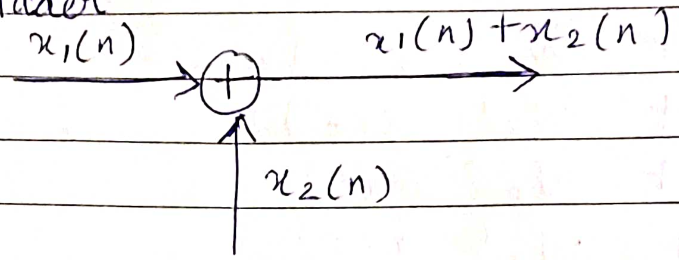
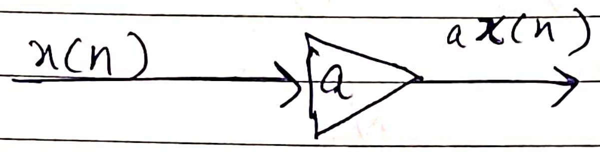
Block Diagram Representation

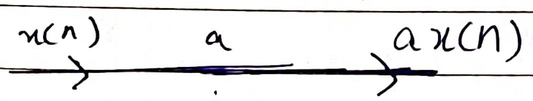
1) Adder



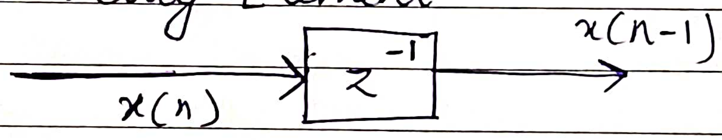
2) Constant Multiplier



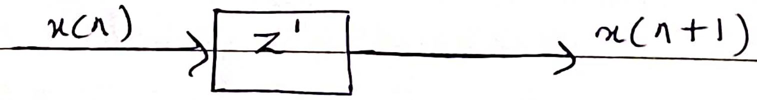
OR



3) Unit Delay Element

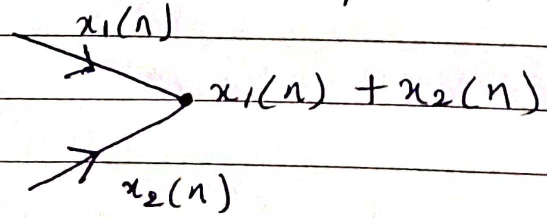


4) Unit Advance Element

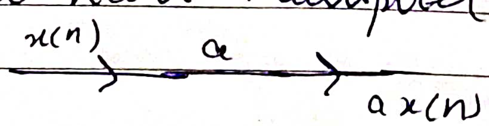


Signal Flow Graph Representation

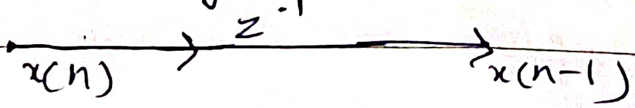
1) Adder



2) Constant Multiplier



3) Unit Delay Element

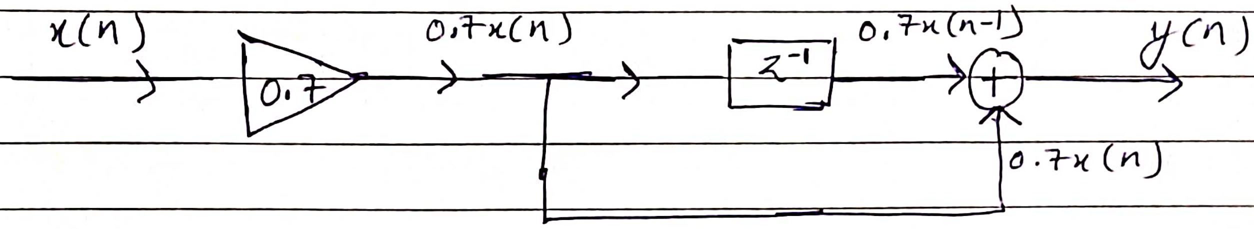


4) Unit Advance Element

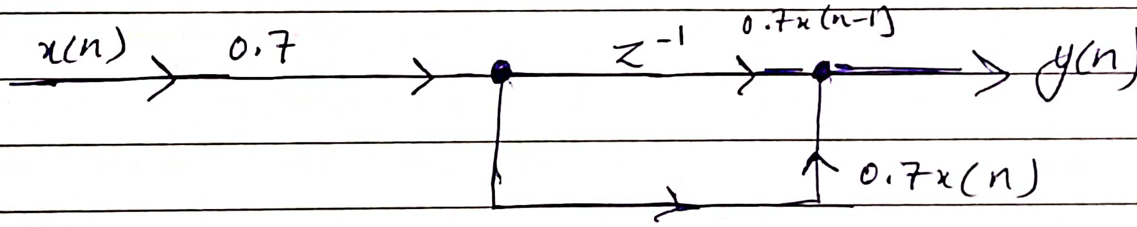
$$x(n) \xrightarrow{z} x(n+1)$$

Que) $y(n) = 0.7\,x(n) + 0.7x(n-1)$

Ans) Block Diagram Representation



Signal Flow Graph Representation



Infinate Impulse Response: [IIR] Filter

EC Academy

→ $y(n)$ depends → $x(n)$ & $x(n-1)$ also on $y(n-1)$

→ Difference eqn

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots + b_M x(n-M) - a_1 y(n-1) - \cdots - a_N y(n-N)$$

→ Transfer fun

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

b_i → $(M+1)$ numerator

a_i → N denominator.

$Y(z)$ & $X(z)$ → Z-Transform of $x(n)$ & $y(n)$

→ Ploes (s) → inside the unit circle → STABLE

→ smaller Filter size.

→ linear phase is not easy

→ Objective → determine the filter numerator & denominator → satisfy Filter specifications

Advantages

→ Easy to design & easy to implement

DisadVantages

→ Non Linear

→ non-stable.

→ Infinate Impulse response.

Subscribe LIKE & Share

Direct Form I Realization of IIR

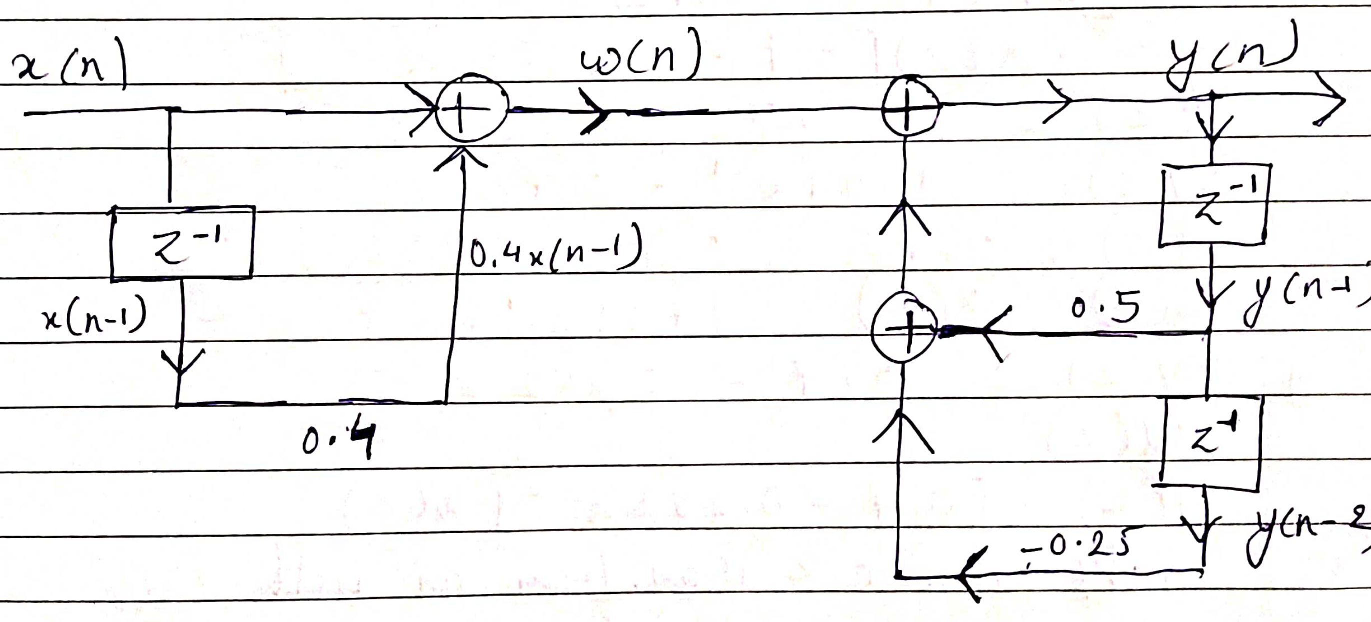
Ques) $y(n) = 0.5\, y(n-1) - 0.25\, y(n-2) + x(n) + 0.4x(n-1)$

Ans) Take x terms together

$x(n) + 0.4\, x(n-1) = \omega(n)$

So, $y(n) = 0.5\, y(n-1) - 0.25\, y(n-2) + \omega(n)$

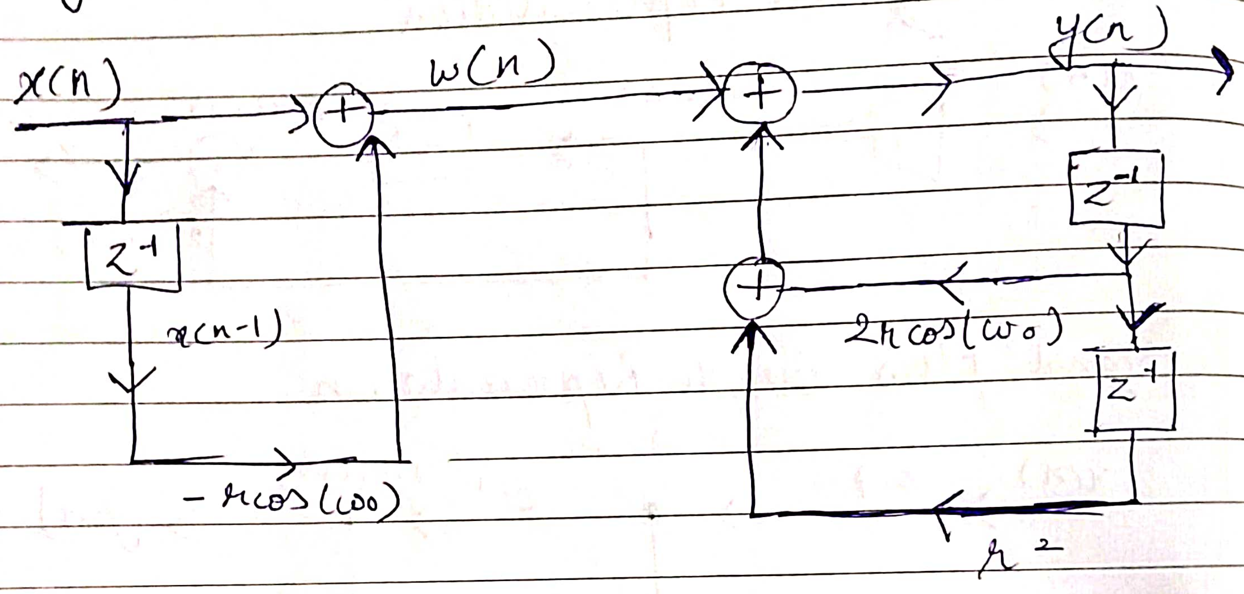
Now Direct form Realization



Ques) $y(n) = 2r\cos(\omega_0)\, y(n-1) + r^2 y(n-2) + x(n) - r\cos(\omega_0)\, x(n-1)$

