# Symmetricities in Fourier Series (Part-1)

**Even Symmetry:** Fourier Series expansion of an even signal does not contain "sine" terms $\Downarrow$ $b_n = 0$

$$x(t) = \underbrace{a_0}_{\text{dc value}} + \underbrace{\sum_{n=1}^{\infty} a_n \cos n\omega_0 t}_{\text{Cosine terms}} + \underbrace{\sum_{n=1}^{\infty} b_n \sin n\omega_0 t}_{\text{Sine terms} \,\to\, 0}$$

$\cos(-t) = \cos t$ — Even

$\sin(-t) = -\sin t$ — odd

$x(t) \rightarrow$ Even $\Rightarrow$ f.s. Exp. will have harmonics of an even signal.

$\boxed{b_n = 0}$

**Odd Symmetry:** Fourier Series of an odd signal contains only "sine" terms

$x(t) \rightarrow$ odd

- $\rightarrow$ f.s. × Even terms $\Rightarrow$ × Cosine terms $\Rightarrow \boxed{a_n = 0}$
- $\rightarrow$ av. Value $= 0 \Rightarrow \boxed{a_0 = 0}$

$\boxed{b_n \neq 0}$

**Half Wave Symmetry:**

**Half Wave Symmetry:** Fourier Series expansion of HWS signal contains only "odd harmonics"

$$x(t) = -x\left(t + \frac{T_0}{2}\right)$$

$$\downarrow \qquad\qquad \downarrow$$

$$C_{n_1} = C_{n_2}$$

$\omega_0 = \frac{2\pi}{T_0}$

$\frac{\omega_0 T_0}{2} = \pi$

$$\cancel{C_{n_1}} = -\cancel{C_{n_1}}\, e^{jn\omega_0 \frac{T_0}{2}}$$

$$1 = -e^{jn\boxed{\omega_0 \frac{T_0}{2}}\,\pi}$$

$$1 = -e^{jn\pi}$$

$$1 + e^{jn\pi} = 0$$

$$1 + e^{(j\pi)n} = 0$$

$$1 + (-1)^n = 0 \Rightarrow n \text{ is an odd int.}$$

$$x(t) \rightleftharpoons C_{n_1}$$

$$x(t - t_0) \rightleftharpoons C_{n_1} e^{-jn\omega_0 t_0}$$

$$t_0 = -\frac{T_0}{2}$$

$$x\left(t + \frac{T_0}{2}\right) \rightleftharpoons C_{n_1} e^{jn\omega_0 \frac{T_0}{2}}$$

$$-x\left(t + \frac{T_0}{2}\right) \rightleftharpoons -C_{n_1} e^{jn\omega_0 \frac{T_0}{2}} = C_{n_2}$$

$n\omega_0 \Rightarrow$ Only odd harmonics

# Symmetricities in Fourier Series (Part-2)

✓ O + HWS:

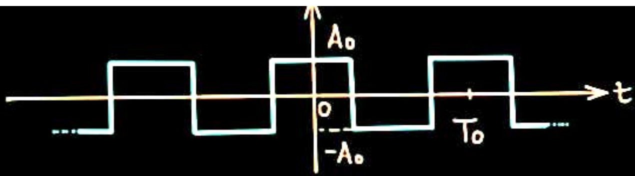
Odd Symm. + HWS
- → only odd harmonics
- → av. value $= 0 = a_0 = C_0$
- → only sine terms $\Rightarrow b_n \neq 0$, $a_n = 0$, $a_0 = 0$

} Only sine terms are present with odd harmonics

✓ E + HWS:

Even Symm. + HWS
- → $a_0 = 0$ & only odd harmonics
- → not have sine terms: $b_n = 0$, $a_n \neq 0$

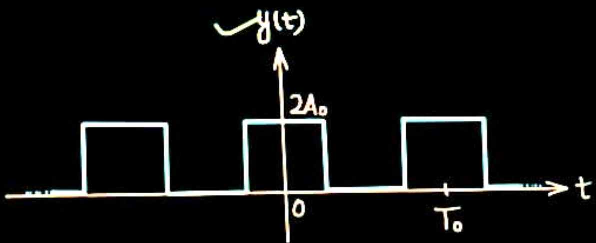
} Only cosine terms are present with odd harmonics




$x(t) = a_1 \cos\omega_0 t + a_3 \cos 3\omega_0 t + a_5 \cos 5\omega_0 t + \cdots$

imp

**Hidden Symmetry:**




a) —

b) —

c) —

d) —

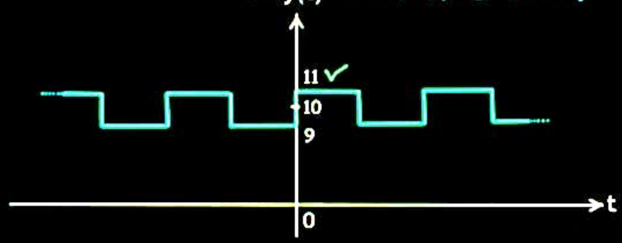
dc term + cosine terms (odd harmonics)

