A signal is a function of independent variables such as time, position, etc. The value of signal at specific value of independent variable is called amplitude

Characterization and Classification of Signals

1) Continuous time and discrete time signal:

Continuous time signal:
If time is continuous, the signal is called continuous time signal. A continuous time signal is defined at every instant of time.

Discrete time signal:
If time is discrete, the signal is called discrete time signal. Discrete time signal is basically a sequence of numbers

2) Analog and Digital Signal

Analog Signal: A continuous time signal with a continuous amplitude is called an analog signal.
eg. speech signal

Digital Signal: A discrete time signal with discrete value amplitudes represented by a finite number of digits is called digital signal eg. signal in CDROM

3) Sampled Data and Quantized boxcar signal:

Sampled Data Signal: A discrete time signal with continuous valued amplitude is called sampled data signal.
eg. signal in switched capacitor circuit

Quantized Boxcar Signal: A continuous time signal with discrete valued amplitude is called quantized boxcar signal. eg signal at fixed level between two instant of clocking.

4) Deterministic and Random Signals:

Deterministic signal: A signal that can be uniquely determined by a well defined process is called deterministic signal.

Random signal: A signal that is generated in random fashion and cannot be predicted is called random signal.

Typical Signal Processing Operations

1) Simple Time Domain Operations

(a) Scaling : Scaling is multiplication of signal with positive or negative constant. If multiplying constant (gain) is greater than one, it is called amplification. If multiplying constant is less than one, operation is called attenuation.

$$x(t) \xrightarrow{\text{Scaling}} \alpha x(t)$$

(b) Delay: Delay operation generates a signal that is a delayed replica of the original signal. For an analog signal $x(t)$, $y(t) = x(t - t_0)$ is the signal obtained by delaying $x(t)$ by amount t_0. If t_0 is negative it is called advance operation

(c) Addition of Signals : $y(t) = x_1(t) + x_2(t) + x_3(t)$

(d) Multiplication : $y(t) = x_1(t)\, x_2(t)$

(e) Integration , $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

(f) Differentiation: $dx(t)/dt$

2) Filtering

The aim is to alter the spectrum according to some given specifications. The system implementing this operation is called filter. The range of frequencies of signal components allowed to pass through the filter is called the passband. The range of frequencies of signal components blocked by filter is called stopband.

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)\, x(\tau)\, d\tau$$

$h(t)$ = impulse response
$y(t)$ = output
In frequency domain

$$Y(j\omega) = H(j\omega)\,X(j\omega)$$

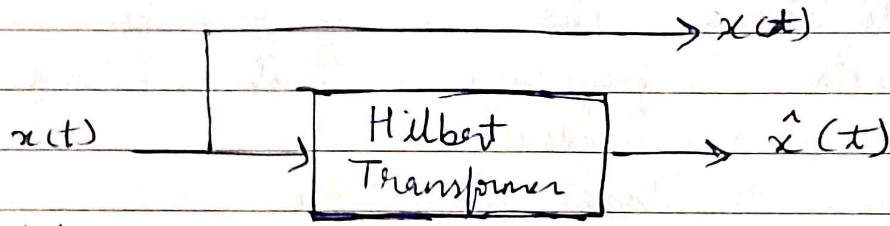
3) Generation of Complex-Valued Signals (Hibert Transformer)
Consider impulse response,

$$H_{HT}(j\omega) = \begin{cases} -j & \omega > 0 \\ j & \omega < 0 \end{cases}$$

$$\hat{X}(j\omega) = H_{HT}(j\omega)\,X(j\omega) = -jX_p(j\omega) + jX_n(j\omega)$$

$$y(t) = x(t) + j\hat{x}(t)$$

$x(t)$ is in phase component of $y(t)$
$\hat{x}(t)$ is quadrature component of $y(t)$



