ANGLE MODULATION (Non-linear Process)

Another way of modulating a sinusoidal carrier wave, in which the angle of the carrier wave is varied according to the information-bearing signal.

NOTE: In Angle modulation, the Amplitude of the carrier wave is maintained constant.

Why Angle Modulation?	
	1) It provide better discrimination against noise & Interference than Amplitude modulation.
	2) Provide us with a practical means of exchanging channel bandwidth for improved noise performance which is not possible in Amplitude modulation.

NOTE: Improvement in noise performance in angle modulation is achieved at the cost of increased system complexity in both the transmitter and receiver.

BASIC DEFINITIONS

Angle modulated wave is expressed as:

$$s(t) = A_c \cos\left[\theta_i(t)\right]$$

→ Modulated Sinusoidal Carrier at time 't'

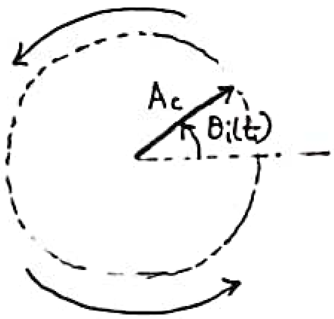
→ Function of the information-bearing signal or message signal

Carrier amplitude

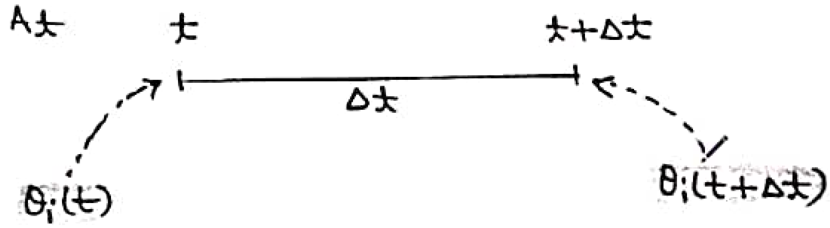
angle at time 't'

Note:- Complete oscillations occur whenever angle $\theta_i(t)$ changes by 2π radian.

Rotating phasor with angular velocity (rad/sec) $\frac{d\theta_i(t)}{dt}$ w.r.t 't'



Consider, small time interval Δt



<u>Average Frequency (Hz)</u>, over a small interval from t to $(t+\Delta t)$ is

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \, \Delta t}$$

$\because$ $s(t) = A\cos\theta$

or $= A\cos\omega t$

or $= A\cos 2\pi f t$

So, $\theta = 2\pi f t$

or $\boxed{f = \frac{\theta}{2\pi t}}$

<u>Instantaneous frequency</u> $f_i(t)$: As $\Delta t \to 0$

So, $\theta = 2\pi f t$

or $\boxed{f = \dfrac{\theta}{2\pi t}}$

Instantaneous frequency $f_i(t)$: As $\Delta t \to 0$

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) = \lim_{\Delta t \to 0} \left[\frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t} \right]$$

$$f_i(t) = \frac{1}{2\pi} \underbrace{\lim_{\Delta t \to 0} \frac{\theta_i(t+\Delta t) - \theta_i(t)}{\Delta t}}_{\equiv \frac{d\theta_i(t)}{dt}}$$

$\equiv \dfrac{d\theta_i(t)}{dt}$ derivative of angle $\theta_i(t)$ w.r.t t

→ Angular velocity of rotating phasor.

So, $\boxed{f_i(t) = \dfrac{1}{2\pi} \dfrac{d\theta_i(t)}{dt}}$

Note: For unmodulated carrier i.e. $m(t) = 0$

$$\theta_c(t) = \boxed{\theta_i(t) = \underbrace{2\pi f_c t} + \phi_c}$$

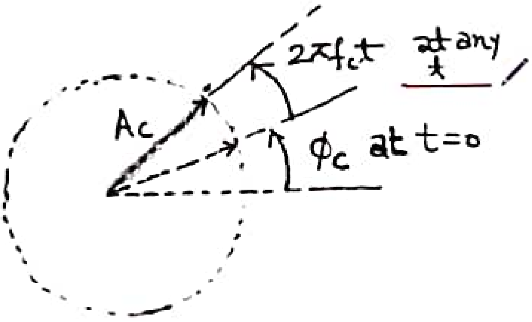
→ Angle of unmodulated

Note: For unmodulated carrier i.e. $m(t) = 0$

$$\theta_c(t) = \boxed{\theta_i(t) = \underbrace{2\pi f_c t}_{} + \phi_c}$$

$2\pi f_c t$ → Constant angular velocity (rad/sec)

ϕ_c → Angle of unmodulated carrier at time $t = 0$



Let, Consider

$$c(t) = A_c \cos \theta_c(t) = A_c \cos[2\pi f_c t + \phi_c] \text{ ---- Unmodulated carrier}$$

$$m(t) = A_m \cos 2\pi f_m t$$

After modulation i.e. Angle modulation.

$$s(t) = A_c \cos \boxed{\theta_i(t)} \longrightarrow \text{Varied with } m(t)$$

Phase Modulation (PM)

Instantaneous angle $\theta_i(t)$ is varied linearly with $m(t)$.

$$\theta_i(t) = \underbrace{2\pi f_c t}_{\text{angle of Unmodulated Carrier}} + \boxed{K_p\, m(t)}$$

K_p: Phase sensitivity factor of the modulator (rad/volt)

Note:- set $\phi_c = 0$ (for convenience)
$m(t) \approx$ voltage waveform. (V)

So, P.M i.e. Phase Modulated wave.

Frequency Modulation (FM)

Instantaneous frequency $f_i(t)$ is varied linearly with $m(t)$.

$$f_i(t) = f_c + \boxed{K_f\, m(t)}$$

f_c: Frequency of Unmodulated Carrier

K_f: Frequency Sensitivity factor of the modulator (Hz/volt)

Now: since

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

Carrier modulator (rad/volt)

> **Note:-** set $\phi_c = 0$ (for convenience)
> $m(t) \approx$ voltage waveform. (V)

So, P.M ie. Phase Modulated wave.

$$\boxed{s(t) = A_c \cos[2\pi f_c t + K_p m(t)]}$$

Carrier factor of the modulator (Hz/volt)

Now : since

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

So, $$\boxed{\theta_i(t) = 2\pi \int_0^t f_i(\tau)\,d\tau}$$

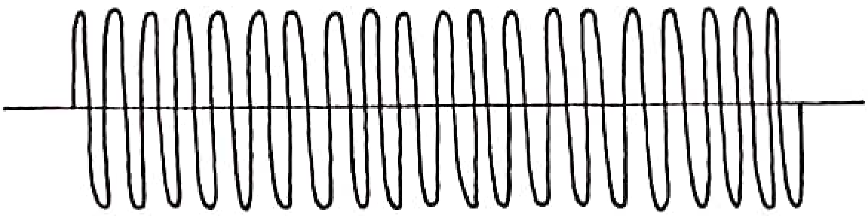
Also

$$\theta_i(t) = \underbrace{2\pi f_c t}_{\text{Angle of Unmodulated Carrier}} + \underbrace{2\pi K_f \int_0^t m(\tau)\,d\tau}_{\text{Increase or decrease in the Instantaneous phase } \theta_i(t) \text{ due to } m(t)}$$

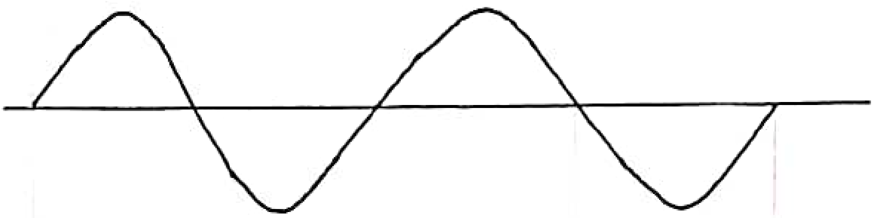
So, FM i.e Frequency Modulated wave.

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\,d\tau\right]}$$

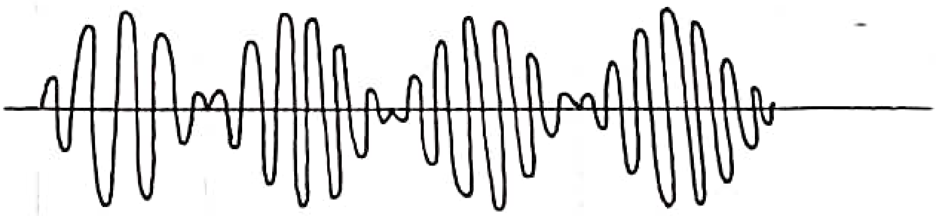
Illustration of AM, PM & FM waves produced by a single tone.



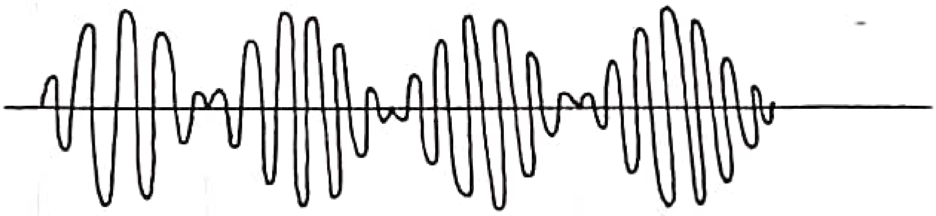
a) Carrier wave $c(t)$



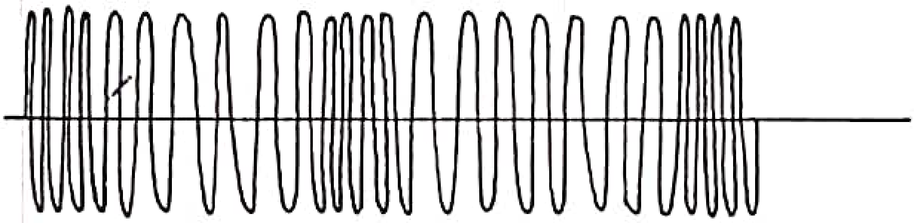
b) Sinusoidal modulating signal $m(t)$



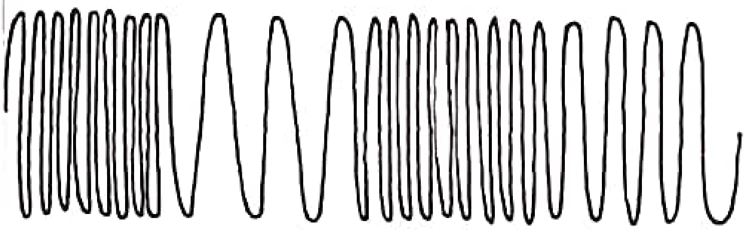
c) AM signal (DSB-SC)



c) AM Signal
(DSB-SC)



d) Phase-modulated
(PM) signal



e) Frequency-modulated
(FM) signal.

Properties of Angle-Modulated waves

1) Consistancy of Transmitted Power :-

$$s(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right] \quad \text{------ PM}$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right] \quad \text{---- FM}$$

Amplitude of PM & FM waves is maintained at a constant value equal to the Carrier amplitude A_c for all time t, irrespective of the sensitivity factor K_p & K_f.

Average transmitted power of angle modulated wave is given as

$$\boxed{P_{av} = \frac{1}{2} A_c^2}$$

Assumed, the load resistor; $R_L = 1\Omega$

2) Non-linearity of the modulation process :-

Both PM & FM waves violates the principle of superposition.

Assume, message signal $m(t)$ is made up of two different components $m_1(t)$ & $m_2(t)$ as

$$m(t) = m_1(t) + m_2(t)$$

Let, $s(t)$, $s_1(t)$ & $s_2(t)$ denote the PM waves produced by $m(t)$, $m_1(t)$ & $m_2(t)$

$$s(t) = A_c \cos\left[2\pi f_c t + K_p\left[m_1(t) + m_2(t)\right]\right]$$

$$s_1(t) = A_c \cos\left[2\pi f_c t + K_p m_1(t)\right]$$

& $$s_2(t) = A_c \cos\left[2\pi f_c t + K_p m_2(t)\right]$$

So, $\boxed{s(t) \neq s_1(t) + s_2(t)}$ ---- Violate the principle of superposition.

3) Irregularity of zero-Crossing :-

zero-crossings are defined as the instants of time at which a waveform changes its amplitude from a positive to negative or any other way.

As shown in waveform, the zero crossing of a PM & FM wave no longer have a perfect regularity across the time scale.

4) Visualization difficulty of message waveform :-

There is the difficulty in visualizing the message waveform in angle-modulated waves due to the non-linear character of PM & FM. While in case of AM, we see the message waveform as the envelope of the modulated wave provided the % modulation is less than 100% as illustrated in above fig.

Relationship Between PM & FM

(rad/volt)

$$s(t) = A_c \cos\left[2\pi f_c t + K_p\, m(t)\right] \quad \text{------ PM}$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right] \text{---- FM}$$

(Hz/volt)

So,

$$K_p\, m(t) \Leftrightarrow 2\pi K_f \int_0^t m(\tau)\, d\tau$$

$$\left(\frac{\text{rad}}{\text{Volt}}\right) m(t) \Leftrightarrow 2\pi \left(\frac{\text{Hz}}{\text{Volt}}\right) \int_0^t m(\tau)\, d\tau$$

$$\left(\frac{\text{rad}}{\text{Volt}}\right) m(t) \Leftrightarrow \left(\frac{\text{rad}}{\text{Volt}}\right) \int_0^t m(\tau)\, d\tau$$

$$\text{(PM)} \ldots\ldots\ m(t) \Leftrightarrow \int_0^t m(\tau)\, d\tau \cdots \text{(FM)}$$

Also

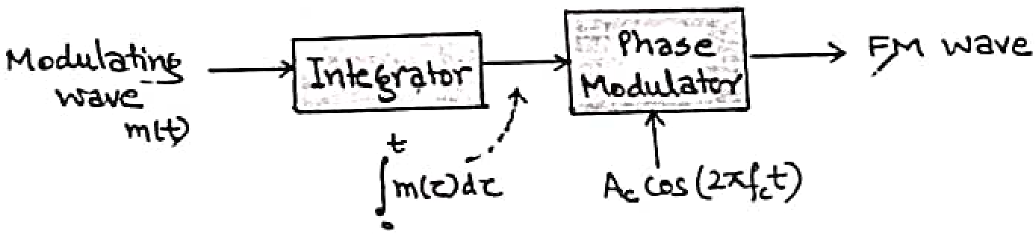
$$\boxed{\begin{array}{c} \dfrac{dm(t)}{dt} = m(t) \\ \text{(PM)} = \text{(FM)} \end{array}}$$

FM wave is viewed as a PM wave, produced by the modulating

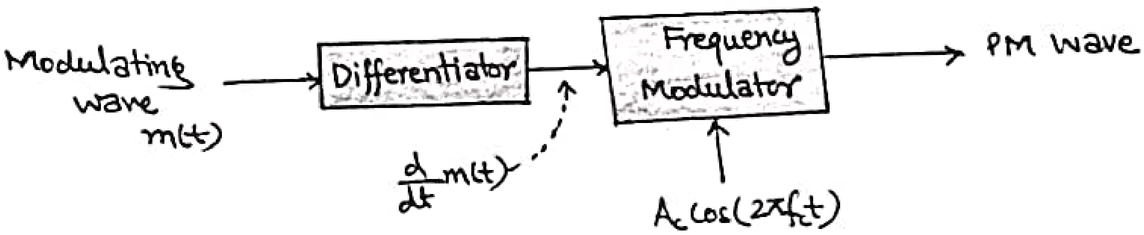
$$(PM) \ldots\ldots m(t) \Leftrightarrow \int_0^t m(\tau)\,d\tau \;\cdots (FM)$$

FM wave is viewed as a PM wave, produced by the modulating wave $\int_0^t m(\tau)\,d\tau$ in place of $m(t)$.

<u>FM wave</u> can be generated by Using <u>Phase modulator</u>.



Also, <u>PM wave</u> can be generated by using <u>Frequency modulator</u>.



SINGLE TONE FREQUENCY MODULATION

The FM signal s(t) is a non-linear function of the modulating signal m(t), which makes frequency modulation a non-linear modulation process.

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right]}$$

Consider, single tone (single frequency) modulating signal :

$$m(t) = A_m \cos(2\pi f_m t)$$

The <u>instantaneous</u> <u>frequency</u> of FM

$$\boxed{f_i(t) = f_c + K_f\, m(t)}$$

$$f_i(t) = f_c + K_f A_m \cos(2\pi f_m t)$$

$$\boxed{f_i(t) = f_c + K_f\, m(t)}$$

$$f_i(t) = f_c + K_f A_m \cos(2\pi f_m t)$$

$\hookrightarrow \Delta f \approx$ Frequency Deviation

{ Representing the maximum departure of the instantaneous frequency (f_i) from f_c. }

Also, $$\boxed{f_i(t) = \frac{1}{2\pi}\frac{d\,\theta_i(t)}{dt}}$$

or $$\theta_i(t) = 2\pi \int_0^t f_i(t)\, dt$$

$$= 2\pi \int_0^t \left[f_c + K_f\, m(t)\right] dt$$

$m(t) \to A_m \cos(2\pi f_m t)$

[frequency (f_i) from f_c.]

Also,
$$\boxed{f_i(t) = \frac{1}{2\pi} \cdot \frac{d\,\theta_i(t)}{dt}}$$

or
$$\theta_i(t) = 2\pi \int_0^t f_i(t)\,dt$$

$$= 2\pi \int_0^t [f_c + K_f \underbrace{m(t)}_{A_m \cos(2\pi f_m t)}]\,dt$$

$$= 2\pi f_c t + \frac{K_f A_m}{f_m} \sin(2\pi f_m t)$$

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$\hookrightarrow \beta \approx$ Deviation ratio / Modulation Index

$$= 2\pi f_c t + \frac{K_f A_m}{f_m} \sin(2\pi f_m t)$$

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$\hookrightarrow \beta \approx$ Deviation ratio / Modulation Index

(9)

$$\therefore \beta = \frac{\Delta f}{f_m}$$

4 $$\boxed{\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)}$$

→ represents the phase deviation of FM wave i.e. max departure of the angle $\theta_i(t)$ from angle $(2\pi f_c t)$ of unmodulated carrier. So measured in radian.

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]}$$ --- FM in terms of β.

f_m

represents the phase deviation of FM wave i.e. max departure of the angle $\theta_i(t)$ from angle $(2\pi f_c t)$ of unmodulated carrier. So <u>measured in radian.</u>

4 $$\boxed{\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)}$$

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]}$$ ... FM in terms of β.

Depend upon the value of 'β'

FM

- NBFM (Narrow Band FM) $\boxed{\beta << 1 \text{ rad}}$
- WBFM (Wide Band FM) $\boxed{\beta >> 1 \text{ rad}}$

Narrow-Band Frequency Modulation (NBFM.)

Consider single tone FM.

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]}$$

$\beta \rightarrow \frac{\Delta f}{f_m} \approx$ Modulation Index.

Using trigonometric Identity

$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

$$s(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)]$$

In case of NBFM , $\beta << 1$ rad.

So, $$\cos\left[\beta \sin(2\pi f_m t)\right] \simeq 1$$

& $$\sin\left[\beta \sin(2\pi f_m t)\right] \simeq \beta \sin(2\pi f_m t)$$

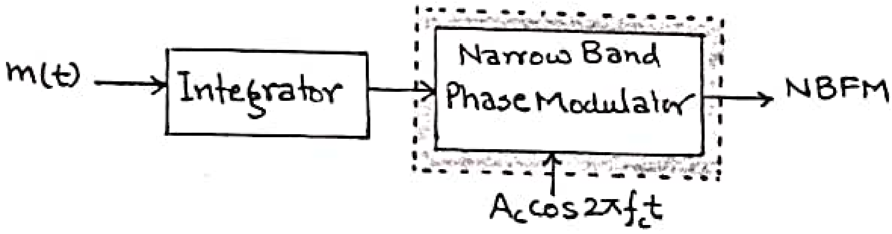
$$\rightarrow s(t) = A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)]$$

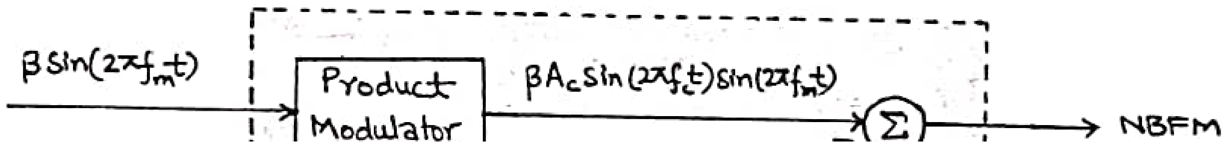
In case of NBFM, $\beta << 1$ rad.

So, $\cos[\beta \sin(2\pi f_m t)] \simeq 1$

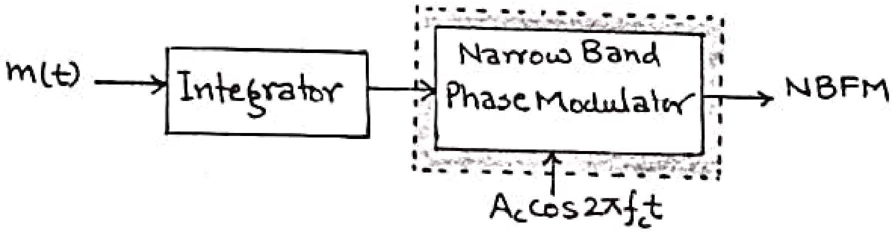
& $\sin[\beta \sin(2\pi f_m t)] \simeq \beta \sin(2\pi f_m t)$

$$\rightarrow \boxed{s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)}$$ Approximate form of NBFM.





$$\boxed{s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)}$$ Approximate form of NBFM.



