Assignment 1

EC3501-S2 (2024-25)

Instructions

Attempt a minimum of 7 questions from Part A, minimum of 5 questions from Part B, and a minimum of 3 questions from Part 3. Note that any one of Q.9 and Q.10 in Part A is compulsory to attempt.

Part A

Q.1 Find the spectrum for the waveform $w(t) = \exp\Big(-\pi(t/T)^2\Big)$ in terms of the parameter T. What can we say about the width of $w(t)$ and $W(f)$ as T increases?

Q.2 Using the convolution property, find the spectrum for

$$w(t) = \sin(2\pi f_1 t) \cdot \sin(2\pi f_2 t).$$

Q.3 If $w(t)$ has the Fourier transform

$$W(f) = \frac{\jmath 2\pi f}{1 + \jmath 2\pi f}$$

find $X(f)$ for the following waveforms:

(a) $x(t) = w(2t + 2)$

(b) $x(t) = \exp(-\jmath t)w(t - 1)$.

Q.4 Find the Fourier transforms for the following waveforms. Plot the waveforms and their magnitude spectra.

(a) $\Pi\left(\frac{t-3}{4}\right)$

(b) 2

(c) $\Delta\left(\frac{t-5}{5}\right)$.

Q.5 Prove that

(a) If $kw(t)$ (k is a constant) is real and an even function of t, $W(f)$ is real.

(b) If $kw(t)$ (k is a constant) is real and an odd function of t, $W(f)$ is imaginary.

Q.6 Show that

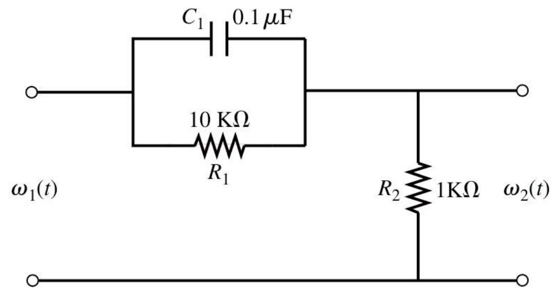
$$\text{sgn}(-f) \longleftrightarrow \frac{1}{\jmath\pi t},$$

that is, $\text{sgn}(-f)$ and $\frac{1}{\jmath\pi t}$ constitute a FT pair.

Q.7 Find the spectrum of $g(t) = A \cdot \Pi(t/T)\sin(2\pi f_0 t)$

Q.8 Plot the amplitude and phase response for the transfer function

$$H(f) = \frac{\jmath 10 f}{5 + \jmath f}.$$



Q.9 Given the filter shown in the above Figure, where $w_1(t)$ and $w_2(t)$ are voltage waveforms,

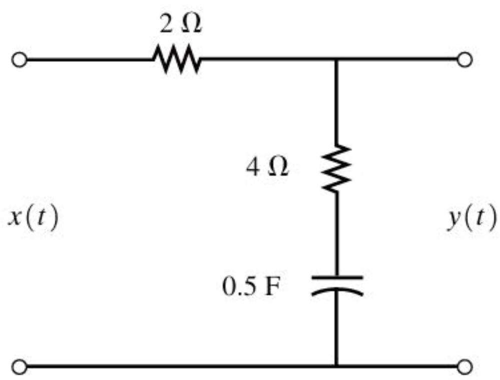
(a) Find the transfer function

(b) Determine the power transfer function.

Q.10 A signal with a PSD of

$$P_x(f) = \frac{2}{(1/4\pi)^2 + f^2}$$

is applied to the network shown in the following figure



(a) Find the PSD for $y(t)$

(b) Find the average normalized power for $y(t)$.

Q.11 Show that the average normalized power of a waveform can be found by evaluating the autocorrelation at $\tau = 0$. That is, $P = R_w(0)$.

