a = 2^{k-1}$

$r = 1/2$

$\Rightarrow S_k = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{2^{k-1}(1-(1/2)^n)}{1/2}$

$= 2^k(1-(1/2)^k)$

$= 2^k - 1$

Let $n-k = 0$

$\Rightarrow n = k$

$\Rightarrow T(n) = 2^n T(n-n) - (2^n - 1)$

$\Rightarrow T(n) = 2^n \cdot 1 - (2^n - 1)$

$\Rightarrow T(n) = 2^n - (2^n - 1)$

Qs-5 What should be time complexity of

```
int i=1, s=1;
while (s<=n)
{ i++; s=s+i;
  printf("#");
}
```

i = 1 2 3 4 5 6 ------

s = 1 + 3 + 6 + 10 + 15 + 21 ------ n —①

~~Also see s~~

s becomes

Sum of s = 1+3+6+10+ ---- + T_n — ①

also s = 1+3+6+10 + ---T_{n-1} + T_n — ②

from ① - ②

0 = 1+2+3+4+ ----- n - T_n

⇒ T_k = 1+2+3+4+ ---- k

⇒ $T_k = \frac{1}{2} k(k+1)$

⇒ for k iterations.

1+2+3+ ----- + k <= n.

⇒ $\frac{k(k+1)}{2}$ <= n.

⇒ $\frac{k^2+k}{2}$ <= n

⇒ $O(k^2)$ <= n

⇒ k = $O(\sqrt{n})$

DOMS

Ques-6 Time complexity of -

```
void fn(int n)
{ int i, count=0;
   for(i=1; i*i<=n; ++i)
       count++                 // O(i)
}
```

as $i^2 <= n$

$\Rightarrow i <= \sqrt{n}$

$i = 1, 2, 3, 4 \ldots \sqrt{n}$

$\sum_{i=1}^{n} 1+2+3+4+ \ldots + \sqrt{n}$.

$\Rightarrow T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$\Rightarrow T(n) = \frac{n \times \sqrt{n}}{2}$

$\Rightarrow T(n) = O(n)$.

Ques-7 Time complexity of :-

```
void fn(int n)
{ int i, j, k, count=0;
  for(i=n/2; i<=n; ++i)
     for(j=1; j<=n; j=j*2)
        for(k=1; k<=n; k=k*2)
            count++;
```

for $k = k*2$

$k = 1, 2, 4, 8, \ldots n.$

$\Rightarrow$ GP $\Rightarrow$ $a = 1$, $r = 2$.

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{1(2^k - 1)}{1}$

$n \Rightarrow 2^k$.

$\Rightarrow \log n = k$

i	j	k
1	log n	log n * log n.
2	log n	log n * log n
⋮	⋮	⋮
n	log n	log n * log n

$\Rightarrow O(n * \log n * \log n)$

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8) Time Complexity of

```
function (int n)
{ int (n==1)
    return;                         // O(1)
    for (i=1 to n)                  // i = 1,2,3,4 --- n => O(n)
    { for (j=1 to n)                // j = 1,2,3,4, --- n => O(n^2)
        { print ("*");
        }
    }
    function (n-3);                 T(n/3)
}
```

$\Rightarrow T(n) = T(n/3) + n^2$

$\Rightarrow a = 1, \quad b = 3, \quad f(n) = n^2$

$c = \log_3 1 = 0$

$\Rightarrow n^0 = 1 \quad < (f(n) = n^2)$

$\Rightarrow T(n) = \Theta(n^2)$

Ques 9 Time complexity of -

```
void function (int n)
{ for (i=1 to n)                      // O(n)
    { for (j=1 ; j<=n ; j=j+i)
          print ("*")                 // O(n)
    }
}
```

~~i = 1, 2, 3, 4, ... n~~

~~1+1+3+4+ ... n~~

for i=1 ⇒ j = 1, 2, 3, 4 ... n = n.

for i=2 ⇒ j = 1, 3, 5, ... n = n/2

for i=3 ⇒ j = 1, 4, 7, ... n = n/3

⋮

for i=n ⇒ j = 1 ... 1

$$\Rightarrow \sum_{j=n} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \cdots + 1$$

$$\Rightarrow \sum_{j=n} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{n}\right]$$

$$\sum_{j=n} n[\log n]$$

$$\Rightarrow T(n) = O[n \log n]$$

Q-10 for functions, n^k & c^n, what is the asymptotic relation between these functions?
assume that $k >= 1$, & $c > 1$ are constant.
Find out the value of c & n_0 for which relation holds

as given n^k & c^n

relation b/w n^k & c^n is

$$n^k = O(c^n).$$

as $n^k \le ac^n$
$\forall n \ge n_0$ & some constant $a > 0$

for $n_0 = 1$
$c = 2$

$\Rightarrow 1^k \le a2^1$

$\Rightarrow n_0 = 1$ & $c = 2$