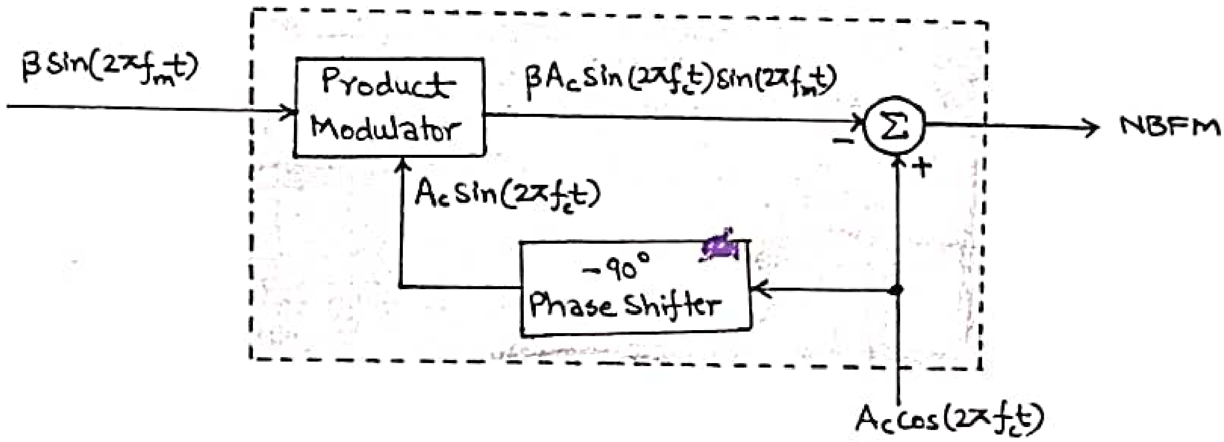


Fig: Block diagram of an Indirect method for Generating a NBFM.

Again; From Approx form of NBFM.

$$s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

Using trigonometric Identity

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$s(t) \simeq A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2} \left\{ \cos[2\pi(f_c+f_m)t] - \cos[2\pi(f_c-f_m)t] \right\}$$

(NBFM)

From Expression of AM.

$$s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \left\{ \cos[\underbrace{2\pi(f_c+f_m)t}_{USB} + \cos[\underbrace{2\pi(f_c-f_m)t}_{LSB} \right\}$$

(AM)

$$\rightarrow s(t) \simeq A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \left\{ \cos\left[2\pi (f_c + f_m)t\right] - \cos\left[2\pi (f_c - f_m)t\right] \right\}$$

(NBFM)

From Expression of AM.

$$s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \left\{ \underbrace{\cos\left[2\pi (f_c + f_m)t\right.}_{USB} + \underbrace{\cos\left[2\pi (f_c - f_m)t\right.}_{LSB} \right\}$$

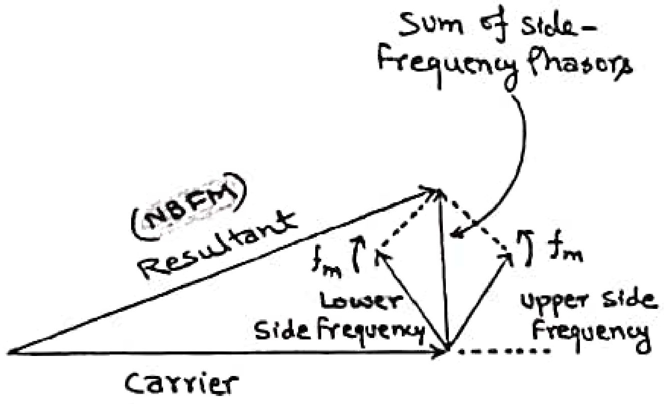
(AM)

<u>Note</u>:- The basic difference between NBFM & AM is that the sign of the LSB frequency in NBFM is reversed.

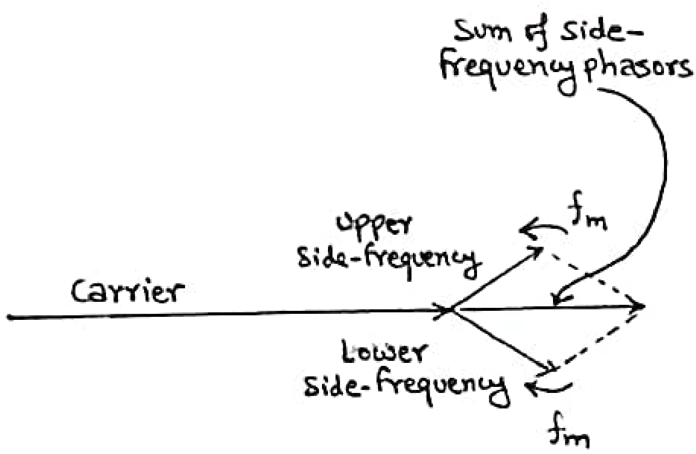
So, In NBFM, The transmission bandwidth

$\boxed{B_T = 2f_m}$ Same as AM signal.

PHASOR COMPARISON OF NARROW-BAND FM & AM WAVES



a) Narrow-band FM wave



b) AM wave

a) Narrow-band FM wave

1) Carrier phasor as reference.

2) The resultant of the two side-frequency phasors is always at right angle to the carrier phasor.

b) AM wave

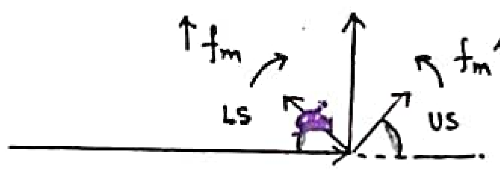
1) Carrier phasor as reference.

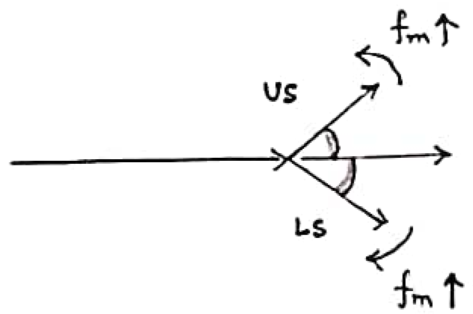
2) The resultant of the two side frequency phasors is always in phase with the carrier phasor.

3) The resultant phasor represent narrowband FM wave i.e. approximately of the same amplitude as Carrier phasor, but out of phase w.r.t it.

3) The resultant phasor represent the corresponding AM wave that has a different amplitude from that of the carrier phasor but always in phase with it.

Note:-





Wide-Band Frequency Modulation (WBFM)

Wide-Band Frequency Modulation (WBFM)

* modulation Index ; $\beta >> 1$ rad.

Consider a single-tone FM wave for an arbitrary value of β.

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]}$$

This FM wave is produced by a sinusoidal modulating wave m(t) that is a periodic function of time 't' only when $f_c = n f_m$ (i.e. integral multiple of modulating frequeny)

Here ; f_c is large enough compared to BW of FM wave.

Next, consider m(t) ≈ sinusoidal modulating wave is a non-periodic function of time.

So,
$$s(t) = \mathrm{Re}\left[A_c\, e^{j[2\pi f_c t + \beta \sin 2\pi f_m t]} \right]$$ .... Real part of complex quantity

$$s(t) = \mathrm{Re}\left[\underbrace{A_c\, e^{j\beta \sin 2\pi f_m t}} \cdot e^{j2\pi f_c t} \right]$$

$$s(t) = \mathrm{Re}\left[\tilde{s}(t)\, e^{j2\pi f_c t} \right] \quad \text{— (1)}$$

where $\tilde{s}(t) = A_c e^{j\beta \sin 2\pi f_m t}$

--> Complex envelope of FM wave $s(t)$

--> Periodic function of time with a fundamental frequency equal to modulating frequency f_m.

So,

Replacing

$$\boxed{t \rightarrow t + \frac{k}{f_m}}$$

So,
$$\tilde{s}(t) = A_c\, e^{j\beta \sin\left[2\pi f_m\left(t + \frac{k}{f_m}\right)\right]}$$

$$= A_c\, e^{j\beta \sin(2\pi f_m t + 2\pi k)}$$

$$\boxed{\tilde{s}(t) = A_c\, e^{j\beta \sin(2\pi f_m)}}$$ ---> Confirm f_m as a fundamental frequency of $\tilde{s}(t)$

Expand Using Fourier Series

$$s(t) = \mathrm{Re}\left[\underbrace{A_c e^{j\beta \ldots}} \cdot e^{j \ldots} \right]$$

$$s(t) = \mathrm{Re}\left[\tilde{s}(t)\, e^{j2\pi f_c t} \right] \quad \text{—①}$$

where $\tilde{s}(t) = A_c e^{j\beta \sin 2\pi f_m t}$

→ Complex envelope of FM wave $s(t)$

→ Periodic function of time with a fundamental frequency equal to modulating frequency f_m.

So,

Replacing

$$\boxed{t \rightarrow t + \frac{k}{f_m}}$$

for some Integer 'k'

So, $\tilde{s}(t) = A_c e^{j\beta \sin\left[2\pi f_m\left(t + \frac{k}{f_m}\right)\right]}$

$= A_c e^{j\beta \sin(2\pi f_m t + 2\pi k)}$

$$\boxed{\tilde{s}(t) = A_c e^{j\beta \sin(2\pi f_m)}}$$

→ Confirm f_m as a fundamental frequency of $\tilde{s}(t)$

Expand Using Fourier Series

$\tilde{s}\ldots \quad j\beta \sin(2\pi f_m)$

$$\tilde{s}(t) = A_c e^{j\beta \sin(2\pi f_m)}$$

$$\boxed{\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j 2\pi n f_m t}} \quad \text{------ ②}$$

where

$$\boxed{c_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{s}(t)\, e^{-j2\pi n f_m t}\, dt} \quad \text{.... complex fourier coefficient}$$

$$C_n = f_m A_c \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{(j\beta \sin \underline{2\pi f_m t} - j2\pi n f_m t)}\, dt$$

Let, Define new variable : $x = 2\pi f_m t$

Let, Define new variable : $x = 2\pi f_m t$

$$C_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{(j\beta \sin x - jnx)} \, dx$$

$dx = 2\pi f_m \, dt$

so, $dt = \dfrac{dx}{2\pi f_m}$

for $t = -\dfrac{1}{2f_m}$, $x = -\pi$

$t = \dfrac{1}{2f_m}$, $x = +\pi$

$$C_n = A_c J_n(\beta)$$

where
$$\boxed{J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \, dx}$$
Complex Notation

n^{th} order Bessel function of First kind & argument β.

From eqⁿ ②

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \, e^{j2\pi n f_m t} \quad \text{---③}$$

From eqⁿ 2

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\, e^{j2\pi n f_m t} \quad \text{---} (3)$$

Substitute (3) in (1)

$$s(t) = \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} A_c J_n(\beta)\, e^{j2\pi n f_m t} \cdot e^{j2\pi f_c t}\right]$$

(2)

$$s(t) = \mathrm{Re}\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi (f_c + n f_m) t}\right]$$

$$\boxed{s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi (f_c + n f_m) t\right]}$$

--- desired form for the fourier series expansion of single tone FM signal for an arbitrary value of β (modulation Index).

$n=-\infty$

$$\boxed{s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi (f_c+nf_m)t\right]}$$

-- desired form for the Fourier series expansion of single tone FM signal for an arbitrary value of β (modulation Index).

Time domain

Fourier Transform.

$$\boxed{S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f-f_c-nf_m) + \delta(f+f_c+nf_m)\right]}$$

... frequency domain

$\because \cos 2\pi f_0 t \rightleftharpoons \frac{1}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$

for any f_0

Shows $S(f)$ contains Infinite number of delta functions spaced at $f = f_c \pm nf_m$

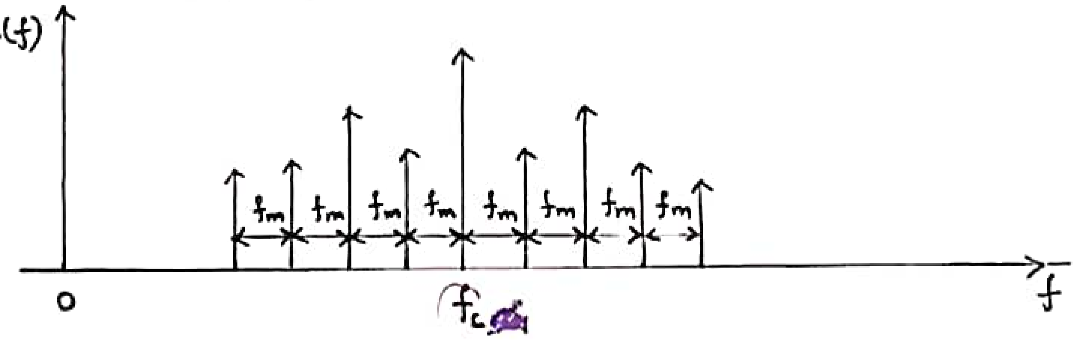
for $n=0, \pm1, \pm2, \ldots$

... infinite number of delta functions spaced at $f = f_c \pm n f_m$

for $n = 0, \pm 1, \pm 2, \ldots$

<u>Note</u>: FM signal is composed of carrier with an Amplitude $\frac{A_c}{2} J_0(\beta)$ & a set of side frequency spaced symmetrically on either side of the carrier at a frequency separation of f_m, $2f_m$, $3f_m$, --- etc.

⊛ The variation of Bessel Function $J_n(\beta)$ determines the Amplitude of various side frequency component of WBFM.



BESSEL FUNCTION

$J_n(\beta) \approx$ n^{th} order Bessel function of First Kind ; where

$\beta \approx$ argument

* Basic Form of Bessel's equation of order 'n' is

$$\beta^2 \frac{d^2y}{d\beta^2} + \beta \frac{dy}{d\beta} + (\beta^2 - n^2)y = 0$$

Solution is defined by power series :

$$\boxed{J_n(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{n+2m}}{m!\,(n+m)!}}$$

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$$\boxed{J_n(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{n+2m}}{m!\,(n+m)!}}$$

$$J_0(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{2m}}{m!\,m!} = 1 - \frac{\beta^2}{1!\,1!\,2^2} + \frac{\beta^4}{2!\,2!\,2^4} - \frac{\beta^6}{3!\,3!\,2^6} + \cdots$$

$$J_1(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{1+2m}}{m!\,(1+m)!} = \frac{\beta}{2} - \frac{\beta^3}{1!\,2!\,2^3} + \frac{\beta^5}{2!\,3!\,2^5} - \cdots$$

$$J_2(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{2+2m}}{m!\,(2+m)!} = \frac{\beta^2}{2!\,2^2} - \frac{\beta^4}{1!\,3!\,2^4} + \frac{\beta^6}{2!\,4!\,2^6} - \cdots$$

Note:-

1) For small value of β.

$$m=0 \quad m!\,(1+m)! \qquad \qquad 1!\,2!\,2 \qquad 2!\,3!\,2$$

$$J_2(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\beta\right)^{2+2m}}{m!\,(2+m)!} = \frac{\beta^2}{2!\,2^2} - \frac{\beta^4}{1!\,3!\,2^4} + \frac{\beta^6}{2!\,4!\,2^6} - \cdots$$

Note:-

1) For small value of β.

$$\boxed{\begin{aligned} J_0(\beta) &\simeq 1 \\ J_1(\beta) &\simeq \frac{\beta}{2} \\ J_n(\beta) &\simeq 0 \quad \text{for } n>1 \end{aligned}} \qquad \boxed{J_n(\beta) \simeq \frac{\beta^n}{2^n\,n!}}$$

2) The function $J_n(\beta)$ may also be expressed in the form of an Integral as:

$$\boxed{J_n(\beta) = \frac{1}{\pi}\int_0^{\pi} \cos(\beta \sin\theta - n\theta)\, d\theta}$$

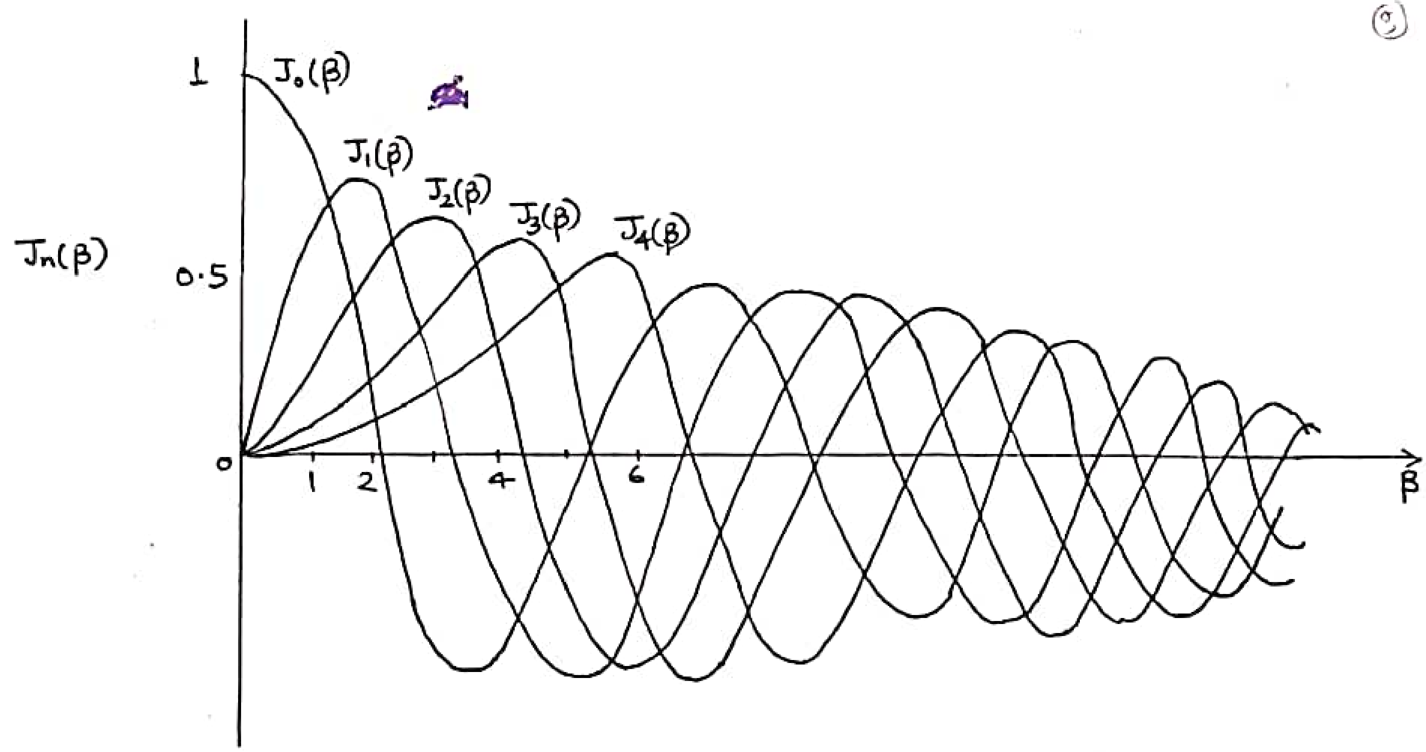

fig: plot of $J_n(\beta)$ Vs β, for varying n.

WBFM spectrum is expressed as:

fig: plot of $J_n(\beta)$ Vs β, for varying n.

WBFM spectrum is expressed as:

$$\boxed{S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m) \right]}$$

Using approximation for Bessel function $J_n(\beta)$,

For small values of β

$$J_0(\beta) \approx 1$$

$$J_1(\beta) \approx \frac{\beta}{2}$$

$$J_n(\beta) \approx 0, \quad n > 1$$

In time domain. WBFM is expressed as:

$$S(t) = A_c \sum^{\infty} J_n(\beta) \cos\left[2\pi (f_c + nf_m)t\right]$$

Using approximation for Bessel function $J_n(\beta)$,

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In time domain. WBFM is expressed as:

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi (f_c + n f_m) t\right]$$

$$\boxed{s(t) \approx A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}\left[\cos 2\pi (f_c + f_m) t - \cos 2\pi (f_c - f_m) t\right]}$$

Same as NBFM.

OBSERVATIONS

1) Special Case : $\beta << 1\,\text{rad}$ (NBFM)

Only $J_0(\beta)$ & $J_1(\beta)$ have significant values.

So, that the FM wave is effectively composed of a carrier and a single pair of side-frequencies at $f_c \pm f_m$.

2) The amplitude of the carrier component varies with β according to $J_0(\beta)$ in case of FM wave.

3) Average power of an FM wave may also be determined using:

2) The amplitude of the carrier component varies with β according to $J_0(\beta)$ in case of FM wave.

3) Average power of an FM wave may also be determined using:

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

$$\boxed{P = \frac{1}{2} A_c^2} \quad \text{where} \quad \boxed{\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1}$$

* Equality property of $J_n(\beta)$ (for arbitrary β)

Transmission Bandwidth of FM waves:

CARSON'S RULE :

In Theory ≈ FM wave contains an infinite number of side-frequencies

so, BW required to transmit FM ≈ ∞

In Practice ≈ FM wave is effectively limited to a finite number of significant side-frequencies, with specified amount of distortion.

Approximate rule ; to determine transmission Bandwidth

$$\boxed{B_T \approx 2(\Delta f + f_m)}$$

Approximate rule ; to determine transmission Bandwidth

$$\boxed{B_T \approx 2(\Delta f + f_m)}$$

$$\boxed{B_T = 2\Delta f\left(1 + \frac{1}{\beta}\right)}$$ Where $\beta = \frac{\Delta f}{f_m}$

Case I: NBFM $(\beta << 1)$ i.e. $\frac{\Delta f}{f_m} << 1$ or $\boxed{\Delta f << f_m}$

$\boxed{B_T = 2 f_m}$ ... Same as AM

Case II: WBFM $(\beta >> 1)$ i.e. $\frac{\Delta f}{f_m} >> 1$ or $\boxed{\Delta f >> f_m}$

$$\boxed{B_T = 2\Delta f}$$

International Regulation for FM :

* Prescribed by CCIR (Consultative Committee for International Radio) followed by commercial FM broadcast station in order to avoid interference.

1) Maximum frequency deviation ± 75 KHz
2) Frequency stability of the carrier ± 2 KHz
3) Allowable bandwidth per channel = 200 KHz

Effect of variation in β on the spectrum of FM wave (WBFM)

"The number of sidebands produced in FM increases with increase in β."