$y(t) = A_0 + x(t)$

# Hidden Symmetry (Examples)

**1) Find y(t)**

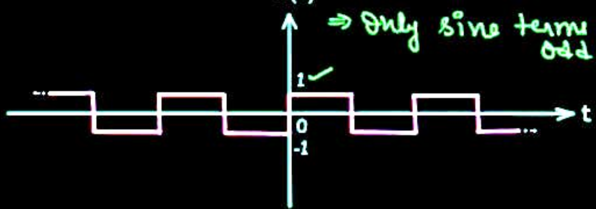
$y(t) \xrightarrow{DAS} HWS\,(x(t))$




Sol :—

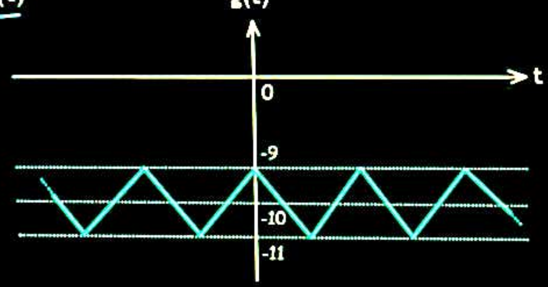
$$-x(-t) = x(t)$$

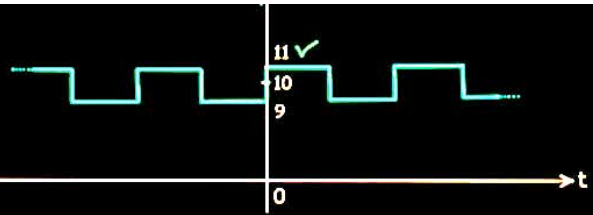
x(t) → HWS + 0
⇒ Only sine terms with odd har.

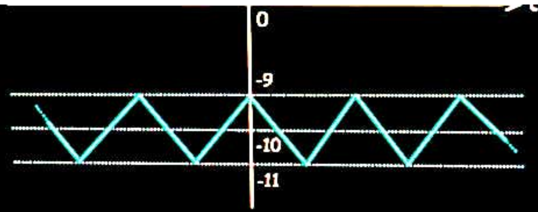



$$x(t) = -10 + y(t)$$

**2) Find g(t)**





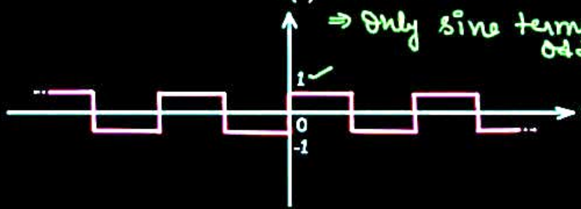




Sol :—

$$-x(-t) = x(t)$$

$x(t) \rightarrow HWS + 0$

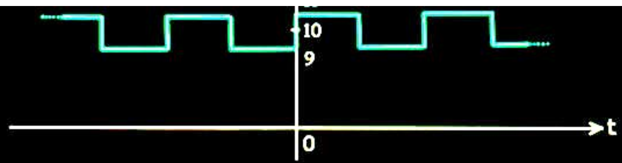
$\Rightarrow$ Only sine terms with odd har.

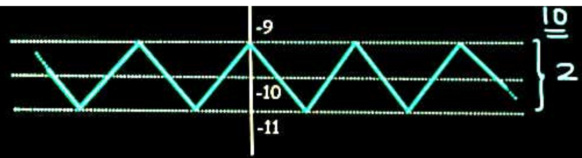



$$x(t) = -10 + y(t)$$

$$y(t) = 10 + x(t)$$

$$y(t) = \underbrace{10}_{a_0} + \underbrace{b_1 \sin \omega_0 t + b_3 \sin 3\omega_0 t + \cdots}_{\substack{b_n \neq 0 \\ a_n = 0}}$$



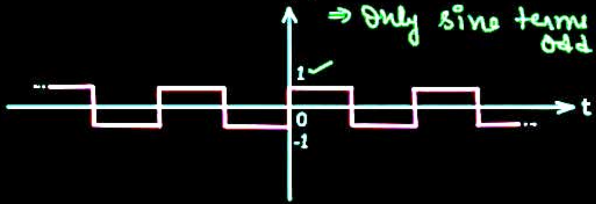

सol :—

$$-x(-t) = x(t)$$

$$f(-t) = f(t)$$

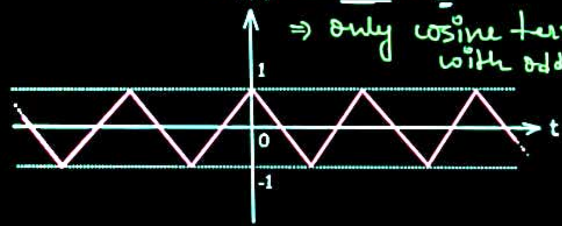
$x(t) \longrightarrow$ HWS + O

$\Rightarrow$ Only sine terms with odd har.




$f(t) \longrightarrow$ HWS + E

$\Rightarrow$ Only cosine terms with odd har.




$$x(t) = -10 + y(t)$$

$$y(t) = 10 + x(t)$$

$$y(t) = \underbrace{10}_{a_0} + \underbrace{b_1 \sin w_0 t + b_3 \sin 3 w_0 t + \cdots}_{\substack{b_n \neq 0 \\ a_n = 0}}$$

$$f(t) = 10 + g(t) \Rightarrow g(t) = -10 + f(t)$$

$$g(t) = \underbrace{-10}_{a_0} + a_1 \cos w_0 t + \underbrace{a_3 \cos 3 w_0 t + \cdots}_{\substack{a_n \neq 0 \\ b_n = 0}}$$