Angle Modulation

In angle modulation frequency or phase of carrier signal changes wrt modulating signal.

Frequency Modulation

- Frequency of carrier signal changes wrt modulating signal
- $y(t) = A_c \cos(w(t)\,t + \phi)$

  $w(t) = F(m(t))$

Phase Modulation

Phase of carrier signal changes wrt modulating signal

$y(t) = A_c \cos(w_c t + \phi(t))$

$\phi(t) = F(m(t))$

~~Advantages of FM over AM~~

- ~~All the transmitted power in FM is useful~~
- ~~Reduced noise~~

Advantages of FM over AM

- Noise reduction
- Efficient use of power
- Improved system fidelity
- Improved signal to noise ratio
- Less radiated power
- Less interference between neighbouring stations

Improved Power Use, System Fidelity and Signal to Noise Ratio

Disadvantages of FM over AM

- Larger bandwidth
- More complicated transmitter and receiver
- Increased cost of transmission and reception
- Infinite number of sidebands

Increased cost, complexity and bandwidth

Application

- Radio Broadcasting
- TV sound transmission
- cellular Radio

Frequency Modulation (FM)

In frequency modulation, frequency of carrier signal changes wrt modulating signal

If we have carrier signal

$$c(t) = E_c \cos(\omega_c t + \phi)$$
$$= E_c \cos(2\pi f_c t + \phi)$$
$$= E_c \cos \theta(t)$$

→ We have modulating signal $m(t)$

Frequency $f_i = F(m(t))$

→ In Frequency Modulation

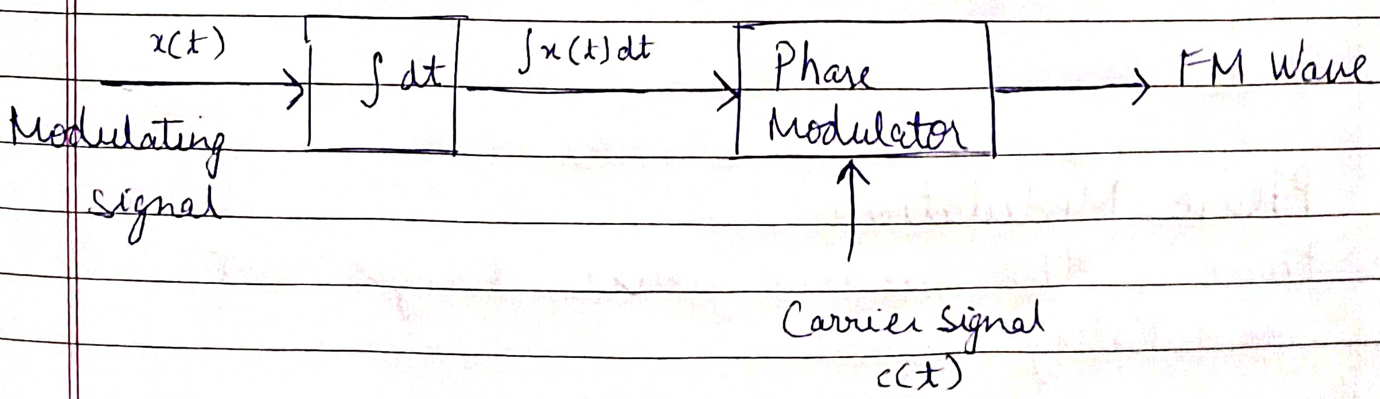
$$\theta(t) = 2\pi f_c t + 2\pi K_f \int_0^t x(t)\, dt$$

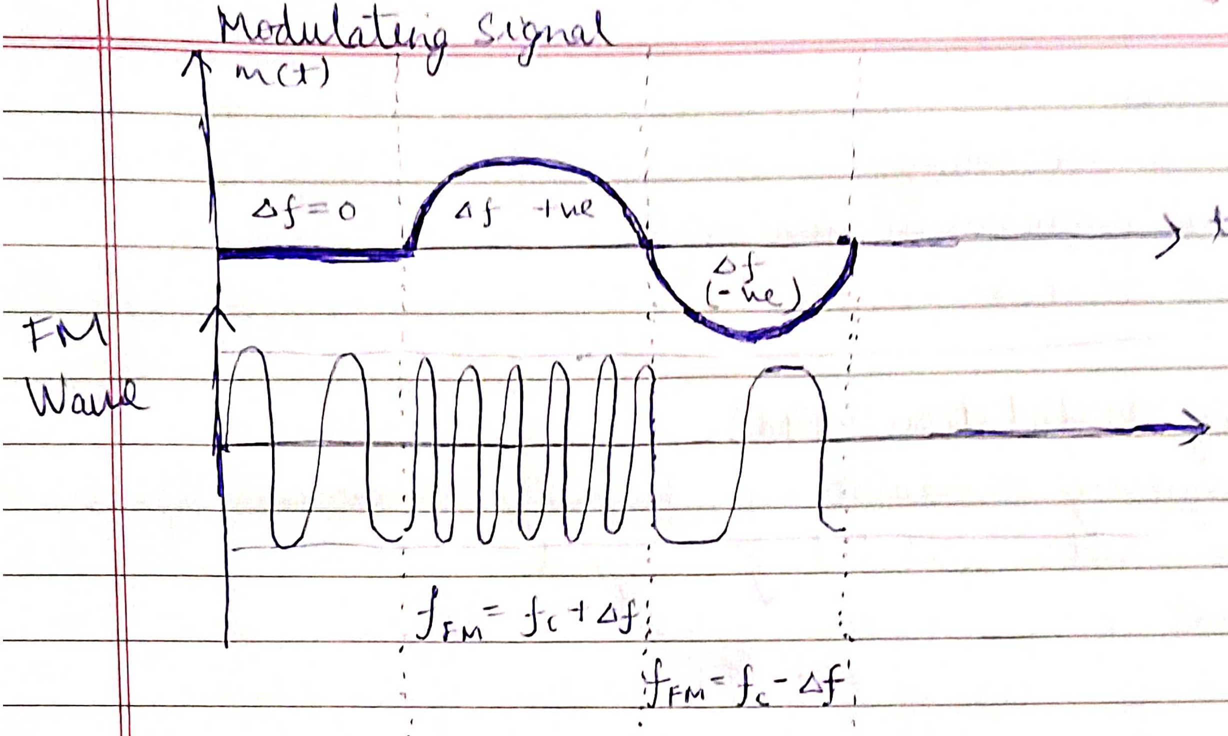
→ Modulated signal

$$y_{FM}(t) = E_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t x(t)\, dt\right)$$

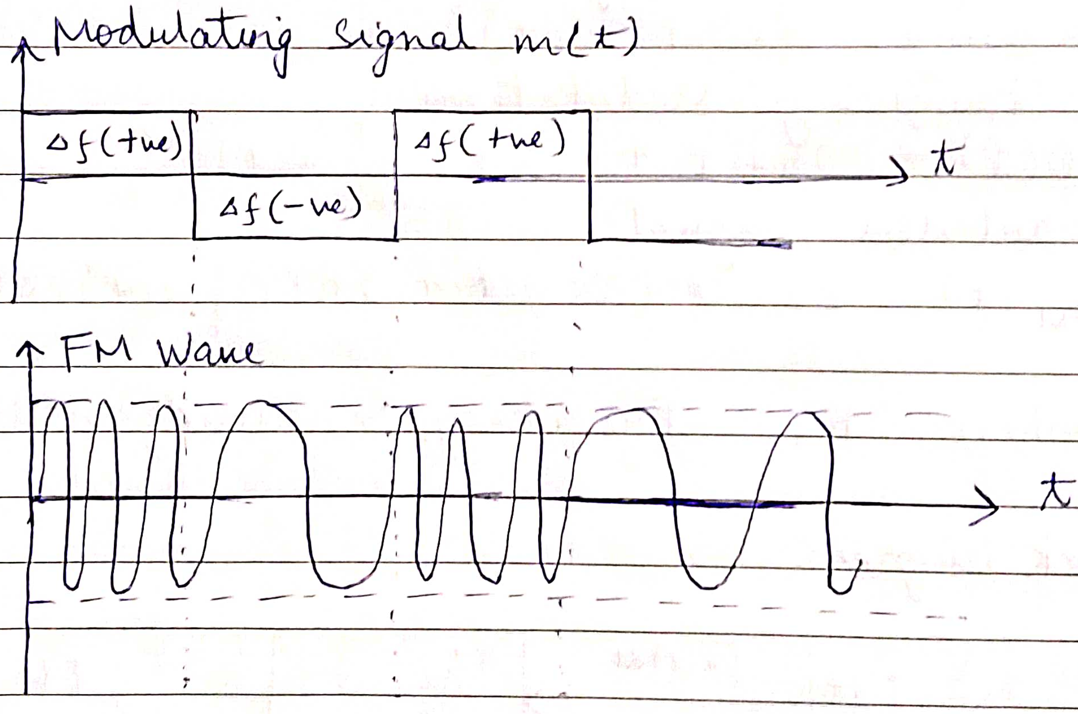
where K_f = Frequency Sensitivity (Hz/volt)

Block Diagram





Δf = frequency deviation



Phase Modulation

Phase of the carrier wave changes wrt modulating signal.