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"The number of sidebands produced in FM increases with increase in β."

$$\boxed{\text{no. of sidebands} \uparrow \ \approx\ \beta \uparrow}$$

Using

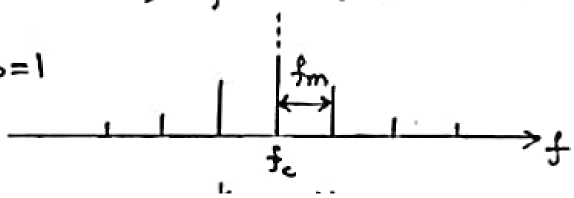
$$\boxed{\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m}}$$

$\beta \uparrow$ Increase

$f_m \downarrow$ keep A_m (fixed)

So, $K_f A_m = \Delta f$ (Constant)

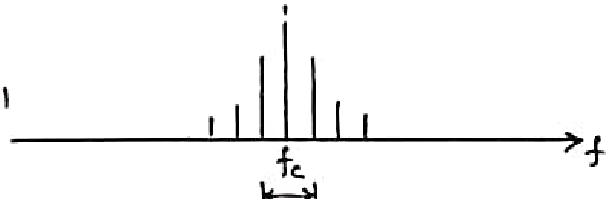
$\beta = 1$



$A_m \uparrow$ keep f_m (fixed)

$K_f A_m \neq$ Constant i.e. Δf

$\beta = 1$



Using $\boxed{\beta = \dfrac{\Delta f}{f_m} = \dfrac{K_f A_m}{f_m}}$

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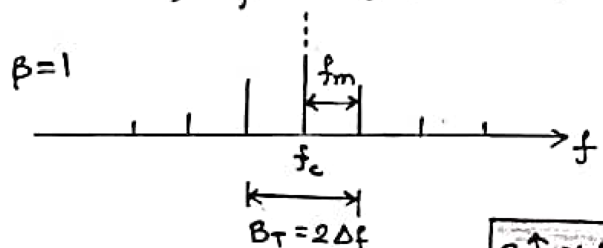
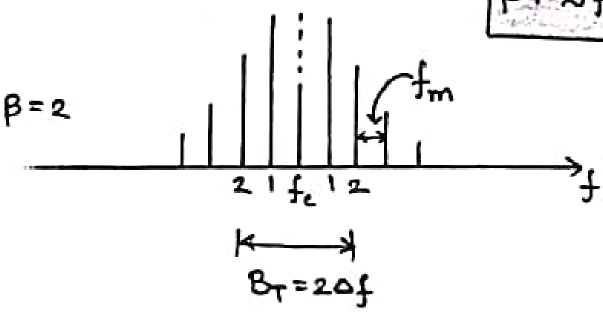


$\boxed{\beta \uparrow \approx f_m \downarrow}$



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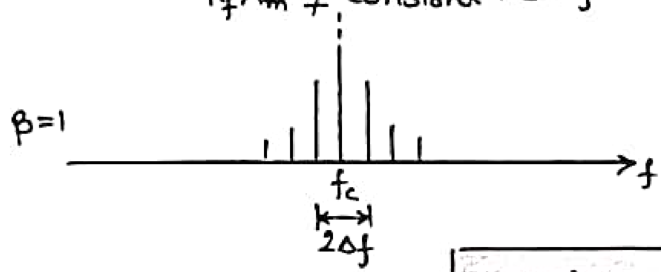
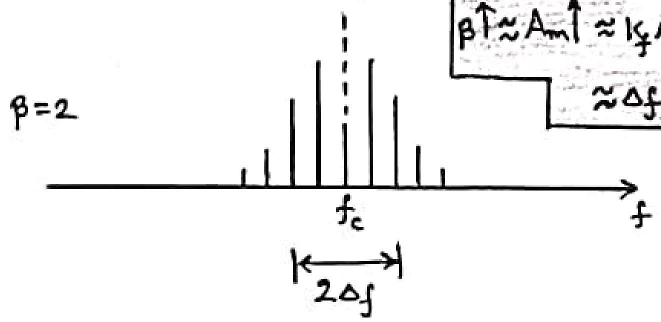
$K_f A_m \neq$ constant i.e. Δf



$\boxed{\beta \uparrow \approx A_m \uparrow \approx K_f A_m \uparrow \approx \Delta f \uparrow}$



GENERATION OF FM WAVES

$$\boxed{f_i(t) = f_c + K_f\, m(t)}$$

Instantaneous Frequency varies linearly with message signal

Two methods → DIRECT METHOD (Reactance Modulator)
→ INDIRECT METHOD.

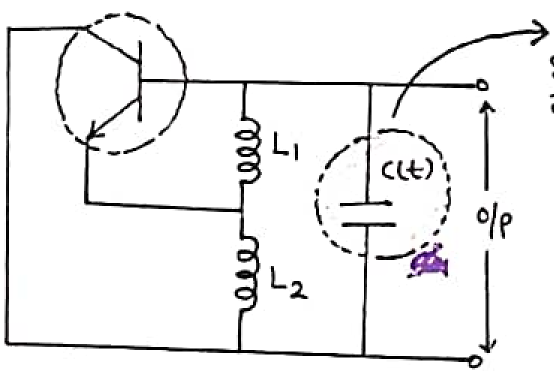
DIRECT METHOD: FM generated by using VCO (voltage controlled Oscillator).

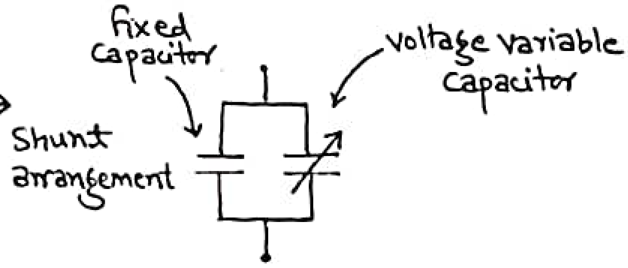
VCO ≈ Sinusoidal oscillator with the reactive element (e.g. capacitive element)

> "The frequency of an oscillator is directly controlled by the messag signal m(t) i.e. voltage signal."

i.e. voltage signal."

The fig. shows an FM generator which uses a <u>Hartley Oscillator</u>.





Assume; c(t) ≈ Instantaneous value of capacitance.

<u>Frequency of oscillation.</u>

$$\boxed{f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\ c(t)}}}$$

$$\boxed{\quad \frac{\quad}{2\pi\sqrt{(L_1+L_2)\ c(t)}}}$$

Here; $\boxed{c(t) = C_0 + \Delta C \cos(2\pi f_m t)}$

C_0 → Total capacitance in absense of modulation

ΔC → Max. change in capacitive value.

(10)

After substitution

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0 + \Delta C \cos 2\pi f_m t)}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}\sqrt{1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)}}$$

⇓

f_0 --- Unmodulated frequency of oscillations

C_0

f_0 --- Unmodulated frequency of oscillations.

$$f_i(t) = f_0 \left[1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t)\right]^{-1/2}$$

If $\Delta C << C_0$, approx eq^n is

$$f_i(t) \simeq f_0 \left[1 - \frac{1}{2} \frac{\Delta C}{C_0} \cos(2\pi f_m t)\right]$$

$$f_i(t) \simeq f_0 \underbrace{- f_0 \frac{\Delta C}{2C_0}}_{\equiv \Delta f \text{ (Assuming)}} \cos(2\pi f_m t)$$

$$\frac{\Delta C}{2C_0} = -\frac{\Delta f}{f_0}$$

{ As increase in capacitance reduce the frequency of oscillation.

$$f_i(t) = f_0 \left[1 - \frac{1}{2}\frac{\Delta C}{C_0} \cos(2\pi f_m t)\right]$$

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{ As increase in capacitance reduce the frequency of oscillation.

$$\boxed{f_i(t) \simeq f_0 + \Delta f \cos(2\pi f_m t)}$$

desired relationship for the instantaneous frequency of the wave.

⊛ Generally, NBFM is generated. To obtain WBFM, frequency multiplier is used.

NBFM $(f_c \pm \Delta f)$ → $\boxed{\text{FREQUENCY MULTIPLIER}}$ → WBFM with center freq. $= Nf_c$

f_c: Center Frequency; Δf: Deviation

deviation $= N\Delta f$

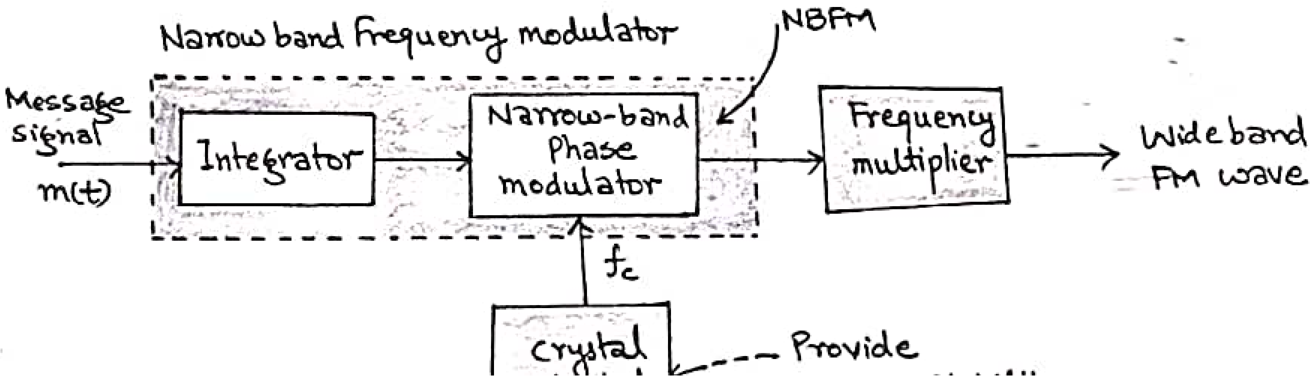
& Modulation Index $= N\beta$

INDIRECT METHOD : ARMSTRONG MODULATOR

"Message signal is first used to produce a NBFM, followed by frequency multiplication to Increase the frequency deviation to the desired level."

<u>Note:-</u> The carrier-frequency stability problem is solved by using highly stable oscillator (i.e. crystal oscillator) in NBFM generation.

⊛ Also called <u>Armstrong wide-band frequency modulator.</u>



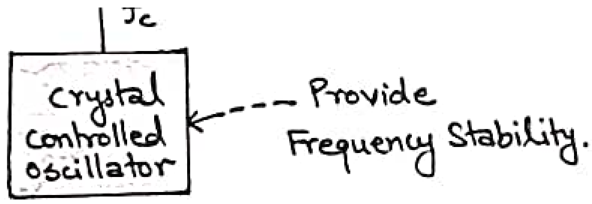


Fig: Indirect method of generating a WBFM using phase modulation technique

⊗ In order to minimize the distortion in Phase modulator, β is kept small, thereby resulting in NBFM wave.

Frequency Multiplier

Consists of a memoryless non-linear device followed by a Band Pass filter.

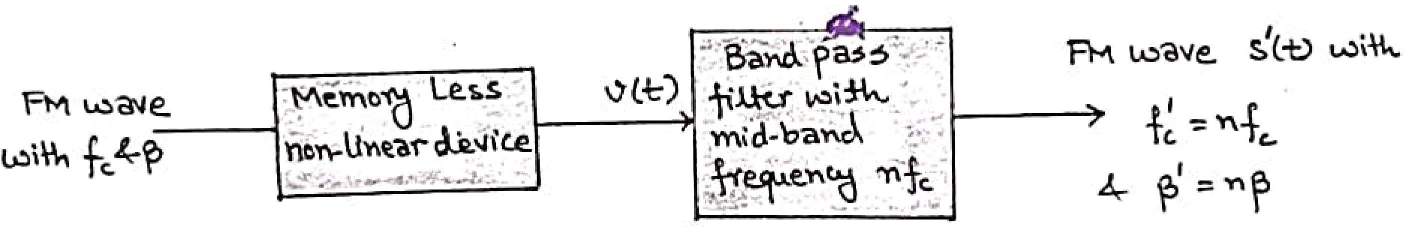


Fig: Block diagram of Frequency multiplier

* Non-linear device being memoryless means it has no energy-storage elements.

Input-Output relation is in the form:

$$v(t) = a_1 s(t) + a_2 s^2(t) + \cdots + a_n s^n(t) \quad -①$$

{ where $a_1, a_2, \ldots, a_n$ are coefficients.
'n' = Highest order of nonlinearity

Here, Input $s(t)$ is FM wave

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]$$

where Instantaneous frequency

$$\boxed{f_i(t) = f_c + k_f m(t)}$$

* After, bandpass filtering of the non-linear device's output $v(t)$,
We have new FM wave defined by

where Instantaneous frequency

$$\boxed{f_i(t) = f_c + K_f m(t)}$$

* After, bandpass filtering of the non-linear device's output $v(t)$, We have new FM wave defined by

$$s'(t) = A_c \cos\left[2\pi f_c'(t) + 2\pi K_f' \int_0^t m(\tau)\, d\tau\right]$$

Whose Instantaneous frequency

$$\boxed{f_i'(t) = nf_c + nK_f m(t)}$$

Here $f_c' = nf_c$
& $K_f' = nK_f$

Note:- The frequency multiplication ratio 'n' is determined by the highest power n in the input-output relationship that characterizing the memoryless non-linear device.

DEMODULATION OF FM SIGNALS

- **Frequency discriminator**
  (Slope detection followed by envelope detection.)
- **Phase-Locked Loop**
  (FM demodulation in Indirect manner)

FREQUENCY DISCRIMINATOR

- This FM demodulator, produce an output voltage linearly dependent on Input frequency.

- Using Slope detection technique ≈ A simple tuned circuit

  Differentiator

  Tuned slightly above or below resonant frequency 'f_0' over a limited range.

Tuned slightly above or below resonant frequency 'f_0' over a limited range.

Differentiator

$$\underset{\text{FM Signal}}{s(t)} \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow \boxed{\text{Envelope Detector}} \longrightarrow \text{Envelope} \propto m(t)$$

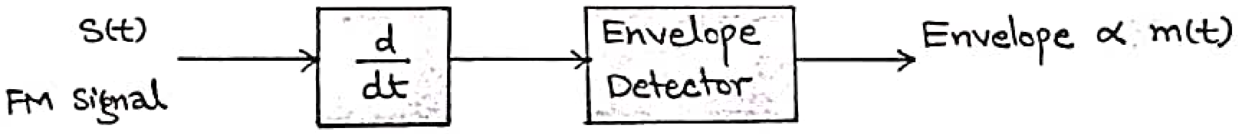
$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right)$$

$$\frac{d}{dt} s(t) = \frac{d}{dt} A_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right)$$

$$= -\sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right) \times \left[2\pi f_c + 2\pi K_f m(t)\right]$$

$$\frac{d}{dt} s(t) = \underbrace{-2\pi A_c\left[f_c + K_f m(t)\right]}_{\text{Envelope.}} \sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right)$$

Both Amplitude & Frequency modulated.



$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right)$$

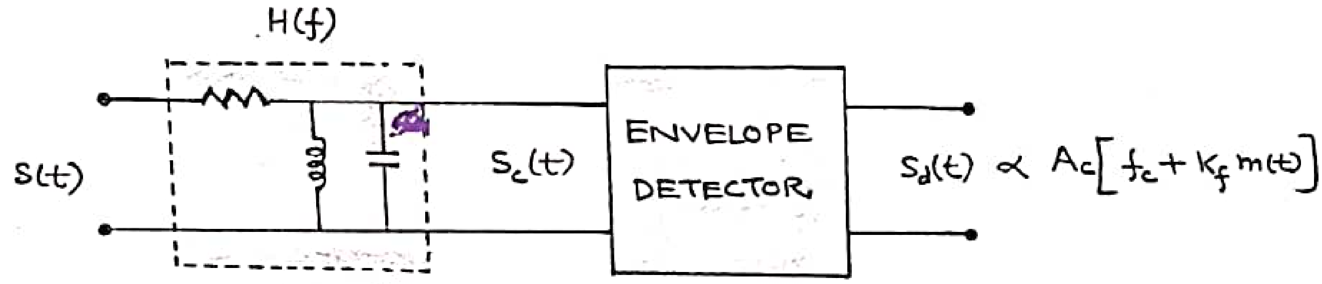
$$\frac{d}{dt} s(t) = \frac{d}{dt} A_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right)$$

$$= -\sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right) \times \left[2\pi f_c + 2\pi K_f\, m(t)\right]$$

$$\frac{d}{dt} s(t) = \underbrace{-2\pi A_c\left[f_c + K_f\, m(t)\right]}_{\text{Envelope.}} \sin\left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau\right)$$

Both <u>Amplitude</u> & <u>Frequency modulated</u>.

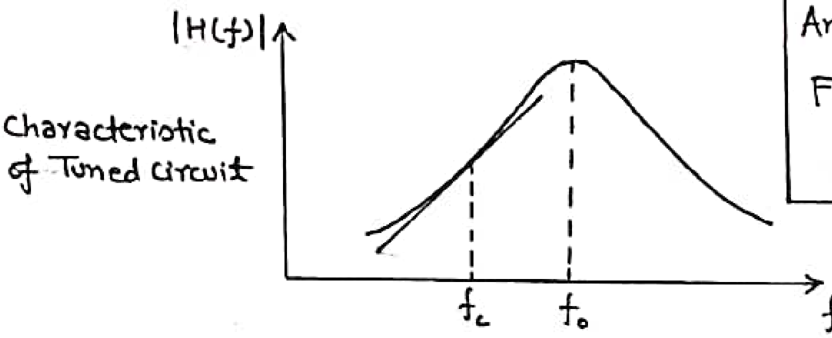
$\boxed{\text{\underline{Note}: } s(t) \approx \text{FM signal with } f_c < f_o \text{ of tuned circuit.}}$

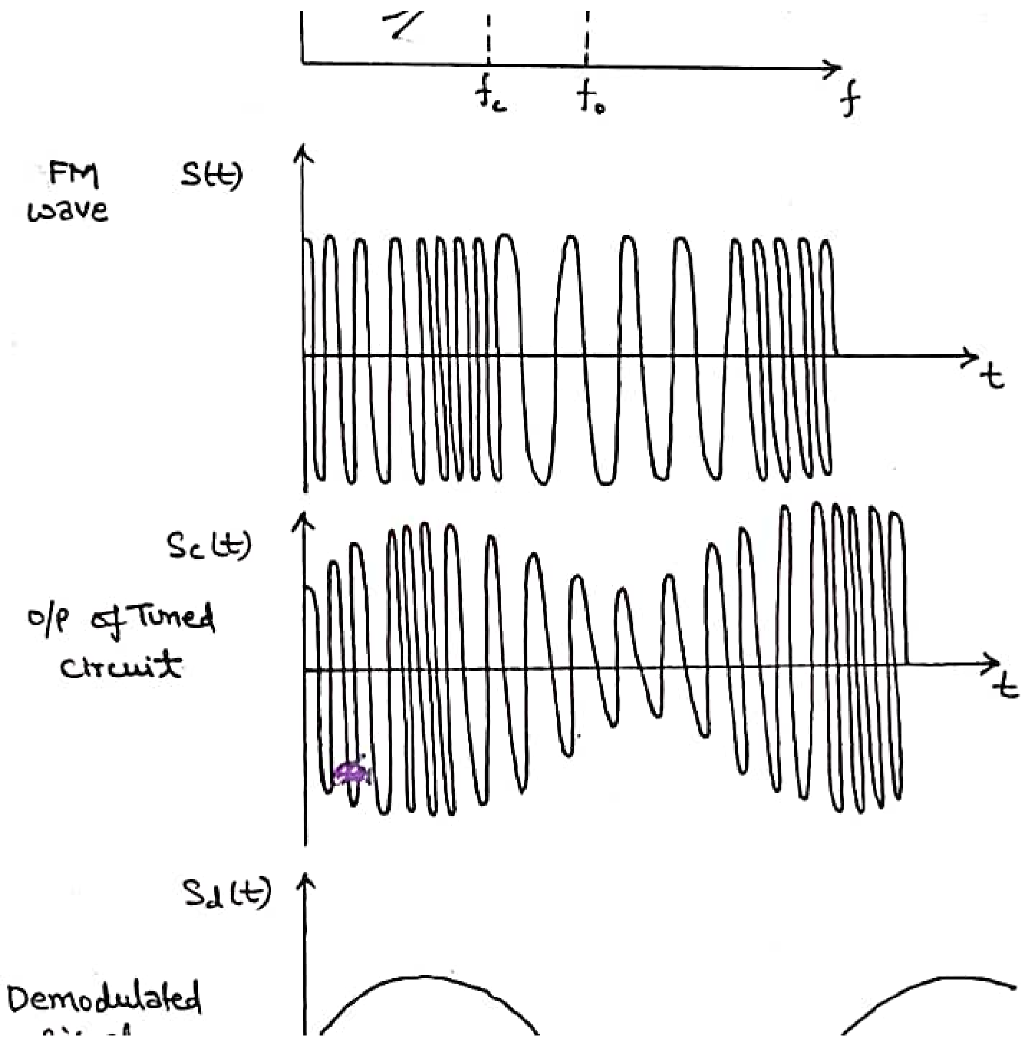


Tuned Circuit
with resonant frequency f_0

Note: $f_i(t)$ of FM swing above or below f_c

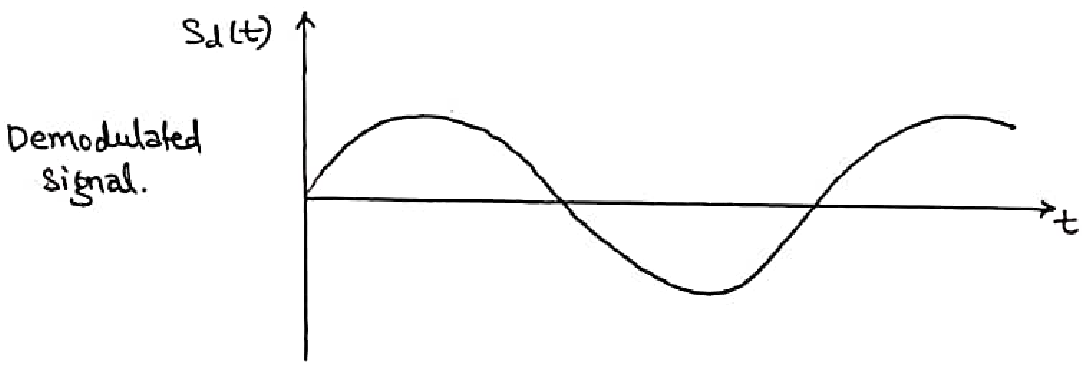
Amplitude ratio of Tuned circuit convert Frequency variation $\xrightarrow{\text{to}}$ Amplitude variation Resulting $s_c(t)$





Extract only the amplitude variation with Envelope detector (with d.c. block) Produce $S_d(t)$ i.e. demodulated Signal.

(with d.c. block)
Produce $S_d(t)$ i.e.
demodulated Signal.



