Tutorial-4 (Solutions)

Q. 1. (a) and

Please refer the book "SIGNALS & SYSTEMS" (2-Ed) by ALAN V. OPPENHEIM, ALAN. S. WILLSKY, S. H. NAWAB

Example : 4.9 ; 4.12.

(b) One can derive :

$$x(t) = e^{-a|t|} \longleftrightarrow X(j\omega) = \frac{2a}{a^2+\omega^2}$$

Thus, from the synthesis equation from the F.P pair:

$$e^{-a|t|} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2a}{a^2+\omega^2} e^{j\omega t}\, d\omega$$

Now, replacing t by $-t$, we obtain :

$$e^{-a|-t|} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2a}{a^2+\omega^2} e^{-j\omega t}\, d\omega.$$

i.e. $$\frac{\pi}{a} e^{-a|t|} = \int_{-\infty}^{\infty} \frac{1}{a^2+\omega^2} e^{-j\omega t}\, d\omega.$$

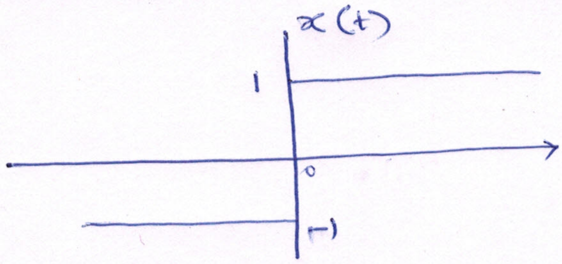
Now, interchanging the names of the variables t & ω:

$$\frac{\pi}{a} e^{-a|\omega|} = \int_{-\infty}^{\infty} \frac{1}{a^2+t^2} e^{-j\omega t}\, dt$$

Thus, we obtain $$\boxed{\frac{1}{a^2+t^2} \xleftrightarrow{F.P.} \frac{\pi}{a} e^{-a|\omega|}}$$

Ans.

Q.1 c)



Here, $x(t) = sgn(t) = \begin{cases} 1 , & t > 0 \\ -1 & t < 0 . \end{cases}$

Now, it can be expressed as:

$$sgn(t) = 2\,u(t) - 1. \quad \text{—} \ (1)$$

$$\frac{d}{dt}\{sgn(t)\} = 2\delta(t) \quad \text{—} \ (2)$$

Let, $x(t) = sgn(t) \longleftrightarrow X(\omega)$

Applying differentiation property on eqn (2):

$$j\omega X(\omega) = 2.$$

$$\Rightarrow \quad X(\omega) = \frac{2}{j\omega}.$$

So, the Fourier transform of the sgn(t) is $\frac{2}{j\omega}$.

Ans.

Q.2 Follow the same book as for Q.1. — Example: 4.18.

Q.3 (a) $$X(\omega_1,\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(-t_1+2t_2)\, u(t_1-1)\, u(2-t_2)\, \exp(-j(\omega_1 t_1+\omega_2 t_2))\, dt_1\, dt_2$$

$$= \int_{1}^{\infty}\int_{-\infty}^{2} \exp(-t_1+2t_2)\exp(-j(\omega_1 t_1+\omega_2 t_2))\, dt_1\, dt_2$$

$$= \int_{1}^{\infty}\int_{-\infty}^{2} \exp\left[-t_1(1+j\omega_1)+t_2(2-j\omega_2)\right] dt_1\, dt_2$$

Integrating it we get:

$$= \frac{e^{-(1+j\omega_1)}}{(1+j\omega_1)} \cdot \frac{e^{2(2-j\omega_2)}}{(2-j\omega_2)} \quad \text{Ans.}$$

(b) $$\int_{-1}^{1}\int_{-1}^{1} \exp(-|t_1|-|t_2|)\exp\left[-j(\omega_1 t_1+\omega_2 t_2)\right] dt_1\, dt_2$$

$$= \int_{-1}^{1} \exp(-|t_1|-j\omega_1 t_1)\, dt_1 \cdot \int_{-1}^{1} \exp(-|t_2|-j\omega_2 t_2)\, dt_2.$$

$$= \left\{\int_{-1}^{0} \exp(t_1-j\omega_1 t_1)\, dt_1 + \int_{0}^{1} \exp(-t_1-j\omega_1 t_1)\, dt_1\right\} \times$$

$$\left\{\int_{-1}^{0} \exp(t_2-j\omega_2 t_2)\, dt_2 + \int_{0}^{1} \exp(-t_2-j\omega_2 t_2)\, dt_2\right\}$$

$$= \frac{4\left[1-\exp\{-(1+j\omega_1)\}\right]\left[1-\exp\{-(1+j\omega_2)\}\right]}{(1+\omega_1^2)\,(1+\omega_2^2)} \quad \text{Ans.}$$

Q.4. Refer the same book as for Q.1 & Q.2 - section 4.3.7.

Q.5. LTI system is described by:

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Taking F.T on both side:

$$j\omega Y(\omega) + 2Y(\omega) = X(\omega)$$

Hence, $H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{2+j\omega}$.

(a) $x(t) = e^{-t} u(t)$

So, $X(\omega) = \frac{1}{1+j\omega}$.

Now, $Y(\omega) = X(\omega) H(\omega) = \frac{1}{1+j\omega} \cdot \frac{1}{2+j\omega}$

$$= \frac{1}{1+j\omega} - \frac{1}{2+j\omega}$$

Therefore, $y(t) = (e^{-t} - e^{-2t}) u(t)$

(b) $x(t) = u(t)$

$$X(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

Again $Y(\omega) = X(\omega) H(\omega)$

$$= \left[\pi\delta(\omega) + \frac{1}{j\omega}\right] \frac{1}{2+j\omega}$$

$$= \pi\delta(\omega) \frac{1}{2+j\omega} + \frac{1}{j\omega(2+j\omega)}$$

$$Y(\omega) = \frac{\pi}{2}\delta(\omega) + \frac{1}{2}\frac{1}{j\omega} - \frac{1}{2}\frac{1}{2+j\omega}$$

$$= \frac{1}{2}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right] - \frac{1}{2}\frac{1}{2+j\omega}$$

Thus $y(t) = \frac{1}{2}u(t) - \frac{1}{2}e^{-2t}u(t)$

$$\boxed{y(t) = \frac{1}{2}\left(1 - e^{-2t}\right)u(t)}$$ Ans.