If we have carrier signal

$$c(t) = E_c \cos(\omega_c t + \phi)$$
$$= E_c \cos(2\pi f_c t + \phi)$$

$= E_c \cos \theta(t)$

→ For phase modulation

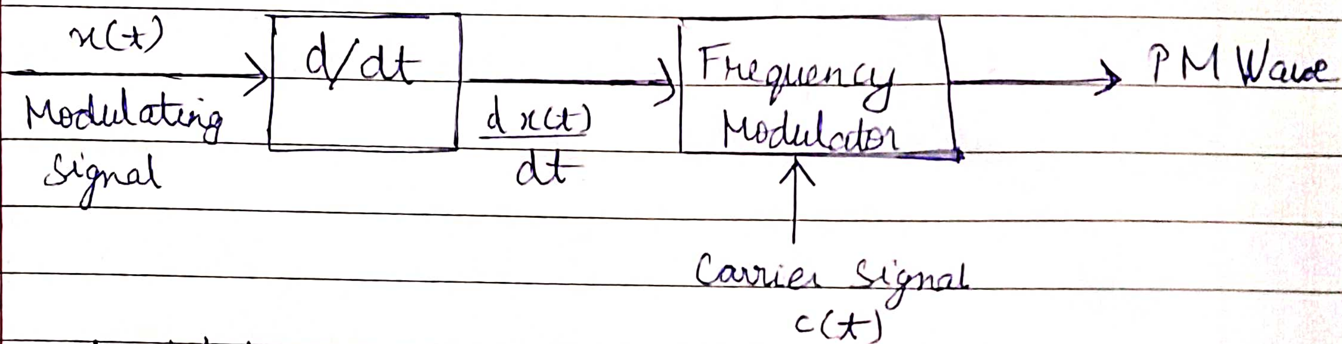
$$\theta(t) = 2\pi f_c t + K_p x(t)$$

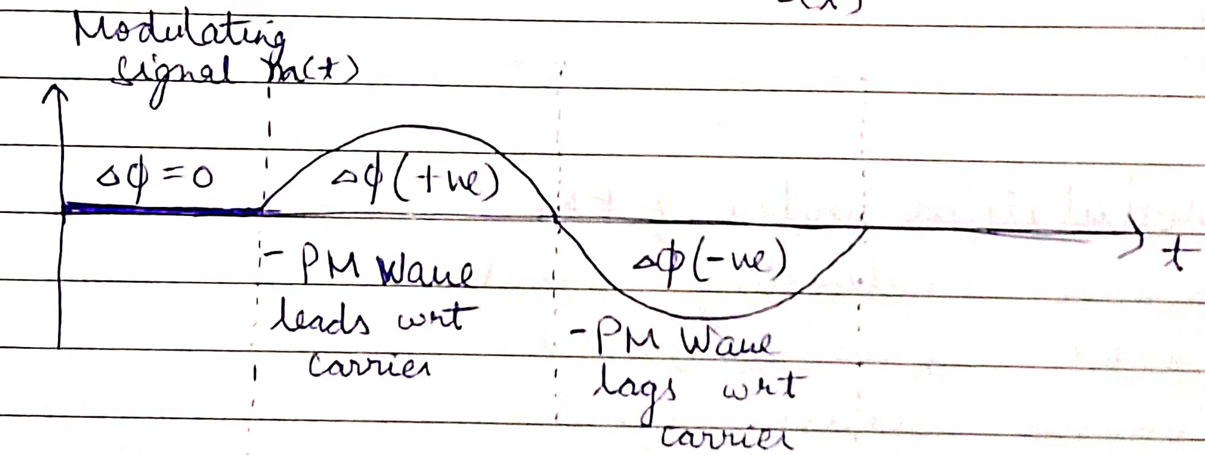
→ PM Wave

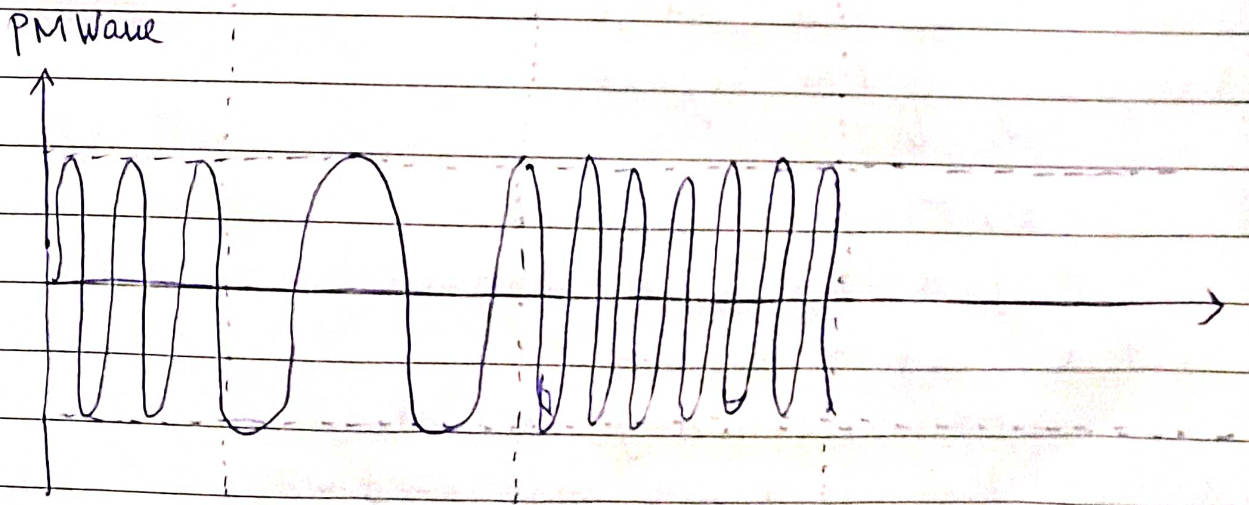
$$\boxed{y_{PM}(t) = E_c \cos(2\pi f_c t + K_p x(t))}$$

where K_p = phase sensitivity in (rad/volt)

Block Diagram

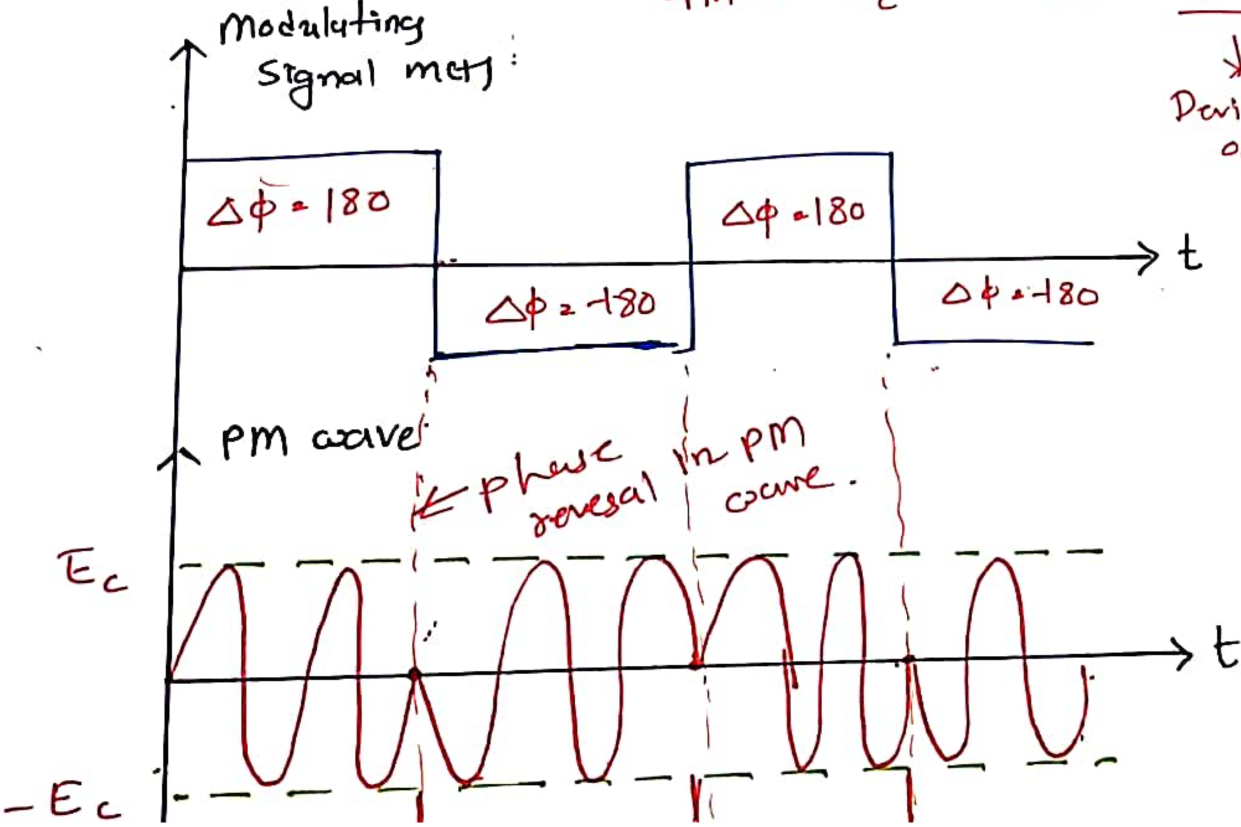






Δϕ = phase deviation

$$s_{pm}(t) = E_c \cos(2\pi f_c t + \underbrace{k_p x(t)}_{\text{Deviation of phase.}})$$



Modulation index of FM

→ Consider modulating signal

$x(t) = E_m \cos(2\pi f_m t)$

→ Consider carrier signal

$= E_c \sin(2\pi f_c t)$

→ Frequency of FM signal

$$F_i(t) = f_c + K_f\, x(t)$$

$$= f_c + K_f E_m \cos(2\pi f_m t)$$

Put $\Delta f = K_f E_m$

$\Delta f =$ frequency deviation

$\Rightarrow F_i(t) = f_c + \Delta f \cos(2\pi f_m t)$

Max freq deviation $= f_c + \Delta f$

Min freq deviation $= f_c - \Delta f$

→ FM Signal

$$y_{FM}(t) = E_c \sin\left(\omega_c t + K_f 2\pi \int^{t} x(t)\, dt\right)$$

$$= E_c \sin\left(\omega_c t + K_f 2\pi \int_0^{t} E_m \cos(2\pi f_m t)\, dt\right)$$

$$= E_c \sin\left(\omega_c t + \frac{K_f \cancel{2\pi} E_m}{\cancel{2\pi} f_m} \sin(2\pi f_m t)\right)$$

$$= E_c \sin\left(\omega_c t + \frac{K_f E_m}{f_m} \sin(2\pi f_m t)\right)$$

$$= E_c \sin\left(\omega_c t + \boxed{\frac{\Delta f}{f_m}} \sin(2\pi f_m t)\right)$$

↳ Modulation Index of FM

Modulation index of FM, $\boxed{m_f = \frac{\Delta f}{f_m}}$

$$\boxed{y_{FM}(t) = E_c \sin\left(\omega_c t + m_f \sin(2\pi f_m t)\right)}$$

$$\text{Deviation Ratio} = \frac{\text{Max Deviation}}{\text{Max Modulating Frequency}}$$

$$\text{\% Modulation of FM} = \frac{\text{Actual freq deviation}}{\text{Max. Allowed deviation}}$$

Spectral Components of Angle Modulated Signal.

- In Angle modulation, there is Freq. & phase modulation.
- Spectral components are identical for freq. & phase modulation.
- For Phase Modulation

$$e(t) = E_c \sin(\omega_c t + m \cos \omega_m t)$$

m ↑ Modulating Index $= \boxed{\frac{\Delta f}{f_m}}$

$$= E_c [\, J_0 \sin \omega_c t + J_1 [\sin(\omega_c t + \omega_m t) + \sin(\omega_c - \omega_m)t] + J_2 [\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t] + J_3 [\sin(\omega_c + 3\omega_m)t + \sin(\omega_c - 3\omega_m)t] + \dots \,]$$

- Here $J_0, J_1, J_2 \dots$ Bessel function. It's value is depending on modulating index. m.

In Angle Modulation,
- For Carrier Signal amplitude is $J_0 E_c$
- For Side-bands amplitude will be $E_c J_1$, $E_c J_2$, $E_c J_3$, ...