Note:- Simple slope detection Technique having two basic problems.

1) Detector responds to spurious amplitude variation of input FM.
2) Range of linear slope is quite small.

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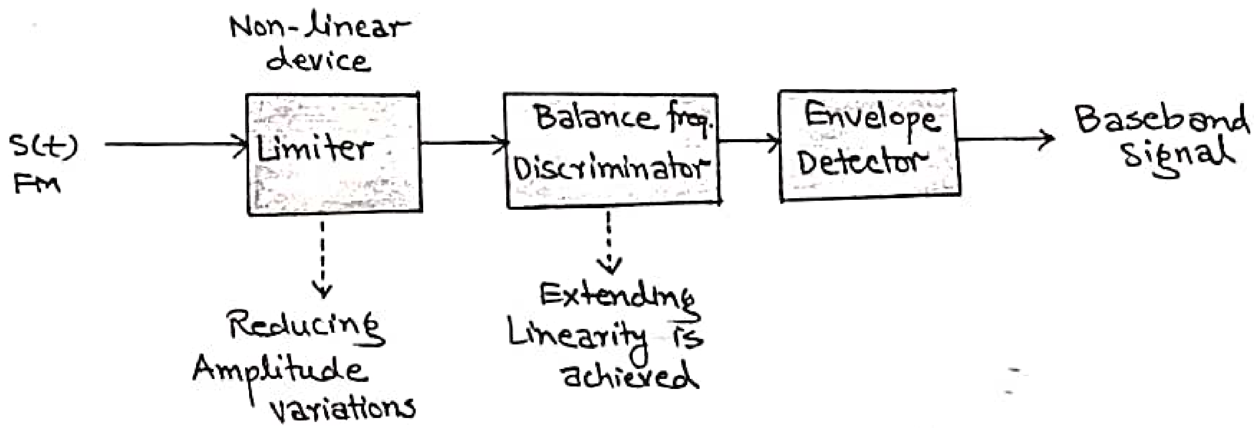
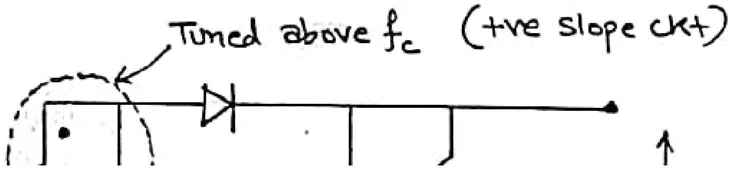
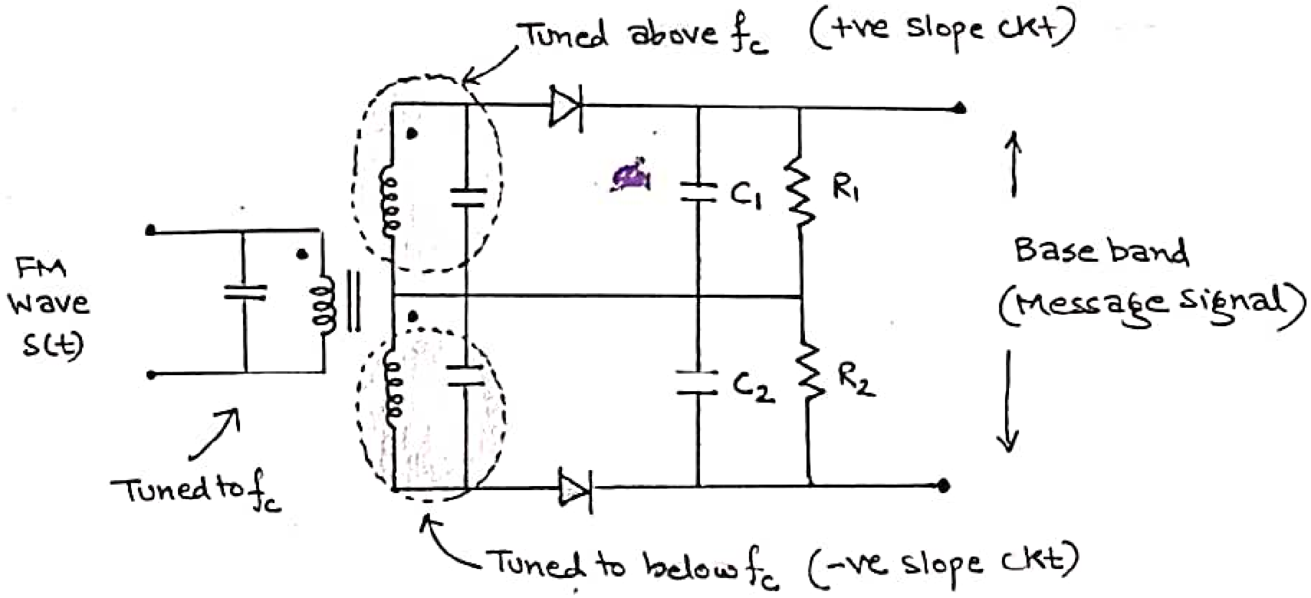
2) Range of linear slope is quite small.



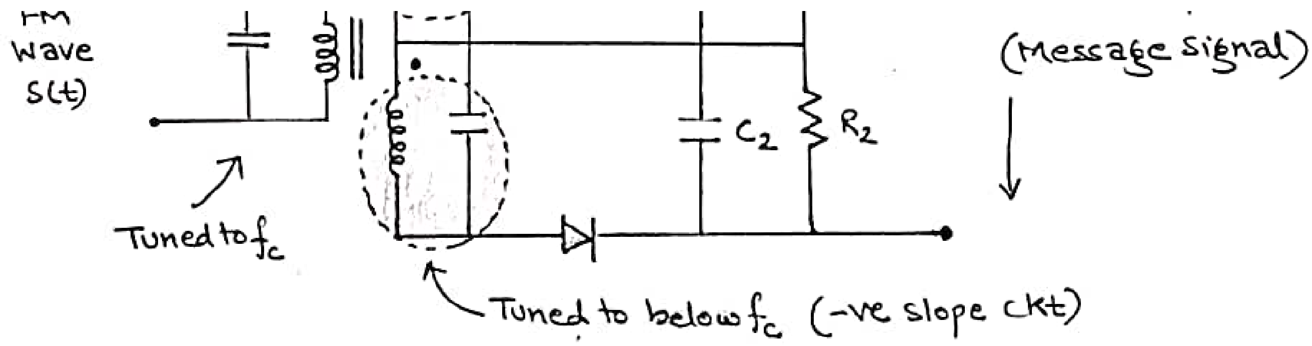


variations



- The output of tuned circuit on the secondary are envelope detected separately ; the difference of the two envelope detected output would be proportional to m(t).

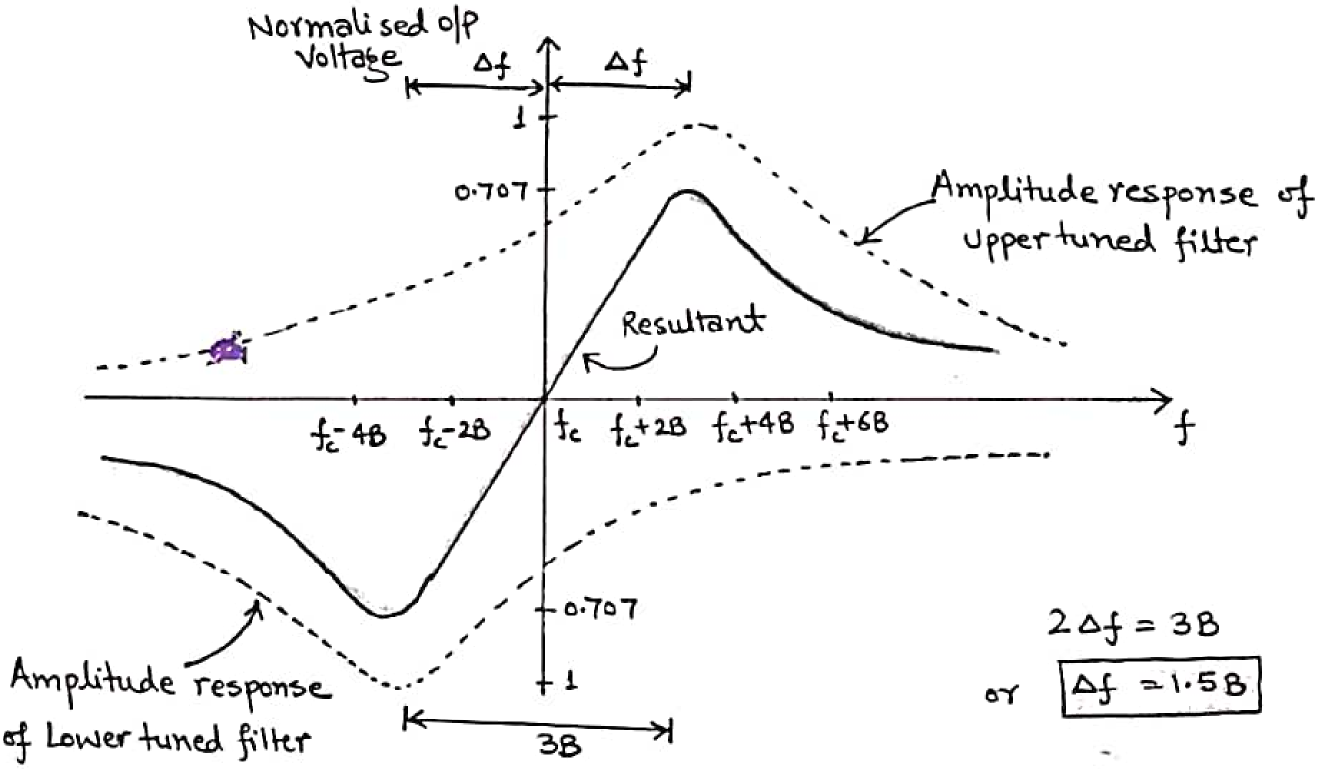
Amplitude variation Difference of these amplitude



⊗ The output of tuned circuit on the secondary are envelope detected separately ; the difference of the two envelope detected output would be proportional to m(t).

So, $f_i(t)$ changes $\approx$ Amplitude variation in opposite directions $\approx$ Difference of these amplitude variation is known as **S - curve**

i.e. Frequency to voltage characteristic

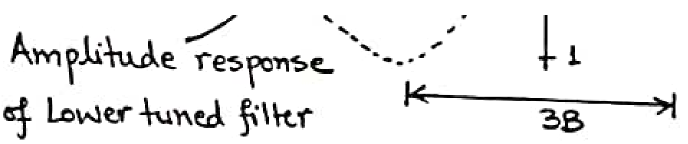


$$2\Delta f = 3B$$

or $\boxed{\Delta f = 1.5B}$

Fig: S-curve

Note:- The width of linear frequency response is about 3B



or $\boxed{\Delta f = 1.5B}$

Fig: S-curve

<u>Note</u>:- The width of linear frequency response is about 3B.
So, the deviation Δf for which the curve is linear & give satisfactory output is given as

$$\boxed{\Delta f = 1.5B}$$

<u>Case</u>: If Δf of input FM is more than $(1.5B)$, distortion occur due to non-linearity of the characteristic.

So, operation of this detector is <u>limited</u> to <u>small deviation only</u>:

PRACTICAL FREQUENCY DEMODULATORS

widely used in past

Case : If Δf of input FM is more than $(1.5B)$, distortion occur due to non-linearity of the characteristic.

So, operation of this detector is limited to small deviation only:

PRACTICAL FREQUENCY DEMODULATORS

- Ratio Detector ≈ Another balanced demodulator widely used in past, offer better protection against carrier amplitude variation.
  - → For many years detectors were standard in almost all FM receivers.

- Zero-crossing detectors ≈ Also used because of advances in digital integrated circuits.
  - ↓ These are frequency counters ≈ measures the instantaneous frequency by the number of zero crossings.

$$\boxed{\text{Rate of zero Crossing} = f_i(t)}$$

(PLL) PHASE LOCKED LOOP

Most widely used method today [closed-loop feedback system]

"Low cost & superior performance especially when SNR is Low"

PLL $\xrightarrow{\text{Used as}}$
- Indirect frequency demodulation
- Carrier synchronization

PLL Consist of.

- **VCO** (Voltage controlled oscillator) [Perform Frequency modulation on its own control signal]
- **Multiplier** [Multiplies incoming FM by the output of VCO]
- **Loop filter** (Low-pass kind) [Remove High frequency component contained in multiplier output.]

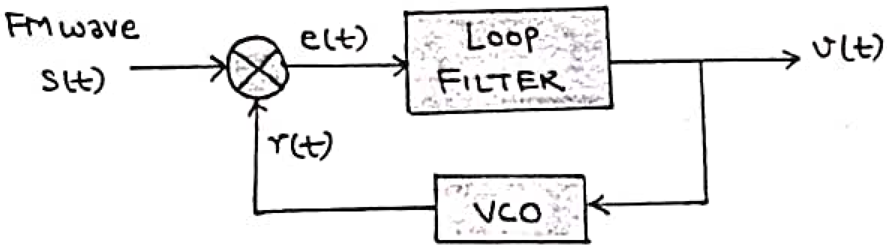


Fig: Block diagram of PLL.

Assume:-

VCO is adjusted such that when the input i.e. control signal $v(t) = 0$; two conditions are satisfied.

1) The free-running frequency of VCO is set to carrier frequency 'f_c'
2) VCO output $r(t)$ has a $90°$ phase shift w.r.t unmodulated 'f_c'.

Suppose, Incoming FM wave is given as

$$s(t) = A_c \sin\left[2\pi f_c t + \phi_1(t)\right]$$

where $\phi_1(t) = 2\pi K_f \int^{t} m(\tau)\,d\tau$

Here
A_c = Carrier Amplitude
K_f = frequency sensitivity of frequency modulator
for Generation

Suppose, Incoming FM wave is given as

$$s(t) = A_c \sin\left[2\pi f_c t + \phi_1(t)\right]$$

where $\phi_1(t) = 2\pi K_f \int_0^t m(\tau)\,d\tau$

Here
A_c = Carrier Amplitude
K_f = frequency sensitivity of frequency modulator responsible for generation of $s(t)$.

Now, define the locally generated FM wave by VCO as

$$r(t) = A_v \cos\left[2\pi f_c t + \phi_2(t)\right]$$

where $\phi_2(t) = 2\pi K_v \int_0^t v(\tau)\,d\tau$

Here
A_v = Amplitude
K_v = Frequency Sensitivity factor of the VCO.

Note: The function of the feedback loop around the VCO is to adjust the angle $\phi_2(t)$ equals $\phi_1(t)$.

the angle $\phi_2(t)$ equals $\phi_1(t)$.

Multiplication of the incoming FM wave $s(t)$ by the locally generated FM wave $r(t)$ produced two components.

$$e(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \times A_v \cos[2\pi f_c t + \phi_2(t)]$$

$\because \sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$

$$e(t) = \frac{A_c A_v}{2}\left\{\sin[\phi_1(t) - \phi_2(t)] + \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]\right\}$$

$$e(t) = K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)] + K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) - \phi_2(t)]$$

(Low-frequency component) $\approx$ difference-frequency term

(High-frequency component) $\approx$ Double-frequency term.

where, $K_m = \frac{1}{2}$ i.e. multiplier gain.

Pass through
Loop filter
(Remove High frequency component)

(Remove High frequency component) ↓

so, $\boxed{e(t) = K_m A_c A_v \sin[\phi_e(t)]}$ where

$$\phi_e(t) = \phi_1(t) - \phi_2(t) \quad \text{---- Phase error}$$

$$\phi_e(t) = \phi_1(t) - 2\pi K_v \int_0^t v(\tau)\,d\tau$$

(19)

Now, when $\phi_e(t) = 0$, the PLL is said to be in <u>Phase-lock.</u>

$\phi_e(t) < 1$ rad, " " " " " " <u>near Phase-Lock.</u>

So, we may use the approximation.

$$\sin[\phi_e(t)] \approx \phi_e(t)$$ --- Accurate within 4%

Provided $\phi_e(t) < 0.5$ radians

So, we may use the appr

$$\sin[\phi_e(t)] \approx \phi_e(t)$$ --- Accurate within 4% provided $\phi_e(t) < 0.5$ radian.

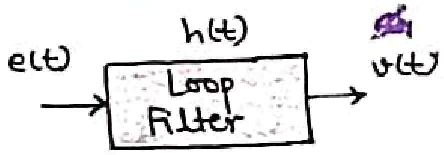
So, $e(t) \cong K_m A_c A_v \phi_e(t)$

$$\boxed{e(t) = \frac{K_0}{K_v}\phi_e(t)}$$

where New parameter $K_0 = K_m K_v A_c A_v$ ⤷ Loop gain parameter of PLL.

The error signal $e(t)$ acts on loop filter produce overall output $v(t)$

Let $h(t)$ = Impulse response of loop filter



$v(t) = e(t) * h(t)$ ---- o/p of LTI system

$$\boxed{v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau}$$

So, equation constitute a linearized feedback model of PLL.

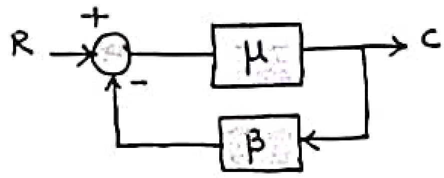
$$\boxed{v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau}$$

→ So, equation constitute a linearized feedback model of PLL.

Using Theorem from "Linear feedback Theory"

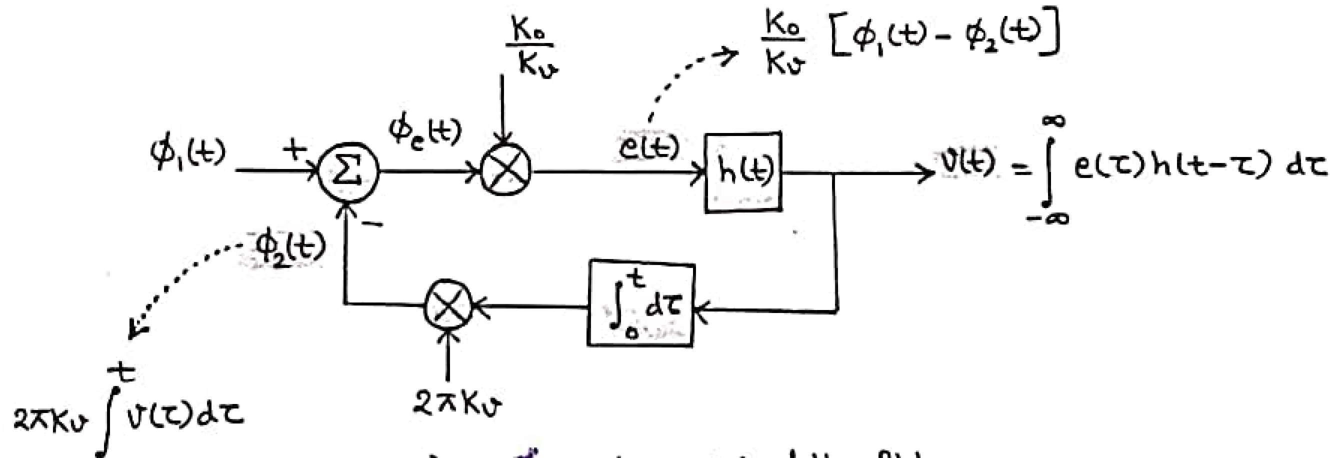
"When the open-loop transfer function of linear feedback system has a large magnitude compared to unity for all frequencies, the closed-loop transfer function of the system is effectively determined by the Inverse of the transfer function of the feedback path."

Example: Negative feedback Amplifier



So, Gain, $A = \dfrac{\mu}{1+\mu\beta} \approx \dfrac{1}{\beta}$

when $\mu\beta \gg 1$ (i.e. open loop gain)



a) Linearized model of the PLL

For the linearized feedback model, We observe three points

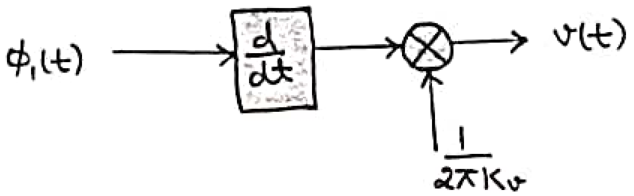
1) Using theorem
   The feedback path is defined by the scaled integrator. Correspondingly, the Inverse of this feedback path is described in the time domain

The feedback path is defined by the ...

the Inverse of this feedback path is described in the time domain by the scaled differentiator.

$$\boxed{v(t) = \frac{1}{2\pi K_v}\left(\frac{d\phi_2(t)}{dt}\right)} \qquad \text{where } \phi_2(t) = 2\pi K_v \int_0^t v(\tau)\,d\tau$$

2) The closed loop time-domain behaviour of PLL is described by the overall output $v(t)$ produced in response to angle $\phi_1(t)$ in the incoming FM wave $s(t)$.



b) Approximate form of model (Assume loop gain; $K_0 >> 1$ of PLL

$K_0 \to K_m K_v A_c A_v$

3) The magnitude of the open-loop transfer function of the PLL is controlled by the loop gain parameter $K_0 = K_m K_v A_c A_v$.

Assuming, that the loop-gain parameter $K_0 >> 1$.

" The closed loop transfer function of PLL is effectively determined by the Inverse of the transfer function of the feedback path."

So, we may relate the overall output $v(t)$ by the approximate formula in terms of $\phi_1(t)$.

$$\boxed{v(t) \approx \frac{1}{2\pi K_v}\left(\frac{d\,\phi_1(t)}{dt}\right)}$$

As, K_0 assume very large $\approx$ make phase error tends to zero
so, Under this condition $\phi_1(t) \approx \phi_2(t)$.

$$v(t) \cong \frac{1}{2\pi K_v}\frac{d}{dt}\left(2\pi K_f \int_0^t m(\tau)\,d\tau\right)$$

$$v(t) = \frac{K_f}{K_v}\, m(t)$$

↳ scaling factor

$\boxed{\text{As, } K_o \text{ assume very large} \approx \text{make phase error tends to zero so, Under this condition } \phi_1(t) \approx \phi_2(t).}$

$$v(t) \cong \frac{1}{2\pi K_v} \frac{d}{dt}\left(2\pi K_f \int_0^t m(\tau)\,d\tau\right)$$

$$v(t) = \frac{K_f}{K_v} m(t)$$

↳ scaling factor

So, $\boxed{v(t) \propto m(t)}$

When system operate in phase lock mode or near phase lock & the loop gain parameter K_o is large compared to unity. Then only, the original message signal $m(t)$ is recovered from $s(t)$ except the scaling factor $\frac{K_f}{K_v}$.

