

# CS669-Pattern Recognition

## Assignment - 4

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# **Bayes Classifier Using GMM On Reduced Dimensions Obtained Using PCA**

**Group 6**

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## 1. Objective :

- 1.1. Build Bayes classifier using Gaussian mixture model (GMM) with 1, 2, 4 and 8 mixtures on the reduced dimensional representations of Dataset-2 obtained using PCA.
- 1.2. Find confusion matrix for each case (for different datasets).
- 1.3. Calculate classification accuracy, precision for every class, mean precision, recall for every class, mean recall, F-measure for every class and mean F-measure.
- 1.4. Plot density contour for each class with data points superposed.
- 1.5. Plot decision region for each pair of classes and together for all classes.

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## 2. Procedure :

- 2.1. Separate given data files of each class into training and test data files. Training data consists of 75% and test data consists of 25% of given data set.
- 2.2. To apply Bayesian classification, assume that data of all classes -follow Gaussian distribution.
- 2.3. Convert all the image data given in dataset 2b into 32 dimension BOVW (Bag of Visual Words) representation.
- 2.4. KMeans and GMM is already applied on this data in assignment 2.
- 2.5. Convert this 32-Dimensional data into l-dimensional data where l is the reduced dimension of the given data.
- 2.6. Use principal component analysis (PCA) to find the reduced dimensions of the given data.
- 2.7. Eigen values and eigen vectors are calculated to find the values of lambda and transformation vector.
- 2.8. Transformation vector is the vector which is when imposed on a d-dimensional data will convert the data into l-dimensional data.
- 2.9. Gaussian Mixture Model (GMM) and KMeans are applied on the corresponding reduced dimensional data.

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**3. Procedure For finding Principal component analysis (PCA) :**

- 3.1. Suppose  $x$  be the given datapoint which is a high dimension ( $d$ ) given data and  $a$  be the datapoint which is converted into some reduced dimension  $l$ .
- 3.2. Now find transformation vector (directional) which will be an orthonormal vector.
- 3.3. Consider  $Y$  be the mean subtracted representation of  $x$  which is the given datapoint.
- 3.4. Reduced dimensional datapoint will be calculated by using the multiplication of transpose of transformation vector and mean subtracted representation of datapoint.
- 3.5. It involve finding  $l$  directions of projection in such a way that it minimizes the error between the original representation and the approximated representation and summed over all the training examples.
- 3.6. With a constraint that each direction of projection should be orthonormal and the error is denoted by  $J$ .
- 3.7. After solving for  $J$  we get a constrained optimization problem to find the minimum of  $J$  with a constraint that each direction of projection should be orthonormal.
- 3.8. Lagrangian method is used to solve this constrained optimization problem.
- 3.9. After solving using lagrangian we get corresponding eigen vectors and corresponding eigen values.

