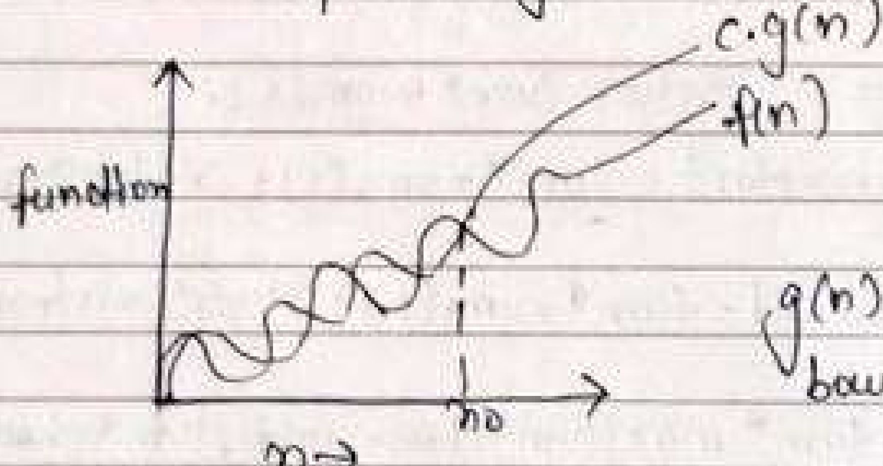
Ques-1: What do you mean by Asymptotic notations. Define different type of notations along with example.

Ans: Asymptotic Notations: Means tending to infinity. They are used to used to tell the complexity when input is very large.

→ Different types of Asymptotic Notations:

1. Big oh (O) Notation:

$f(n) = O(g(n))$



g(n) is "tight" upper bound of f(n)

$f(n) = O(g(n))$ iff $f(n) \le c.g(n)$ $\forall$ $n \ge n_0$ and some constant, c

Example:

```
for(i=1; i<=n; i++)
{  print("*");  —— O(1)
}
```

$\Rightarrow T(n) = O(n)$

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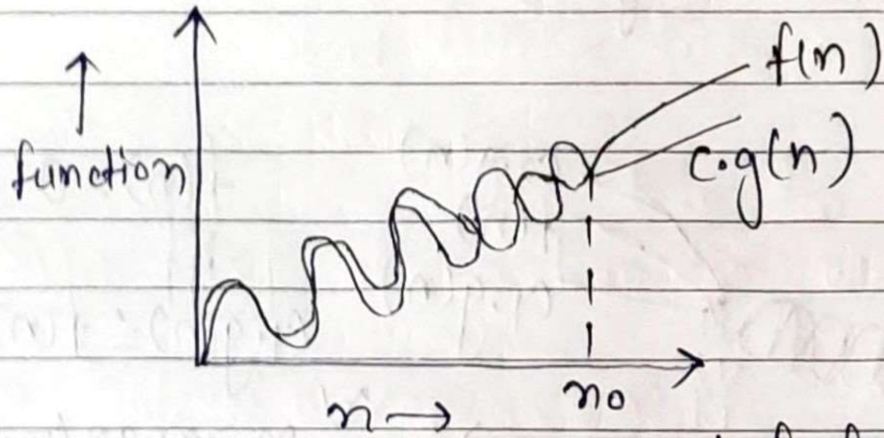
2. Big Omega (Ω):

$$f(n) = \Omega(g(n))$$



g(n) is "tight" lower bound of f(n)

$$f(n) = \Omega(g(n))$$

iff $f(n) \geq c \cdot g(n)$

$\forall$ $n \geq n_0$, and some constant $c > 0$

Example:

$$f(n) = 2n^2 + 3n + 5 \quad , \quad g(n) = n^2$$

$$0 \leq c \cdot g(n) \leq f(n)$$

$$0 \leq c \cdot n^2 \leq 2n^2 + 3n + 5$$

$$c \leq 2 + \frac{3}{n} + \frac{5}{n^2}$$

On putting $n = \infty$, $\frac{3}{n} \to \infty$, $\frac{5}{n^2} \to \infty$

$$\Rightarrow c = 2$$

$$\Rightarrow 2n^2 \leq 2n^2 + 3n + 5$$

On putting $n = 1$

$$2 \leq 2 + 3 + 5$$

$$2 \leq 10 \quad \text{True.}$$

$$\Rightarrow \boxed{c = 2, \quad n = n_0 = 1}$$

$$0 \leq 2n^2 \leq 2n^2 + 3n + 5$$

$$\therefore f(n) = \Omega(n^2)$$

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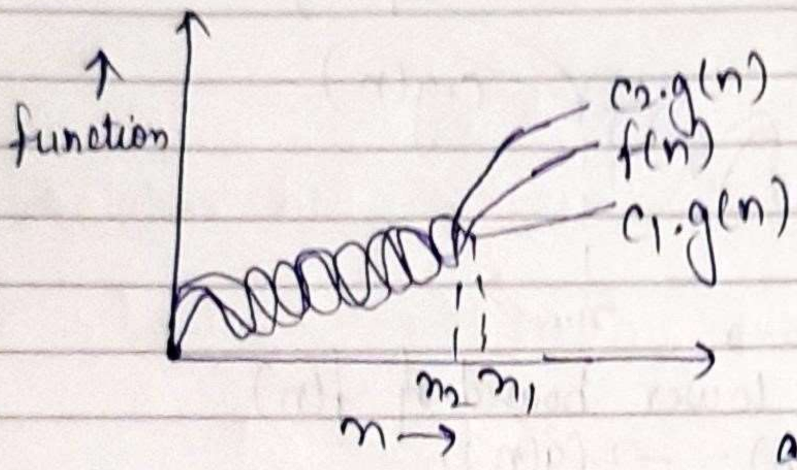
3. **Big Theta (Θ):**

$$f(n) = \Theta(g(n))$$



$f(n) = \Theta(g(n))$ iff

$c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$

$\forall \; n \ge \max(n_1, n_2)$

and some constant $c_1 > 0$ and $c_2 > 0$

Example: $f(n) = 10\log_2 n + 4$, $g(n) = \log_2 n$

$$f(n) \le c_2 \cdot g(n)$$

$$\Rightarrow 10\log_2 n + 4 \le 10\log_2 n + \log_2 n$$

$$10\log_2 n + 4 \le 11\log_2 n$$

$$c_2 = 11$$

$$\rightarrow 4 \le 11\log_2 n - 10\log_2 n$$

$$4 \le \log_2 n$$

$$16 \le n$$

Here

$$\forall \; n \ge 16$$

$$n_2 = 16$$

$$\& \; c_2 = 11$$

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$f(n) \geq c_1 \cdot g(n)$

$10 \log_2 n + 4 \geq 1 \log_2 n$

$c_1 = 1,\ n > 0$

$\Rightarrow n_1 = 1 \Rightarrow n_0 = \max(n_1, n_2) \Rightarrow n_0 = 16$

$\Rightarrow \log_2 n \leq 10 \log_2 n + 4 \leq 11 \log_2 n$

$c_1 = 1,\ c_2 = 11$

$\Rightarrow \Theta(\log_2 n)$

4. <u>Small oh (o)</u>:

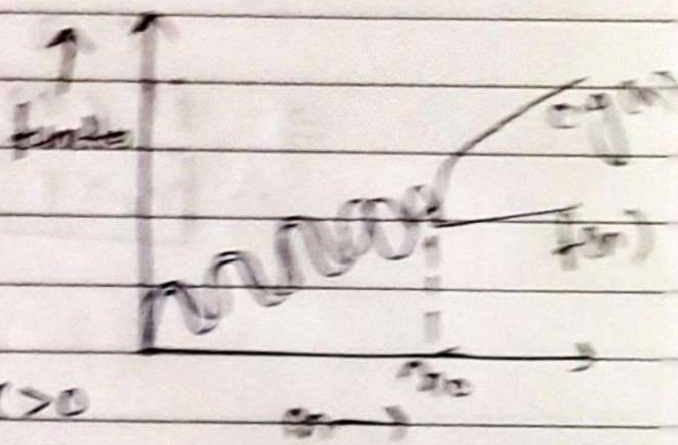
$f(n) = o(g(n))$

$g(n)$ is upper bound of $f(n)$

$f(n) = o(g(n))$

iff $f(n) < c \cdot g(n)$

$\forall\, n > n_0$ and $\forall$ constant, $c > 0$



5. <u>Small omega (ω)</u>:

$f(n) = \omega(g(n))$

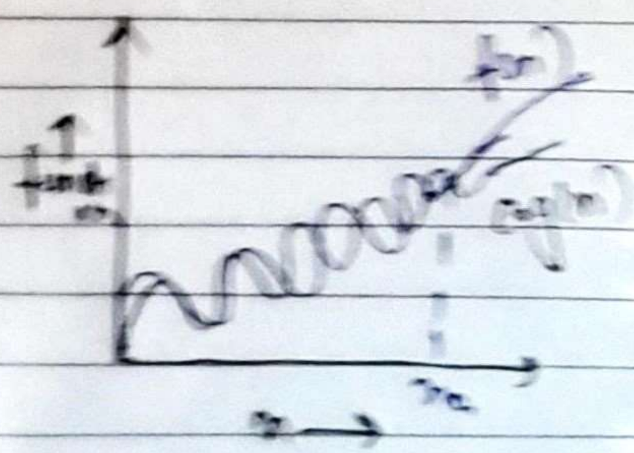
$g(n)$ is lower bound of $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c \cdot g(n)$