PULSE MODULATION

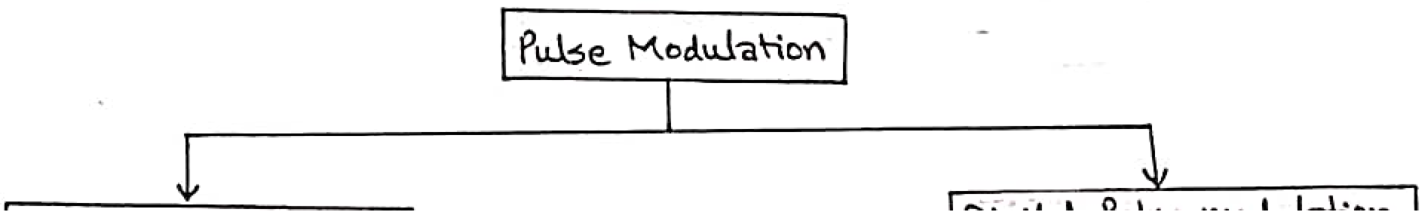
TRANSITION FROM ANALOG TO DIGITAL COMMUNICATION

Last Lecture: Continuous wave (CW) Modulation.

"Some parameter (Amplitude, Frequency & Phase) of a sinusoidal carrier wave is varied continuously in accordance with the message signal."

Pulse Modulation

"Some parameters (Amplitude, duration or position) of a pulse train is varied in accordance with the message signal."



Analog Pulse modulation

Digital Pulse modulation.

carrier wave ≈ Periodic Pulse train

Some characteristic features of each pulse (e.g. amplitude, duration or position) is varied in continuous manner in accordance with the corresponding sample value of message signal.

↓

Information is transmitted in analog form but transmission takes place at discrete times

(Continuous wave process)

Message signal is represented in a form that is discrete in both time & amplitude.

(sampling + Quantization)

↓

Transmission in digital form as a sequence of coded pulses

(No CW process)

(Continuous wave process)

Pulse Modulation

Analog Pulse modulation

Ex: PAM (Pulse amplitude Modulation)
PPM (Pulse Position Modulation)

$\boxed{\text{SAMPLING PROCESS}}$

Digital Pulse modulation

Ex: PCM (Pulse code modulation)
DM (Delta Modulation)
DPCM (Differential Pulse code modulation)

$\boxed{\text{SAMPLING + QUANTIZATION PROCESS}}$

Note: Quantization provides a representation of message signal that is discrete in both time & Amplitude.

Advantages of digitization of Analog sources / Digital systems.

1) Digital systems are less sensitive to noise than analog.

2) With digital systems, it is easier to integrate different services, For Ex: Video & the accompanying sound track, into the same transmission scheme.

3) Digital circuits are less sensitive to physical effects such as vibration & temperature.

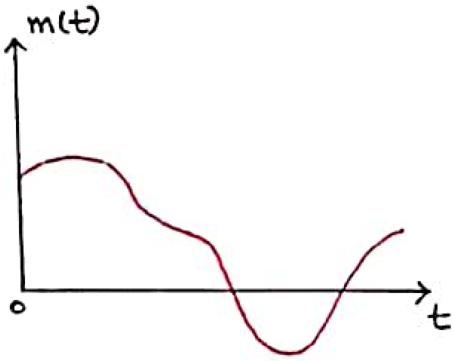
4) Compression of large data signal is possible only in digital communication.

5) Storage & signal manipulation is easier because of advanced DSP techniques and digital computer systems.

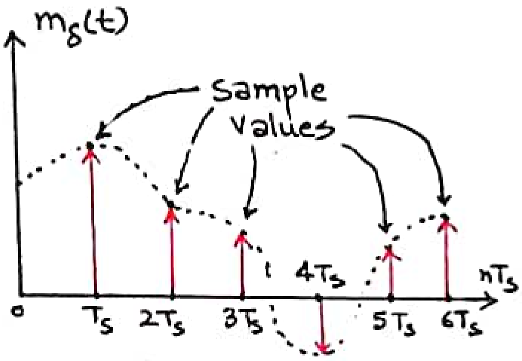
SAMPLING PROCESS

Continuous analog signal $\xrightarrow[t = nT_s]{}$ Discrete analog signal

$m(t)$ $\quad$ T_s = sampling period $\quad$ $m(nT_s) = m(n)$



Original signal.



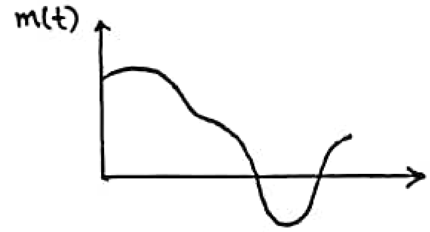
Sampled signal

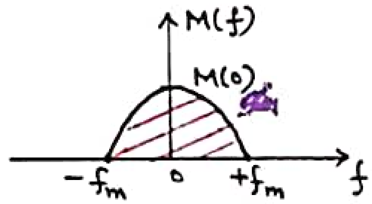
corresponding sequence of samples are uniformly spaced corresponding sequence of samples are uniformly spaced in time.

Sampling Instants

i.e T_s, $2T_s$, $3T_s$ .....

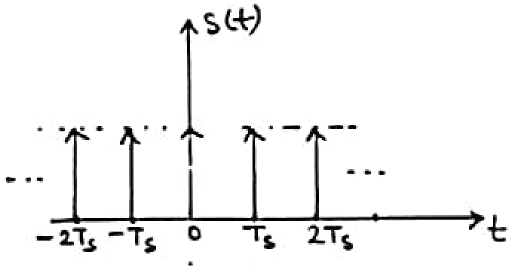
<u>Modulating signal</u> ---- $m(t)$ → Band limited signal
→ Max. modulating frequency, f_m or ω_m

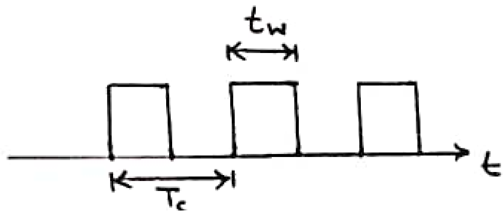




<u>Carrier signal</u> or <u>Sampling Signal</u>

$s(t)$ → Impulse Train (Ideally)
→ Pulse Train (Practically)





Modulating signal ---- $m(t)$ --> Band limited signal
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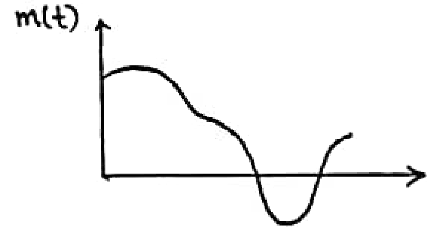
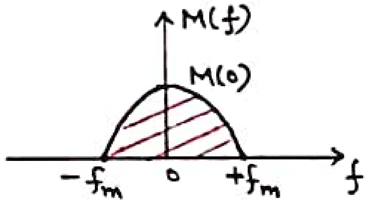
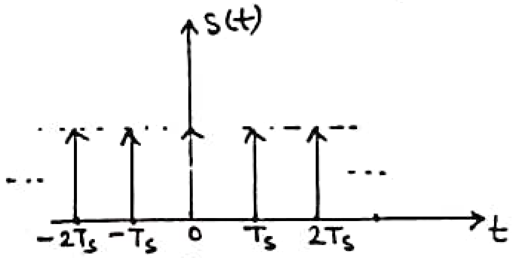
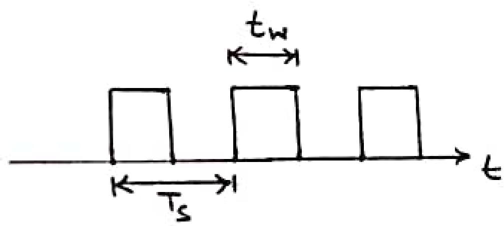




Carrier signal or Sampling Signal
$s(t)$ --> Impulse Train (Ideally)
--> Pulse Train (Practically)



T_s = sampling Interval



$$\boxed{f_s = \frac{1}{T_s}}$$

Sampling Frequency

Sampling rate.

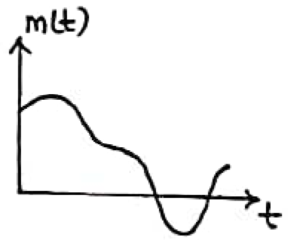
$m(t)$

$m_s(t)$

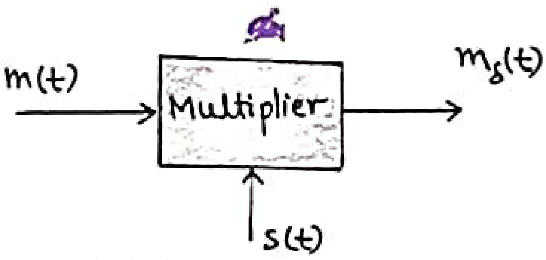
Interval

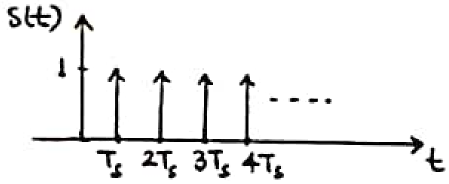
Sampling Frequency

Sampling rate.

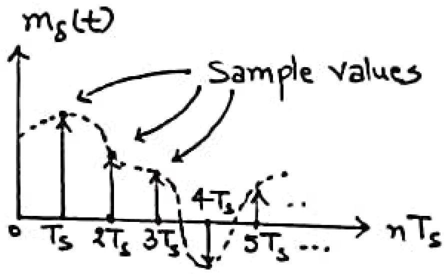


Analog Signal





Unit Impulse Train.



sampled signal
or discrete signal

Sampled at uniform rate, Once every T_s seconds. Called Instantaneous (Ideal) Sampling.

SAMPLING THEOREM

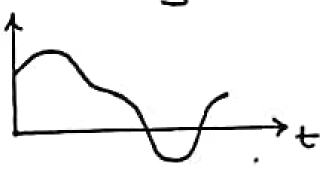
In order to recover original Band limited signal from its sampled version, Called instantaneous (ideal) Sampling.

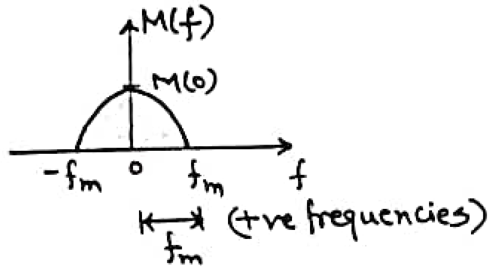
SAMPLING THEOREM

In order to recover original Band limited signal from its sampled version, the sampling frequency must always greater than or equal to twice of the maximum modulating frequency component (f_m).

$$\boxed{f_s \geq 2f_m} \text{ -- Nyquist Rate.}$$

Consider, Baseband signal --- m(t)

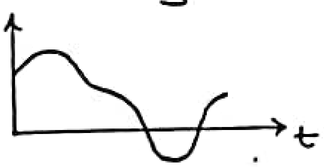


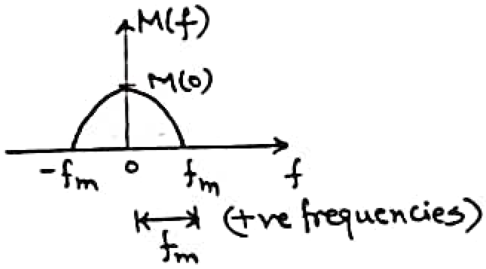


& sampled signal. $\boxed{s(t) = \sum^{\infty} \delta(t-nT_s)}$ $\boxed{S(f) = f_s \sum^{\infty} \delta(f-nf_0)}$

$$\boxed{f_s \geq 2f_m}$$ -- Nyquist Rate.

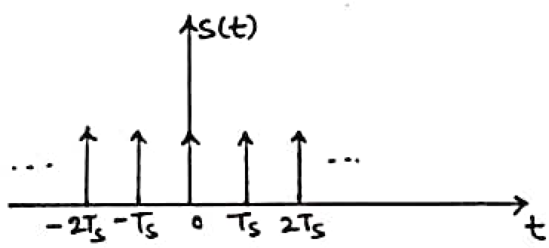
Consider, Baseband signal --- $m(t)$

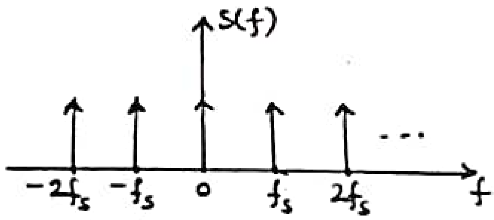




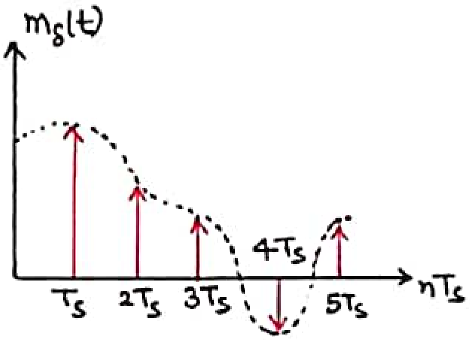
& sampled signal; $\boxed{s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)}$

$$\boxed{S(f) = f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0)}$$

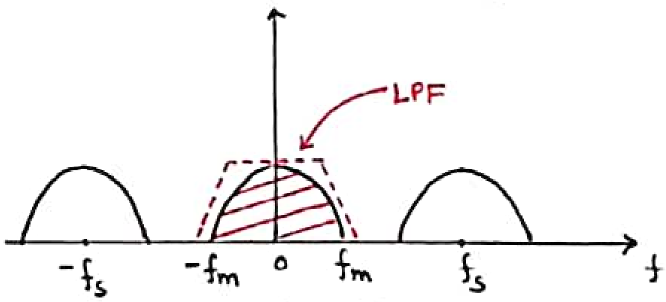




$f_s = \frac{1}{T_s}$

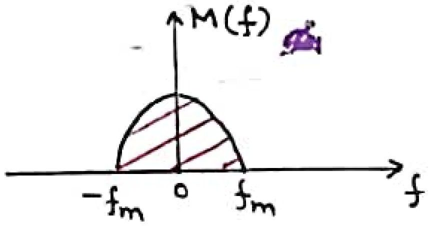


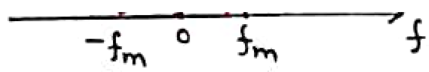
$M_\delta(f)$ --- sampled signal



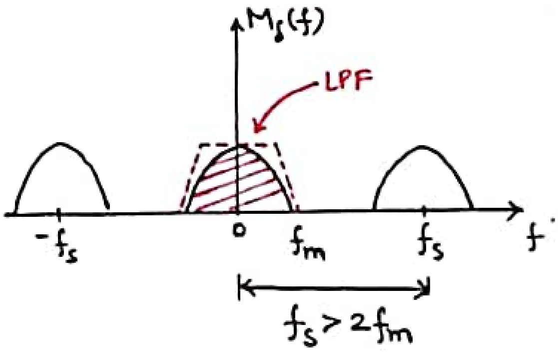
$\boxed{f_s \geq 2f_m}$

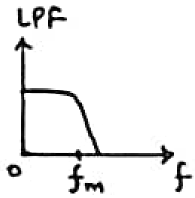
At the Receiver
LPF used with
cut-off freq = f_m



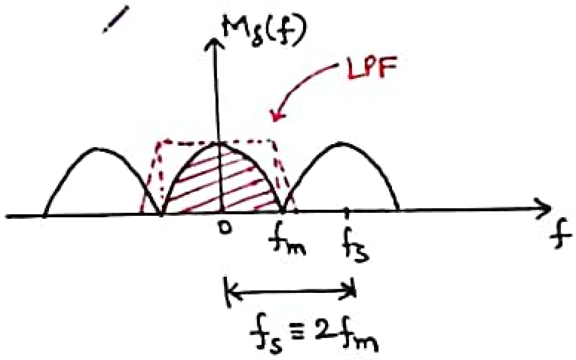


Case I $\boxed{f_s > 2f_m}$

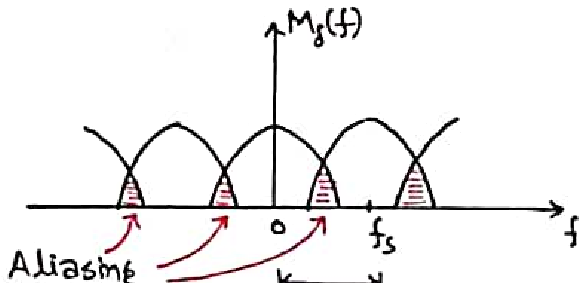


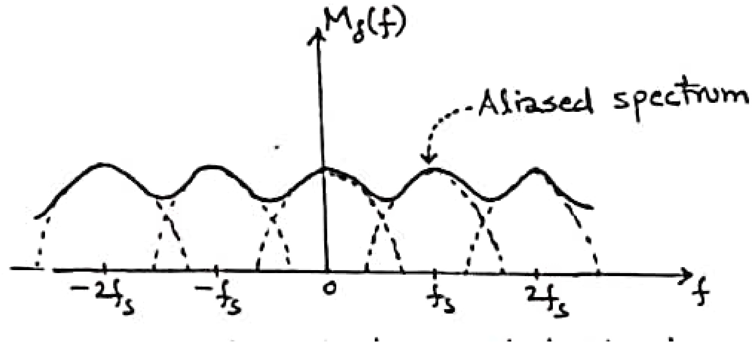


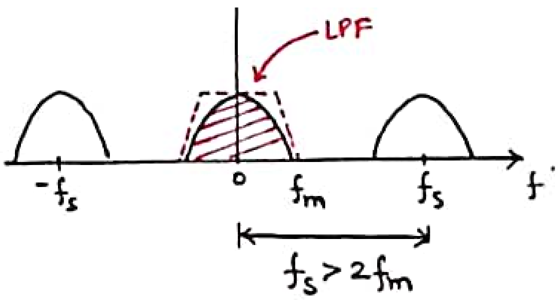
Case II $\boxed{f_s = 2f_m}$

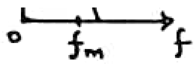


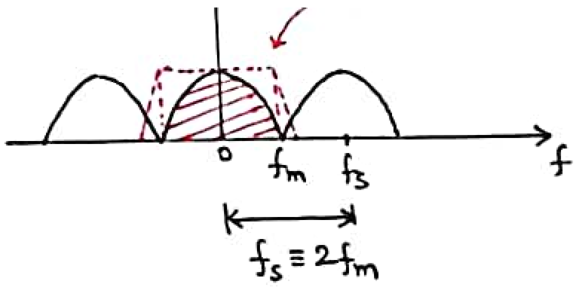
Case III $\boxed{f_s < 2f_m}$



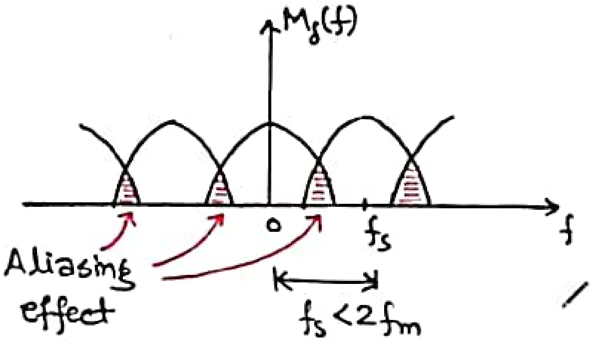




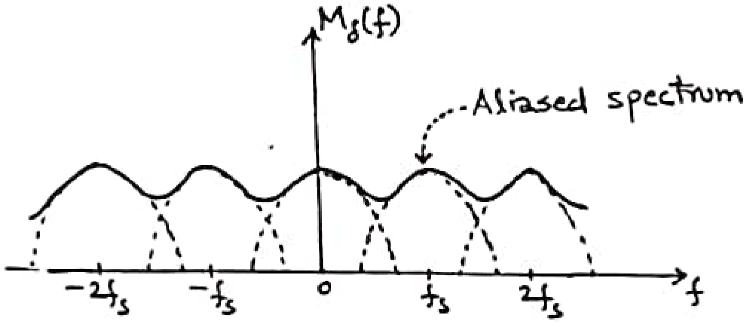




<u>case III</u> $\boxed{f_s < 2f_m}$



Never Recovered $m(t)$



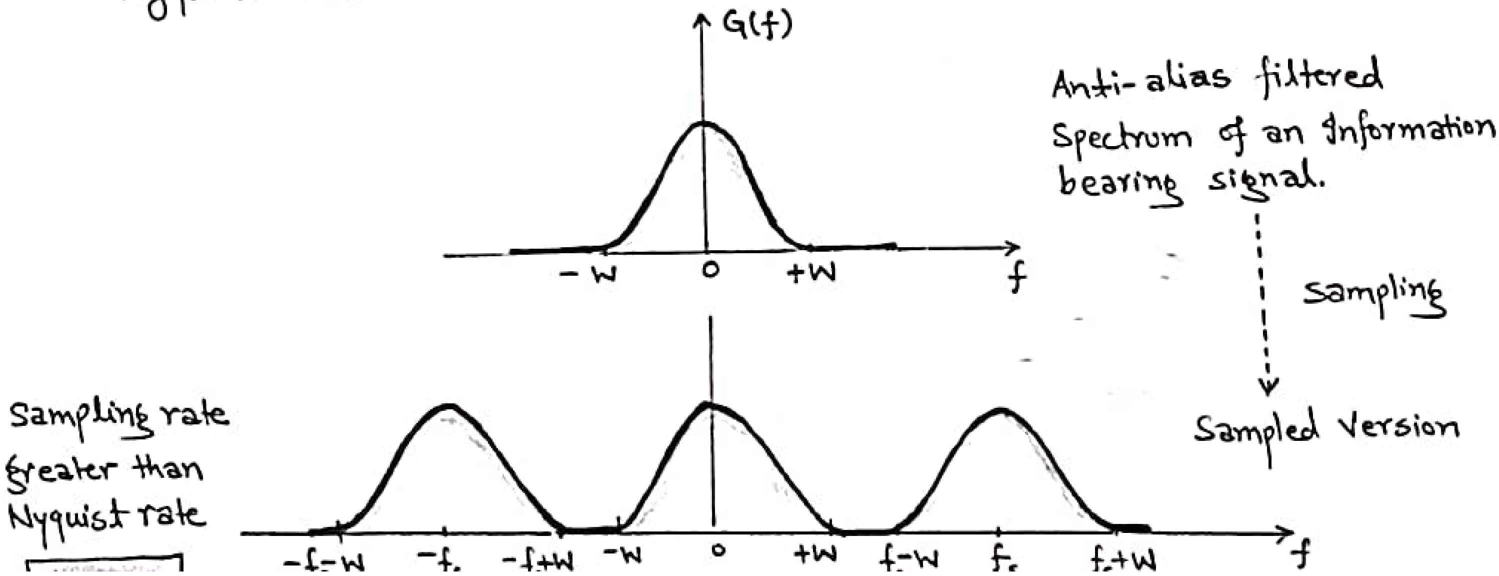
Spectrum of an <u>Undersampled</u> Version exhibiting the <u>aliasing Phenomenon</u>.

