Tutorial – 3 (Solutions)

Q.1

$$x_1(t) = \begin{cases} 2A & 0 \le t \le T_0/2 \\ 0 & T_0/2 < t \le T_0 \end{cases} \quad \text{[Considering one period]}$$

$$\therefore\ C_{01} = \frac{1}{T}\int_T x(t)\,dt = \frac{2A}{T_0}\int_0^{T_0/2} dt = A$$

Now, $$C_{k1} = \frac{2A}{T_0}\int_0^{T_0/2} e^{-jk\omega_0 t}\,dt = \frac{A}{jk\pi}\left[1 - e^{-jk\pi}\right]$$

$$\Rightarrow C_{k1} = \frac{A}{jk\pi}\left[1 - (-1)^k\right]$$

This implies that $\rightarrow$ $C_{k1} = 0$ for $k_1 = 2m$ (even)

$$C_{k1} = \frac{2A}{jk\pi} = \frac{2A}{j\pi(2m+1)} \quad \text{for } k_1 = 2m+1.$$

Hence, $$\boxed{C_{01} = A\ ;\quad C_{2m} = 0\ ;\quad C_{2m+1} = \frac{2A}{j\pi(2m+1)}}$$

Thus, we can write: $$x_1(t) = A + \sum_{m=-\infty}^{+\infty} \frac{2A}{j\pi(2m+1)} \cdot e^{j(2m+1)\omega_0 t} \quad \text{—— (1)}$$

* It can be verified that:

$$x_2(t) = x_1(t) - A.$$

$$\Rightarrow x_2(t) = \frac{2A}{j\pi}\sum_{m=-\infty}^{+\infty} \frac{1}{2m+1} e^{j(2m+1)\omega_0 t} \quad \text{—— (2)}$$

Thus $$\boxed{C_{02} = 0\ ;\quad C_{k2} = C_{2m+1} = \frac{2A}{j\pi(2m+1)}}$$

* Now, $$x_3(t) = \sum_{k_3} C_{k3}\, e^{j\omega_0 k_3 t} \quad \text{—— (3)}$$

$$x_3'(t) = \sum_{k_3} C_{k3}\, j\omega_0 k_3\, e^{j\omega_0 k_3 t} \quad \text{—— (4)}$$

Contd.

If we observe carefully we can observe the relationship between $x_2(t)$ & $x_3(t)$:

$$\frac{2}{T_0} x_2(t) = x_3'(t) \quad \left[x_3'(t) = \text{1st derivative w.r.t } t \right].$$

$$\Rightarrow x_2(t) = \frac{2A}{j\pi} \sum_{m=-\alpha}^{+\alpha} \frac{1}{2m+1} e^{j(2m+1)w_0 t}$$

So, $$x_3'(t) = \frac{4A}{j\pi T_0} \sum_{m=\alpha}^{+\alpha} \frac{1}{2m+1} e^{j(2m+1)w_0 t} \quad \text{—⑤}$$

If we compare equation ④ & ⑤ we get:-

$$C_{k_3} j W_0 k_3 = \frac{4A}{j\pi(2m+1) T_0} \quad [k_3 = 2m+1]$$

$$C_{k_3} = \frac{-4A}{\pi(2m+1)^2 w_0 T_0} = \frac{-4A}{2\pi^2 (2m+1)^2}$$

$$\boxed{C_{k_3} = \frac{-2A}{\pi^2 (2m+1)^2}}$$

Similarly C_{03} can be calculated.

Q.2 Please refer the book "Signals & Systems"

– A.V. OPPENHEIM; A.S. WILLSKY; S.H. NAWAB.

Example - 3.14.

Q.3 $x(t) = \sum_{k=-3}^{+3} a_k e^{j2\pi t}$.

where, $a_0 = 1$, $a_1 = a_{-1} = \frac{1}{4}$, $a_2 = a_{-2} = \frac{1}{2}$, $a_3 = a_{-3} = \frac{1}{3}$.

Thus. $x(t) = 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t$.

Again, $h(t) = e^{-t} u(t)$.

$$H(j\omega) = \int_0^\infty e^{-\tau} e^{-j\omega\tau} d\tau = \frac{1}{1+j\omega}.$$

Thus, $\omega_0 = 2\pi$.

So, $y(t) = \sum_{k=-3}^{+3} b_k e^{jk2\pi t}$

where $b_k = a_k H(jk2\pi)$

So, $b_0 = 1$, $b_1 = \frac{1}{4}\left(\frac{1}{1+j2\pi}\right)$, $b_{-1} = \frac{1}{4}\left(\frac{1}{1-j2\pi}\right)$.

$b_2 = \frac{1}{2}\left(\frac{1}{1+j4\pi}\right)$, $b_{-2} = \frac{1}{2}\left(\frac{1}{1-j4\pi}\right)$

$b_3 = \frac{1}{3}\left(\frac{1}{1+j6\pi}\right)$, $b_{-3} = \frac{1}{3}\left(\frac{1}{1-j6\pi}\right)$

Q.4 Please refer the same book as for Q.2.

– Section 3.11.1

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