Bandwidth Requirement in angle modulation as per Carson's rule.

- Modulation Index $m = \frac{\Delta f}{f_m}$ where, Δf = deviation in freq. f_m = Max freq in modulating signal.
- Modulation Index has three categories
  - low (less than 1)
  - Medium (in bet.^{n} 1 to 10)
  - high (greater than 10).
- Minimum Bandwidth

  $BW = 2f_m \quad (m < 1)$
- For high Modulation Index

  $BW = 2\Delta f \quad (m > 10)$

  - (Δf Deviation in freq.).

- As per Carson's rule, minimum Bandwidth

  $\boxed{BW = 2[\Delta f + f_m].}$ ✓
- This rule accomodates almost 98% of transmitted power.

Bandwidth Requirement in angle modulation as per Carson's rule.

For AM signal, Bandwidth = 2fm

where, Δf = deviation in freq.

f_m = Max freq in modulating signal.

For SSBSC, Bandwidth = fm

– low (less than 1)

– As per Carson's rule, minimum Bandwidth

For VSB signal, Bandwidth = fv + fm

$$\boxed{BW = 2[\Delta f + f_m]}$$

– high (greater than 10).

– Minimum Bandwidth

$BW = 2f_m \quad (m < 1)$

– This rule accomodates almost 98% of transmitted power.

So, we can say, angle modulation has higher bandwidth then Amplitude modulation but it has greater noise immunity.

– (Δf Deviation in freq.)

Average Power for Angle Modulation

- As per phase Modulation

$$\boxed{e(t) = E_c \sin(\omega_c t + m \cos \omega_m t)}$$

$$= E_c [J_0 \sin \omega_c t +$$
$$J_1 (\sin(\omega_c + \omega_m)t + \sin(\omega_c - \omega_m)t) +$$
$$J_2 (\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m t)) +$$
$$J_3 (\sin(\omega_c + 3\omega_m)t + \sin(\omega_c - 3\omega_m)t) + \dots]$$

- Power with Carrier

$$P_c = \frac{E_c^2}{2R}$$ | where R is load Resistance.

- Carrier signal has amplitude $E_c J_0$.
- Amplitude with sidebands $E_c J_1$, $E_c J_2$, $E_c J_3$, ...
- For Modulated Spectrum $E_0 = E_c J_0$.

$$E_1 = E_c J_1$$
$$E_2 = E_c J_2$$

- Power of modulated signal :

$$P = P_0 + \underline{P_1 + P_1} + \underline{P_2 + P_2} + \underline{P_3 + P_3} + \dots$$

$$= P_0 + 2P_1 + 2P_2 + 2P_3 + \dots$$

$$= \frac{E_0^2}{2R} + 2\left[\frac{E_1^2}{2R}\right] + 2\left[\frac{E_2^2}{2R}\right] + 2\left[\frac{E_3^2}{2R}\right] + \dots$$

$$= \frac{1}{R}\left[\frac{E_0^2}{2} + E_1^2 + E_2^2 + E_3^2 + \dots\right].$$

Narrow Band Frequency Modulation (NBFM).

- It is used to improve spectrum efficiency.
- It is used in voice communication & in Radio settings.
- modulating Index for NBFM is less than 1.

$$\boxed{m \leq 1} \quad , \quad m = \frac{\Delta f}{f}$$

- FM expression is given by

$$e(t) = E_c \cos\left(\omega_c t + \boxed{\frac{k_f E_m}{\omega_m}} \sin \omega_m t\right)$$

Here $m = \dfrac{k_f E_m}{\omega_m}$

$$e(t) = E_c \cos(\omega_c t + m \sin \omega_m t)$$

As per $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

$$e(t) = E_c \left[\cos \omega_c t \, \underline{\cos(m \sin \omega_m t)} - \sin \omega_c t \, \underline{\sin(m \sin \omega_m t)}\right].$$

If m is very small $\begin{cases} \rightarrow \cos(m \sin \omega_m t) \approx 1 \\ \rightarrow \sin(m \sin \omega_m t) \approx m \sin \omega_m t \end{cases}$

- Modulating index ...

$$\boxed{m \le 1}, \quad m = \frac{\Delta f}{f}$$

- FM expression is given by

$$e(t) = E_c \cos\left(\omega_c t + \boxed{\frac{k_f E_m}{\omega_m}} \sin \omega_m t\right)$$

Here $m = \dfrac{k_f E_m}{\omega_m}$

$$e(t) = E_c \cos(\omega_c t + m \sin \omega_m t)$$

As per $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

$$e(t) = E_c [\cos \omega_c t \; \underline{\cos(m \sin \omega_m t)} - \sin \omega_c t \; \underline{\sin(m \sin \omega_m t)}].$$

If m is very small
- $\rightarrow \cos(m \sin \omega_m t) \approx 1$
- $\rightarrow \sin(m \sin \omega_m t) \approx m \sin \omega_m t$

$$e(t) = \boxed{E_c \cos \omega_c t - E_c m \sin \omega_c t \sin \omega_m t}$$

$$\boxed{= E_c \cos \omega_c t - k_f a(t) \sin \omega_c t}$$

where $a(t) = \int m(t)\, dt$

$$= \boxed{\underset{\text{Carrier.}}{E_c \cos \omega_c t} + \underset{\text{USB}\ \left[\frac{E_c m}{2}\right]}{\frac{E_c m}{2} \cos(\omega_c + \omega_m)t} - \underset{\text{LSB.}\ \left[\frac{E_c m}{2}\right].}{\frac{m E_c}{2} \cos(\omega_c - \omega_m)t}}$$

- $\rightarrow$ LSB is 180, out of phase w.r.t USB.
- $\rightarrow m \le 1$
- $\rightarrow$ NBFM has only two side bands.
- $\rightarrow$ Amplitude of LSB & USB is $\dfrac{E_c m}{2}$
- $\rightarrow \Delta f$ is limited here with NBFM.
- $\rightarrow m = \dfrac{k_f E_m}{\omega_m} = \dfrac{\Delta f}{f_m}$.

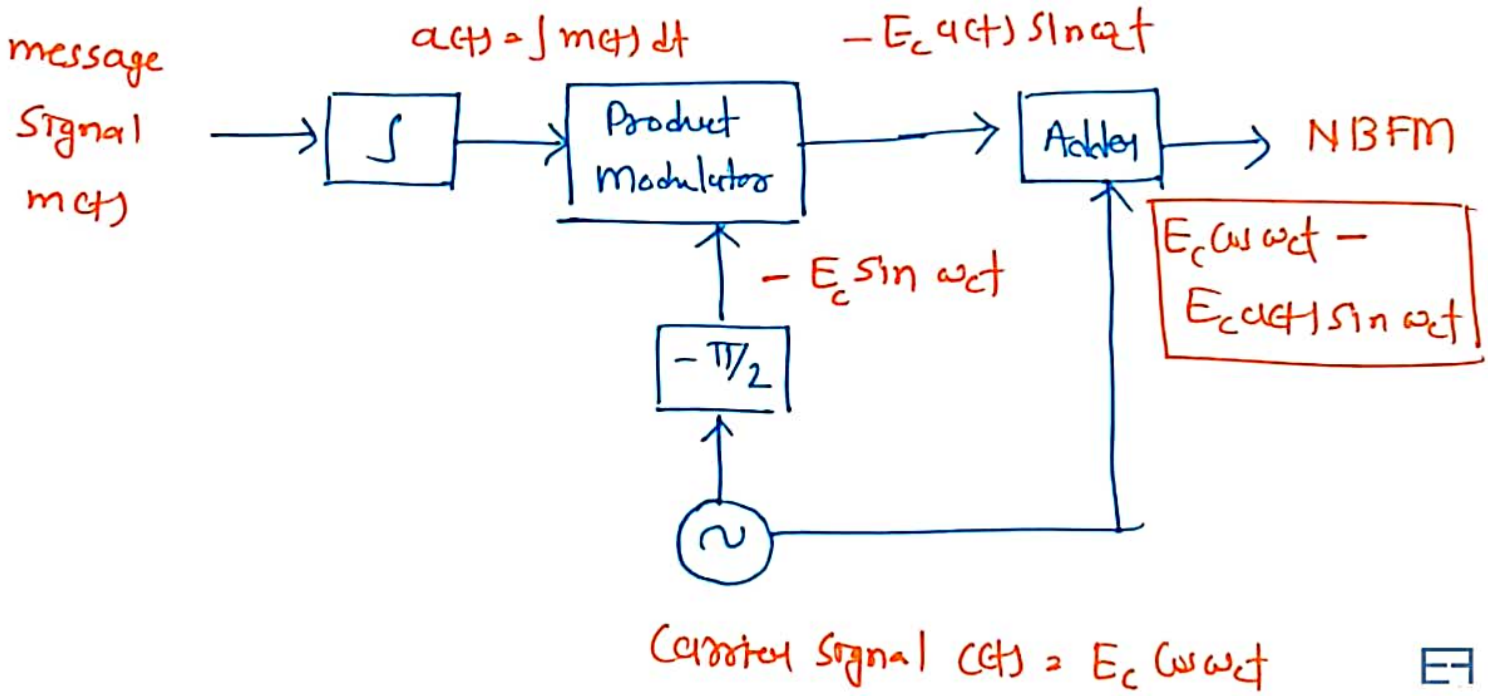
NBFM generation

- Narrowband FM signal is given by

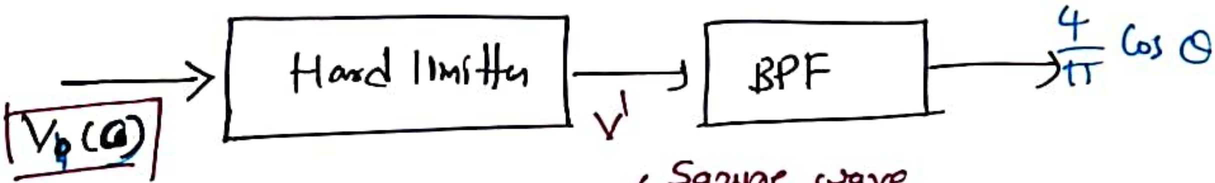
$$y_{FM}(t) = E_c(\underbrace{\cos \omega_c t}_{\text{carrier}} - k_f \underbrace{a(t) \sin \omega_c t}_{\text{Side band}})$$