Ans) $x(n) - r\cos(\omega_0)\, x(n-1) = w(n)$

$y(n) = 2r\cos(\omega_0)\, y(n-1) + r^2 y(n-2) + w(n)$



Direct Form 2 Realization of IIR Filter

Ques) $y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$

Ans) Take z Transform on both sides

$Y(z) = -0.1 z^{-1} Y(z) + 0.72 z^{-2} Y(z) + 0.7 X(z) - 0.252 z^{-2} X(z)$

Take $Y(z)$ on one side and $X(z)$ on other

$Y(z)[1 + 0.1 z^{-1} - 0.72 z^{-2}] = X(z)[0.7 - 0.252 z^{-2}]$

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

$$\frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)} = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

$\Rightarrow$ $$\frac{Y(z)}{W(z)} = 0.7 - 0.252 z^{-2}$$

$Y(z) = [0.7 - 0.252 z^{-2}]\, W(z)$

Take inverse z Transform on both sides

$$y(n) = 0.7\,w(n) - 0.252\,w(n-2) \quad \text{— ①}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+0.1z^{-1}-0.72z^{-2}}$$

$$X(z) = \left[1+0.1z^{-1}-0.72z^{-2}\right] W(z)$$

Take Inverse Z Transform on both sides

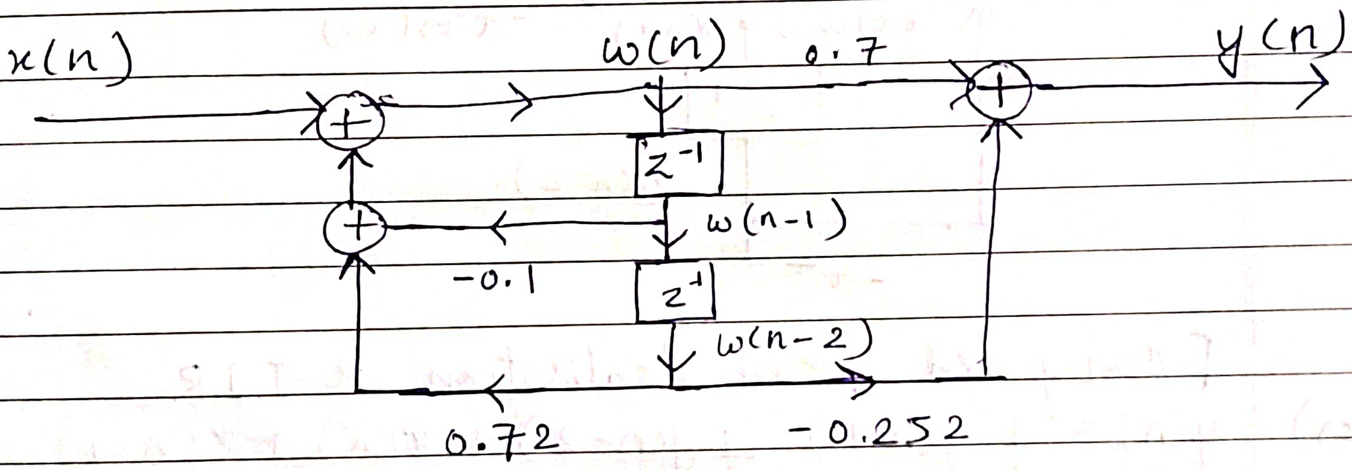
$$x(n) = w(n) + 0.1\,w(n-1) - 0.72\,w(n-2)$$

$$\Rightarrow w(n) = x(n) - 0.1\,w(n-1) + 0.72\,w(n-2) \quad \text{— ②}$$

$$\Rightarrow y(n) = 0.7\,w(n) - 0.252\,w(n-2) \quad \text{— ①}$$

$$\cancel{x(n) = w(n) + 0.1\,w(n-1) - 0.72\,w(n-2)}$$

$$w(n) = x(n) - 0.1\,w(n-1) + 0.72\,w(n-2) \quad \text{— ②}$$



Ques) $y(n) = 2r\cos(\omega_0)\,y(n-1) - r^2 y(n-2) + x(n) - r\cos(\omega_0)\,x(n-1)$

Ans) $Y(z) = 2r\cos(\omega_0) z^{-1} Y(z) - r^2 z^{-2} Y(z) + X(z) - r\cos(\omega_0) z^{-1} X(z)$

$$\Rightarrow Y(z)\left[1 - 2r\cos(\omega_0) z^{-1} + r^2 z^{-2}\right] = \left[1 - r\cos(\omega_0) z^{-1}\right] X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1 - r\cos(\omega_0) z^{-1}}{1 - 2r\cos(\omega_0) z^{-1} + r^2 z^{-2}} = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)}$$

$$\frac{Y(z)}{W(z)} = 1 - r\cos(\omega_0) z^{-1}$$

$$\Rightarrow y(n) = w(n) - r\cos(\omega_0)\,w(n-1) \quad \text{— ①}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - 2r\cos(\omega_0) z^{-1} + r^2 z^{-2}}$$

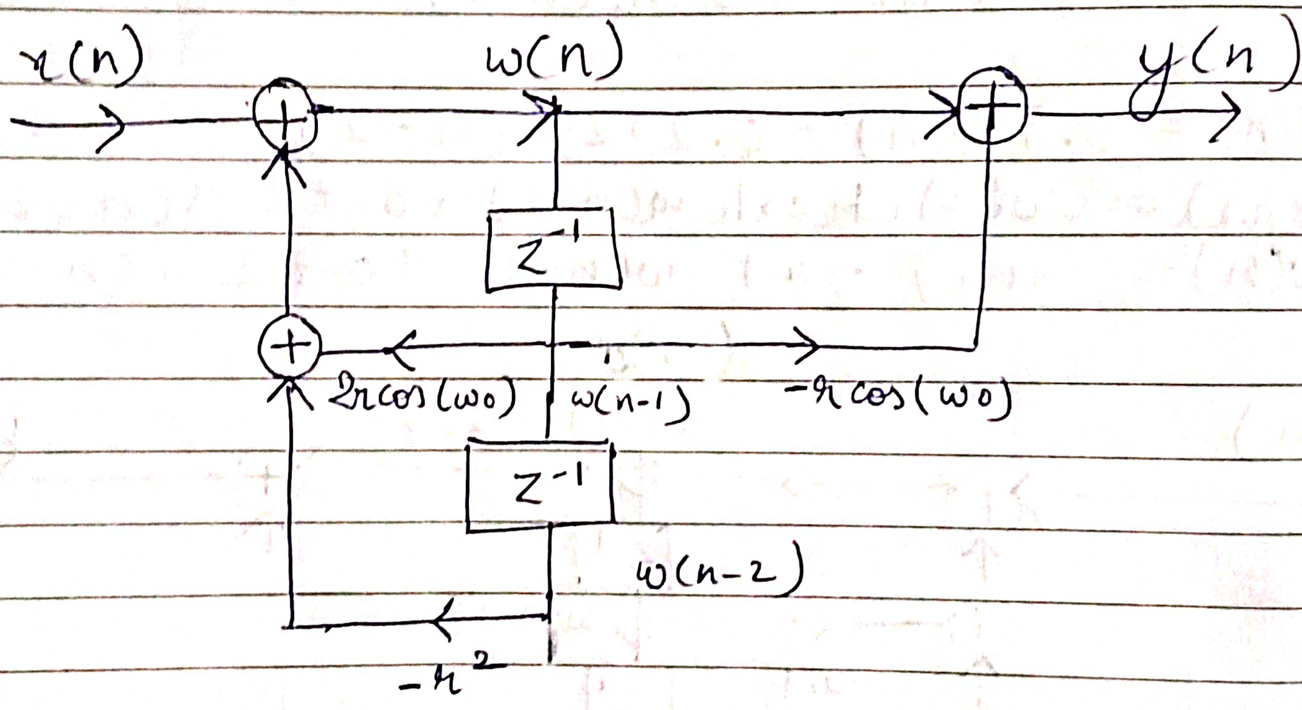
$X(z) = W(z)[1 - 2r\cos(\omega_0) z^{-1} + r^2 z^{-2}]$

$x(n) = \omega(n) - 2r\cos(\omega_0)\, \omega(n-1) + r^2 \omega(n-2)$

$\omega(n) = x(n) + 2r\cos(\omega_0)\, \omega(n-1) - r^2 \omega(n-2)$

$\rightarrow$ $y(n) = \omega(n) - r\cos(\omega_0)\, \omega(n-1)$

$\omega(n) = x(n) + 2r\cos(\omega_0)\, \omega(n-1) - r^2 \omega(n-2)$



Transposed Form Realization

$$y(n) = \frac{1}{2} y(n-1) - \frac{1}{4} y(n-2) + x(n) + x(n-1)$$

Steps

→ First Draw the Direct Form II, form signal Flow Graph.