Q.12 (a) Find the Hilbert transform of $\cos(2\pi f_m t)$.
(b) If the Hilbert transform of a signal $x(t)$ is represented by $x_h(t)$, then what would the Hilbert transform of $x_h(t)$ in terms of $x(t)$?

Part B

Q.1 An AM transmitter is modulated with an audio testing signal given by $m(t) = 0.3 \sin \omega_1 t + 0.6 \cos \omega_2 t$, where $f_1 = 1000$ Hz, $f_2 = 1000\sqrt{2}$ Hz, and $A_c = 100$. Assume that the AM signal is fed into a 50-Ω load.

(a) Sketch the AM waveform.

(b) What is the modulation percentage (modulation index)?

(c) Evaluate and sketch the spectrum of the AM waveform.

Q.2 For the AM signal given in Problem Q1,

(a) Evaluate the average power of the AM signal.

(b) Evaluate the peak envelope power (PEP) of the AM signal. (*Refer to Example SA4-4 on page 323 of Couch's book.*)

Q.3 A DSB-SC signal is modulated by $m(t) = \cos \omega_1 t + 2 \cos 2\omega_1 t$, where $\omega_1 = 2\pi f_1$, $f_1 = 1000$ Hz, and $A_c = 1$.

(a)Write an expression for the DSB-SC signal and sketch a picture of this waveform.

(b) Evaluate and sketch the spectrum for this DSB-SC signal.

(c) Find the value of the average (normalized) power.

(d) Find the value of the PEP (normalized).

Q.4 Assume that transmitting circuitry restricts the modulated output signal to a certain peak value, say, A_p, because of power-supply voltages that are used and because of the peak voltage and current ratings of the components. If a DSB-SC signal with a peak value of A_p is generated by this circuit, show that the sideband power of this DSB-SC signal is four times the sideband power of a comparable AM signal having the same peak value A_p that could also be generated by this circuit.

Q.5 The signal $x(t) = 4 \sin(\pi/2\, t)$ is transmitted by DSB-AM with $\mu = 1$. Draw the phasor diagram. What is the minimum amplitude of the carrier such that phase reversals don't occur?

Q.6 a) Explain the working principle of envelope detector in AM demodulation.

b) For an AM signal $s(t) = 2\cos(4400\pi t) + 8\cos(4200\pi t) + 2\cos(4000\pi t)$,

i. Provide canonical AM representation.

ii. Find the respective message signal frequency, carrier signal frequency, and modulation idex.

iii. Compute sideband power and the power efficiency.

Q.7 For message signal $x(t) = B\,\mathrm{sinc}(\pi Bt)$ with $B = 1000$ Hz and carrier signal of frequency 10 kHz, do the following tasks

(a) Sketch spectrum of signal $x(t)$ and DSB-SC signal $2x(t)\cos(2\pi f_c t)$.

(b) Find the LSB spectrum by suppressing the USB spectrum from the DSB-SC spectrum.

(c) Find the corresponding time domain representation of the LSB spectrum.

Q.8 An SSB-AM transmitter is modulated with a sinusoid $m(t) = 10\cos\omega_1 t$, where $\omega_1 = 2\pi f_1$, $f_1 = 1000$ Hz, and $A_c = 1$.

(a) Evaluate $m_h(t)$, the Hilbert transform of $m(t)$.

(b) Find the expression for a lower SSB signal.

(c) Find the RMS value of the SSB signal.

(d) Find the peak value of the SSB signal.

(e) Find the normalized average power of the SSB signal.

(f) Find the normalized PEP of the SSB signal.

Q.9 A USSB transmitter is modulated with the pulse

$$m(t) = \frac{\sin(\pi a t)}{\pi a t}$$

(a) Prove that

$$m_h(t) = \frac{\sin^2[(\pi a/2)t]}{(\pi a/2)t}$$

(b) Plot the corresponding USSB signal waveform for the case of $A_c = 1$, $a = 2$, and $f_c = 20$ Hz.