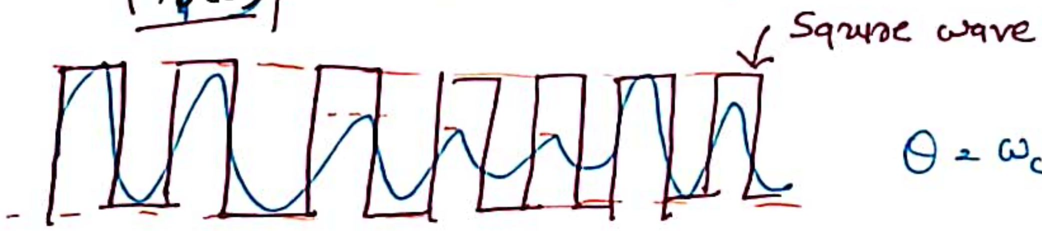
where, $a(t) = \int m(t)\, dt$

$$|k_f a(t)| << 1$$


message
signal
m(t)
$a(t) = \int m(t)\,dt$
$-E_c\, a(t) \sin \omega_c t$
$\int$
Product
Modulator
Adder
NBFM
$-E_c \sin \omega_c t$
$-\pi/2$
$E_c \cos \omega_c t - E_c a(t) \sin \omega_c t$
Carrier signal $c(t) = E_c \cos \omega_c t$

→ By band pass limitter we resolve issue of amplitude variation





$$\theta = \omega_c t + K_f(2\pi)\int m(t)\,dt$$

$$V' = \begin{cases} 1 & \cos\theta > 1 \\ -1 & \cos\theta < 1 \end{cases}$$

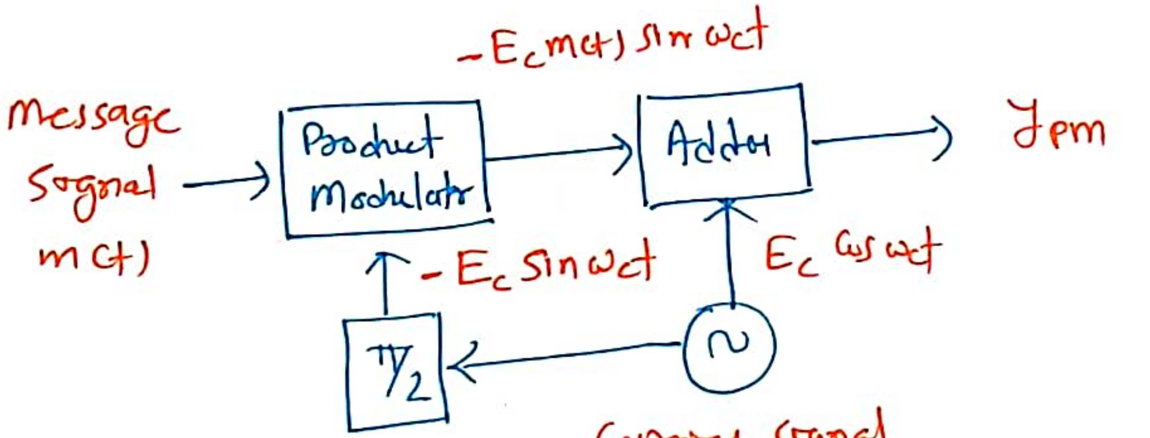
$$\rightarrow V_o(\theta) = \boxed{\frac{4}{\pi}\left[\cos\theta\right.} - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta + \ldots\Big]$$

NBPM generation

- Narrowband PM signal is given by

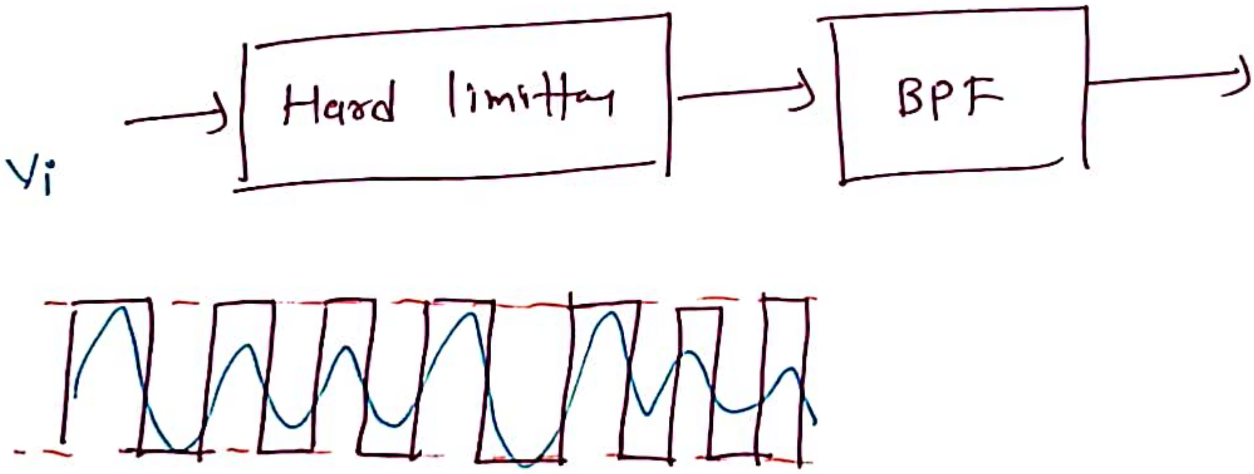
$$y_{pm} = E_c \left(\underbrace{\cos \omega_c t}_{\text{Carrier}} - \underbrace{K_p m(t) \sin \omega_c t}_{\text{Sideband}} \right)$$

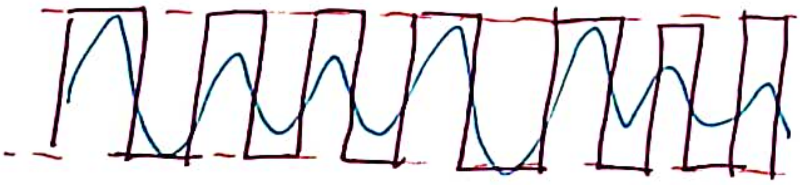
where, $|K_p m(t)| << 1$

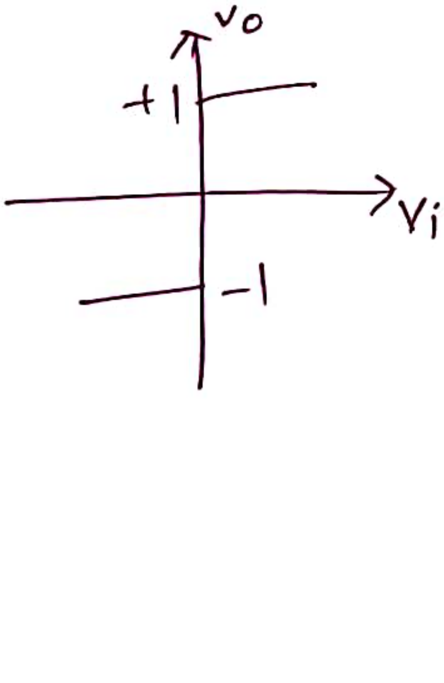


-Ecm(t) sin ωct
Message signal m(t)
Product Modulator
Adder
Ypm
-Ec sin ωct
Ec cos ωct
π/2
Carrier signal
c(t) = Ec cos ωct .

→ Band pass limitter







$$v_o = \begin{cases} +1 & \cos\theta > 0 \\ -1 & \cos\theta < 0 \end{cases}$$

$$v_o = \frac{4}{\pi}\left(\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta - \frac{1}{7}\cos 7\theta + \dots\right)$$

Parameters	WBFM	NBFM
Modulation Index	$m_f >> 1$	Less than 1 or slightly greater than 1
Maximum deviation	75 KHz	5 KHz
Freq. range of modulating signal	30 Hz to 15 KHz	30 Hz to 3 KHz

Maximum modulation Index	5 to 2500	Slightly greater than 1
Bandwidth	15 times than NBFM.	- Same AM

Applications	- Entertainment broadcasting. - high quality music transmission	- FM mobile communication for police wireless, Ambulance - For speech transmission.

FM generation by Direct Method

→ FM signal

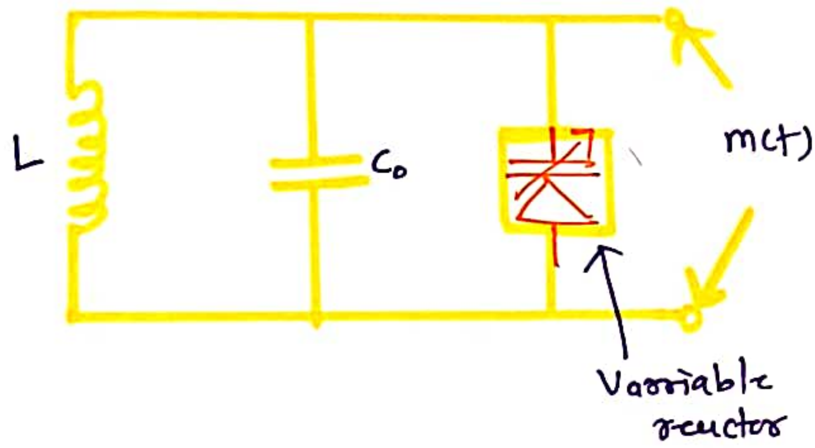
$$v_{FM}(t) = E_c \cos(\omega_c t + k_f \int m(t)\, dt)$$

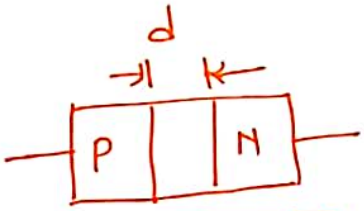
where

$$\omega(t) = \omega_c + k_f m(t)$$

→ In Voltage controlled Oscillator (VCO), the freq. is controlled by external Voltage.