4) Amplitude Modulation :
In amplitude modulation, the amplitude of high frequency sinusoidal signal $A\cos(\omega_0 t)$ called carrier signal is varied by the low-frequency band-limited signal $x(t)$, called modulating signal

$$y(t) = A\,x(t)\cos(\omega_0 t)$$

Amplitude Modulation can be achieved by

$$y(t) = A[1 + m\,x(t)]\cos(\omega_0 t)$$

The portion of AM signal between ω_0 and $\omega_0 + \omega_m$ is called upper sideband whereas portion between ω_0 and $\omega_0 - \omega_m$ is called lower sideband
~~Due to presence of two sidebands and absence of carrier component in modulated~~
When two sidebands are present and carrier component is absent in modulated signal, the process is called

double sideband suppressed carrier modulation (DSB-SC)
when carrier is also present, the process is called double sideband modulation (DSB)
To increase the capacity of transmission medium, either the upper or lower sideband of modulated signal is transmitted, the process is called single sideband modulation (SSB).

5) Multiplexing and Demultiplexing:
For an efficient utilization of wideband transmission channel, many narrow-bandwidth low frequency ~~channels~~ signals are combined to form a composite wideband signal that is transmitted as a single signal. The process is called multiplexing. The recovery of original narrow bandwidth low frequency signal at receiving end is called demultiplexing.

6) Quadrature Amplitude Modulation
This method uses DSB modulation to modulate two different signals so that they both occupy the same bandwidth.

7) Signal Generation:
Oscillator is a signal generator that generates sinusoidal signal

Example of Typical Signal

1) Electrocardiography (ECG) Signal
Electrical activity of heart is represented by ECG signal. ECG trace is a periodic waveform. The waveform is generated by an electrical impulse originating at the sinoatrial node in right atrium of heart.

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2) Electroencephalogram (EEG) Signal
The electrical activity of individual neuron in brain is represented by EEG signal. Bandwidth of EEG ranges from 0.5 to 100Hz with amplitude 2 to 100 mV.

3) Seismic Signals: They are caused by movement of rocks resulting from an earthquake. The ground movement generates elastic waves that propogates through the earth in all direction.

4) Speech Signals: A speech signal is created by exciting the vocal tract using either quasi-periodic puffs of air or by creating turbulant air flow in vocal tract. Thus digital signal processing is used in speech processing.

5) Musical Sound Signal: Multiple vibrations in single instrument generate musical sound. Electronic synthesizer is an example of modern signal processing technique.

6) Time Series: Finite extent signals like annual yield of crop, total monthly export, etc that occurs in business and engineering are called time series.

7) Images: An image is a 2D signal whose intensity at any point is a function of two spatial variables eg. photo, video, radar and sonar images.

Typical Signal Processing Applications

1) Sound Recording Applications

(a) Compressor and Limiters: These devices are used for compression of dynamic range of audio signal.

(b) Expander and Noise Gates: It is an amplifier with two ~~gate levels~~ gain levels

(c) Equalizer and Filters: It is used to modify the frequency response of a recording or monitoring channel.

(d) Noise Reduction System: It consists of two parts. First part provides compression during recording mode while second part provides complementary expansion during playback mode.

(e) Delay and Reverberation system:

(f) Special Effects

2) Telephone Dialing Applications.

In telephones, pressing each button generates unique set of two tone signals that are processed at telephone office to identify the pressed number.

3) FM Stereo Applications

For wireless transmission of a signal occupying a lower-frequency range, it is necessary to transform the signal to a high frequency range by modulating it onto a high frequency carrier. At the receiver end, modulated signal is demodulated to recover the low frequency signal.

4) Musical Sound Synthesis:

Four methods of musical sound synthesis are: wavetable synthesis, spectral modelling synthesis, nonlinear synthesis, physical modelling synthesis.

5) Echo Cancellation in Telephone Network

Time Domain Representation

Signals are represented as sequence of numbers called samples.

