

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

- A. 0.3875
- B. 0.2676
- C. 0.5
- D. 0.6987

Ans. (B) –0.2676

Explanation :-

- $\mu = 45\text{min}, \sigma = 8\text{min}$
- $\mu = 10\text{min after} = 45 + 10 = 55$
- Car will be ready in $1\text{hr} = 60\text{min}(x)$
- $Z = (x - \mu) / \sigma$
- $60 - 55 / 8$
- 0.625
- $1 = \text{stats.norm.cdf}(z)$
- $1 = \text{stats.norm.cdf}(0.625)$
- 0.2659
- P value for z score is 0.2659 approx to 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.

- A. More employees at the processing center are older than 44 than between 38 and 44.

Ans. False.

Explanation—

- $\mu = 38$
- $\sigma = 6$
- Z score $\mu = 38$
- $\sigma = 6$
- Z score $= (X - \mu) / \sigma$
- Z score for 44 $= (44 - 38) / 6 = 1$

- In python- stats.norm.cdf(1) => 84.13 %
- People above 44 age = 100 - 84.13 = 15.87%
- Given 400 clerical employees, that is calculating 15.87% of 400=336.52 out of 400
- 400 - 336.52
- 63.49≈63(approx)
- Z score for 38 = (38 - 38)/6 = 0
- In python- stats.norm.cdf(0) => 50%

Hence, People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. Zscore for 30 = (30 - 38)/6 = -1.33 = 9.15 % ≈ 36 out of 400

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans.

- As both are independent normal random variables, X_1+X_2 is normal with $N(\mu_1+ \mu_2, \sigma^2_1+ \sigma^2_2)$. And $2X_1$ will just scale the normal distribution by 2 times

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

- A. 90.5, 105.9
- B. 80.2, 119.8
- C. 22, 78
- D. 48.5, 151.5
- E. 90.1, 109.9

Ans. 48.5, 151.5

Explanation;–

- $p(a < x < b) = 0.99, \mu=100, \sigma = 20$
- To Find:
- Identify symmetric values for the standard normal distribution such that the area enclosed is 0.99

- Using Python
- `stats.norm.interval (0.99, 100, 20)`

Two values symmetric about mean for the given standard normal distribution are [48.5,151.5]

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
- A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
- Refer **Yash Bhosale** Assignment – 2 Python File
Range is Rs (99.00810347848784, 980.9918965215122) in Millions.
- B. Specify the 5th percentile of profit (in Rupees) for the company
- Refer **Yash Bhosale** Assignment – 2 Python File
5th percentile of profit (in Million Rupees) is 170.0.
- C. Which of the two divisions has a larger probability of making a loss in a given year?
- Refer **Yash Bhosale** Assignment – 2 Python File
Probability of Division 1 making a loss $P(X < 0)$
0.047
Probability of Division 2 making a loss $P(X < 0)$
0.040