IC 260: Signals and Systems (Feb - June, 2017) - End Sem (Marks: 30)

Name: Roll no.:

For questions 1 - 10, provide very short and to-the-point answers.

1. (a) State an equation stating the condition for stability of a system $h(t)$. 0.5 mark
(b) Based on this condition, discuss in brief what part of the complex s-plane should necessarily be included in the ROC of the Laplace transform. 1.5 mark

2. A signal $\cos(4000\pi t)$ is sampled with a frequency of 6 KHz. Plot the continuous frequency spectrum of the sampled signal with proper labels on the x and y axis. 2 marks

3. Provide an example for sampling where the Nyquist criteria is not satisfied. You are expected to sketch the original spectrum, and the spectrum after sampling, and provide the sampling frequency. 2 marks

4. If the Nyquist rate for sampling a signal $x(t)$ is ω, what is the Nyquist rate for $x^2(t)$. Why ? 2 marks

5. If the Nyquist rate for sampling a signal $x(t)$ is ω, what is the Nyquist rate for $x(t)cos(\omega_o t)$. Why ? 2 marks

6. Show that the DTFT of any discrete-time signal is periodic. State the period. 2 marks

7. A right-sided but a non-causal signal is to be transformed to a causal signal by performing some operation. 2 mark
a) Suggest such an operation.
b) How would it affect the ROC ?

8. What is the condition on the poles of Laplace transform for a system which is both stable and causal. Why ? 2 mark
(Hint: Consider your answer in question 1)

9. Prove the time-shifting property of Fourier transform using the convolution property. 2 marks

10. For a discrete-time signal, considering the DTFT, if the region close to $\omega = 0$ corresponds to low-frequency, the region close to which value of ω corresponds to high-frequency ?
1 mark

11. Given the frequency response of a system as

$$H(j\omega) = \frac{j\omega + 2}{-\omega^2 + 4j\omega + 3}$$

(a) Find the impulse response $h(t)$. 3 marks
(b) Provide the differential equation between input and output. 1 mark

12. A causal and stable LTI system has an input $x[n] = (4/5)^n u[n]$ and the corresponding output is $y[n] = n(4/5)^n u[n]$.
(a) Determine the frequency response $H(e^{j\omega})$ for the system S. 2 marks
(b) Determine a difference equation relating the input and output. 1 mark

13. Given a Laplace transform $X(s)$ as

$$X(s) = \frac{s-1}{s^2+3s+2}$$

Provide the various possible time-domain functions which can correspond to this Laplace transform. Justify your answers. 4 marks