→ Interchange input and output

→ Reverse the direction of All Branches

→ summing points become branching points

→ Branching points become summing points

$-r^2$

Transposed Form Realisation of IIR

Ques) $y(n) = \frac{1}{2} y(n-1) - \frac{1}{4} y(n-2) + x(n) + x(n-1)$

Ans) (1) Draw the direct form 2 diagram

$$Y(z) = \frac{1}{2} z^{-1} Y(z) - \frac{1}{4} z^{-2} Y(z) + X(z) + z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{z^{-1}}{2} + \frac{z^{-2}}{4}\right] = X(z)\left[1 + z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1 - \frac{z^{-1}}{2} + \frac{z^{-2}}{4}} = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)}$$

$$\frac{Y(z)}{W(z)} = 1+z^{-1}$$

classmate
Date
Page

$$Y(z) = [1 + z^{-1}]W(z)$$

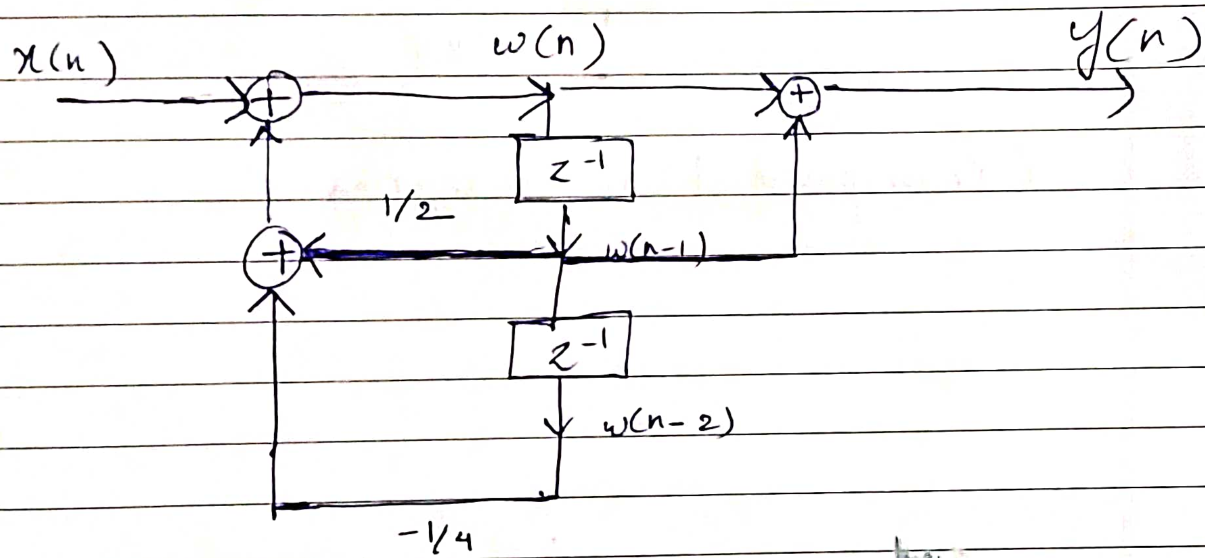
$$y(n) = w(n) + w(n-1) \quad — ①$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - \frac{z^{-1}}{2} + \frac{z^{-2}}{4}}$$

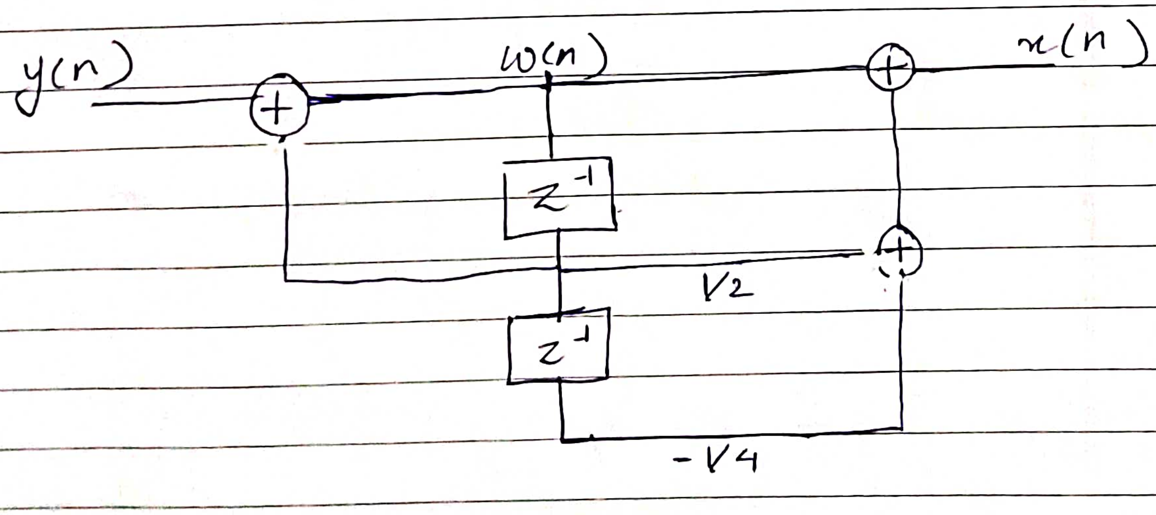
$$X(z) = W(z)\left[1 - \frac{z^{-1}}{2} + \frac{z^{-2}}{4}\right]$$

$$x(n) = w(n) - \frac{1}{2}w(n-1) + \frac{1}{4}w(n-2)$$

$$w(n) = x(n) + \frac{1}{2}w(n-1) - \frac{1}{4}w(n-2) \quad — ②$$



② Interchange the input and output

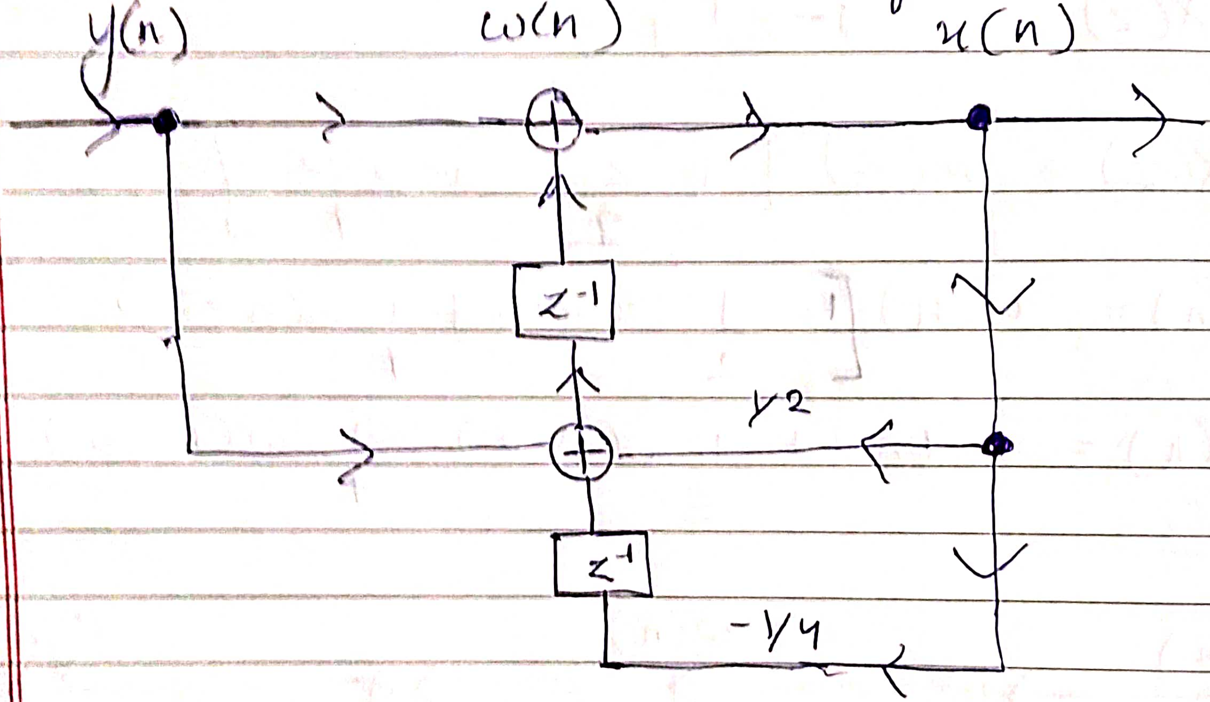


classmate

Date

Page

(3) Convert summing points to branching points and branching points to summing points and reverse the direction of arrows



(Transposed Form Realization

Cascade Form Realization of IIR Filters

Ques) $y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1)$

Ans) $Y(z) = \frac{3}{4} z^{-1} Y(z) - \frac{1}{8} z^{-2} Y(z) + X(z) + \frac{1}{3} z^{-1} X(z)$

$$Y(z)\left[1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right] = X(z)\left[1 + \frac{1}{3} z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{z^{-1}}{3}}{1 - \frac{3}{4} z^{-1} + \frac{z^{-2}}{8}}$$

$$H(z) = \frac{1 + \frac{z^{-1}}{3}}{1 - \frac{3 z^{-1}}{4} + \frac{z^{-2}}{8}}$$

Split denominator into two terms

Date ______ Page ______

$$H(z) = \frac{1 + \frac{z^{-1}}{3}}{\left(1 - \frac{z^{-1}}{2}\right)\left(1 - \frac{z^{-1}}{4}\right)}$$

$$H(z) = H_1(z)\, H_2(z)$$

$$H_1(z) = \frac{1 + \frac{z^{-1}}{3}}{\left(1 - \frac{z^{-1}}{4}\right)} \qquad H_2(z) = \frac{1}{\left(1 - \frac{z^{-1}}{2}\right)}$$