#### 4. Observations :

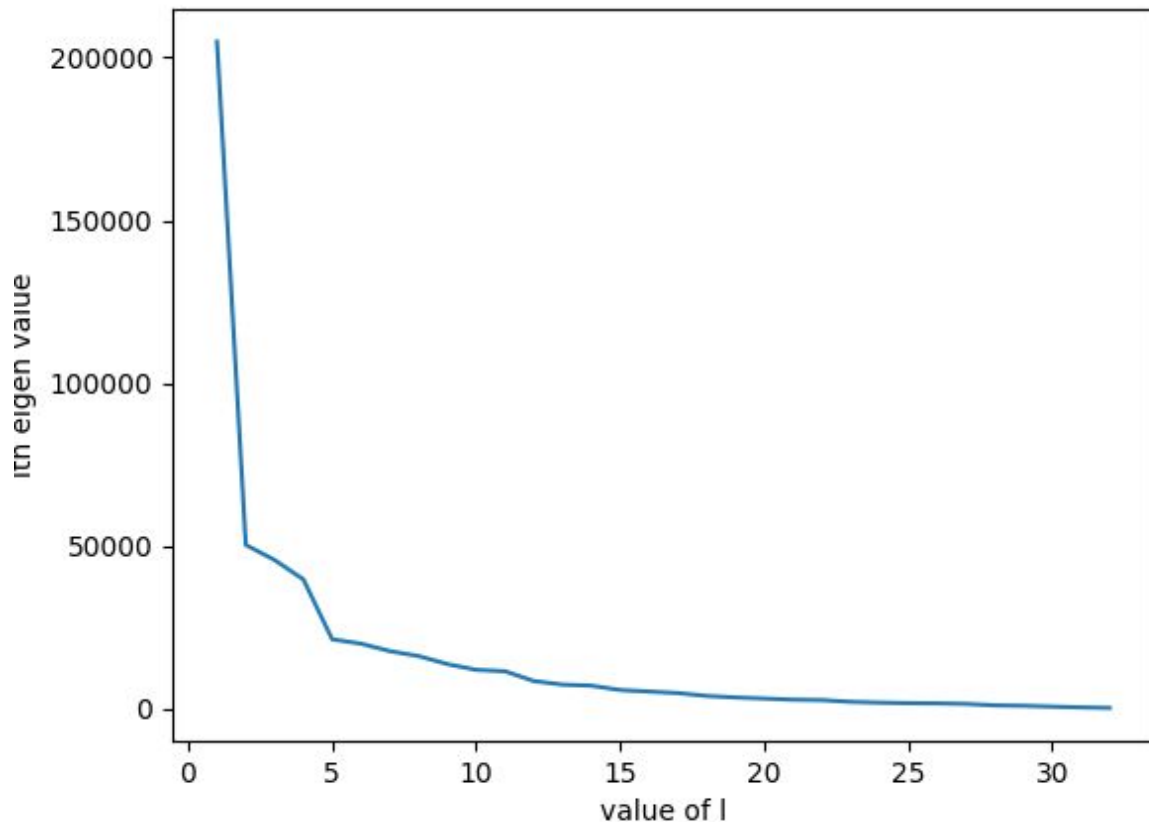


Fig : 1 Plot of Eigen Values in decreasing order

#### 32 Eigen Values in decending order are :

204911.42371983654, 50274.91291991595, 45639.69295370778,  
39675.362010630124, 21268.430981828078, 19931.34138831108,  
17634.54680157979, 16124.52098115722, 13660.63758481213,  
11895.450541444494, 11460.41927652893, 8453.085282423832,  
7352.235022807325, 7027.20930673038, 5704.391257423952,  
5232.30867694192, 4760.287042049027, 3879.8117750317324,  
3409.3178648779353, 3077.767521890299, 2722.0924997674806,  
2597.8594734935054, 2073.458682076009, 1830.7543815684292,  
1663.6154336300363, 1592.2603827135706, 1436.4033748555703,

985.1642662749077, 853.9220119100451, 600.1328490193212,  
350.0996571833756, 183.31203744581555

**a) K = 1, L = 1: Accuracy : 62.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	9	2	39
Class 2	1	41	8
Class 3	5	1	44

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.18	0.82	0.88	0.6266
Precision	0.6	0.9318	0.4835	0.8448
F-Measure	0.2769	0.8723	0.6241	0.5911

**b) K = 1, L = 2: Accuracy : 38.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	16	0	34
Class 2	2	1	47
Class 3	9	0	41

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.32	0.02	0.82	0.3866
Precision	0.5925	1.0	0.3360	0.6428
F-Measure	0.4155	0.0392	0.4767	0.3105

**c) K = 1, L=3: Accuracy : 35.3333333333 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	11	0	39
Class 2	1	2	47
Class 3	10	0	40

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.22	0.04	0.8	0.3533
Precision	0.5	1.0	0.3174	0.6058
F-Measure	0.3055	0.0769	0.4545	0.2790

**d) K = 1, L=4: Accuracy : 34.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	8	0	42
Class 2	0	0	50
Class 3	6	0	44

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.16	0.0	0.88	0.3466
Precision	0.5714	nan	0.3235	nan
F-Measure	0.25	nan	0.4731	nan

e) **K = 1, L=5: Accuracy : 33.3333333333 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	0	0	50
Class 2	0	0	50
Class 3	0	0	50

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.0	0.0	1.0	0.3333
Precision	nan	nan	0.3333	nan
F-Measure	nan	nan	0.5	nan

f) **K = 2, L = 1: Accuracy : 56.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	13	2	35
Class 2	2	28	20
Class 3	5	1	44

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.26	0.56	0.88	0.5666
Precision	0.65	0.9032	0.4444	0.6658
F-Measure	0.3714	0.6913	0.5906	0.5511

g) **K = 2, L = 2: Accuracy : 50.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	29	0	21
Class 2	1	1	48
Class 3	3	1	46