Discrete time signals are generated by periodically sampling a continuous time signal $x_a(t)$ at uniform time intervals

$$x[n] = x_a(t)\big|_{t=nT} = x_a(nT)$$

$$n = \ldots, -2, -1, 0, 1, 2, \ldots$$

Spacing T between two consecutive samples is called sampling interval. Reciprocal of sampling interval is called sampling frequency $F_T = 1/T$

Operations on Sequences

1) Elementary Operations

(a) Modulation: Let $x[n]$ and $y[n]$ be two known sequences. Taking product of sample values of these two sequences we get new sequence $w_1[n]$:

$$w_1[n] = x[n] \cdot y[n]$$

(b) Process is called modulation and device used is called modulator.

(b) Multiplication: $w_2[n]$ is generated by multiplying $x[n]$ with scalar A:

$$w_2[n] = A x[n]$$

Device used for multiplication is called multiplier

(c) Addition: $w_3[n] = x[n] + y[n]$

Device used is called adder

(d) Time shifting: $w_4[n] = x[n-N]$

When $N>0$, it is called delaying operation

if $N<0$, it is called advancing operation

(e) Time Reversal Operation: Also called folding operation. $w_6[n] = x[-n]$

2) Combination of Elementary Operations

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

3) Convolution Sum: Let $x[n]$ and $h[n]$ denote two sequences. Convolution sum $y[n]$ is given by

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

OR

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

$y[n] = x[n] \circledast h[n]$

4) Sampling Rate Alteration: It is used to generate new sequence with a sampling rate higher or lower than that of a given sequence. If $x[n]$ is a sequence with sampling rate F_T Hz and it is used to generate another sequence $y[n]$ with sampling rate F'_T Hz then sampling rate alteration ratio is given by:

$$\boxed{R = \frac{F'_T}{F_T}}$$

~~The Sampling Process~~
~~Let t be time variable of continuous time signal and n be time variable of discrete time signal.~~
~~Then~~

The Sampling Process
Time variable t of continuous time signal is related to time variable n of discrete time signal only at discrete time instants t_n given by

$$t_n = nT = \frac{n}{F_T} = \frac{2\pi n}{\omega_T}$$

F_T = sampling frequency
ω_T = sampling angular frequency

If continuous time signal is

$$x_a(t) = A\cos(2\pi f_0 t + \phi) = A\cos(\omega_0 t + \phi)$$

corresponding discrete time signal is given by

$$x[n] = A\cos(\omega_0 nT + \phi)$$

$$= A\cos\left(\frac{2\pi\omega_0}{\omega_T} n + \phi\right)$$

$$= A\cos(\omega n + \phi)$$

where, $\omega = \frac{2\pi\omega_0}{\omega_T} = \omega_0 T$

ω = normalized angular frequency of discrete time signal $x[n]$

It's unit is radians per sample

* The phenomenon of continuous time sinusoidal signal of higher frequency acquiring the identity of a sinusoidal sequence of lower frequency after sampling is called aliasing

* Continuous time signal $x_a(t)$ can be represented uniquely by it's sampled version $\{x[n]\}$ if sampling frequency ω_T is chosen to be greater than 2 times the highest frequency contained in $x_a(t)$. This condition is called Sampling Theorem and it is used to prevent aliasing.

$\omega_T \geq 2\omega_m$ Nyquist Condition

$2\omega_m$ is called Nyquist Rate

If $\omega_T > 2\omega_m$ Oversampling

$\omega_T < 2\omega_m$ Undersampling

$\omega_T = 2\omega_m$ Critical Sampling

Discrete Time System

Discrete Time system is used to process a given sequence called input sequence to generate another sequence called output sequence with more desirable properties or to extract certain information from input signals.

$$\underset{\text{Input Sequence}}{x[n]} \longrightarrow \boxed{H(\cdot)} \longrightarrow \underset{\text{Output sequence}}{y[n] = H(x[n])}$$

Schematic Representation of Discrete Time system

Classification of Discrete Time System

1) Linear System : For a linear system, if $y_1[n]$ and $y_2[n]$ are responses of input sequences $x_1[n]$ and $x_2[n]$, then for an input

$$x[n] = \alpha x_1[n] + \beta x_2[n]$$

$$y[n] = \alpha y_1[n] + \beta y_2[n]$$

2) Shift Invariant System : If $y_1[n]$ is the response to an input $x_1[n]$, then the response to an input $x[n] = x_1[n-n_0]$ is simply $y[n] = y_1[n-n_0]$

It is also called time invariance property.