Now we realize $H_1(z)$ and $H_2(z)$ separately using direct form 2 realization and combine them

$$H_1(z) = \frac{1 + \frac{z^{-1}}{3}}{1 - \frac{z^{-1}}{4}}$$

$$\frac{Y_1(z)}{W_1(z)} \times \frac{W_1(z)}{X_1(z)} = \frac{1 + \frac{z^{-1}}{3}}{\left(1 - \frac{z^{-1}}{4}\right)}$$

$$\frac{Y_1(z)}{W_1(z)} = 1 + \frac{z^{-1}}{3}$$

$$Y_1(z) = W_1(z) + \frac{z^{-1}}{3} W_1(z)$$

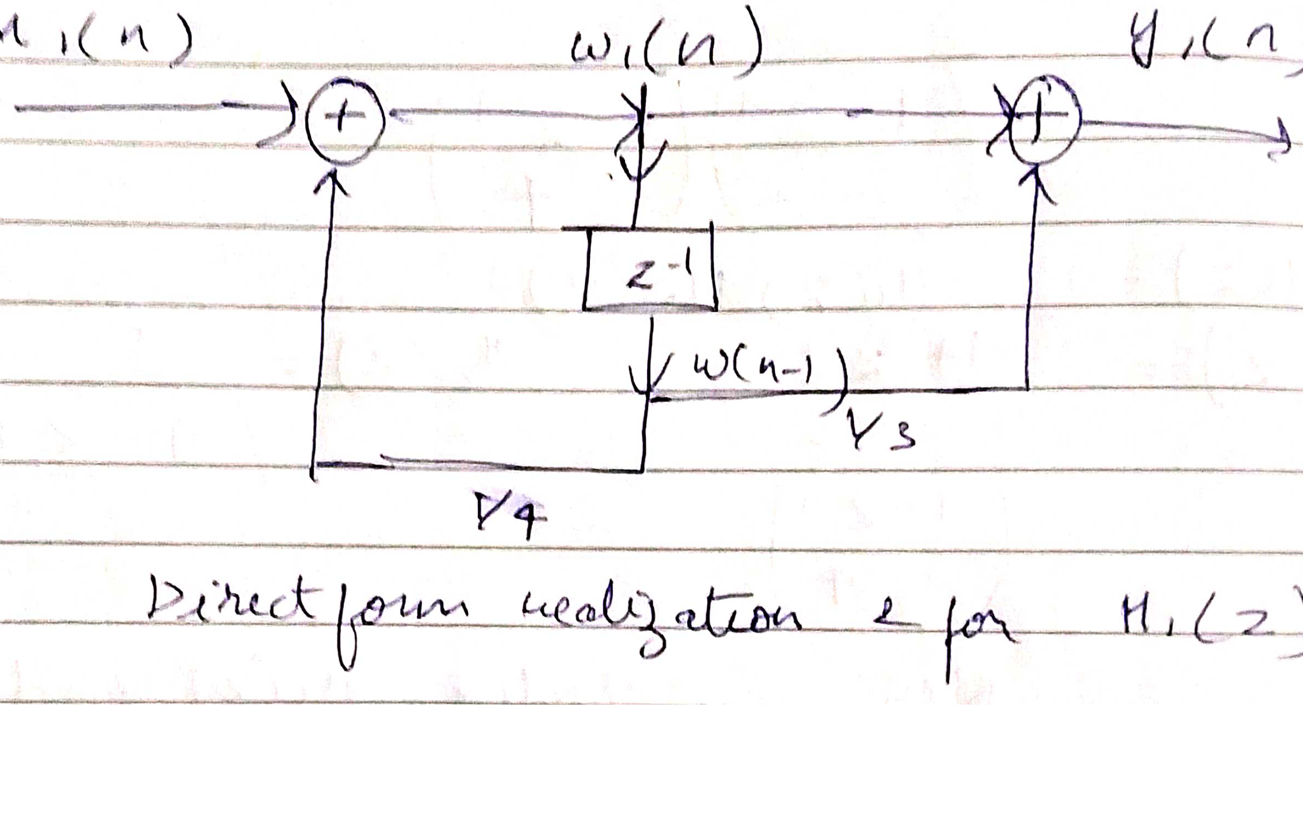
$$y_1(n) = w_1(n) + \frac{1}{3} w_1(n-1)$$

$$\frac{W_1(z)}{X_1(z)} = \frac{1}{1 - \frac{z^{-1}}{4}}$$

$$X_1(z) = W_1(z) - \frac{1}{4} z^{-1} W_1(z)$$

$$x(n) = w_1(n) - \frac{1}{4} w_1(n-1)$$

$$w(n) = x(n) + \frac{1}{4} w_1(n-1)$$



Direct form realization 2 for $H_1(z)$

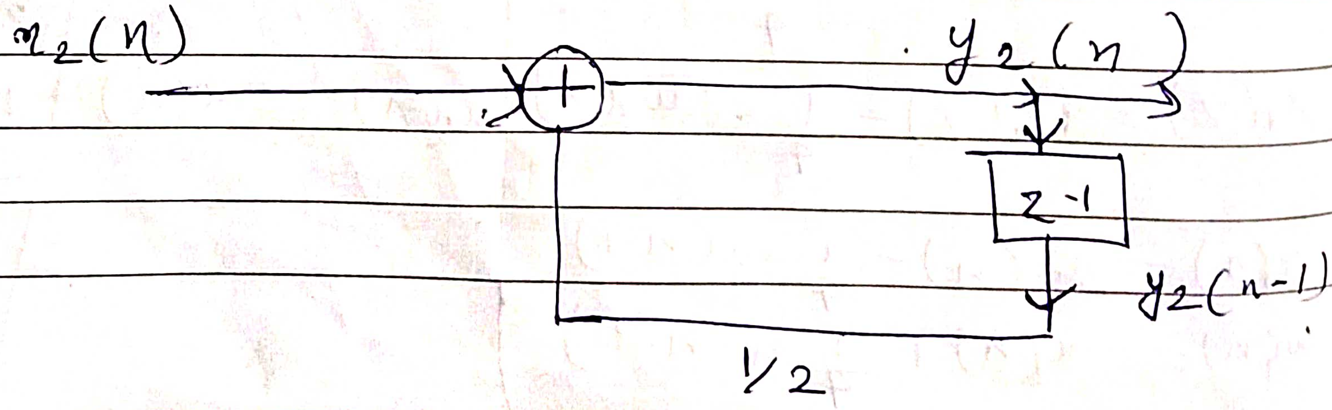
$$\frac{Y_2(z)}{X_2(z)} = \frac{1}{\left(1 - \frac{z^{-1}}{2}\right)}$$

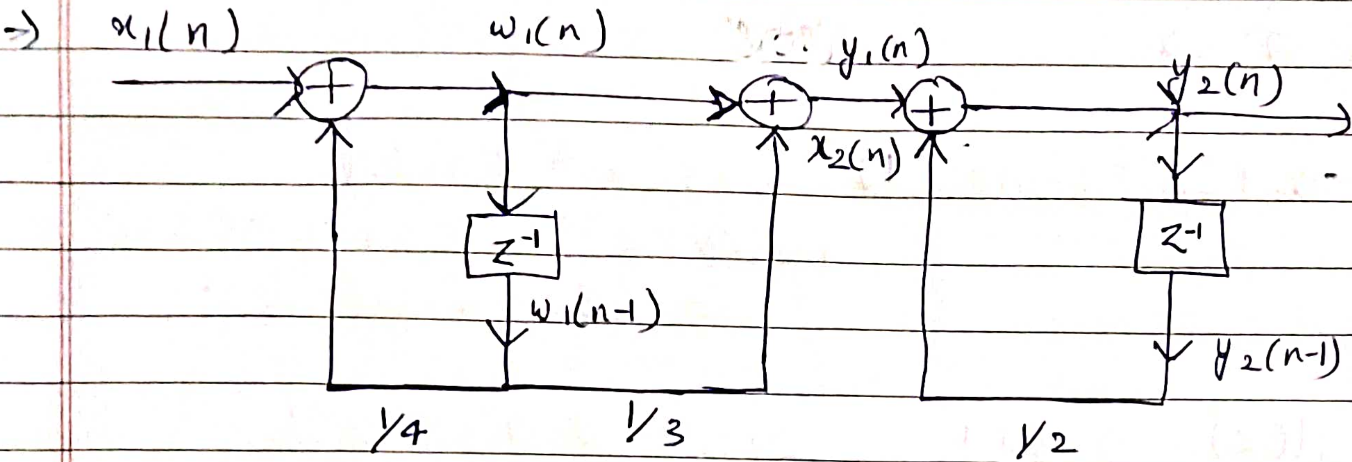
[$\because$ There is no numerator so, no need of $W_2(z)$]

$$Y_2(z)\left[1 - \frac{z^{-1}}{2}\right] = X_2(z)$$

$$y_2(n) - \frac{1}{2} y_2(n-1) = x_2(n)$$

$$y_2(n) = \frac{1}{2} y_2(n-1) + x_2(n)$$





[Cascade Form Realization]

Parallel Form Realization of IIR Filter

Ques) $y(n) = -0.1y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$

$$Y(z) = -0.1 z^{-1} Y(z) + 0.72 z^{-2} Y(z) + 0.7 X(z) - 0.252 z^{-2} X(z)$$

$$\Rightarrow Y(z)\left[1 + 0.1 z^{-1} - 0.72 z^{-2}\right] = X(z)\left[0.7 - 0.252 z^{-2}\right]$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}} = H(z)$$

$$H(z) = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

We need to represent $H(z)$ as

$$H(z) = c + H_1(z) + H_2(z) + \dots$$

$$
\begin{array}{r|l}
 & 0.35 \\
\hline
-0.72z^{-2}+0.1z^{-1}+1 & -0.252z^{-2}+0.70 \\
 & \mp 0.252z^{-2} \pm 0.35 \pm 0.035z^{-1} \\
\hline
 & 0.35 - 0.035z^{-1}
\end{array}
$$

$$H(z) = 0.35 + \frac{0.35 - 0.035z^{-1}}{-0.72z^{-2}+0.1z^{-1}+1}$$

$$\frac{0.35-0.035z^{-1}}{-0.72z^{-2}+0.1z^{-1}+1}$$

$$= \frac{0.35-0.035z^{-1}}{(1+0.9z^{-1})(1-0.8z^{-1})} = \frac{A}{(1+0.9z^{-1})} + \frac{B}{(1-0.8z^{-1})}$$

$$\Rightarrow A(1-0.8z^{-1}) + B(1+0.9z^{-1}) = 0.35 - 0.035z^{-1}$$

$$\Rightarrow A+B = 0.35$$

$$-A\times(0.8) + B(0.9) = -0.035$$

$$\Rightarrow A = (0.35-B)$$

$$(-0.35+B)(0.8) + B(0.9) = -0.035$$

$$(1.7)B = -0.035 + 0.35\times 0.8$$

$$1.7B = 0.35(-0.1+0.8)$$

$$\boxed{B = 0.144} \qquad \boxed{A = 0.206}$$

$$\text{So, } H(z) = C + H_1(z) + H_2(z)$$

$$H(z) = 0.35 + \left(\frac{0.206}{1+0.9z^{-1}}\right) + \frac{0.144}{(1-0.8z^{-1})}$$