1) Use Hartley or Colpitt Oscillators
2) Useing an Operational ampr and hysteretic comparator [Schimitt trigger circuit]
3) Using a Saturable core reactor.





$$\boxed{C = \varepsilon_0 \varepsilon_r \frac{A}{d}}$$

→ generated freq reactor

$$\omega = \frac{1}{\sqrt{LC}}$$

→ $C = C_0 - k\,m(t)$

→ $$\omega = \frac{1}{\sqrt{L(C_0 - k\,m(t))}}$$

$$= \frac{1}{\sqrt{LC_0\left(1 - \frac{k\,m(t)}{C_0}\right)}}$$

$$= \frac{1}{\sqrt{LC_0}}\left(1 - \frac{k\,m(t)}{C_0}\right)^{-1/2}$$

ω_c

→ $$\omega = \omega_c\left(1 - \frac{k\,m(t)}{C_0}\right)^{-1/2}$$

If $\frac{k\,m(t)}{C_0} << 1$

→ $$\omega = \omega_0\left(1 + \frac{k\,m(t)}{2C_0}\right)$$

$$= \omega_0 + \frac{k\omega_0}{2C_0}\,m(t).$$

$\leftarrow k_f$

$$= \omega_0 + k_f\,m(t).$$

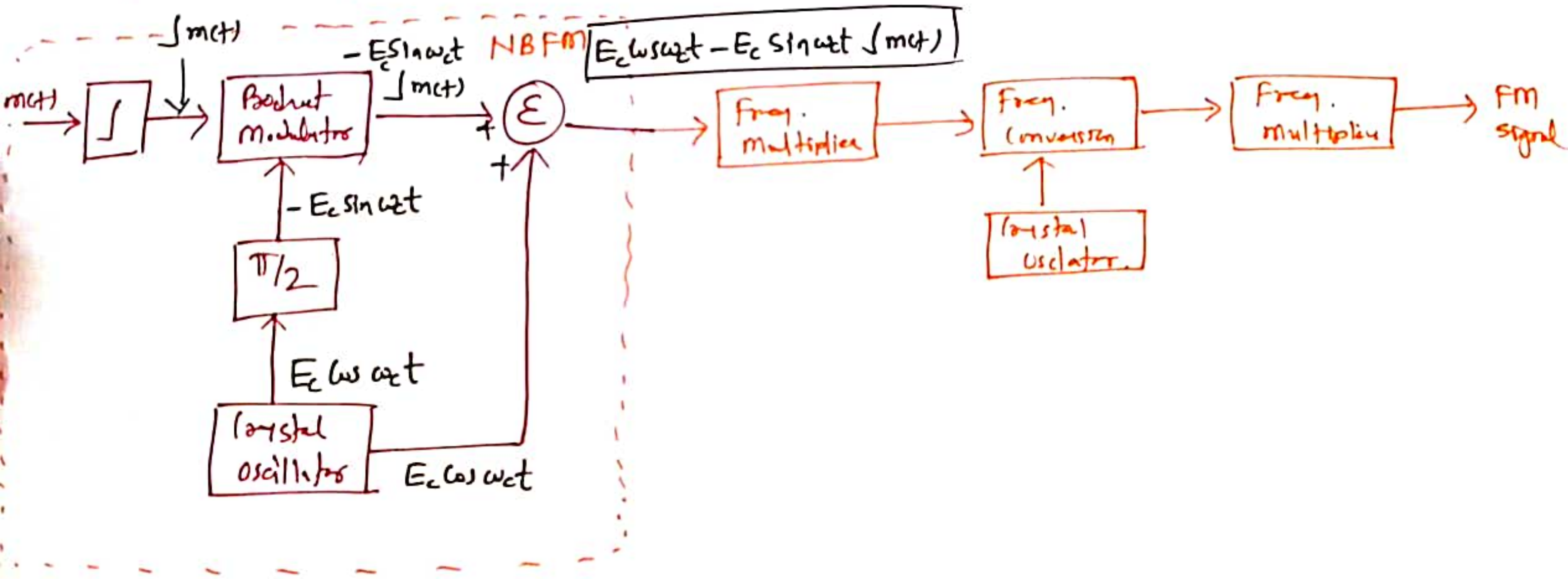
Advantages

- It generates sufficient freq. deviation
- It requires little freq. multiplication

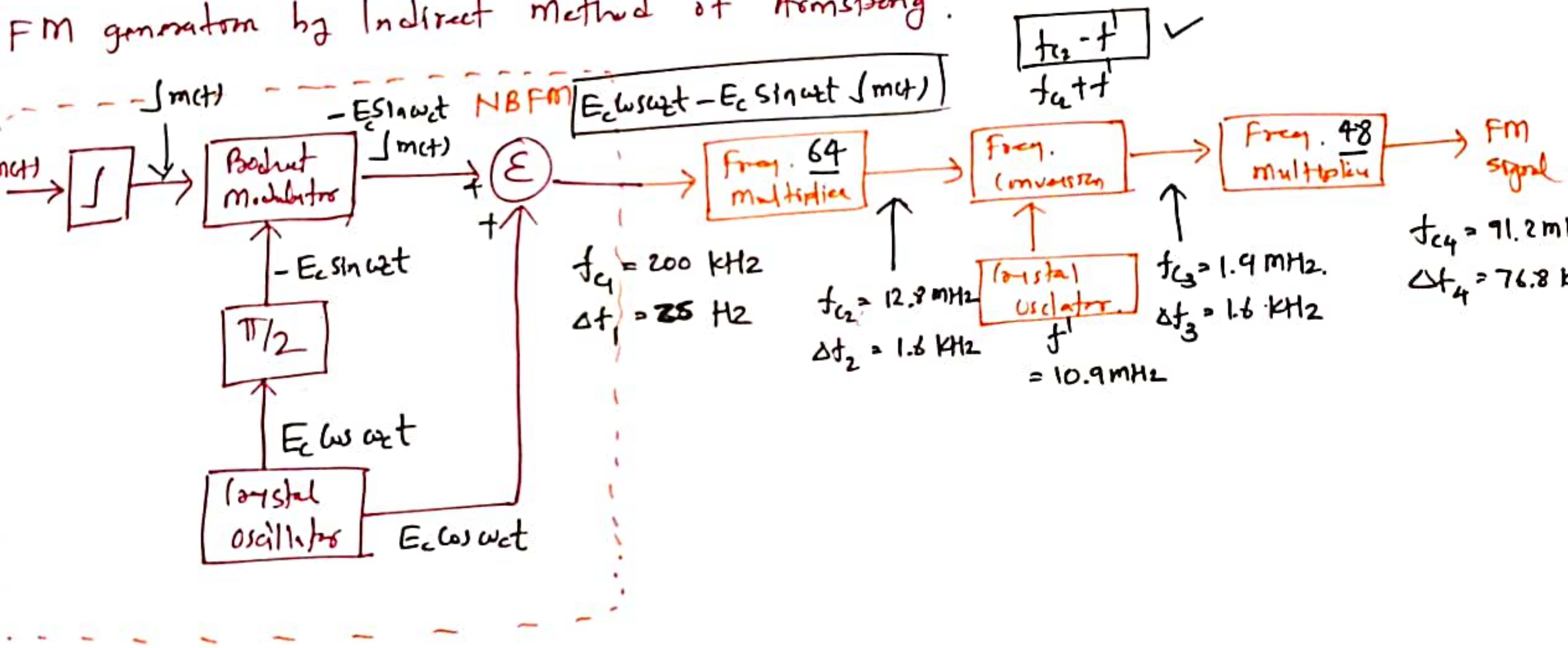
Disadvantage

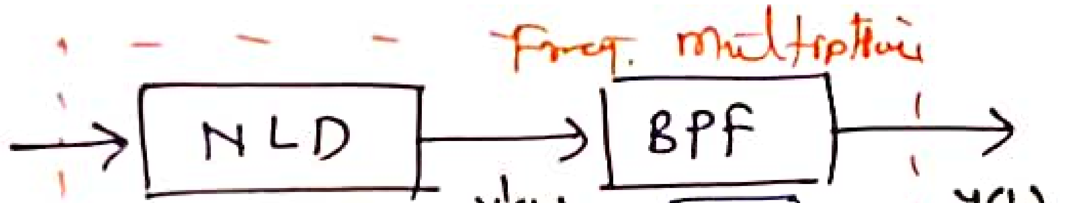
- Poor freq. stability

FM generation by Indirect Method of Armstrong.



FM generation by Indirect method of Armstrong.





$x(t) = \cos\theta = \cos\omega_c t$

- $y'(t) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \ldots$

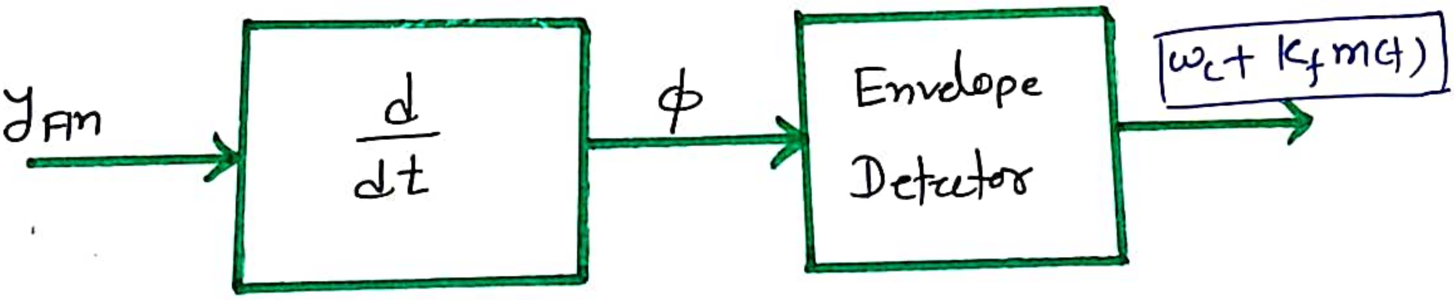
$\quad = a_0 + a_1 \cos\theta + a_2 \cos^2\theta + a_3 \cos^3\theta + \ldots$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\cos^3\theta = \frac{1}{4}(\cos\theta + \cos 3\theta).$

- $y'(t) = b_0 + b_1 \cos\theta + b_2 \cos 2\theta + b_3 \cos 3\theta + b_4 \cos 4\theta + \ldots$
- $\boxed{y(t) = b_6 \cos 6\omega_c t}$