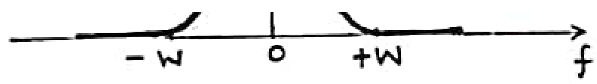
To avoid the aliasing in Practice:

To avoid the aliasing in Practice :

1) Before sampling , a low-pass anti-alias filter is used to attenuate high frequency component of message signal (that is not essential to the information bearing signal.).

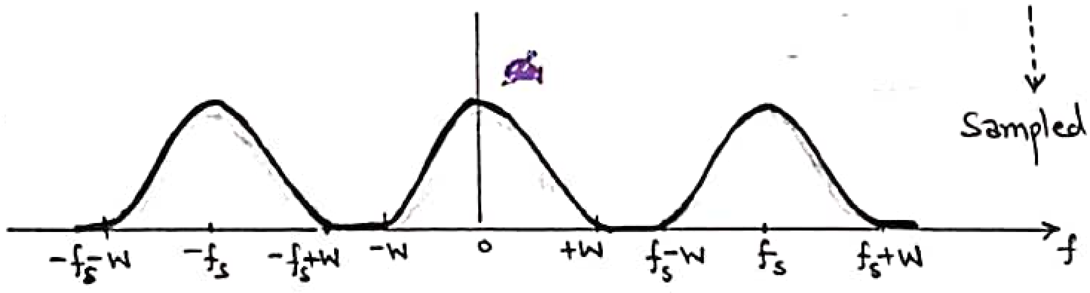
2) The filtered signal is sampled at a rate slightly higher than the Nyquist rate.





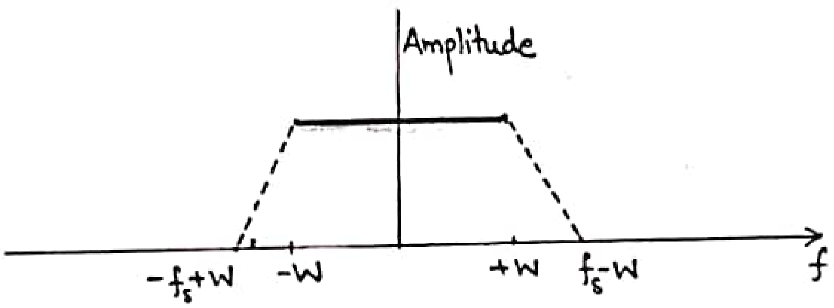
sampling

Sampled Version



Sampling rate greater than Nyquist rate

$\boxed{f_s > 2W}$

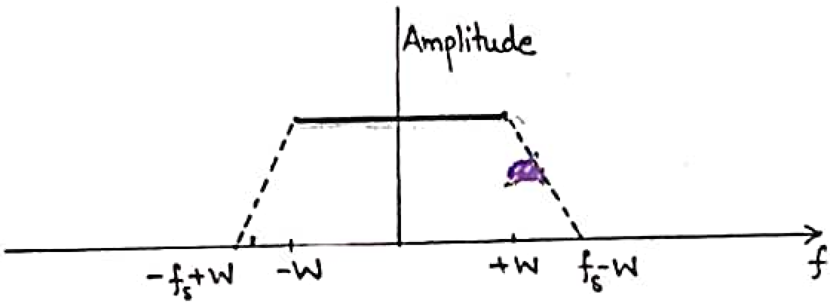


Ideal amplitude response of the Reconstruction filter

Reconstruction filter :- Low-pass kind with a passband (-w to w), (determined by anti-alias filter itself)

$\boxed{f_s > 2W}$

$-f_s-W \quad -f_s \quad -f_s+W \quad -W \quad 0 \quad +W \quad f_s-W \quad f_s \quad f_s+W \quad f$



Ideal amplitude response of the Reconstruction filter

Reconstruction Filter :- Low-pass kind with a passband ($-W$ to W), (determined by anti-alias filter itself)

--> Non-zero Transition band (W to f_s-W) for +ve frequencies where f_s = sampling rate.

--> Non-zero Transition band (W to $f_s - W$) for +ve frequencies
where f_s = sampling rate.

Disadvantage of Ideal Sampling :-

1) Due to very narrow samples, the transmitted power is very Small. Thus, the Ideally sampled pulses may got lost in the noise.
2) Practically impossible to have pulses of width approaching zero.

Note:- Ideal sampling used only to prove sampling Theorem.

Practical Sampling Techniques

Duration of sampling pulses is finite & amplitude of pulses is also finite.

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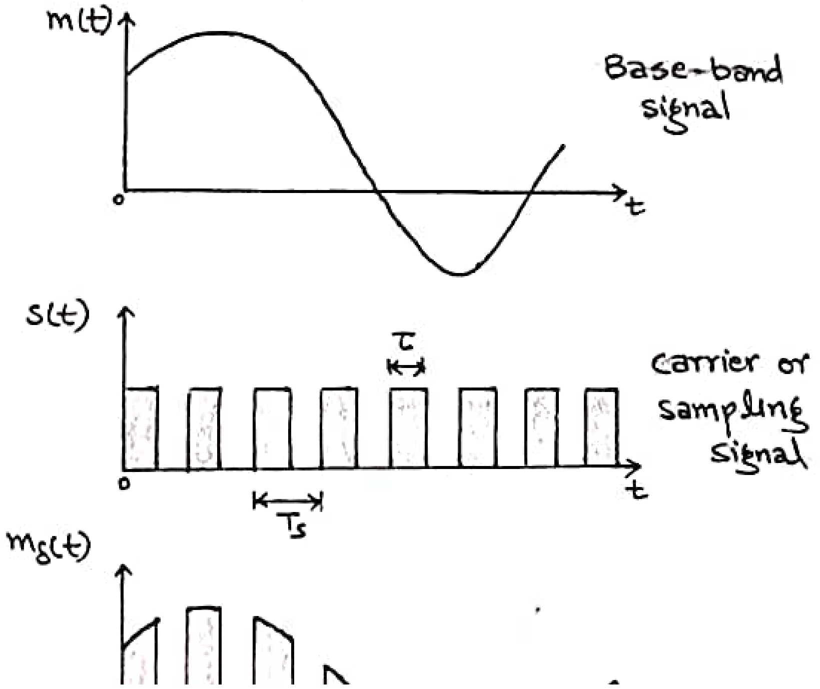
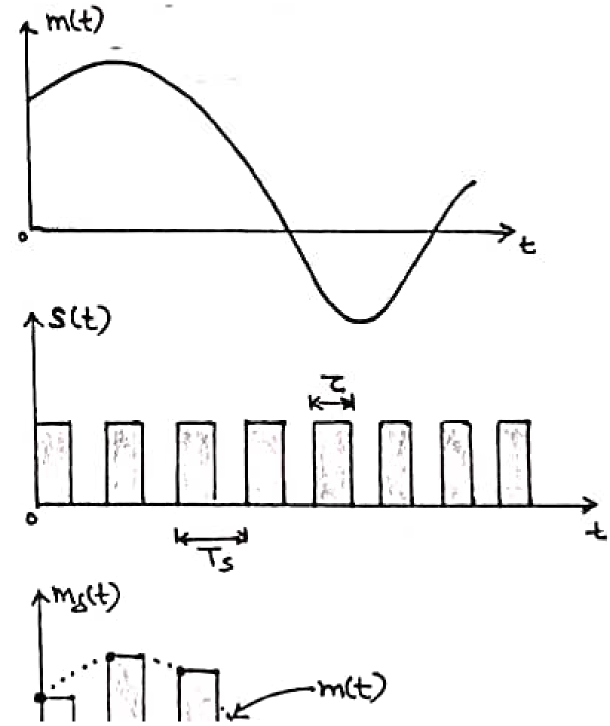
Duration of sampling pulses is finite & amplitude of pulses is also finite.

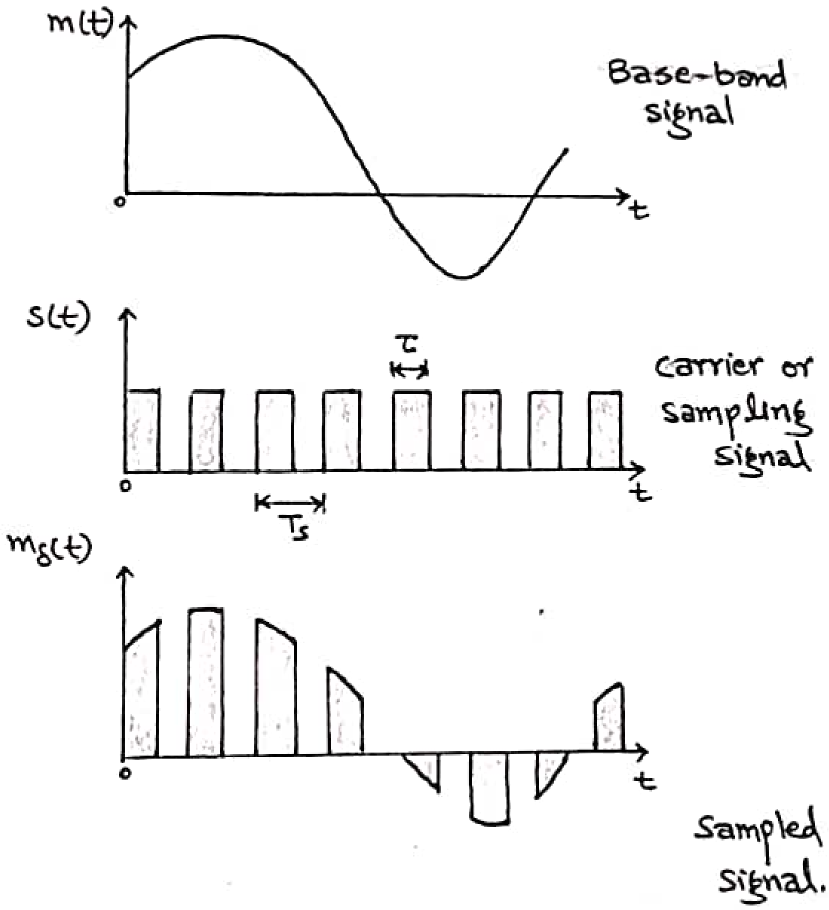
1) Natural Sampling



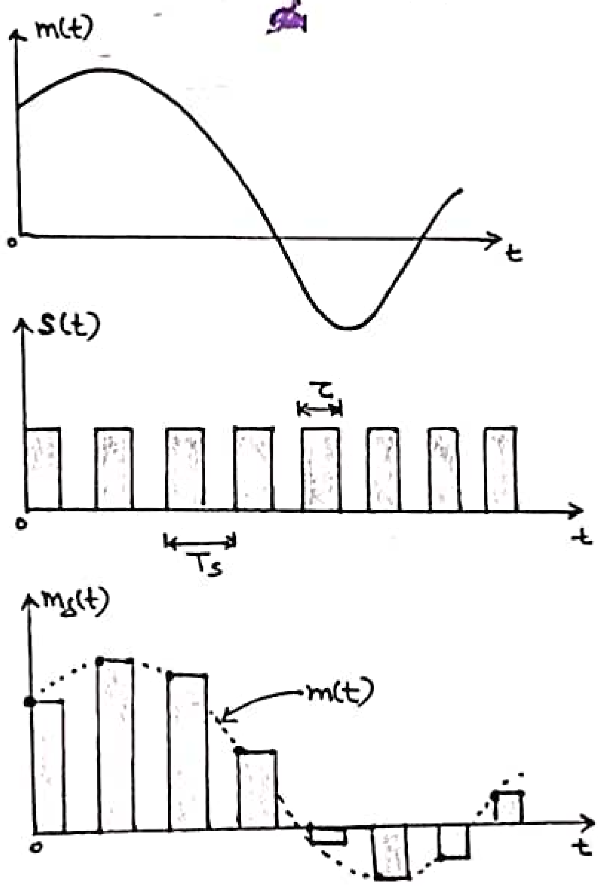
2) Flat-Top Sampling.



1) <u>Natural Sampling</u>

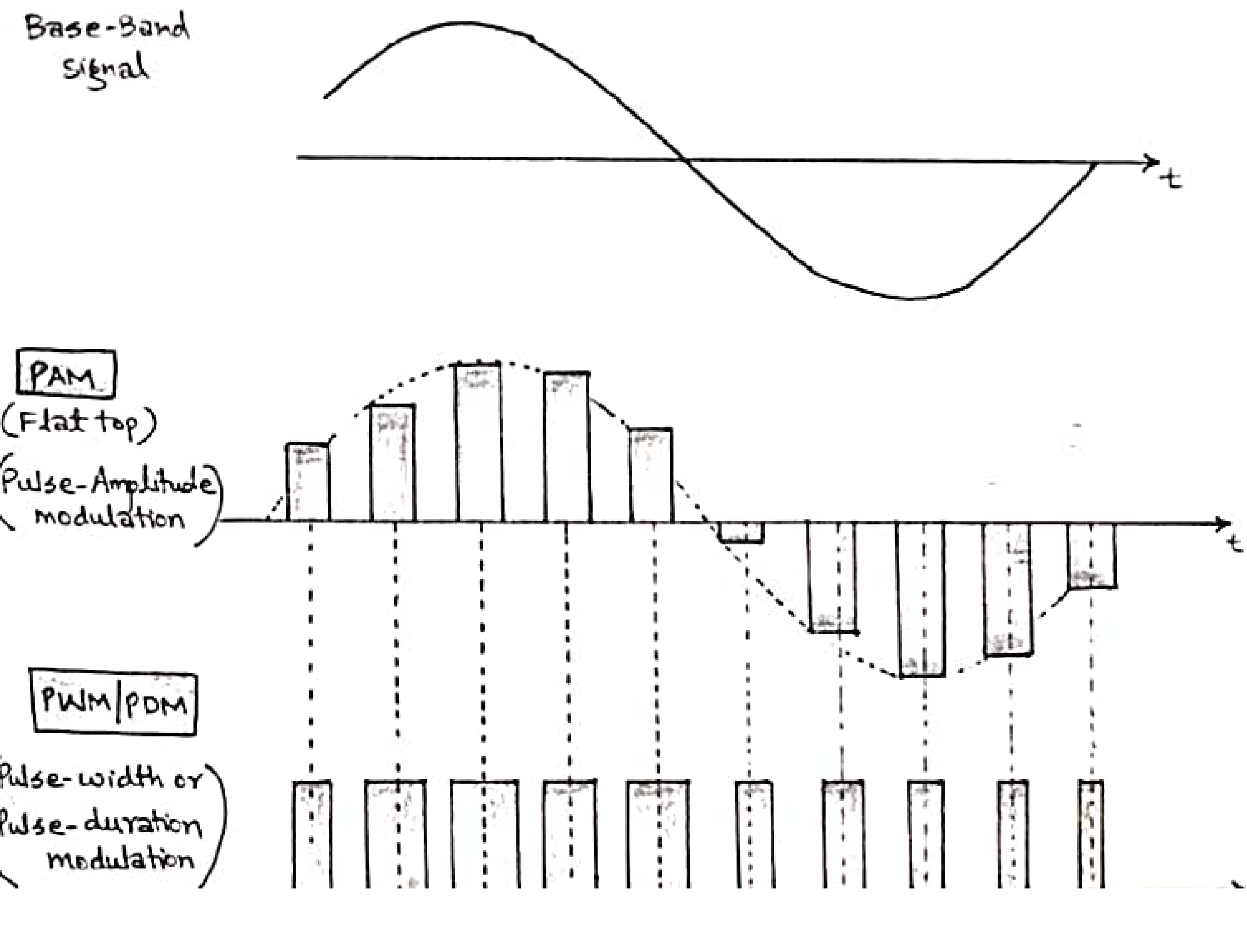


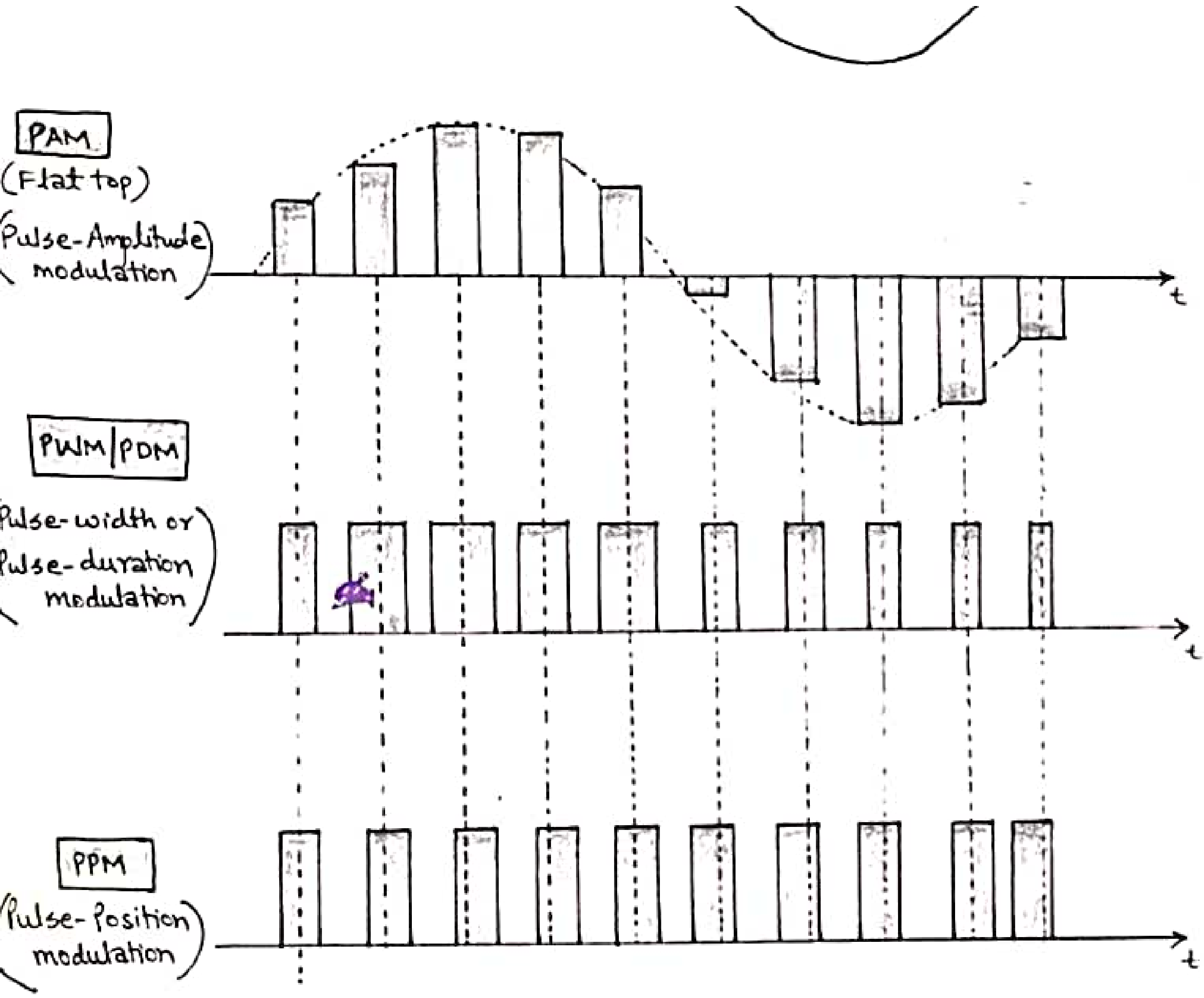
2) <u>Flat-Top Sampling.</u>



ANALOG PULSE MODULATION

(CW Modulation)

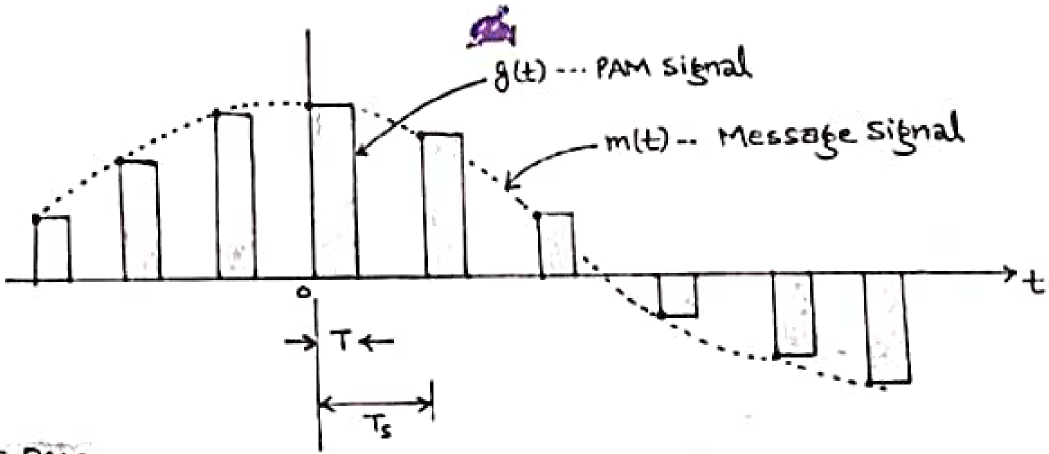



PAM
(Flat top)
(Pulse-Amplitude modulation)
t
PWM/PDM
(Pulse-width or Pulse-duration modulation)
t
PPM
(Pulse-Position modulation)
t

PULSE - AMPLITUDE MODULATION (PAM)

The amplitudes of regular spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal.