$$\begin{array}{r|l} & \quad 0.35 \\ \hline -0.72z^{-2}+0.1z^{-1}+1 & -0.252z^{-2}+0.70 \\ & \mp 0.252z^{-2} \pm 0.35 \pm 0.035z^{-1} \\ \hline & \quad 0.35 - 0.035z^{-1} \end{array}$$

$$H(z) = 0.35 + \frac{0.35 - 0.035z^{-1}}{-0.72z^{-2}+0.1z^{-1}+1}$$

$$\frac{0.35-0.035z^{-1}}{-0.72z^{-2}+0.1z^{-1}+1}$$

$$= \frac{0.35-0.035z^{-1}}{(1+0.9z^{-1})(1-0.8z^{-1})} = \frac{A}{(1+0.9z^{-1})} + \frac{B}{(1-0.8z^{-1})}$$

$$\Rightarrow A(1-0.8z^{-1}) + B(1+0.9z^{-1}) = 0.35 - 0.035z^{-1}$$

$$\Rightarrow A+B = 0.35$$

$$-A \times (0.8) + B(0.9) = -0.035$$

$$\Rightarrow A = (0.35-B)$$

$$(-0.35+B)(0.8) + B(0.9) = -0.035$$

$$(1.7)B = -0.035 + 0.35 \times 0.8$$

$$1.7B = 0.35(-0.1+0.8)$$

$$\boxed{B = 0.144} \qquad \boxed{A = 0.206}$$

So, $H(z) = C + H_1(z) + H_2(z)$

$$H(z) = 0.35 + \left(\frac{0.206}{1+0.9z^{-1}}\right) + \frac{0.144}{(1-0.8z^{-1})}$$

$$\frac{Y_1(z)}{X_1(z)} = \frac{0.206}{(1+0.9z^{-1})}$$

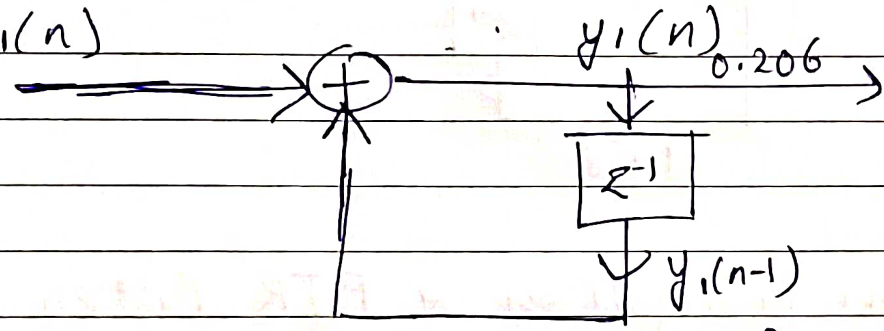
No need to take 0.206 as we can multiply it later in Direct Form 2 output

$$\frac{Y_1(z)}{X_1(z)} = \frac{1}{(0.9z^{-1}+1)}$$

$$X_1(z) = (1+0.9z^{-1})\, Y_1(z)$$

$$x_1(n) = y_1(n) + 0.9\, y_1(n-1)$$

$$y_1(n) = x_1(n) - 0.9\, y_1(n-1)$$

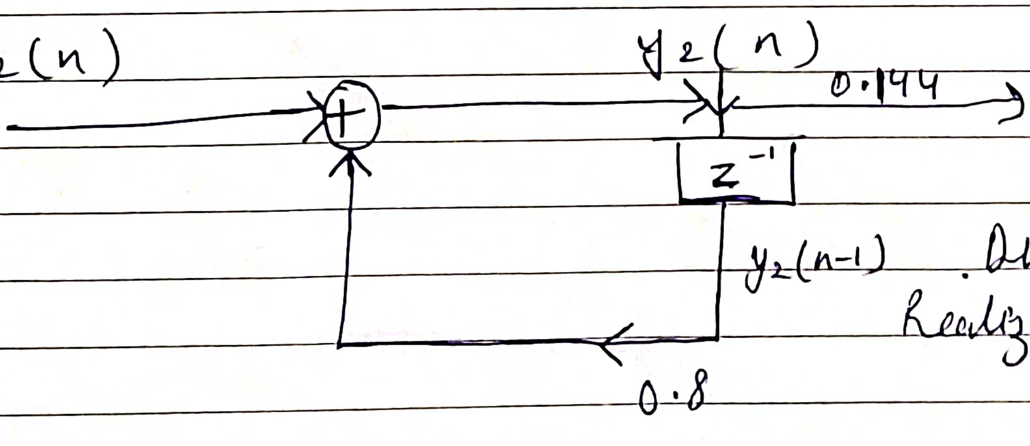


Direct Form 2 Realization of $H_1(z)$

$$\frac{Y_2(z)}{X_2(z)} = \frac{1}{1-0.8z^{-1}}$$

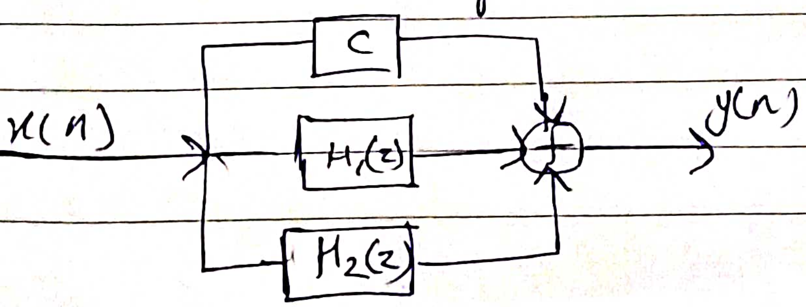
$$y_2(n) - 0.8\, y_2(n-1) = x_2(n)$$

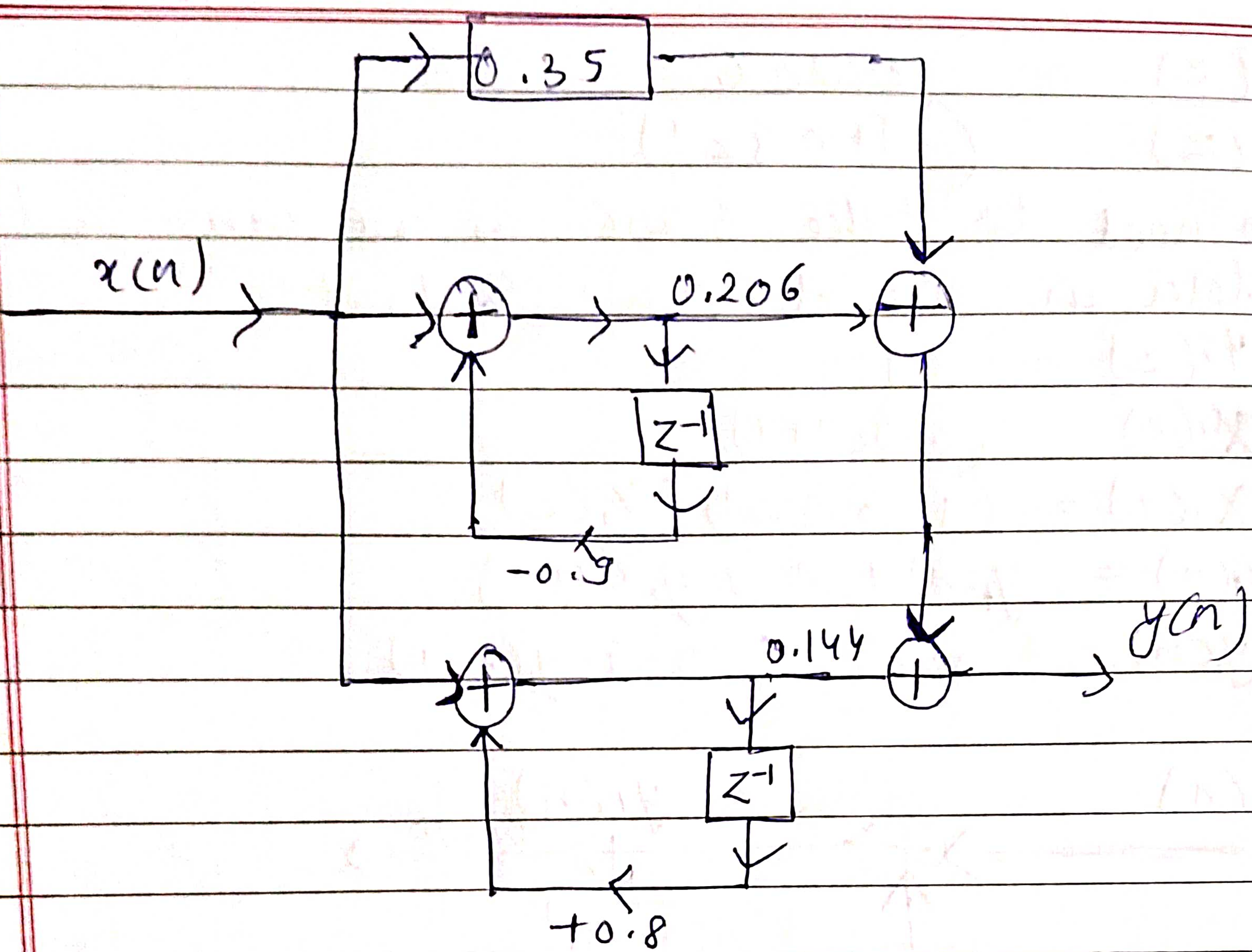
$$y_2(n) = 0.8\, y_2(n-1) + x_2(n)$$



Direct Form 2 Realization of $H_2(z)$

Structure of Parallel Form is





0.35
x(n)
0.206
Z-1
-0.9
0.144
y(n)
Z-1
+0.8

Introduction – Design of FIR Filter

→ Filter → Impulse response is finite.

→ output → depends only on present and (or) past values.

→ Applications → where linear phase is important.

Ex:- Data transmission, Speech processing, Correlation processing, Interpolation.

Characteristics:

(i) Impulse response → Finite length.

(ii) non-recursive FIR filter → stable.

(iii) phase distortion of Frq response can be eliminated by FIR filter.

(iv) Implement a recursive FIR Filters

(v) Effect of Start-up transient have small duration.

(vi) Quantization noise can be made negligible

Advantages:

→ stable

→ Can be realized in both recursive & non-recursive.

→ exact linear phase.

→ Flexible.

→ low sensitive to Quantization noise.