FM demodulation & FM classification of detection



→ FM signal

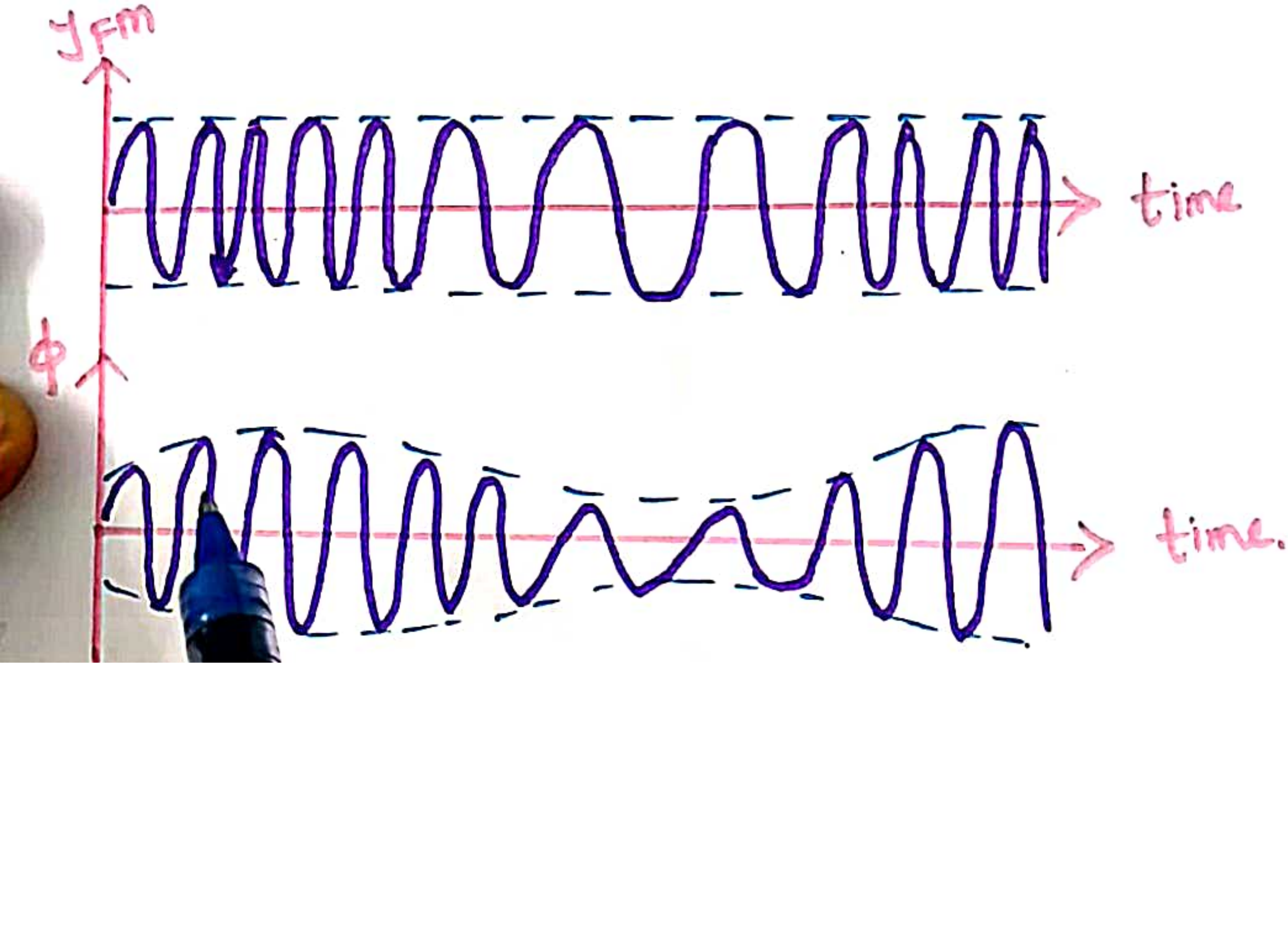
$$y_{FM}(t) = E_c \cos(\omega_c t + K_f \int m(t)\, dt)$$

→ After differentiation

$$\boxed{\omega_c > K_f m(t)}$$

$$\phi = \frac{d\, y_{FM}(t)}{dt} = -E_c \underline{[\omega_c + K_f m(t)]} \sin(\omega_c t + K_f \int m(t)\, dt)$$

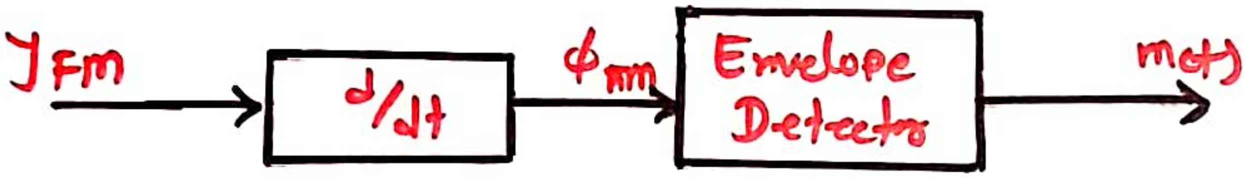
AM signal [Envelope].



FM detection Classification

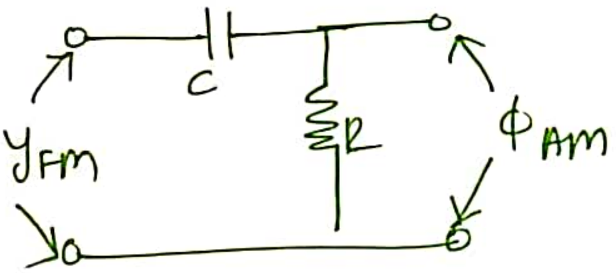
- Freq. Discriminator
  - Simple slope detector
  - Balanced slope detector
- Phase Discriminator
  - Foster Seeley
  - Ratio Detector
- PLL
- Zero crossing detector.

FM Demodulation by Frequency Discrimination by Single Slope Method.



→ differentiator transfer function $H(f) \propto f$

→ differentiator could be made by HPF

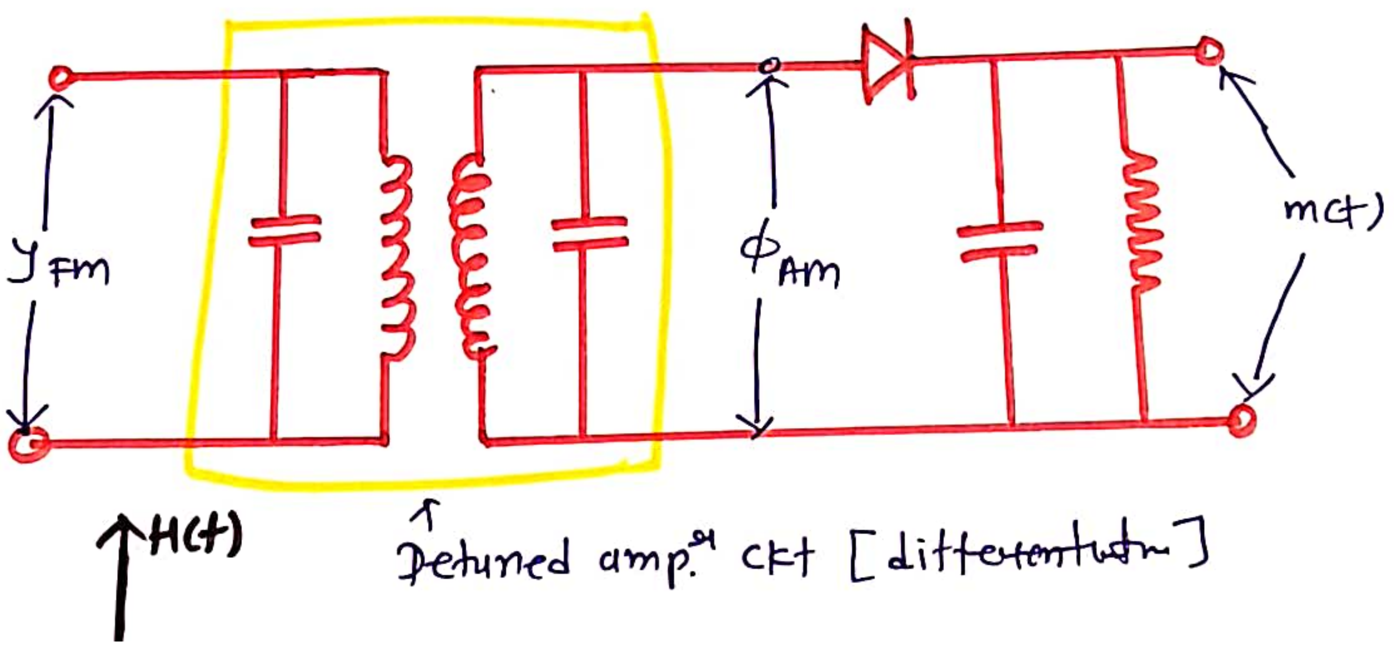


$$H(f) = \frac{R}{R + 1/j\omega C} = \frac{jR\omega C}{jR\omega C + 1}$$

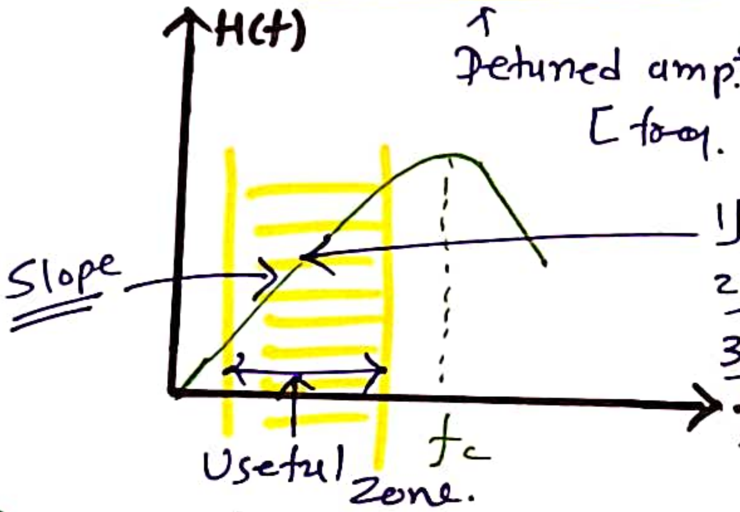
→ If $R\omega C \ll 1$

$$H(f) \approx j\omega RC \propto \underline{\underline{\omega}}$$

→ Here differentiator $\frac{dy_{FM}}{dt}$ = Slope



Detuned ampr ckt [differentiator]



Detuned amp^n ckt [differentiator]

[freq. discriminator]

1) linearity of TF

2) Do not use N.L. Zone

3) due to slope it freq function as d/dt

DrawBacks

- Useful range is limited
- It does not eliminate the amplitude variation and o/p is sensitive to any amplitude variation

FM Demodulation, Frequency Discriminator by Balanced Slope Detection

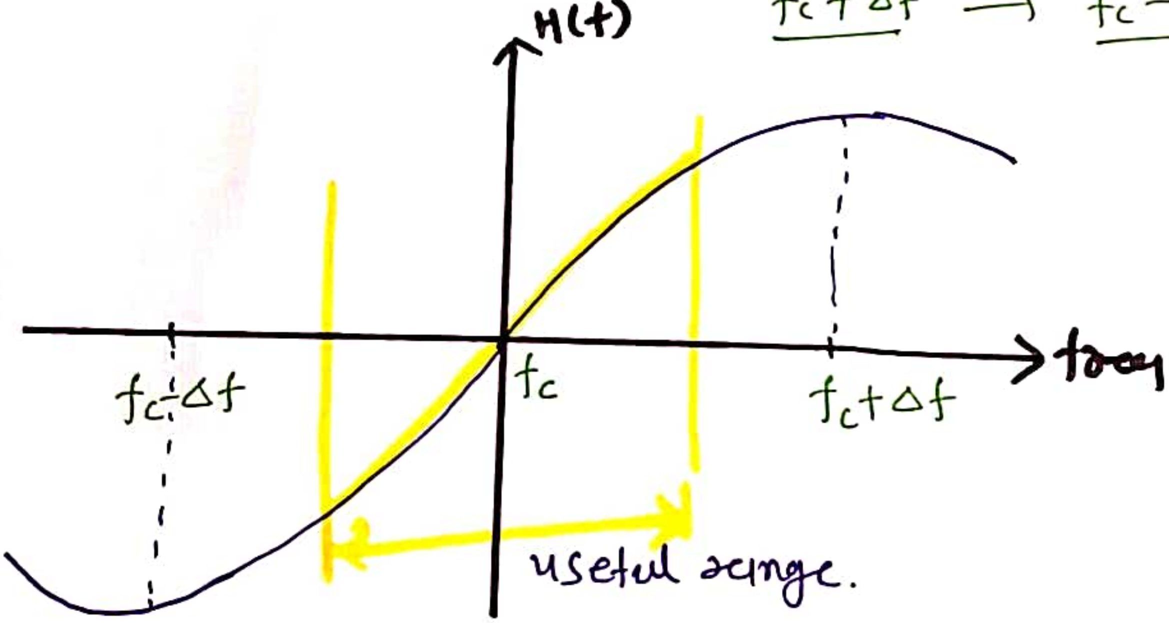
- In single slope detection, tuning range was limited. So to increase tuning range. we provide staggered tuning.