$\forall\, n > n_0$

and $\forall\, c > 0$



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Ques-2: What should be the time complexity of:
for (i = 1 to n) { i = i * 2; }

Sol^n

values of $i = \underbrace{1, 2, 4, 8, 16, \ldots\ldots n}_{K \text{ terms}}$

As this is a G.P with $a = 1$, $r = 2$

Now, K^{th} term :- $t_K = ar^{K-1}$

$$n = 1 \cdot 2^{K-1}$$

$$n = 2^{K-1}$$

taking $\log_2$ on to the both sides.

$$\Rightarrow \log_2 n = \log_2 2^{K-1}$$

$$\log_2 n = (K-1)\log_2 2$$

$$\log_2 n = K - 1 \Rightarrow K = 1 + \log_2 n \quad [\because \log_2 2 = 1]$$

$\therefore$ Time complexity $T(n) = O(K)$

$$= O(1 + \log_2 n)$$

$$= O(\log_2 n).$$

Ques-3:

$T(n) = \{ 3T(n-1)$ if $n > 0$, otherwise $1 \}$

Sol^n.

$$T(n) = 3T(n-1) \text{ ------ } (1)$$

put $n = n-1$ in eq^n (1)

$$T(n-1) = 3T(n-1-1)$$

$$T(n-1) = 3T(n-2) \text{ ---- } (2)$$

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Put value of $T(n-1)$ from eqⁿ ② in eqⁿ ①

$$T(n) = 3[3T(n-2)]$$
$$T(n) = 9T(n-2) \text{ ------ } ③$$

Put $n = n-2$ in eqⁿ ①

$$T(n-2) = 3T(n-3) \text{ ---- } ④$$

Put value of $T(n-2)$ in eqⁿ ③

$$T(n) = 3[9T(n-3)]$$
$$T(n) = 27T(n-3) \text{ --- } ⑤$$

On Generalising eqⁿ ⑤

$$T(n) = 3^k T(n-k)$$

Put $n-k = 0$

$$\Rightarrow T(n) = 3^k T(0)$$
$$= 3^k \quad (\because T(0) = 1)$$

$$\therefore T(n) = O(3^n)$$

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Ques-4. $T(n) = \{2T(n-1) - 1$ if $n > 0$, otherwise $1\}$

Sol^n.

$$T(n) = 2T(n-1) - 1 \quad \text{---} \ (1)$$

Put $n = n-1$ in eq^n (1)

$$T(n-1) = 2T(n-1-1) - 1$$

$$T(n-1) = 2T(n-2) - 1 \quad \text{---} \ (2)$$

Put value of $T(n-1)$ from (2) in (1)

$$T(n) = 2[2T(n-2) - 1] - 1$$

$$T(n) = 4T(n-2) - 2 - 1 \quad \text{---} \ (3)$$

Put $n = n-2$ in eq^n (1)

$$T(n-2) = 2T(n-3) - 1$$

Put value of $T(n-2)$ in eq^n (3)

$$T(n) = 4[2T(n-3) - 1] - 2 - 1$$

$$T(n) = 8T(n-3) - 4 - 2 - 1$$

On Generalising

$$T(n) = 2^K T(n-K) - 2^{K-1} - 2^{K-2} - \dots - 1$$

Put $n - K = 0 \Rightarrow n = K$, $T(0) = 1$ (Given)

$$T(n) = 2^n T(0) - 2^{n-1} - 2^{n-2} - \dots - 1$$

$$= 2^n - \underbrace{[2^{n-1} + 2^{n-2} - \dots + 1]}_{K \text{ terms}}$$

$$\Rightarrow \quad a = 2^{n-1}, \quad r = 1/2$$

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$$\text{Sum of GP} = \frac{2^{n-1}\left[1-\left(\frac{1}{2}\right)^{n-1}\right]}{1-1/2}$$

$$= 2^n - 2$$

$$\Rightarrow T(n) = \cancel{2^n} - [\cancel{2^n} - 2] = 2$$

$$= O(2)$$

$$\boxed{T(n) = O(1)}$$

Que-5 What should be the time complexity of-

```
int i=1, s=1;
while (s<=n) {
    i++; s=s+i;
    printf("#");
}
```

Solⁿ:

i	s
1	1
2	3
3	6
!	10
!	15
!	!
!	n
n	k times

$$S = \underbrace{1, 3, 6, 10, 15, \ldots n}_{k \text{ terms}}$$

k^{th} term, $t_k = t_{k-1} + k$

$$k = t_k - t_{k-1} \quad \text{---} \quad (1)$$

$$\Rightarrow k = n - t_{k-1}$$

loop runs k-times

Time complexity $= O(1+1+1+n-t_{n-1})$

but $t_{n-1} = C$ (constant)