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.58	0.02	0.92	0.5066
Precision	0.8787	0.5	0.4	0.5929
F-Measure	0.6987	0.0384	0.5575	0.4316



**h) K = 2, L = 3: Accuracy : 44.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	20	1	29
Class 2	1	2	47
Class 3	5	0	45

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.4	0.04	0.9	0.4466
Precision	0.7692	0.6666	0.3719	0.6025
F-Measure	0.5263	0.0754	0.5263	0.3760

**i) K = 2, L = 4: Accuracy : 38.0 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	10	1	39
Class 2	0	0	50
Class 3	3	0	47

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.2	0.0	0.94	0.38
Precision	0.7692	0.0	0.3455	0.3716
F-Measure	0.3174	0.0	0.5053	0.2742

**j) K = 2, L = 5: Accuracy : 44.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	15	0	35
Class 2	0	2	48
Class 3	0	0	50

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.3	0.04	1.0	0.4466
Precision	1.0	1.0	0.3759	0.7919
F-Measure	0.4615	0.0769	0.5464	0.3616

**k) K = 4, L = 1: Accuracy : 75.3333333333 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	34	2	14
Class 2	1	37	12
Class 3	7	1	42

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.68	0.74	0.84	0.7533
Precision	0.8095	0.925	0.6176	0.7840
F-Measure	0.7391	0.8222	0.7118	0.7577

**l) K = 4, L = 2: Accuracy : 52.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	38	0	12
Class 2	1	0	49
Class 3	9	0	41

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.76	0.0	0.82	0.5266
Precision	0.7916	nan	0.4019	nan
F-Measure	0.7755	nan	0.5394	nan

**m) K = 4, L = 3: Accuracy : 49.3333333333 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	30	0	20
Class 2	0	1	49
Class 3	7	0	43

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.6	0.02	0.86	0.4933
Precision	0.8108	1.0	0.3839	0.7315
F-Measure	0.6896	0.0392	0.5308	0.4199

**n) K = 4, L = 4: Accuracy : 42.0 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	21	0	29
Class 2	1	1	48
Class 3	9	0	41

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.42	0.02	0.82	0.42
Precision	0.6774	1.0	0.3474	0.6749
F-Measure	0.5185	0.0392	0.4880	0.3486

**o) K = 4, L = 5: Accuracy : 36.6666666667 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	4	0	46
Class 2	0	1	49
Class 3	0	0	50

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.08	0.02	1.0	0.3666
Precision	1.0	1.0	0.3448	0.7816
F-Measure	0.1481	0.0392	0.5128	0.2333

**p) K = 8, L = 1: Accuracy : 71.3333333333 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	40	2	8
Class 2	1	35	14
Class 3	17	1	32

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.8	0.7	0.64	0.7133
Precision	0.6896	0.9210	0.5925	0.7344
F-Measure	0.7407	0.7954	0.6153	0.7171

**q) K = 8, L = 2: Accuracy : 33.3333333333 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	0	0	50
Class 2	0	0	50
Class 3	0	0	50

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.0	0.0	1.0	0.3333
Precision	nan	nan	0.3333	nan
F-Measure	nan	nan	0.5	nan

**r) K = 8, L = 3: Accuracy : 36.0 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	40	0	10
Class 2	2	1	47
Class 3	37	0	13

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.8	0.02	0.26	0.36
Precision	0.5063	1.0	0.1857	0.5640
F-Measure	0.6201	0.0392	0.2166	0.2920

s) **K = 8, L = 4: Accuracy : 33.3333333333 %**

Confusion Matrix

	Class 1	Class 2	Class 3
Class 1	0	0	50
Class 2	0	0	50
Class 3	0	0	50

Analysis

	Class 1	Class 2	Class 3	Mean
Recall	0.0	0.0	1.0	0.3333
Precision	nan	nan	0.3333	nan
F-Measure	nan	nan	0.5	nan

## 5. Conclusion

- 1) From the above data, it can be seen that as we increase the reduced dimension( $l$ ) accuracy decreases in most of the cases.
- 2) In some cases, value of precision and recall is nan because zero number of test examples are classified in one or more classes.
- 3) For larger values of  $K$ , results are not that good because chances of only one data point in each cluster increases due to which the determinant becomes zero and nan values occurs in covariance matrix.
- 4) It is observed from above results that most of the test examples are classified in class 3.
- 5) It can be observed from the plot of eigen values that after certain no. of dimensions difference between two consecutive eigen values becomes very less.