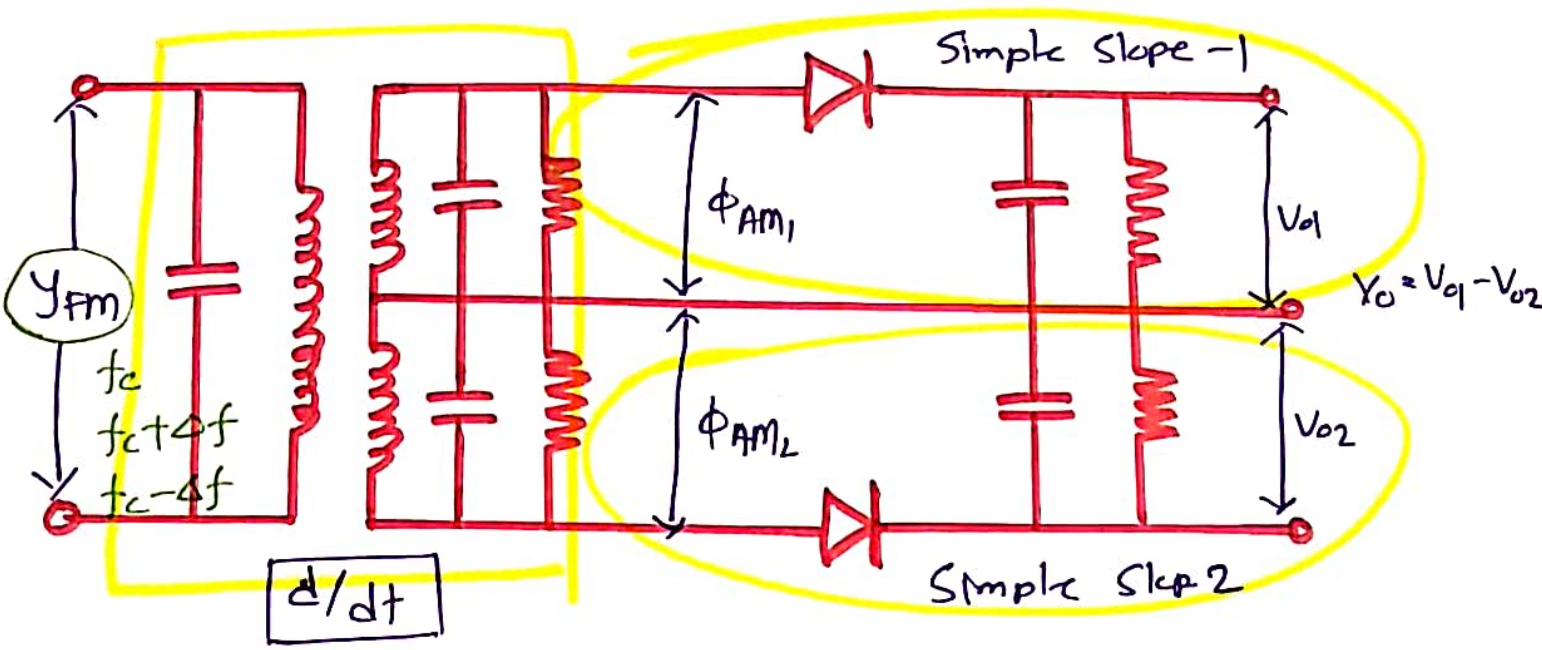
$\uparrow H(f)$ $\qquad$ $\underline{f_c + \Delta f} \longrightarrow \underline{f_c - \Delta f}$

Balanced Slope Detection

- In single slope detection, tuning range was limited. So to increase tuning range. we provide staggered tuning.

$\underline{f_c + \Delta f} \longrightarrow \underline{f_c - \Delta f}$





→ Case-1, $f_c (f_m) \rightarrow V_{o1} = V_{o2} \rightarrow V_o = 0$

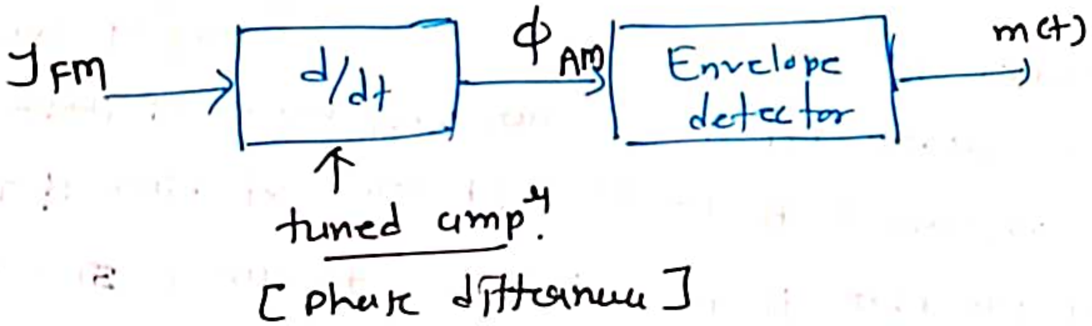
→ Case-2, $f_{FM} \rightarrow f_c + \Delta f \rightarrow V_{o1} > V_{o2} \rightarrow V_o = +Ve$

→ Case-3, $f_{FM} \rightarrow f_c - \Delta f \rightarrow V_{o2} > V_{o1} \rightarrow V_o = -Ve$.

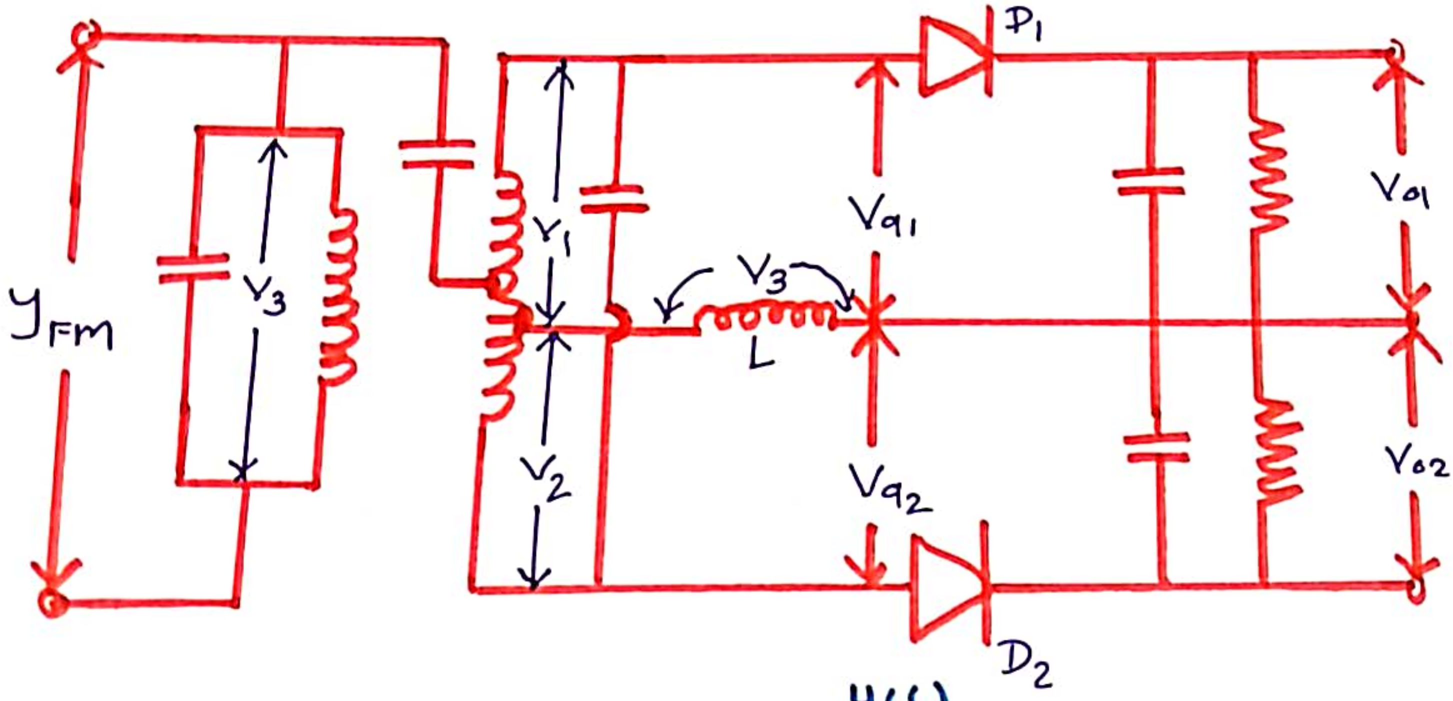
Draw Backs

- The linear characteristics is limited to a small freq. deviation
- The tuned O/P is not ~~purely~~ purely band limited. hence, the low pass RC filter of envelope detector introduced distortion.
- The discriminator characteristics depend critically on the amount of detuning of resonant circuit.