$\therefore$ Time complexity $= O(3+n-C)$

$= O(n)$

Que-7: Time Complexity of

```
void function (int n)
{ int i,j,k count=0;
  for (i= n/2; i<=n; i++)
     for (j=1; j<=n; j=j*2)
        for(k=1, k<=n; k=k*2)
          count++
}.
```

Solⁿ

$i \to n/2, \frac{n+2}{2}, \frac{n+4}{2}, \frac{n+6}{2}, \text{-----}$ upto n

$= \frac{n+0\times2}{2} + \frac{n+1\times2}{2} + \frac{n+2\times2}{2}, \text{---}$ upto n

General form.

$$t_k = \frac{n+k*2}{2}$$

total terms $= k+1$

$$t_{k+1} = n$$

$$\Rightarrow \frac{n+(k+1)*2}{2} = n$$

$$n+2k+2 = 2n$$

$$2k = n-2$$

$$k = \frac{n}{2} - 1$$

i	j	k
$n/2$	$\log_2 n$ times	$(\log_2 n)^2$
$\frac{n+2}{2}$	$\log_2 n$ times	$(\log_2 n)^2$
⋮	⋮	
n	$\log_2 n$ times	$(\log_2 n)^2$
$(\frac{n}{2}-1)$ times		

$$\Rightarrow \left(\frac{n}{2}-1\right)(\log_2 n)^2$$

$$= O\left(\frac{n}{2}\log_2^2 n - \log_2 n\right)$$

$$= O(n\log^2 n)$$

Que-6 Time complexity of -

```
void function(int n) {            — O(1)
O(1) — int i, count = 0;
    for (i=1; i*i<=n; i++)
        count++; — O(1)
}.
```

Sol^n.

$i*i$

1^2

2^2

3^2

4^2

⋮

n

$$i*i = \underbrace{1^2, 2^2, 3^2, 4^2, \ldots n}_{K \text{ terms}}$$

$\Rightarrow$ Kth term, $t_K = K^2$

$K^2 = n$

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$$\text{Time complexity} = O(1+1+1+n^{1/2})$$
$$= O(n^{1/2})$$
$$= O(\sqrt{n})$$

Ques-8: Time complexity of

```
function(int n) {
    if (n==1) return;        — O(1)
    for (i=1 to n) {         — O(n)
        for (j=1 to n) {     — O(n)
            printf("*");     — O(1)
        }
    }
    function(n-3);
```

Sol^n: for function call

$$\underbrace{n, n-3, n-6, n-9 \ldots\ldots 1}_{K \text{ terms}}$$

AP with $d = -3$, $a = n$

$a_n = a+(n-1)d$

$1 = n+(K-1)(-3)$

$\frac{1-n}{(-3)} = K-1$

$K-1 = \frac{n-1}{3}$

$K = \frac{n-1+3}{3}$

$\boxed{K = \frac{n+2}{3}}$

3,

Hence function have a recursive call $\frac{n+2}{3}$ times

$\Rightarrow$ Time Complexity $= \left(\frac{n+2}{3}\right)(n)(n)$

$= O(n^3)$

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Ans-9 Time Complexity of

```
void function(int n) {
    for(i=1 to n) {
        for(j=1; j<=n; j=j+i)
            printf("*");
    }
}
```

Sol^n for $i=1 \rightarrow j = 1, 2, 3, 4, \ldots n = n$

for $i=2 \rightarrow j = 1, 3, 5, 7, \ldots n = n/2$

for $i=3 \rightarrow j = 1, 4, 7, \ldots n = n/3$

for $i=n \Rightarrow j = 1, \ldots, n = 1$

$\Rightarrow \sum_{j=n}^{1} n + n/2 + n/3 + n/4 + \ldots + 1$

$\sum_{j=n}^{1} n\left[1 + 1/2 + 1/3 + \ldots + 1/n\right]$

$\sum_{j=n}^{1} n\log n$

$T(n) = [n\log n]$

$T(n) = O(n\log n)$

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Ques-10 for the functions, n^k and c^n, what is the asymptotic notation relationship between these functions. Assume that k>=1 and c>1 are constants. find out the value of c and n_0 for which relation holds.

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Sol:-

As given n^K and c^n

relation b/w n^K and c^n is $\boxed{n^K = O(c^n)}$

as $n^K \leq a c^n$ $\quad \forall\, n \geq n_0$ for a constant $a>0$

for $n_0 = 1$

$c = 2$

$\Rightarrow \quad 1^K \leq a 2^1$

$\therefore \quad \boxed{n_0 = 1, \text{ and } \ c = 2}$