PAM ≈ Similar to Flat-Top sampling.



GENERATION OF PAM.

Involve two operations [Sample & Hold]

1) Instantaneous Sampling of m(t) every T_s second.

Sampling rate ; $\boxed{f_s = \frac{1}{T_s}}$ ... according to sampling Theorem.

2) Lengthening the duration of each sample, (so, occupy some finite value T).

2) Lengthening the duration of each sample, (so, occupy some finite value).

→ To avoid the use of an excessive channel bandwidth.

$\because$ Bandwidth $\propto \frac{1}{T}$ where T = Pulse duration.

Analysis of SAMPLE & HOLD FILTER

PAM signal is express as:

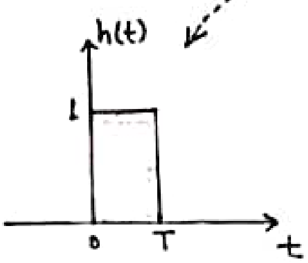
$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t-nT_s)}$$

Where

T_s = sampling period

$m(nT_s)$ = sample value of $m(t)$ at $t = nT_s$.

$h(t)$ = Standard rectangular pulse of unit amplitude & duration T.

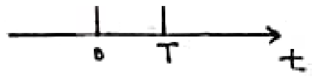


From definition of Ideal Sampling; ... $\boxed{m_\delta(t) = m(nT_s) \times s(t)}$ → unit Impulse train.

→ Baseband signal

$m(t)$ & $s(t)$ $t = nT_s$

Here.



From definition of Ideal Sampling ; ... $\boxed{m_\delta(t) = m(nT_s) \times s(t)}$ → unit Impulse train. → Baseband signal

$m(t)$ & $s(t)$ $\quad t = nT_s$

Here,

$$\boxed{s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)} \quad \& \quad \boxed{m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t - nT_s)}$$

where, $\delta(t - nT_s)$ ... time shifted delta function

After modification i.e. $\boxed{m_\delta(t) * h(t)}$

convolution of $m_\delta(t)$ & $h(t)$ ← Std. rectangular pulse.

$\boxed{\text{PAM signal}}$

So, $$m_\delta(t) * h(t) = \int_{-\infty}^{\infty} m_\delta(\tau)\, h(t-\tau)\, d\tau$$

Using $$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left\{ \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(\tau - nT_s) \right\} h(t-\tau)\, d\tau$$

So, $m_\delta(t) * h(t) = \int_{-\infty}^{\infty} m_\delta(\tau) h(t-\tau)\, d\tau$

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left\{ \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(\tau - nT_s) \right\} h(t-\tau)\, d\tau$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s) \underbrace{\int_{-\infty}^{\infty} \delta(\tau - nT_s)\, h(t-\tau)\, d\tau}_{\text{using shifting property of delta function.}}$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t - nT_s)$$

$$\because \int_{-\infty}^{\infty} \delta(\tau) h(t-\tau)\, d\tau = \delta(t) * h(t) = h(t)$$

$$\int_{-\infty}^{\infty} \delta(\tau - nT_s) h(t-\tau) = \delta(t - nT_s) * h(t) = h(t - nT_s)$$

$m_\delta(t) * h(t) = g(t)$

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t - nT_s)}$$

→ PAM.

So, PAM signal ; $g(t) = m_\delta(t) * h(t)$

So, PAM signal ; $g(t) = m_\delta(t) * h(t)$

$$\big\downarrow \text{Fourier Transform}$$

Here,

$m_\delta(t) \longleftrightarrow M_\delta(f)$

$h(t) \longleftrightarrow H(f)$

$$G(f) = M_\delta(f) \cdot H(f)$$

$$\boxed{G(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)\, H(f)}$$

Here:

$$\boxed{m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t - nT_s)}$$

$$\big\downarrow \text{F.T.}$$

$$\boxed{M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)}$$

where, $f_s = \frac{1}{T_s}$

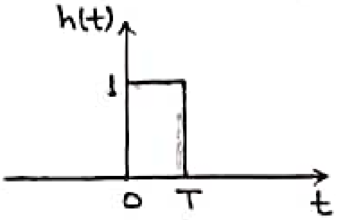
Now, Consider

$h(t)$ = std. rectangular pulse of

where, $f_s = \frac{1}{T_s}$

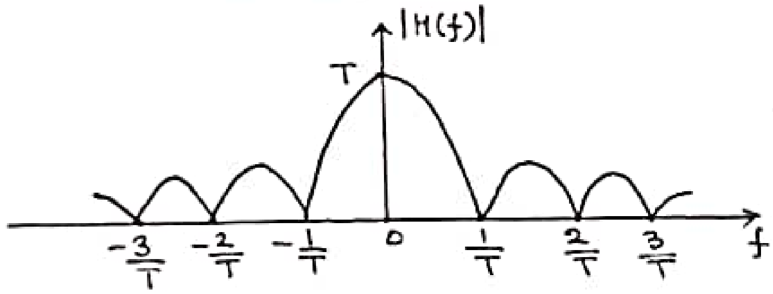
Now, Consider

$h(t)$ = std. rectangular pulse of Unit amplitude & duration T.



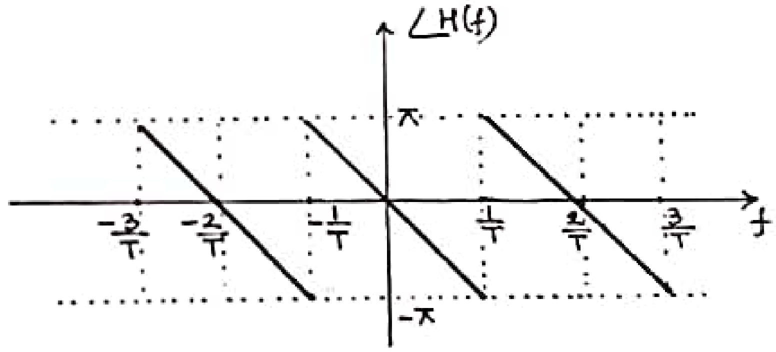
F.T.

$$\boxed{H(f) = T\,\text{sinc}(fT)\,e^{-j\pi f T}}$$



$|H(f)|$ ---- Introduction of

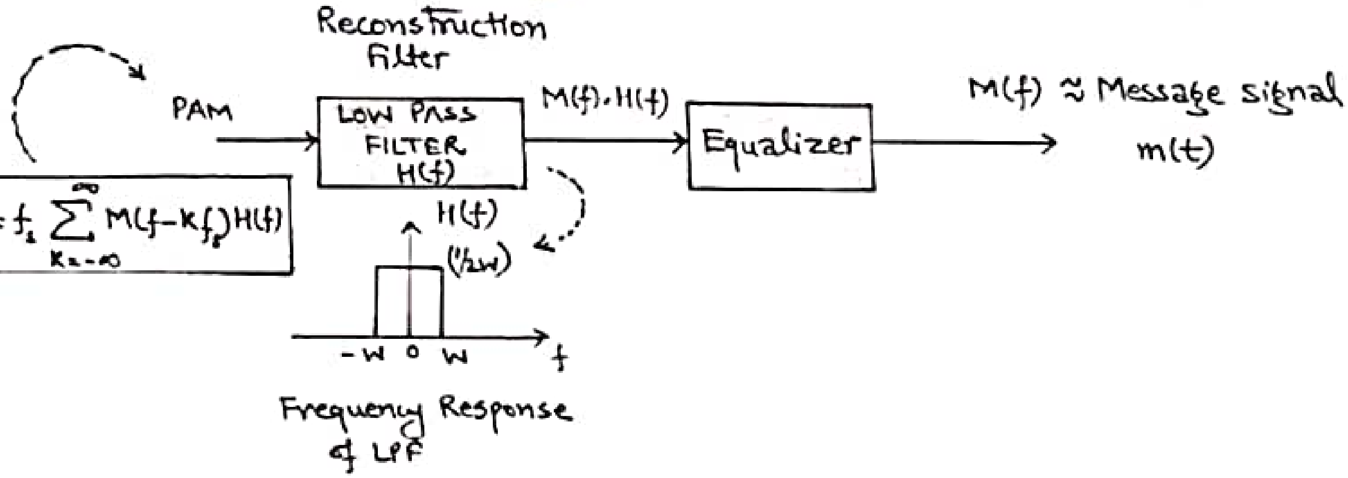
1. Amplitude distortion
2. Delay of $\frac{T}{2}$



Distortion caused by the use of PAM (based on flat-top sampling) to transmit $m(t)$, referred to as

$\boxed{\text{Aperture Effect}}$

DETECTION OF PAM / To recover original message signal m(t).



$$G(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f)$$

<u>Note</u>:- Assume, m(t) --- Bandlimited to Bandwidth W
& sampling rate $f_s > 2W$.

Aperture Effect and its Equalization

" Distortion ≈ <u>Aperture Effect</u> CORRECTED using <u>EQUALIZER</u> "

$\boxed{\text{Equalization}}$ ≈ <u>Decreasing</u> the <u>losses</u> of the reconstruction filter within

Aperture Effect and its Equalization

"Distortion ≈ Aperture Effect CORRECTED using EQUALIZER"

Equalization ≈ Decreasing the losses of the reconstruction filter within Bandwidth.

{ As, Frequency of Reconstruction filter ↑ Increase in such a manner as to compensate the Aperture Effect.

Note: 1) Amplitude response of Equalizer (Ideal)

$$\boxed{\frac{1}{|H(f)|} = \frac{1}{T\,\text{sinc}\,(fT)} = \frac{\pi f}{\sin(\pi f T)}}$$

2) In practice, the amount of equalization is usually SMALL.

3) For Duty cycle $\boxed{\frac{T}{T_s} \leq 0.1}$ ≈ Amplitude distortion < 0.5%

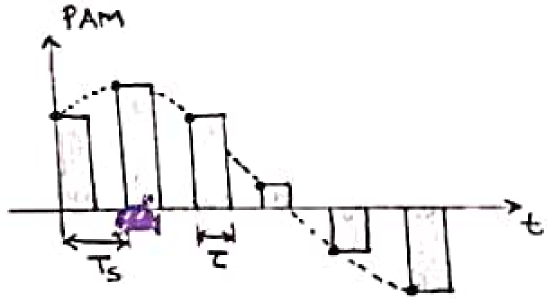
No requirement of Equalization.

Transmission Bandwith of PAM signal

Let τ = width of each pulse in Flat-top sampled PPM.

T_s = Duration between adjacent samples.

Here, $\boxed{\tau << T_s}$ — ①



Here, $T_s = \frac{1}{f_s}$ where f_s = sampling frequency.

According to sampling theorem

For proper recovery of signal $\boxed{f_s \geq 2W}$, where W= message bandwidth

so, $\frac{1}{T_s} \geq 2W$ or $\boxed{T_s \leq \frac{1}{2W}}$

From eqn ①

$$\tau << \frac{1}{2W}$$

or

$$W << \frac{1}{2\tau}$$

Note:-

According to sampling theorem

For proper recovery of signal $\boxed{f_s \geq 2W}$, where W= message bandwidth

so, $\frac{1}{T_s} \geq 2W$ or $\boxed{T_s \leq \frac{1}{2W}}$

From eqⁿ ① $\tau << \frac{1}{2W}$

or $W << \frac{1}{2\tau}$

<u>Note</u>:-

① To transmit & receive the PAM signal without signal distortion, the transmission bandwidth B_T need to satisfy the following equation.

$$\boxed{B_T \geq \frac{1}{2\tau} >> W}$$

② B_T should be as small as possible.

As $B_T \downarrow \approx \tau \uparrow$ But this will Increase "<u>aperture effect</u>"

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As $B_T \downarrow \approx \tau \uparrow$ But this will Increase "aperture effect"

Effect of Noise on PAM

When PAM signal travels over a communication channel, noise gets added as shown in Fig.

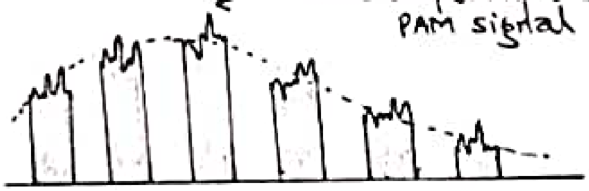


Fig: Effect of noise on PAM signal.

Note:- ① Information is contained in the amplitude, the noise will contaminate the information.

② Noise performance of PAM system is very poor.

PWM and PPM system have a better noise performance.

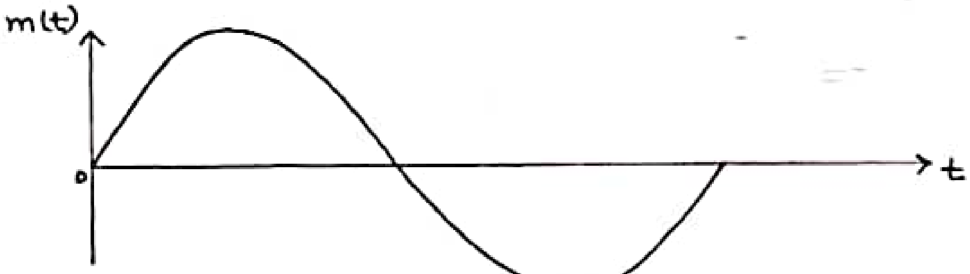
PULSE - POSITION MODULATION

As in PAM = Pulse amplitude is the variable parameter.

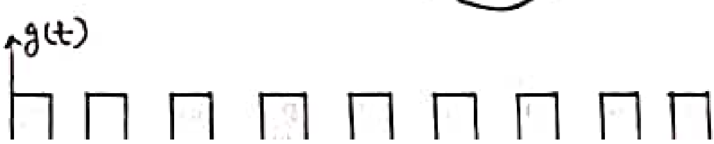
PDM i.e. Pulse duration modulation

- The samples of the message signal are used to vary the duration of the Individual pulses.
- Also, Called (PWM) Pulse width modulation or Pulse length modulation.
- "The modulating signal may vary with time of occurance of the leading edge, the trailing edge or both edge of the pulses.

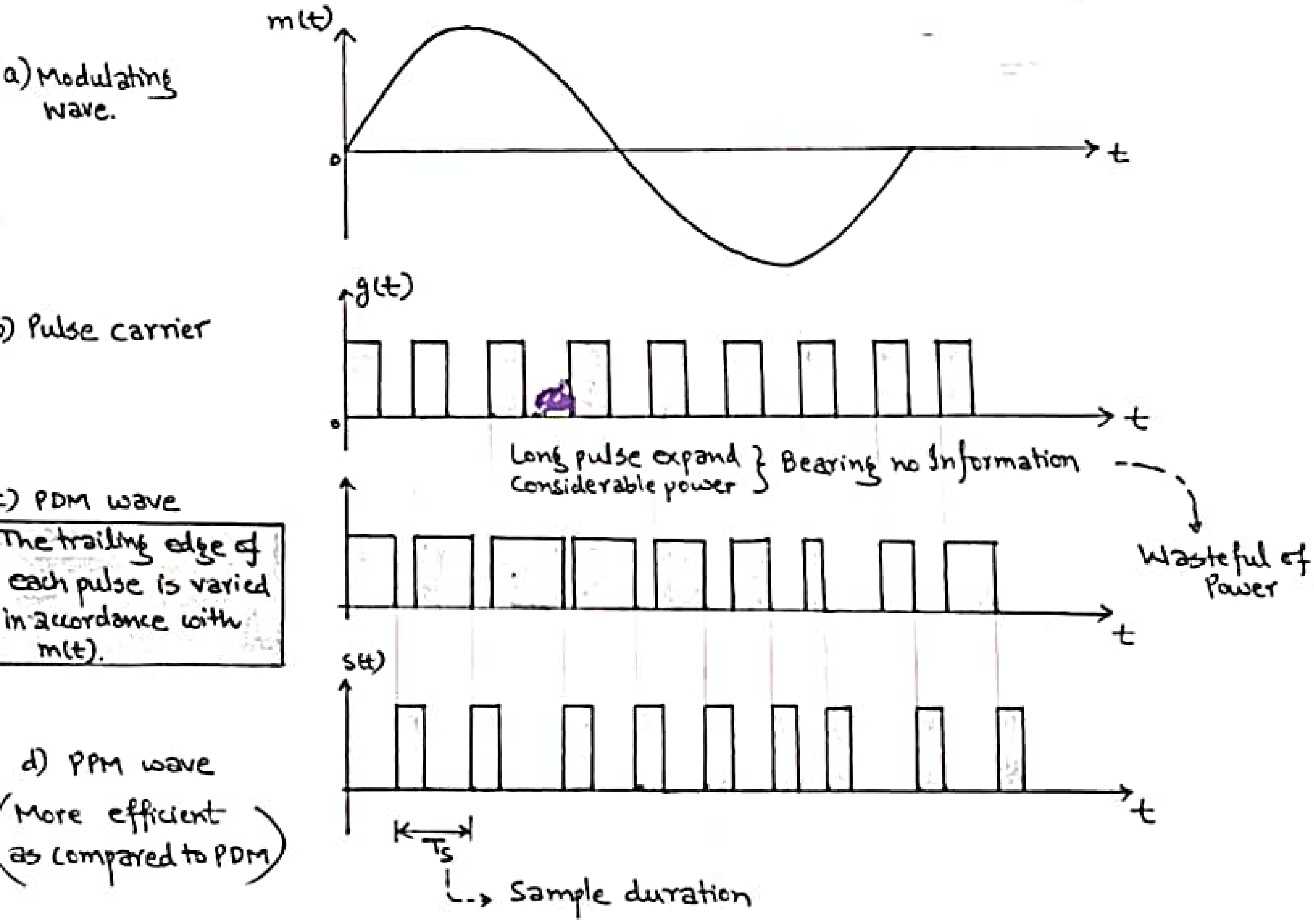
a) Modulating wave.



b) Pulse carrier



----> The modulating signal
of the leading edge, the trailing edge or both edge
of the pulses.



Using the sample $m(nT_s)$ of message signal $m(t)$ to modulate the position of n^{th} pulse. We obtain PPM signal.

sec

$$\boxed{s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - K_p m(nT_s))}$$

Where, K_p = Sensitivity factor of the pulse-position modulator (in seconds per volt)

$g(t)$ = standard pulse of Interest.

$m(nT_s)$ = sampled value of $m(t)$ at $t = nT_s$ .... (volt)

<u>Sufficient condition for PPM</u> "Different pulses constituting the PPM must be <u>strictly</u> <u>non overlapping</u>"

$$\boxed{g(t) = 0 \quad , \quad |t| > \left(\frac{T_s}{2}\right) - K_p |m(t)|_{max}}$$

Requires

$$\boxed{K_p |m(t)|_{max} < \left(\frac{T_s}{2}\right)}$$

$$\boxed{g(t) = 0 \quad , \quad |t| > \left(\frac{T_s}{2}\right) - K_p|m(t)|_{max}}$$

Requires

$$\boxed{K_p|m(t)|_{max} < \left(\frac{T_s}{2}\right)}$$

Note:-

1) As $K_p|m(t)|_{max} \xrightarrow{\text{close to}} \left(\frac{T_s}{2}\right)$ i.e Half the sampling duration.

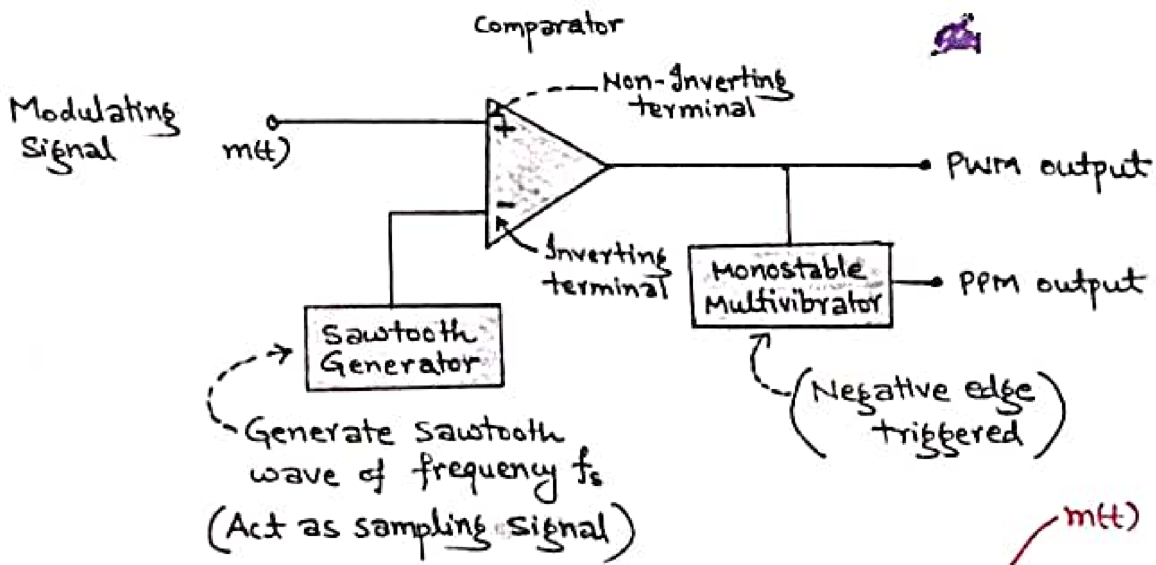
standard pulse $g(t)$ ≈ become Narrow

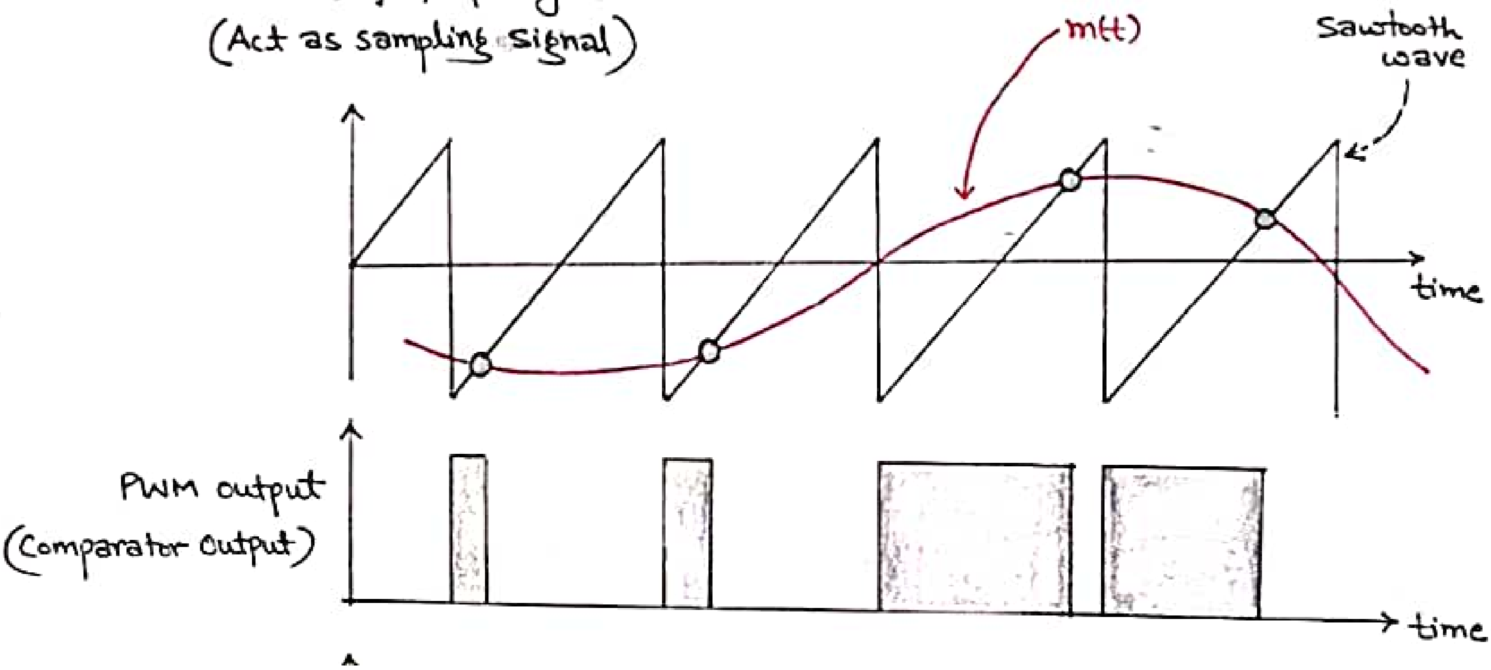
≈ Ensure that the Individual pulse of PPM signal $s(t)$ do not Interfere with each other.

