IC 260 — SIGNALS & SYSTEMS

TUT - 2.

Q. 1. From the i/p - o/p expressions, we can see e^{-2t} term remains as it is from the i/p to o/p.

Only one function does this thing — delta function.

[If you convolute any fn with delta function the convolution result remains the same as input fn].

The form of $h(t)$ [impulse response of the system]. will be $h(t) = \left[c_1 \delta(t) + c_2 e^{\lambda t} \right] u(t)$

$$\Rightarrow \left[c_1 \delta(t) + c_2 e^{\lambda t} \right] u(t) * \left(2e^{-2t} \right) u(t) = \left[7e^{-2t} - 6e^{-3t} \right] u(t)$$

L.H.S

$$\Rightarrow \left[c_1 \delta(t) \right] u(t) * \left(2e^{-2t} \right) u(t) + \left[c_2 e^{\lambda t} \right] u(t) * \left(2e^{-2t} \right) u(t).$$

$$\Rightarrow \left[2c_1 e^{-2t} \right] u(t) + \left[2c_2 \frac{e^{\lambda t} - e^{-2t}}{\lambda + 2} \right] u(t)$$

$$\Rightarrow \left\{ \left[2c_1 e^{-2t} - \frac{2c_2 e^{-2t}}{\lambda + 2} \right] + \left[\frac{2c_2 e^{\lambda t}}{\lambda + 2} \right] \right\} u(t) = \left[7e^{-2t} - 6e^{-3t} \right] u(t)$$

Comparing L.H.S and R.H.S we get

$$\lambda = -3, \quad c_2 = 3 \quad \text{and} \quad c_1 = 1/2.$$

So, the impulse response of the system is:

$$h(t) = \left[\frac{1}{2} \delta(t) + 3e^{-3t} \right] u(t)$$

Ans.

Q.2 Please refer the book "SIGNALS & SYSTEMS" (2 Ed.)

- ALAN V. OPPENHEM, ALAN S. WILLSKY, S. H. NAWAB.

- section: 2.3.8. (The Unit Step Response of an LTI system).

$$h(t) = s'(t).$$

So, the response for any arbitary input $x(t)$ is-

$$y(t) = x(t) * s'(t)$$

Q.4 Please refer the same book as for Q.2.

- Example 2.4.

Q.3 $(D^2+5D+6)\,y(t) = (D+1)\,x(t)$.

Given $y_0(0^-)=2$, $\dot{y}_0(0^-)=-1$; Find Response for $x(t)=e^{-2t}u(t)$

Here, $Q(D)=D^2+5D+6$; $P(D)=D+1$.

Characteristic polynomial ~~$(D^2+5D+6)=($~~

$$\lambda^2+5\lambda+6=(\lambda+3)(\lambda+2)$$

The roots are $\lambda=-3,-2$.

So, $y_0(t) = c_1 e^{-3t} + c_2 e^{-2t}$ — ①

Using the initial conditions, we get $c_1=-3$, $c_2=5$.

Thus, $\boxed{y_0(t) = -3e^{-3t} + 5e^{-2t}}$ — ②

So, $y_n(t) = \hat{c}_1 e^{-3t} + \hat{c}_2 e^{-2t}$ — ③

Since $N=2$: $y_n(0)=0$ & $\dot{y}_n(0)=1$.

Using these conditions in eqn ③

$$\hat{c}_1=-1;\quad \hat{c}_2=1.$$

So, $\boxed{y_n(t) = \left[-e^{-3t}+e^{-2t}\right]}$ — ④

Again, we know $h(t) = b_0\delta(t) + [P(D)\,y_n(t)]\,u(t)$

Since, order of $Q(D)$ > order of $P(D)$

$$b_0=0.$$

So, $h(t) = \left[(D+1)\left(-e^{-3t}+e^{-2t}\right)\right]u(t)$

$$= \left[3e^{-3t}-2e^{-2t}-e^{-3t}+e^{-2t}\right]u(t)$$

$$\boxed{h(t) = \left[2e^{-3t}-e^{-2t}\right]u(t)}$$

Since e^{-2t} term is present in i/p, resonance will be