→ Efficiently realized in H/w.

Disadvantages:

→ complex

→ requires more filter co-efficients to be stored.

→ Long duration impulse response require large amount of processing.

→ narrow transition band FIR filter requires more arithmetic operations & H/w components ↳ Costly.

EC Academ
Ec A

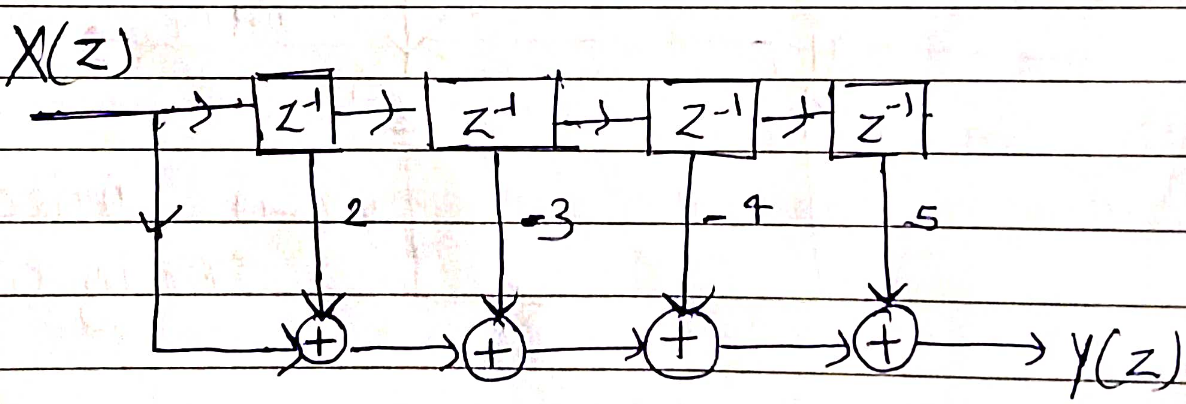
Direct Form Realization of FIR Filter

Ques) $H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

Ans) $\frac{Y(z)}{X(z)} = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

$Y(z) = X(z)[1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}]$

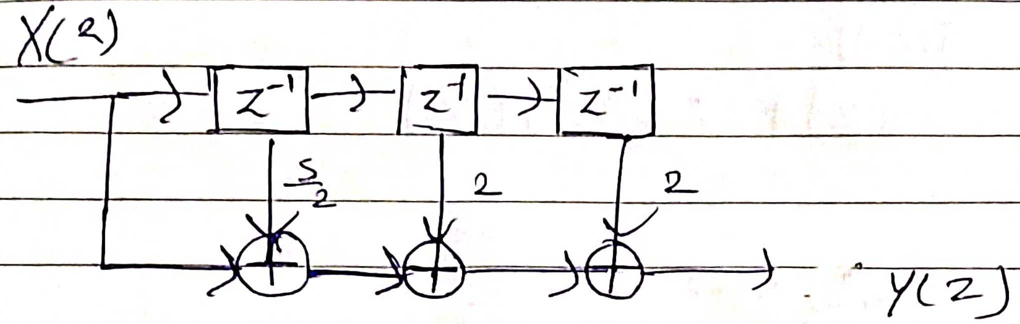
$Y(z) = X(z) + 2z^{-1}X(z) - 3z^{-2}X(z)$
$- 4z^{-3}X(z) + 5z^{-4}X(z)$



Ques) $H(z) = 1 + \frac{5}{2}z^{-1} + 2z^{-2} + 2z^{-3}$

Ans) $\frac{Y(z)}{X(z)} = 1 + \frac{5}{2}z^{-1} + 2z^{-2} + 2z^{-3}$

$$Y(z) = X(z) + \frac{5}{2} z^{-1} X(z) + 2z^{-2} X(z) + 2z^{-3} X(z)$$



Cascade Form Realization of FIR Filter

In cascade form realization question, transfer function must be in the form of product of smaller transfer function.

Qu) $H(z) = \underbrace{(1+2z^{-1} - z^{-2})}_{H_1(z)}\underbrace{(1+z^{-1} - z^{-2})}_{H_2(z)}$

$$\frac{Y_1(z)}{X_1(z)} = (1+2z^{-1} - z^{-2})$$

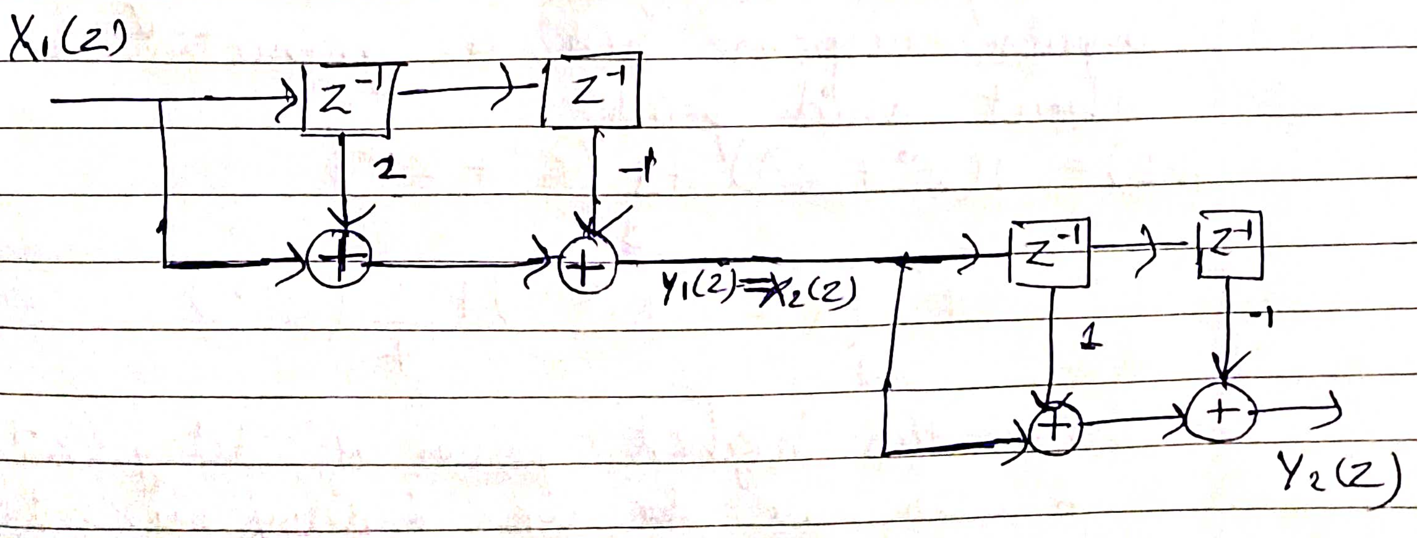
$$Y_1(z) = X_1(z) + 2z^{-1} X_1(z) - z^{-2} X_1(z) \quad \text{— (1)}$$

$$\frac{Y_2(z)}{X_2(z)} = (1+z^{-1} - z^{-2})$$

$$Y_2(z) = X_2(z) + z^{-1} X_2(z) - z^{-2} X_2(z) \quad \text{— (2)}$$

General structure of Cascade Form is

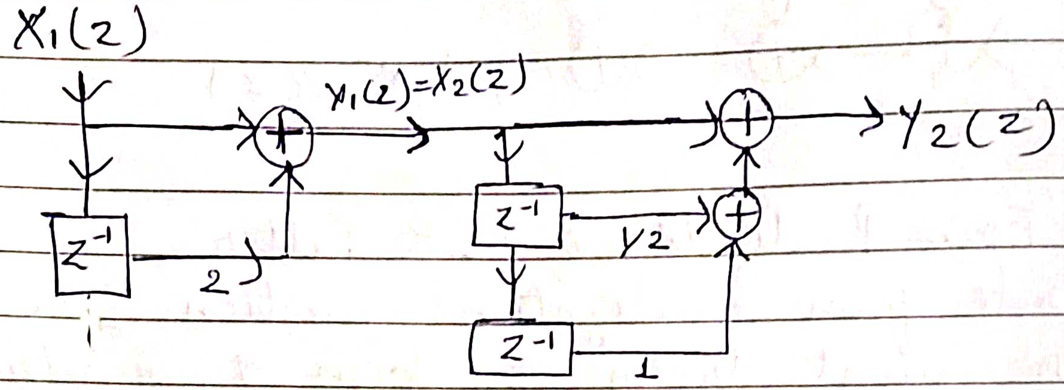
$$X_1(z) \longrightarrow Y_1(z) = X_2(z) \longrightarrow Y_2(z)$$



Ques) $H(z) = (1+2z^{-1})\left(1+\frac{1}{2}z^{-1}+z^{-2}\right)$

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = 1+2z^{-1}$$

$$Y_1(z) = X_1(z) + 2z^{-1}X_1(z)$$



$$\frac{Y_2(z)}{X_2(z)} = 1+\frac{z^{-1}}{2}+z^{-2}$$

$$Y_2(z) = X_2(z) + \frac{1}{2}z^{-1}X_2(z) + z^{-2}X_2(z)$$

Linear Phase Realization of FIR Filter

Advantage: Number of Multiplier is reduced

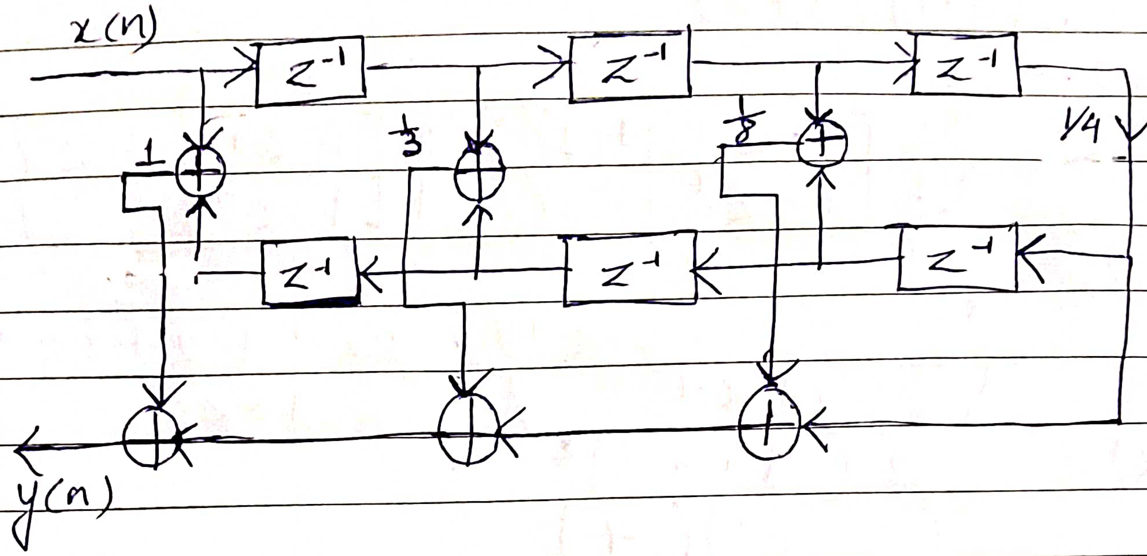
Ques) $H(z) = 1 + \frac{z^{-1}}{3} + \frac{z^{-2}}{8} + \frac{z^{-3}}{4} + \frac{z^{-4}}{8} + \frac{z^{-5}}{3} + z^{-6}$

