Lattice Form Representation of Digital Filters

eg. $$H(z) = \frac{1}{1 + \frac{2}{5}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{3}z^{-3}}$$

Ans) Highest power of z^{-1} is 3
We need to find K_1, K_2, K_3

$$H(z) = \frac{1}{\underset{a_3(0)}{1} + \underset{a_3(1)}{\frac{2}{5}}z^{-1} + \underset{a_3(2)}{\frac{3}{4}}z^{-2} + \underset{a_3(3)}{\frac{1}{3}}z^{-3}}$$

Formula :

(i) $$a_i(i) = K_i$$

(ii) $$a_{m-1}(i) = \frac{a_m(i) - K_m \cdot a_m(m-i)}{(1 - K_m^2)}$$

Now, $a_3(0) = 1$

$a_3(1) = 2/5$

$a_3(2) = 3/4$

$a_3(3) = \boxed{1/3 = K_3}$

$$K_2 = a_2(2) = \frac{a_3(2) - a_3(3)\, a_3(1)}{1 - a_3^2(3)}$$

$$K_2 = a_2(2) = \frac{(3/4) - (1/3)(2/5)}{1 - 1/9}$$

$$\boxed{K_2 = 0.69375}$$

$$K_1 = a_1(1) = \frac{a_2(1) - a_2(2)\, a_2(1)}{1 - a_2^2(2)}$$

$$a_2(1) = \frac{a_3(1) - K_3 \cdot a_3(2)}{1 - K_3^2}$$

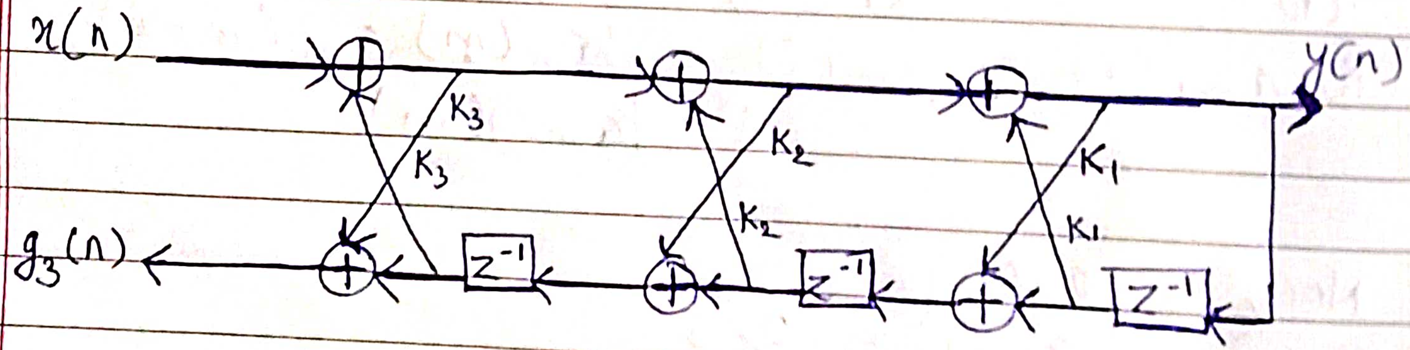
$$a_2(1) = \frac{\frac{2}{5} - \frac{1}{\cancel{3}}\left(\frac{\cancel{3}}{4}\right)}{1 - \frac{1}{9}}$$

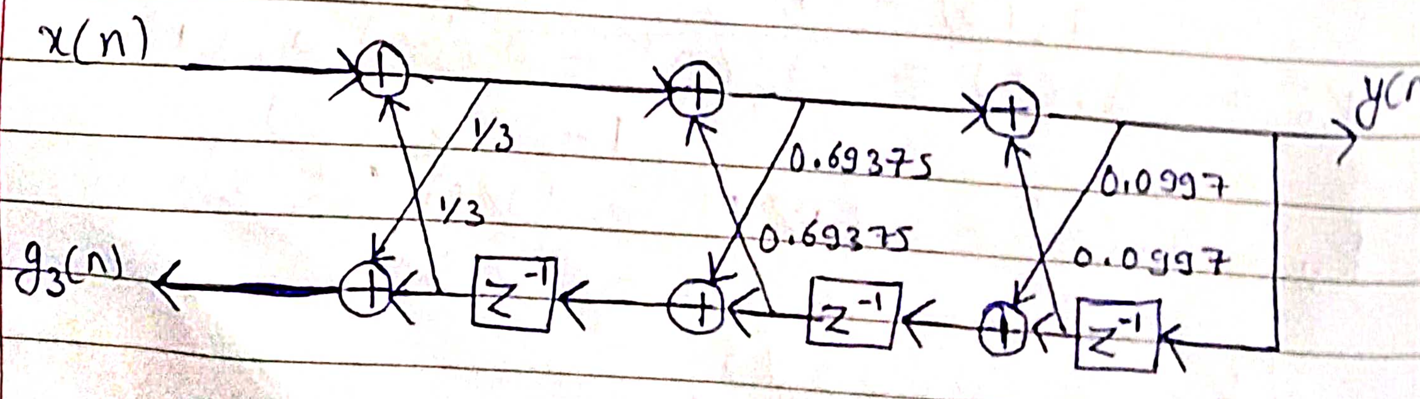
$$a_2(1) = 0.16875$$

$$K_1 = \frac{0.16875 - (0.69375)(0.16875)}{1 - (0.69375)^2}$$

$$K_1 = \frac{0.16875 - 0.117}{0.5187} = \frac{0.05175}{0.5187}$$

$$\boxed{K_1 = 0.0997}$$





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eg. $$H(z) = \frac{1 + 2z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{4}z^{-2}}$$

Highest power of z^{-1} is 2
We need to find K_2, K_1

$a_2(0) = 1$

$a_2(1) = 3/4$

$\boxed{a_2(2) = 1/4 = K_2}$

$$K_1 = a_1(1) = \frac{a_2(1) - K_2\, a_2(1)}{1 - K_2^2}$$

$$K_1 = \frac{\frac{3}{4} - \frac{1}{4}\left(\frac{3}{4}\right)}{1 - \frac{1}{16}} = \frac{\frac{12-3}{16}}{\frac{15}{16}} = \frac{9}{15} = \frac{3}{5}$$

$\boxed{K_1 = 0.6}$