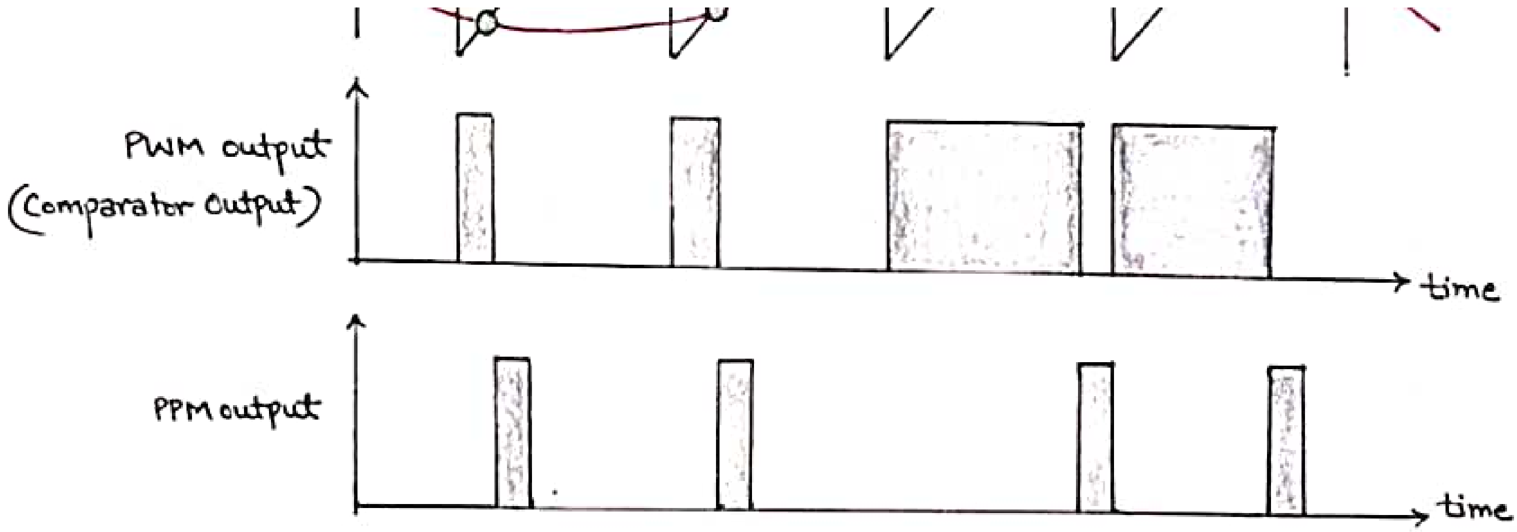
2) Above condition fulfill the requirement of the recovery of the sampled signal $m(nT_s)$ perfectely.

→ Also, If the message signal $m(t)$ is strictly band limited; then it follows the sampling theorem & original message signal $m(t)$ can be recovered from PPM signal $s(t)$ without distortion.

GENERATION OF PWM and PPM SIGNALS







Note:

1) The comparator signal output remain HIGH as long as Instantaneous amplitude of m(t) is HIGHER than ramp signal. This gives rise to PWM signal.

2) Leading edges of PWM signal always generated at fixed time Instants, but the trailing edge will depend on Instantaneous amplitude of m(t). Hence, PWM signal is said to be trail edge modulated PWM.

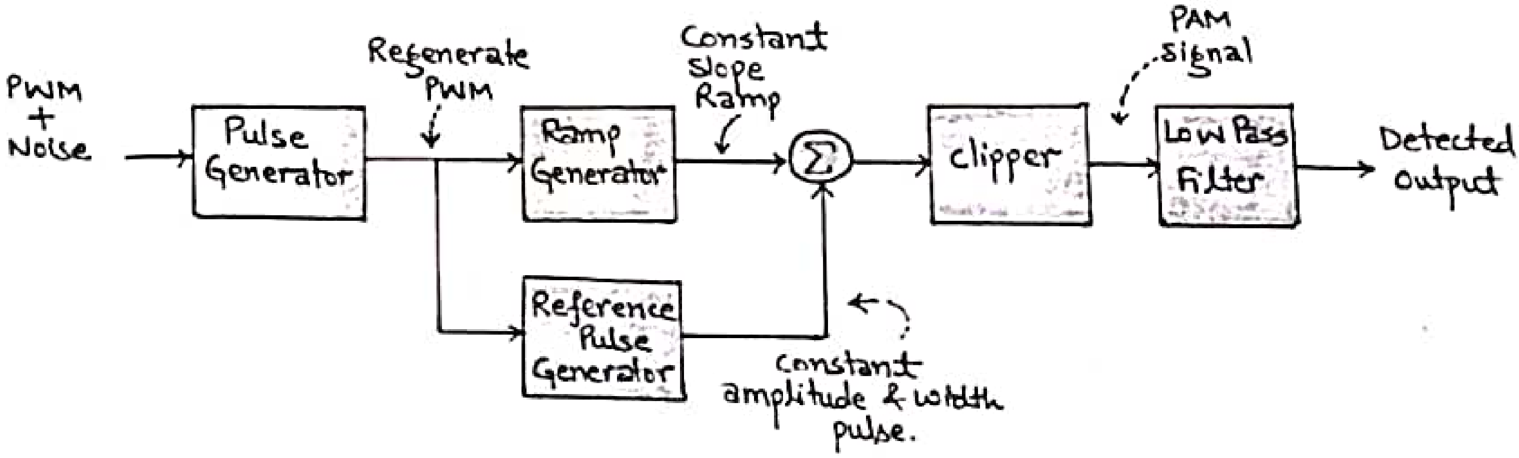
3) Corresponding to each trailing edge of PWM signal, the monostable output goes HIGH for a fixed time decided by its own RC component.

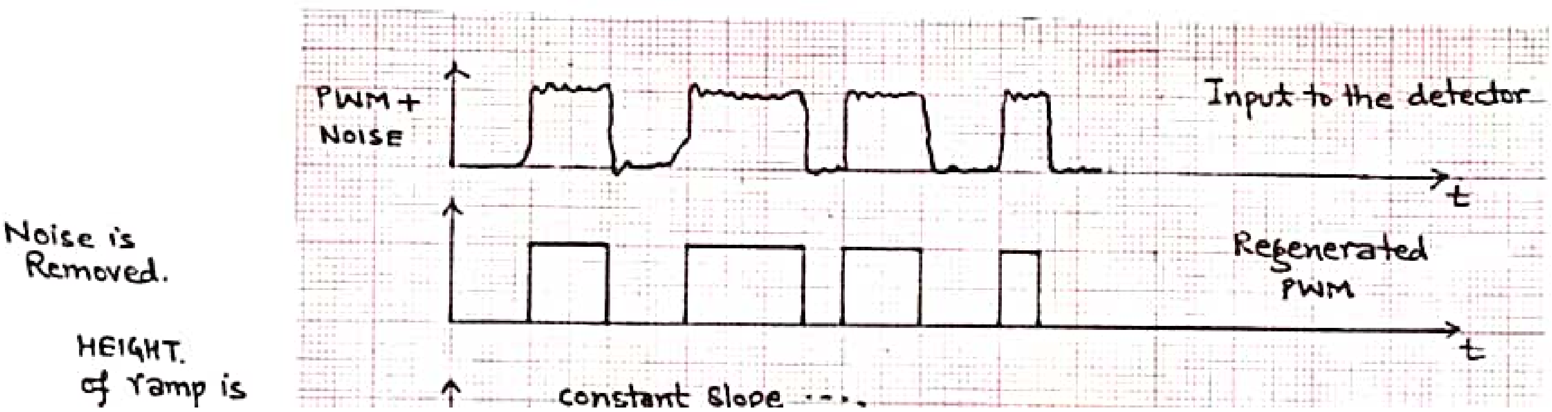
4) PPM pulse keep shifting in proportion to PWM trailing edge.

5) All the PPM pulses have the same width & amplitude. The Information is conveyed via. changing position of the pulses.

Demodulation of PWM and PPM signal

Detection of PWM signals



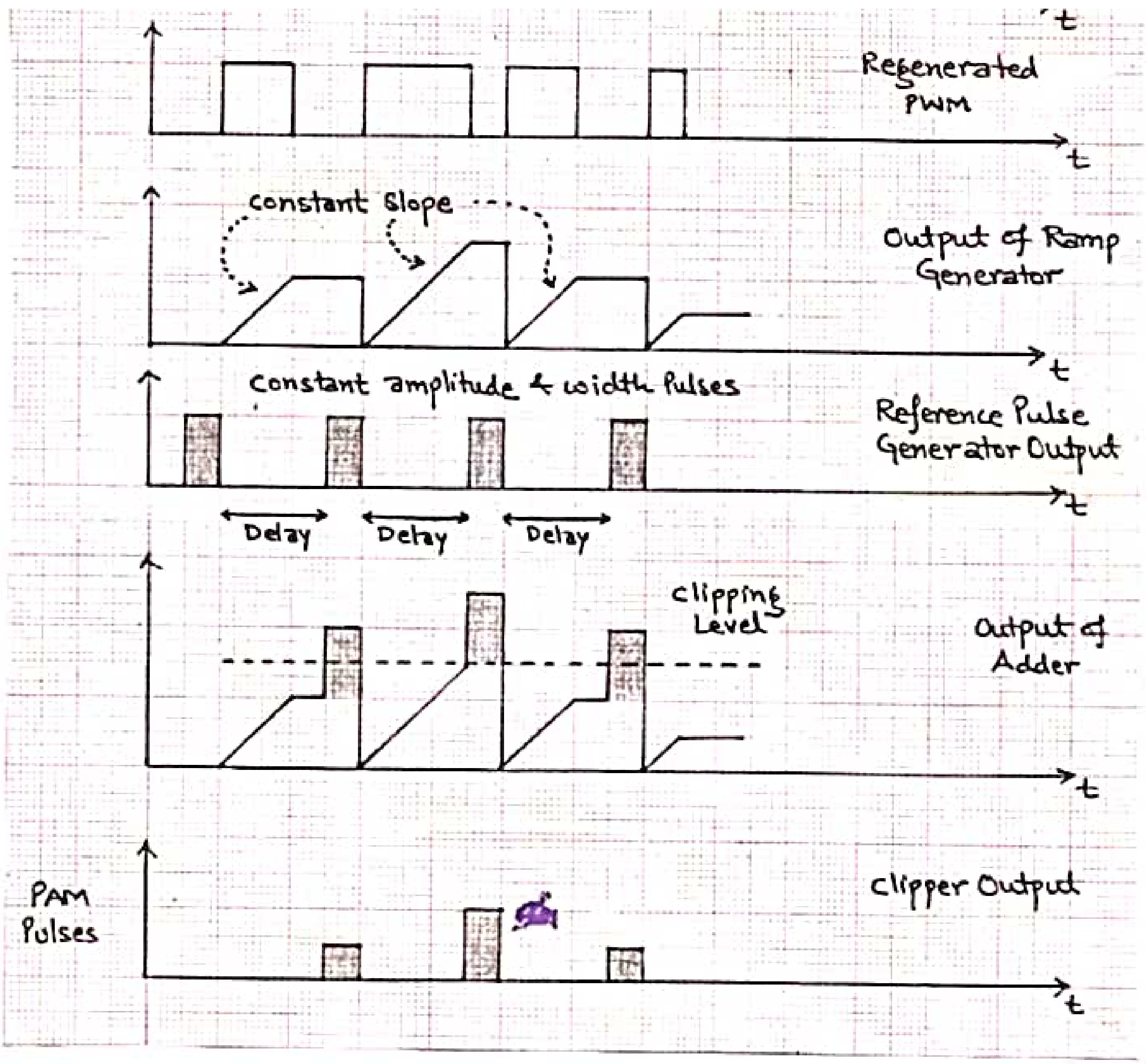


Noise is Removed.

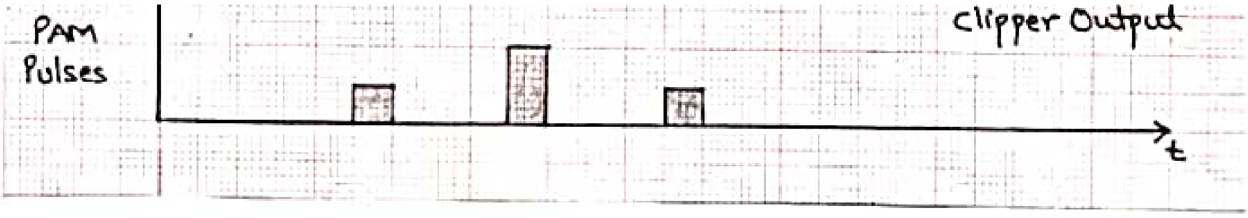
HEIGHT of ramp is roportional to the width of the PWM pulses.

Clipped off at a hreshold level to generate PAM signal.

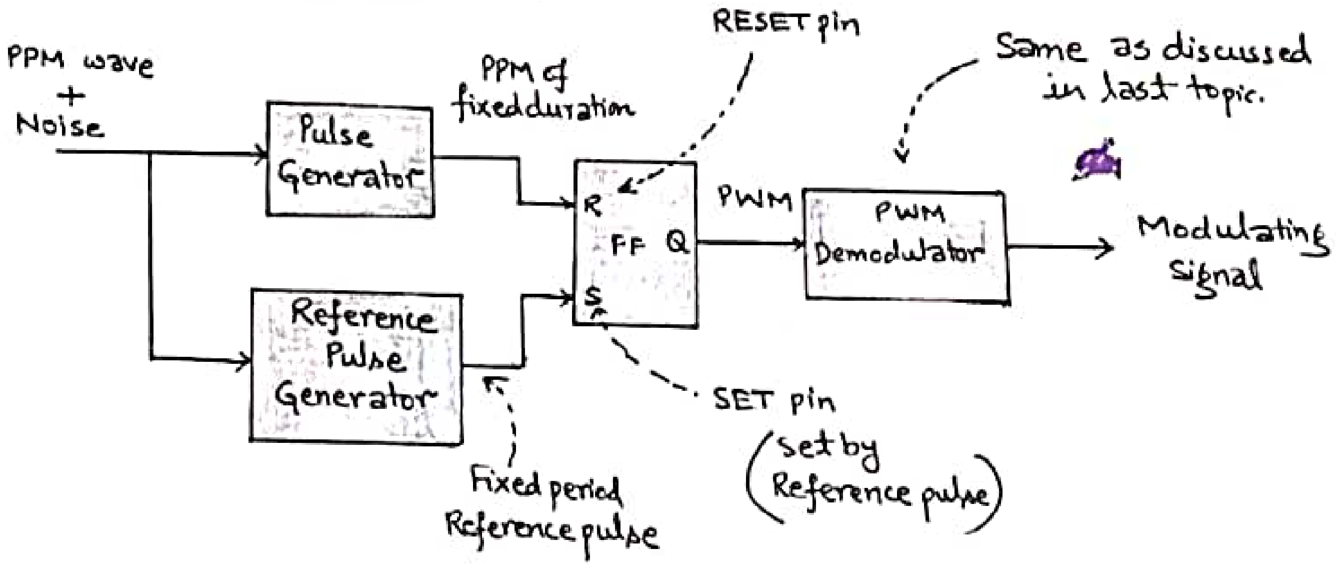
ow Pass filter is Used to recover the riginal modulating signal back from PAM signal.



Used to recover the original modulating signal back from PAM signal.



Detection of PPM signal.



Note: Due to set and reset applied to the flip-flop, we get PWM signal at its output. This PWM signal can be demodulated using the PWM demodulator.

Difference between AM and FM

Below is the table of AM versus FM:

S.No.	Parameters	AM	FM
1.	Full-form	Amplitude modulation	Frequency modulation
2.	Origin	The AM method of audio transmission was successfully carried out in the mid-1870s.	FM radio was developed in the United States in the 1930s by Edwin Armstrong.
3.	Modulating differences	In AM, a radio wave known as the "carrier" or "carrier wave" is modulated in amplitude by the signal that is to be transmitted.	In FM, a radio wave known as the "carrier" or "carrier wave" is modulated in frequency by the signal that is to be transmitted.
4.	Constant parameters	The frequency and phase remain the same.	The amplitude and phase remain the same.
5.	Quality	AM has poorer sound quality, and a lower bandwidth but is cheaper. It can be transmitted over long distances as it has a lower bandwidth, which is why it can hold more stations available in any frequency range.	FM is less affected by interference, but FM signals are impacted by physical barriers. They have a better sound quality due to higher bandwidth.

6.	Frequency range	AM radio ranges from 535 to 1700 kHz or up to 1200 bits per second.	FM radio ranges in a higher spectrum from 88.1 to 108.1MHz or up to 1200 to 2400 bits per second.
7.	Bandwidth BW	BW is much less than FM. B.W. = 2 fm	BW is large. Hence a wide channel is required. B.W. = 2 x (δ + fm)
8.	Bandwidth requirements	Bandwidth is less than FM or PM and doesn't depend upon the modulation index. The bandwidth requirement is twice the highest modulating frequency.	Bandwidth requirement is greater and depends upon the modulating. The bandwidth requirement is twice the sum of the modulating signal frequency and the frequency deviation.
9.	The frequency required for broadcasting	In AM radio broadcasting, if the modulating signal has a bandwidth of 15 kHz, then the bandwidth of an amplitude- The modulated signal is 30 kHz.	Let's say, if the frequency deviation is 75kHz and the modulating signal frequency is 15kHz, the bandwidth required is 180kHz.

10.	No of Sidebands	The number of sidebands is constant and equal to 2.	The number of sidebands having significant amplitude depends upon the modulation index
11.	Zero crossings in modulating signal	Equidistant	Not equidistant
12.	Complexity	AM transmitters and receivers are less complex than FM and PM, but synchronization is needed in the case of SSBSC carriers.	FM (or PM) transmitters are more complex than AM because the variation of modulating signal has to be converted and detected from the corresponding variation in frequencies.
13.	Noise	AM receivers are very less susceptible to noise because noise affects the amplitude, which is where information is stored in AM signals.	FM receivers are better immune to noise and it is possible to decrease noise by further deviation.
14.	Efficiency	Power is wasted in transmitting the carrier.	All transmitted power is useful so that's why FM is very efficient.

15.	Application	MW (Medium wave), SW (short wave) band broadcasting, video transmission in T.V.	Broadcasting FM, audio transmission on T.V.

Difference between AM, FM, and PM

S.No.	Parameters	FM	AM	PM
1.	Definition	Frequency modulation is a technique of modulation, in which the frequency of the carrier varies in accordance with the amplitude of the modulating signal. The amplitude and phase are constant.	Amplitude modulation is a technique of modulation in which the amplitude of the carrier wave varies in accordance with the amplitude of the modulating signal. The frequency and phase are constant.	Phase modulation is a technique of modulation in which the phase of the carrier wave varies in accordance with the amplitude of the modulating signal. The amplitude and frequency are constant.
2.	Noise	Noise immunity of FM is superior to AM and PM.	AM receivers are very susceptible to noise.	Noise immunity is better than AM but not FM.

3.	Function	The frequency of the carrier wave deviates as per the voltage of the modulating signal input.	The amplitude of a carrier wave in AN diverges as per amplitude or voltage of modulating signal input.	A phase of the carrier wave varies as per the voltage of modulating signal input.
4.	Constant parameter	The amplitude of the carrier wave is kept changeless.	The frequency of the carrier wave is kept invariable.	The amplitude of the carrier wave is kept changeless.
5.	Types	Digital FM types: FSK, GFSK, offset PSK, etc.	AM types: DSB-SC, SSB, VSB, etc.	Digital PM types: QPSK, BPSK, QAM (the combination of amplitude and phase, modulation).
6.	Waveforms	(Image will be Uploaded soon)	(Image will be Uploaded soon)	(Image will be Uploaded soon)

For a radio signal to carry audio or other information for broadcasting or for two-way radio communication, signals must be modulated or changed in some way. Though we have several ways in which a radio signal may be modulated, one of the easiest is to change its amplitude in line with variations of the sound. Here, we discussed three types of modulation, viz: FM, AM, and PM, which will help you understand the basics of the modulation along with the difference between each.