3) Causal System : The n_0th output sample $y[n_0]$ depends only on input samples $x[n]$ for $n \leq n_0$ and does not depend on input samples $n > n_0$

4) Stable System : A discrete time system is called stable if and only if for every bounded input, the output is also bounded.

If, $|x[n]| < B_x$

then, $|y[n]| < B_y$

B_x, and B_y are finite positive constants

5) Passive and Lossless Systems : A discrete time system is passive if, for every finite energy input sequence $x[n]$, the output sequence $y[n]$ has, at most, the same energy, that is

$$\sum_{n=-\infty}^{\infty} |y[n]|^2 \leq \sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$$

If the above inequality is satisfied with an equal sign for every input sequence, the discrete-time system is said to be lossless.

Time-Domain Characterization of LTI Discrete-Time System

A linear time invariant (LTI) discrete time system satisfies both linearity and time invariance properties

1) Input Output Relationship

By knowing the impulse response, we can compute output of the system to any arbitrary input.
Consider an arbitrary input sequence

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

The response $y[n]$ to $x[n]$ is given by

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\,h[k] \qquad \text{OR}$$

which is convolution sum of sequences $x[n]$ and $h[n]$.

2) Tabular Method of Convolution Sum Computation

Consider evaluation of convolution of sequence $\{g[n]\}$, $0 \le n \le 3$, with sequence $\{h[n]\}$, $0 \le n \le 2$ generating the sequence $y[n] = g[n] \circledast h[n]$ $\quad 0 \le n \le 5$

n:	0	1	2	3	4	5
$g[n]$:	$g[0]$	$g[1]$	$g[2]$	$g[3]$		
$h[n]$:	$h[0]$	$h[1]$	$h[2]$			
	$g[0]h[0]$	$g[1]h[0]$	$g[2]h[0]$	$g[3]h[0]$		
		$g[0]h[1]$	$g[1]h[1]$	$g[2]h[1]$	$g[3]h[1]$	
			$g[0]h[2]$	$g[1]h[2]$	$g[2]h[2]$	$g[3]h[2]$
$y[n]$:	$y[0]$	$y[1]$	$y[2]$	$y[3]$	$y[4]$	$y[5]$

where

$$y[0] = g[0]h[0]$$

$$y[1] = g[1]h[0] + g[0]h[1]$$

$$y[2] = g[2]h[0] + g[1]h[1] + g[0]h[2]$$

$y[3] = g[3]h[0] + g[2]h[1] + g[1]h[2]$

$y[4] = g[3]h[1] + g[2]h[2]$

$y[5] = g[3]h[2]$

* Convolution of two sided sequences using Tabular method

$g[n] = \{3, \underset{\uparrow}{-2}, 4\}$ $\quad h[n] = \{\underset{\uparrow}{4}, 2, -1\}$

$g[n]$:			3	$-2\bullet$	4
$h[n]$:			$4\bullet$	2	-1
			-3	2	-4
		6	-4	8	
	12	-8	16		
$y[n]$:	12	$-2\bullet$	9	10	-4

$y[n] = \{12, \underset{\uparrow}{-2}, 9, 10, -4\}$

3) Stability Condition in Terms of Impulse Response

LTI discrete time system is Bounded in Bounded - out (BIBO) stable if and only if it's impulse response $h[n]$ is absolutely summable, that is

$$S = \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

eg. Consider a causal LTI system with impulse response

$$h_1[n] = \alpha^n u[n]$$

$$S = \sum_{n=-\infty}^{\infty} |\alpha^n u[n]| = \sum_{n=0} |\alpha^n|$$

$$= \sum_{n=0}^{\infty} |\alpha|^n = \frac{1}{1-|\alpha|}$$

for $|\alpha| < 1$

So $S < \infty$ for $|\alpha| < 1$ for which the system is BIBO stable. For $|\alpha| \geq 1$, the system is unstable