Ans) $h(n) = \left\{1, \frac{1}{3}, \frac{1}{8}, \frac{1}{4}, \frac{1}{8}, \frac{1}{3}, 1\right\}$

Linear phase realization exists only when impulse response $h(n)$ is symmetrical about mid point

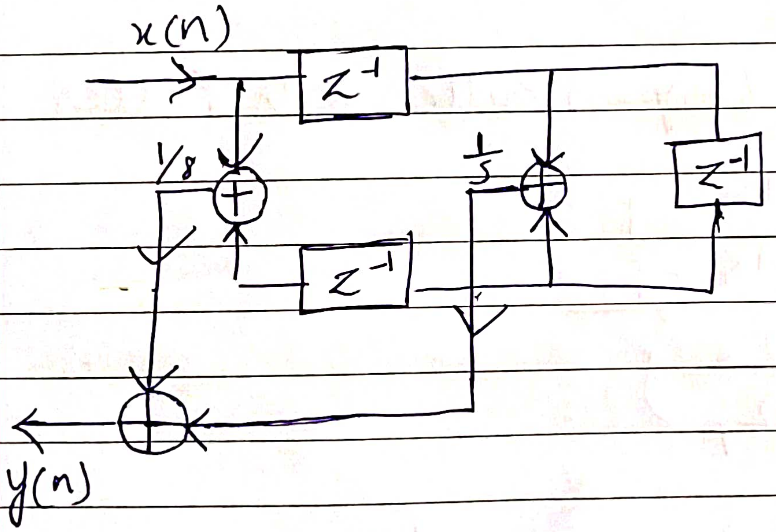
$$H(z) = 1(z^{0}+z^{-6}) + \frac{1}{3}(z^{-1}+z^{-5}) + \frac{1}{8}(z^{-2}+z^{-4}) + \frac{1}{4}(z^{-3})$$

Since the highest power of delay (z^{-1}) is 6. So we use 6 delay elements.



Ans) $H(z) = \frac{1}{8} + \frac{1}{5} z^{-1} + \frac{1}{5} z^{-2} + \frac{1}{8} z^{-3}$

$H(z) = \frac{1}{8}\left(z^{0} + z^{-3}\right) + \frac{1}{5}\left(z^{-1} + z^{-2}\right)$



Bilinear Transformation

Used to convert transfer function from analog domain to digital domain.

$H(s) \rightarrow H(z)$

Procedure : Replace s with $\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$$s \rightarrow \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

Ques) $H(s) = \frac{2}{(s+1)(s+2)}$, with $T = 1$ second

Ans) $H(z) = \frac{2}{\left\{\frac{2}{1}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+1\right\}\left\{\frac{2}{1}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+2\right\}}$

$H(z) = \frac{2}{\left(\frac{2-2z^{-1}+1+z^{-1}}{1+z^{-1}}\right)\left(\frac{2-2z^{-1}+2+2z^{-1}}{1+z^{-1}}\right)}$

$H(z) = \frac{2(1+z^{-1})^2}{(3-z^{-1})(4)}$

$H(z) = \frac{1}{2}\frac{(1+z^{-1})^2}{(3-z^{-1})}$

$H(z) = \frac{1}{6}\frac{(1+z^{-1})^2}{\left(1-\frac{z^{-1}}{3}\right)}$

Finding Order of Lowpass Butterworth Filter

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

Ques) Given $\alpha_p = 1\,dB$ $\alpha_s = 30\,dB$
$\Omega_p = 200$ rad/sec
$\Omega_s = 600$ rad/sec. Find order

Ans) $$N \geq \frac{\log\sqrt{\frac{10^{0.1\times 30}-1}{10^{0.1\times 1}-1}}}{\log\left(\frac{600}{200}\right)}$$

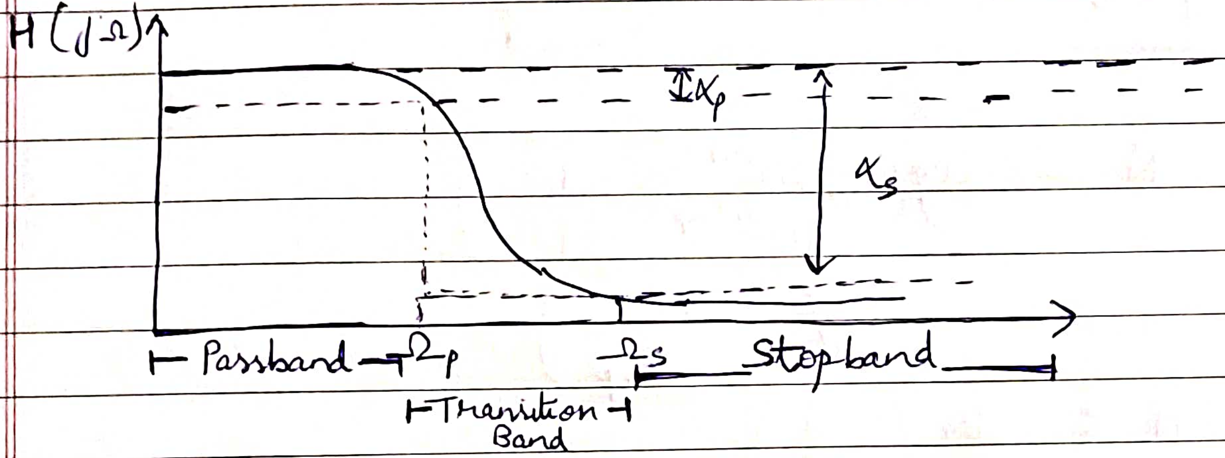
$$N \geq \frac{\log\sqrt{\frac{10^3-1}{10^{0.1}-1}}}{\log(3)}$$

$N \geq 3.758$

$\Rightarrow \boxed{N=4}$

α_p = passband attenuation
α_s = stopband attenuation
Ω_s = stopband ~~end~~ edge frequency in analog domain
Ω_p = passband edge frequency in analog domain
ω_s = stopband edge frequency in digital domain
ω_p = passband edge frequency in digital domain

In analog domain,



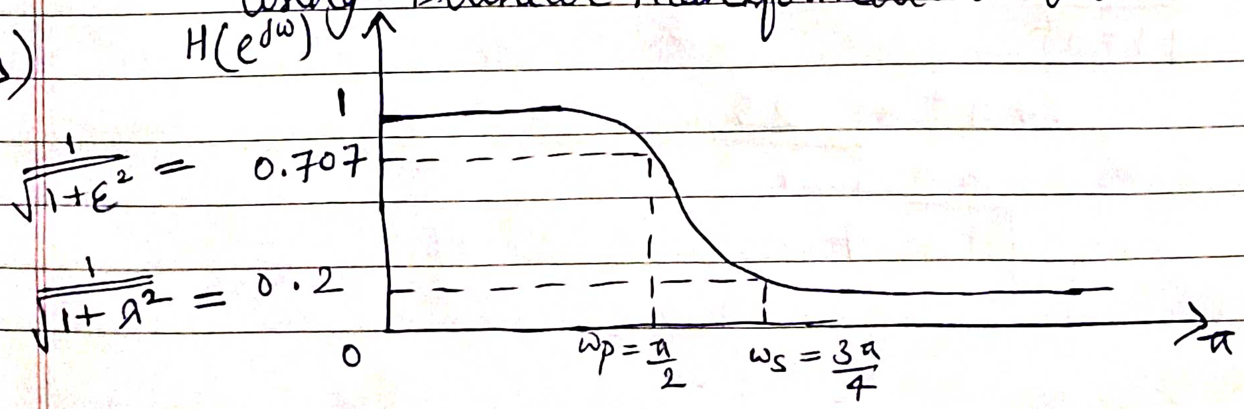
Butterworth Filter Design using Bilinear Transformation

Ques) Design a butterworth filter satisfying the condition $0.707 \leq |H(e^{j\omega})| \leq 1$ for $0 \leq \omega \leq \frac{\pi}{2}$

and $|H(e^{j\omega})| \leq 0.2$ for $\frac{3\pi}{4} \leq \omega \leq \pi$

using Bilinear Transformation with $T = 1$ second

Ans)



$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right)$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right)$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\pi}{4}\right) = 2 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{3\pi}{8}\right) = 2\tan\left(\frac{3\pi}{8}\right)$$

$$\boxed{\Omega_s = 4.824 \text{ rad/sec}}$$

$$N \geq \frac{\log\left(\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}\right)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log\left(\frac{\lambda}{\epsilon}\right)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.707 = \frac{\sqrt{2}}{2}$$

$$\frac{1}{1+\varepsilon^2} = \frac{2}{4} = \frac{1}{2}$$

$$1+\varepsilon^2 = 2$$

$$\boxed{\varepsilon = 1}$$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2 = \frac{1}{5}$$

$$1+\lambda^2 = 25$$

$$\lambda = \sqrt{24}$$

$$\boxed{\lambda = 4.89}$$

Date
Page

$$N \geq \frac{\log\left(\frac{4.89}{1}\right)}{\log\left(\frac{4.824}{2}\right)}$$

$$N \geq \frac{\log(4.89)}{\log(2.412)}$$

$N \geq 1.8 \Rightarrow \boxed{N=2}$

Order(N)	Denominator of H(s)
1	$s+1$
2	$s^2+\sqrt{2}\,s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.847s+1)$