Q.10 A USSB-AM signal is modulated by a rectangular pulse train:

$$m(t) = \sum_{n=-\infty}^{\infty} \prod\left[\frac{t - nT_0}{T}\right]$$

Here, $T_0 = 2T$.
(a) Find the expression for the spectrum of the SSB-AM signal.
(b) Sketch the magnitude spectrum, $|S(f)|$.

Q.11 A modulated signal is described by the equation

$$s(t) = 5\cos[\big(2\pi \times 10^6 t + 5\cos(2\pi \times 500t)\big)]$$

Find each of the following:

(a) Percentage of AM.

(b) Normalized power of the modulated signal.

(c) Carrier signal frequency and message signal frequency.

(d) Frequency modulation index and FM bandwidth.

<u>Part C</u>

Q.1 Find the Fourier transform of AM, NBFM, and NBPM signals considering $x(t) = \cos(400\pi t)$, carrier amplitude, $A_c = 2$, AM sensitivity, $k_a = 1/2$, FM sensitivity, $k_f = 10$, and PM sensitivity, $k_p = 10/\pi$. Also draw the corresponding line spectra with proper annotations.

Q.2 Design an Armstrong indirect FM modulator to generate an FM carrier with a carrier frequency of 94.0 MHz and frequency deviation, $\Delta f = 50$ kHz. A narrowband FM generator is available at a carrier frequency of 200 kHz and an adjustable frequency deviation Δf in the range of 20 to 40 Hz. The stock room also has an oscillator with an adjustable frequency in the range of 2 to 15 MHz. There are also plenty of frequency doublers and triplers available.

Q.3 An FM signal has sinusoidal modulation with a frequency of $f_m = 15\,\text{kHz}$ and a modulation index of $\beta = 5.0$.

Find each of the following:

(a) Find the transmission bandwidth by using Carson's rule.

(b)What percentage of the total FM signal power lies within the Carson rule bandwidth?

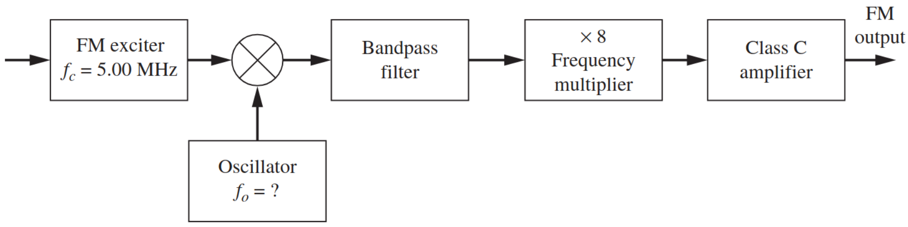
Q.4 An FM transmitter has the block diagram shown in Fig. The audio frequency response is flat over the 20-Hz-to-15-kHz audio band. The FM output signal is to have a carrier frequency of 100.1 MHz and a peak deviation of 75 kHz.

Find each of the following:

(a) Find the bandwidth and center frequency required for the bandpass filter.

(b) Calculate the frequency f_0 of the oscillator.

(c) What is the required peak deviation capability of the FM exciter?



Q.5 A carrier $s(t) = 100\cos(2\pi \times 10^6 t)$ of an FM transmitter is modulated with a tone signal. For this transmitter, a 1-V (RMS) tone produces a deviation of 30 kHz. Determine the amplitude and frequency of all FM signal components (spectral lines) that are greater than 1% of the unmodulated carrier amplitude if the modulating signal is $m(t) = 2.5\cos(3\pi \times 10^4 t)$.

Definitions

- The ***peak envelope power (PEP)*** is the average power that would be obtained if $|s(t)|$ were to be held constant at its peak value, where $s(t)$ is the modulated signal.
- The ***normalized PEP*** for canonical AM is given by

$$P_{PEP} = \frac{1}{2}[\max|s(t)|]^2 = \frac{1}{2}A_c^2[1+\max x(t)]^2$$