1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Sol^n.

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$$a \geq 1, \; b > 1$$

On comparing

$$a = 3, \; b = 2, \; f(n) = n^2$$

Now,

$$C = \log_b a = \log_2 3 = 1.584$$

$$n^c = n^{1.584} < n^2$$

$$\therefore \; f(n) > n^c$$

$$\therefore \; T(n) = \theta(n^2)$$

2. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

Sol^n:

$$a \geq 1, \; b > 1$$

$$a = 4, \; b = 2, \; f(n) = n^2$$

$$c = \log_2 4 = 2$$

$$\therefore \; n^c = n^2 = f(n) = n^2$$

$$\therefore \; T(n) = \theta(n^2 \log_2 n)$$

5. $T(n) = 16T\left(\frac{n}{4}\right) + n$

Sol^n: $a = 16, \; b = 4$

$$f(n) = n$$

$$c = \log_4 16 = \log_4 (4)^2 = 2$$

$$n^c = n^2$$

$$f(n) < n^c$$

$$\therefore \; T(n) = \theta(n^2)$$

3. $T(n) = T\left(n/2\right) + 2^n$

Sol^n: $\quad a = 1$

$$b = 2$$

$$f(n) = 2^n$$

$$c = \log_b a = \log_2 1 = 0$$

$$n^c = n^0 = 1$$

$$f(n) > n^c$$

$$T(n) = \theta(2^n)$$

4. $T(n) = 2^n T\left(n/2\right) + n^n$

Sol^n: $\quad a = 2^n$

$$b = 2, \; f(n) = n^n$$

$$c = \log_b a = \log_2 2^n$$

$$= n$$

$$n^c \Rightarrow n^n$$

$$\therefore \; f(n) = n^c$$

$$\therefore \; T(n) = \theta(n^2 \log_2 n)$$

6. $T(n) = 2T\left(n/2\right) + n\log n$

Sol^n: $\quad a = 2, \; b = 2$

$$f(n) = n\log n$$

$$c = \log_2 2 = 1$$

$$\therefore \; n^c = n^1 = n$$

Since, $n\log n > n$

$$\therefore \; f(n) > n^c$$

$$\therefore \; T(n) = \theta(n\log n)$$

7. $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

Sol^n: $a = 2, \ b = 2, \ f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $\dfrac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

---

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Sol^n: $a = 2, \ b = 4, \ f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

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9. $T(n) = 0.5 T\left(\frac{n}{2}\right) + 1/n$

Sol^n: $a = 0.5, \ b = 2$

Since acc. to Master theorem

$a \geq 1$, but here a is 0.5

so we cannot apply master theorem.

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10. $T(n) = 16 T\left(\frac{n}{4}\right) + n!$

Sol^n: $a = 16, \ b = 4, \ f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

Now, $n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \theta(n!)$

---

11. $4T\left(n/2\right) + \log n$

Sol^n:

$a = 4, \ b = 2, \ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

Since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n^c)$

$\quad = \theta(n^2)$

---

12. $T(n) = \text{sqrt}(n) T\left(n/2\right) + \log n$

Sol^n: $a = \sqrt{n}, \ b = 2$

$\therefore c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(f(n))$

$\quad = \theta(\log(n))$

13. $T(n) = 3T(n/2) + n$

Sol^n $a = 3, b = 2, f(n) = n$

14. $T(n) = 3T(n/3) + sqrt(n)$

Sol^n $a = 3, b = 3$

$c = \log_b a = \log 3 = 1$

19. $T(n) = 4T(n/2) + n\log n$

Sol^n $a = 4, b = 2, f(n) = \dfrac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore \dfrac{n}{\log n} < n^2$

$\therefore T(n) = \theta(n^2)$

20. $T(n) = 64T(n/8) - n^2\log n$

Sol^n $a = 64, b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$\therefore n^c = n^2$

$\therefore n^2\log n > n^2$

$\therefore T(n) = \theta(n^2\log n)$

21. $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$\Rightarrow n^{1.7712} < n^2$

$\therefore T(n) = \theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

Sol^n $a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \theta(n(2 - \cos n))$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$