Here $N=2$

So, $$H(s) = \frac{1}{s^2+\sqrt{2}s+1}$$

Replace s with $\frac{s}{\Omega_c}$

where $\Omega_c = \frac{\Omega_p}{\epsilon^{1/N}}$

Here $\Omega_p = 2$ and $\epsilon = 1$

So $s \to \frac{s}{2}$

$$H(s) = \frac{1}{\left(\frac{s}{2}\right)^2+\sqrt{2}\left(\frac{s}{2}\right)+1}$$

$$H(s) = \frac{1}{\frac{s^2}{4}+\frac{s}{\sqrt{2}}+1}$$

Now, we apply bilinear transformation, to convert it into digital domain

$$S \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$S \to \frac{2}{1}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$H(z) = \frac{1}{\frac{1}{4}\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + \frac{1}{\sqrt{2}}\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right] + 1}$$

$$H(z) = \frac{1}{\frac{1}{4}\left[4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2\right] + \frac{1}{\sqrt{2}}\, 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 1}$$

$$= \frac{(1+z^{-1})^2}{(1-z^{-1})^2 + \sqrt{2}(1-z^{-2}) + (1+z^{-1})^2}$$

$$= \frac{(1+z^{-1})^2}{2 + 2z^{-2} + \sqrt{2} - \sqrt{2}z^{-2}}$$

$$= \frac{(1+z^{-1})^2}{2(1+z^{-2}) + \sqrt{2}(1-z^{-2})}$$

Impulse Invariance Method to design IIR Filter (Analog Filter to Digital Filter)

1. Find $H_a(s)$ – Analog Filter Transfer Function
2. Select sampling period (T)
3. $$H_a(s) = \sum_{K=1}^{N} \frac{C_K}{S - P_K}$$

(Sum of single pole filters)

P_K = pole

4. $$H(z) = \sum_{K=1}^{N} \frac{C_K}{1 - e^{P_K T} z^{-1}}$$

Ques) Find analog $H(z)$ using impulse invariance method ($T = 1$ second)

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Ans)
$$\frac{2}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$= \frac{(2A+B) + (A+B)s}{(s+1)(s+2)}$$

$A+B=0 \qquad 2A+B = 2$

$2A - A = 2$

$\boxed{A=2} \qquad \boxed{B=-2}$

$$H(s) = \frac{2}{(s+1)} + \frac{(-2)}{(s+2)}$$

$P_K = -1, -2$

So, $$H(s) = \frac{2}{1-e^{-1}z^{-1}} + \frac{(-2)}{1-e^{-2}z^{-1}}$$

Chebyshev Filter Design

Ques) Given the specification $\alpha_p = 3\,dB$, $\alpha_s = 16\,dB$, $f_p = 1\,KHz$ and $f_s = 2\,KHz$. Determine the order of filter using Chebyshev Approximation. Find $H(s)$

Ans) $\alpha_p = 3\,dB \qquad \alpha_s = 16\,dB$

$f_p = 1\,KHz \qquad f_s = 2\,KHz$

$\Omega_p = 2\pi \times 1000 = 2000\pi$ rad/sec

$\Omega_s = 2\pi \times 2000 = 4000\pi$ rad/sec

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)} = \frac{\cosh^{-1}\sqrt{\dfrac{10^{1.6}-1}{10^{0.3}-1}}}{\cosh^{-1}(2)}$$

$$\cosh^{-1}x = \ln\left(x + \sqrt{x^2-1}\right)$$

$N \geq 1.91$

$\Rightarrow \boxed{N=2}$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

$$\lambda = \sqrt{10^{0.1\alpha_s} - 1}$$

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2} + 1}$$

$$N \geq \frac{\cosh^{-1}\left(\frac{\lambda}{\epsilon}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\boxed{\mu = 2.414}$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$\boxed{a = 910\pi} \qquad \boxed{b = 2197\pi}$$

Poles

$$s_K = a\cos\phi_K + j\, b\sin\phi_K$$
$$K = 1, 2, \ldots, N$$

$$\phi_K = \frac{\pi}{2} + \left[\frac{2K-1}{2N}\right]\pi \qquad K = 1, 2, \ldots, N$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} = 135^\circ$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = 225^\circ$$

$$~~s_K~~ \quad s_1 = a\cos\phi_1 + jb\sin\phi_1$$

$$s_1 = 910\pi\cos(135^\circ) + j(2197\pi)\sin(135^\circ)$$

$$s_1 = -643.46\pi + j\,1554\pi$$

$S_2 = a \cos\phi_2 + j b \sin\phi_2$

$S_2 = (910a)\cos(225) + j(2197a)\sin(225)$

$S_2 = -643.46a - j\,1554a$

$$H(s) = \frac{\text{Numerator}}{\text{Denominator}}$$

$\text{Denominator} = (s-S_1)(s-S_2)\ldots(s-S_N)$

$$H(s) = \frac{\text{Numerator}}{(s+643.46a-j1554a)(s+643.46a+j1554a)}$$

To find numerator

1. If N is even

   Put $s=0$ in denominator ~~and~~ then divide that value by $\sqrt{1+\varepsilon^2}$

~~Here N=2 is even~~

2. If N is odd

   Put $s=0$ in denominator to get the numerator value.

Here $N=2$ is even

~~Numerator~~ $\sqrt{1+\varepsilon^2} = \sqrt{2} = 1.414$

$$\text{Numerator} = \frac{(643.46a - j1554a)(643.46a + j1554a)}{\sqrt{2}}$$

$$H(s) = \frac{\dfrac{(643.46a - j1554a)(643.46a + j1554a)}{\sqrt{2}}}{(s+643.46a-j1554a)(s+643.46a+j1554a)}$$

Impulse Response of Linear phase FIR Filter:

Symmetric:

$$h(n) = h(\underset{\downarrow}{M}-1-n)\ ;\ 0 \le n \le M-1$$

order

Let $M = 8 \rightarrow$ even

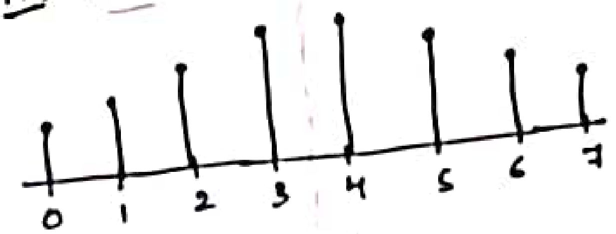
$$\therefore h(n) = h(8-1-n)\ ;\ 0 \le n \le 7$$

$n=0$ $h(0) = h(8-1-0) = h(7)$

$n=1$ $h(1) = h(8-1-1) = h(6)$

$n=2$ $h(2) = h(8-1-2) = h(5)$

$n=3$ $h(3) = h(8-1-3) = h(4)$



$M=9$

Anti Symmetric:

$$h(n) = -h(M-1-n)\ ;\ 0 \le n \le M-1$$

Let $M = 8$

$$h(n) = -h(8-1-n)\ ;\ 0 \le n \le 7$$

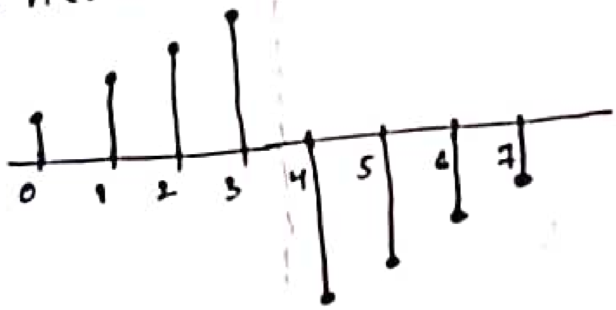
$$\therefore h(n) = -h(8-1-0) = -h(7)$$

$n=0$ $h(0) = -h(8-1-1) = -h(6)$

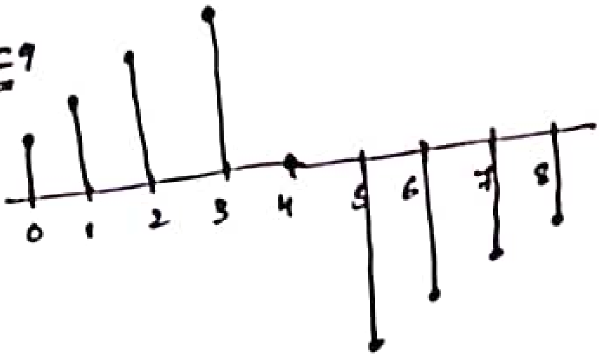
$n=1$ $h(1) = -h(8-1-2) = -h(5)$

$n=2$ $h(2) = -h(8-1-3) = -h(4)$

$n=3$ $h(3) = -h(8-1-3) = -h(4)$



$M=9$



Frequency Response of Linear Phase FIR Filter

$$h(n) \xleftrightarrow{DTFT} H(\omega)$$

Frq. response of FIR filter.

$$H(\omega) = \underbrace{H_r(\omega)}_{\text{real part of } H(\omega)} e^{j\theta(\omega)}$$

(i) Symmetric Impulse response with M is even:

$$H_r(\omega) = \sum_{n=0}^{\frac{M}{2}-1} 2\,h(n) \cos\left[\omega\left(n - \frac{M-1}{2}\right)\right]$$

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) ; & H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi ; & H_r(\omega) < 0 \end{cases}$$

(ii) Symmetric Impulse response with M is odd:

$$H_r(\omega) = h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} 2\,h(n) \cos\left(\omega\left[n - \frac{M-1}{2}\right]\right)$$

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) ; & H_r(\omega) > 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi ; & H_r(\omega) < 0 \end{cases}$$

(iii) Anti Symmetric Impulse response with M is even:

$$H_r(\omega) = 2 \sum_{n=0}^{\frac{M}{2}-1} h(n) \sin\left(\omega\left[\frac{M-1}{2} - n\right]\right)$$

$$\theta(\omega) = \begin{cases} \frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right) ; & H_r(\omega) > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) ; & H_r(\omega) < 0 \end{cases}$$

(iv) Anti Symmetric Impulse response with M is odd:

$$H_r(\omega) = 2 \sum_{n=0}^{\frac{M-3}{2}} h(n) \sin\left(\omega\left[\frac{M-1}{2} - n\right]\right)$$

$$\theta(\omega) = \begin{cases} \frac{\pi}{2} - \omega\left(\frac{M-1}{2}\right) ; & H_r(\omega) > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{M-1}{2}\right) ; & H_r(\omega) < 0 \end{cases}$$