Foster Seeley Phase Discriminator



FM Demodulation by Foster-Seeley Method of phase discrimination



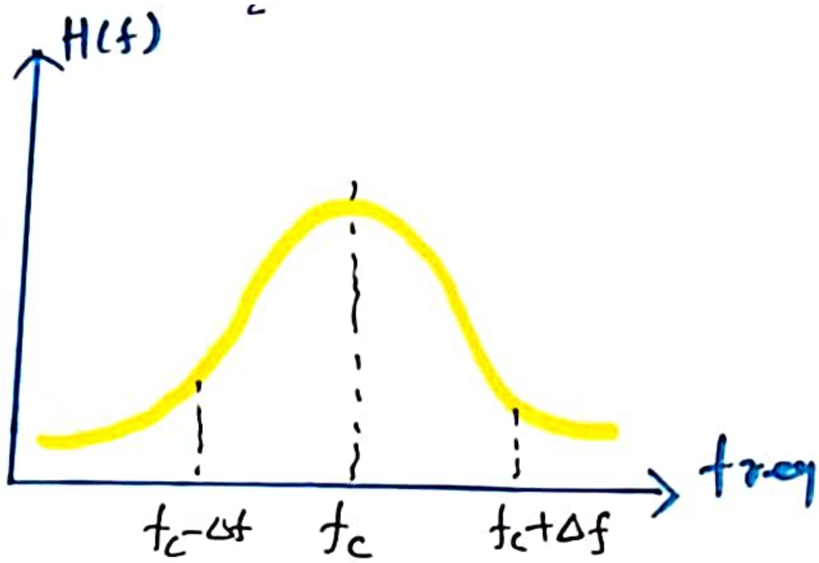
- H(f)

→ V_1 and V_2 are out of phase to each other

→ O/p Voltage $V_0 = |V_{02}| - |V_{01}|$

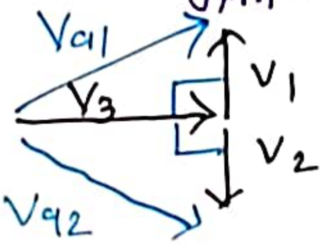
$V_{a1} = V_3 + V_1$

$V_{a2} = V_3 - V_2$



→ Case-I, at Resonance

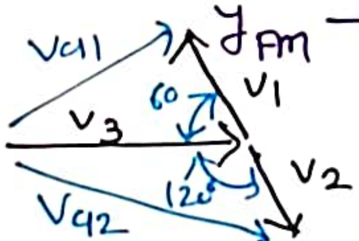
$f_{FM} \rightarrow f_c$



→ $|V_{a1}| = |V_{a2}|$

→ $V_0 = |V_{01}| - |V_{02}| = 0$

→ Case-II , at off Resonance



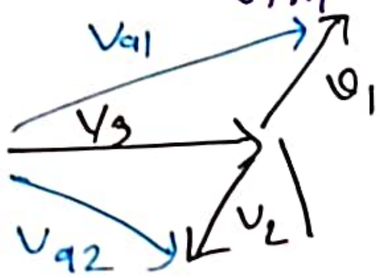
$y_{FM} \to f_c + \Delta f$

→ $|V_{a1}| < |V_{a2}|$

→ $V_0 = |V_{02}| - |V_{01}| = +Ve$

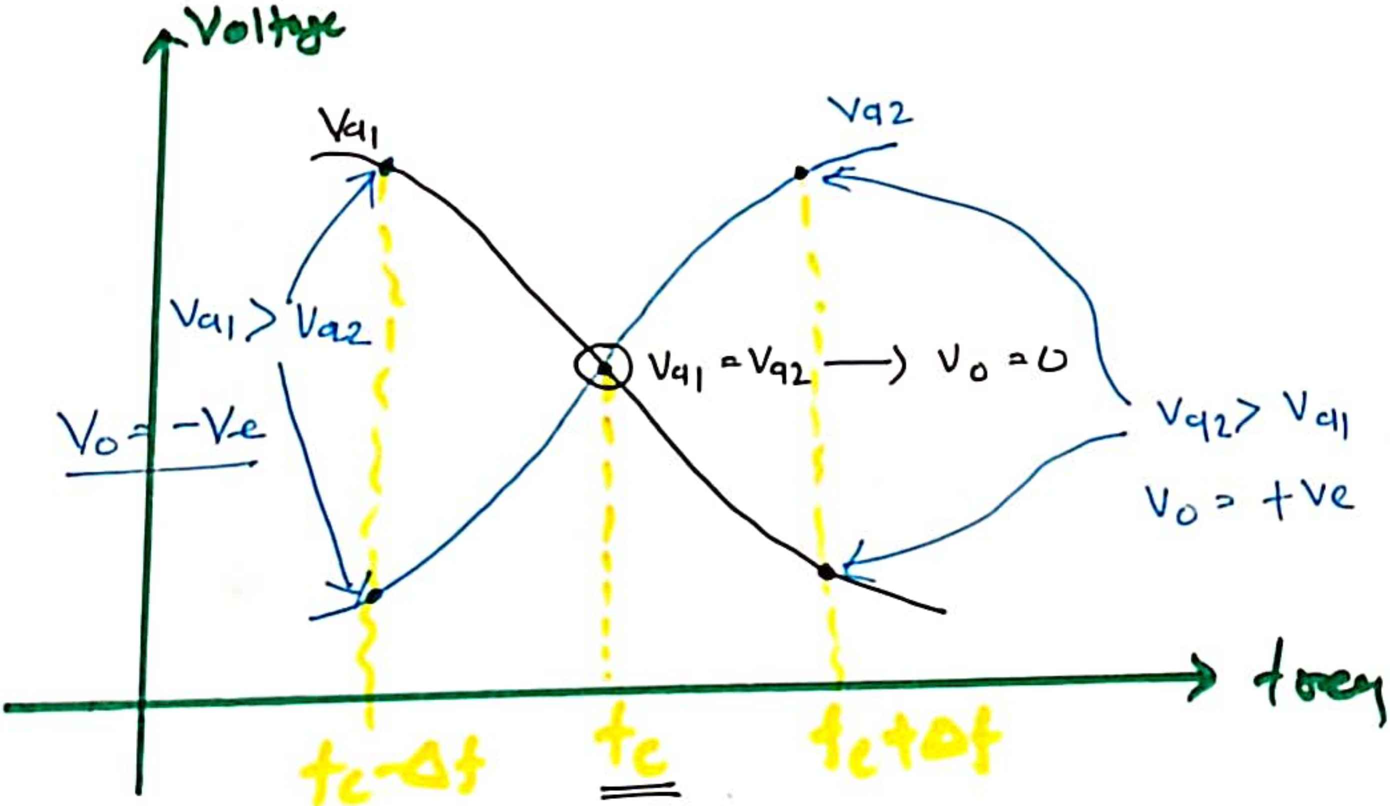
→ Case-III at off Resonance

$y_{FM} \to f_c - \Delta f$



→ $|V_{a2}| < |V_{a1}|$

→ $V_0 = |V_{02}| - |V_{01}| = -Ve.$



Voltage
Va1
Va2
Va1 > Va2
Vo = -Ve
Va1 = Va2 → Vo = 0
Va2 > Va1
Vo = +Ve
freq
fc-Δf
fc
fc+Δf

Advantages

1) Offers good level of Performance and resonance linearity

2) Easy to construct using discrete components L, C

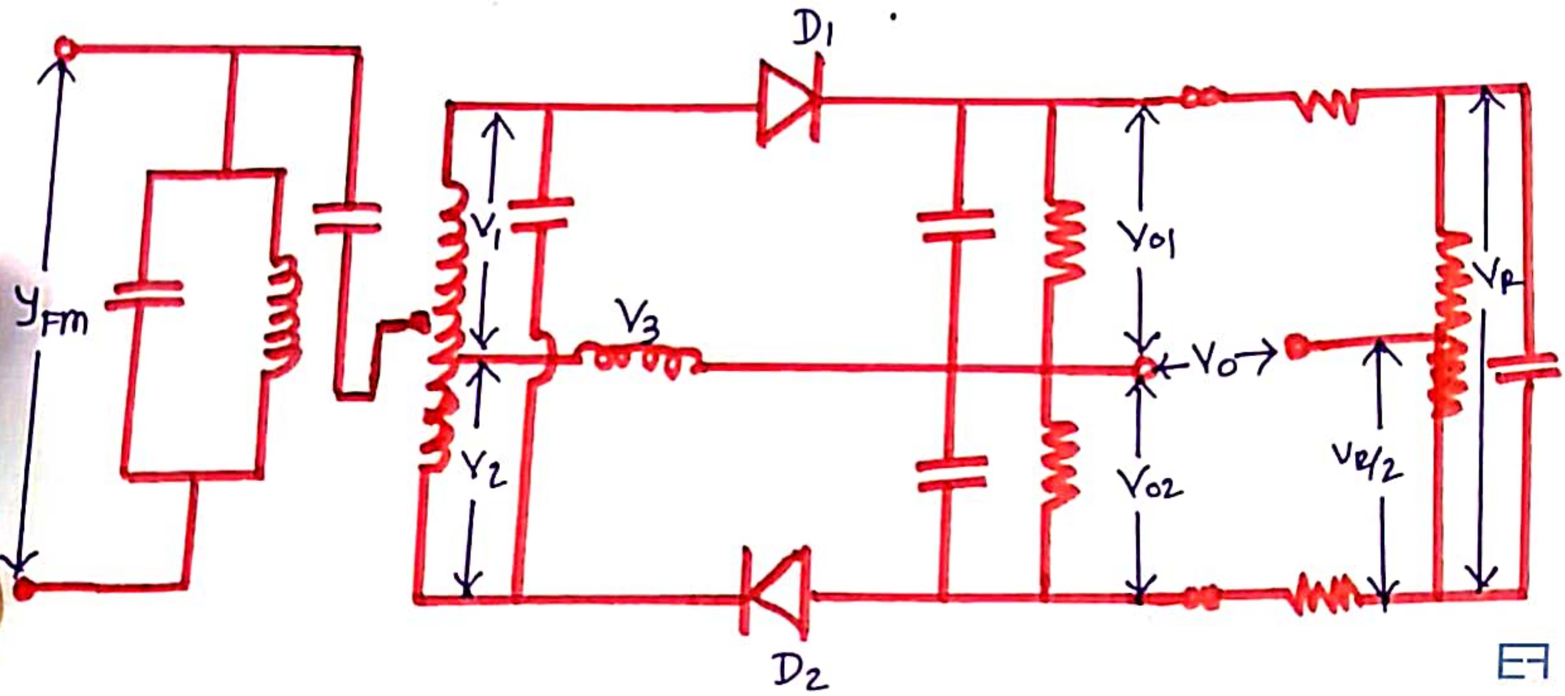
Drawbacks

1) Not Suitable to use IC technology

2) High Cost

3) Noise immunity

Radio Detector, FM Demodulation, Phase Discrimination Method



→ O/p of Radio detector $V_o = \frac{1}{2}$ (V_o Foster seeley)

→ $V_R = V_{o1} + V_{o2}$ — 1

→ $V_o = V_{o2} - V_R/2$ — 2

→ From 1 & 2

$$V_o = V_{o2} - \frac{V_{o1} + V_{o2}}{2} = \frac{V_{o2} - V_{o1}}{2} = \left(\frac{V_o}{2} \text{ Foster seeley} \right)$$

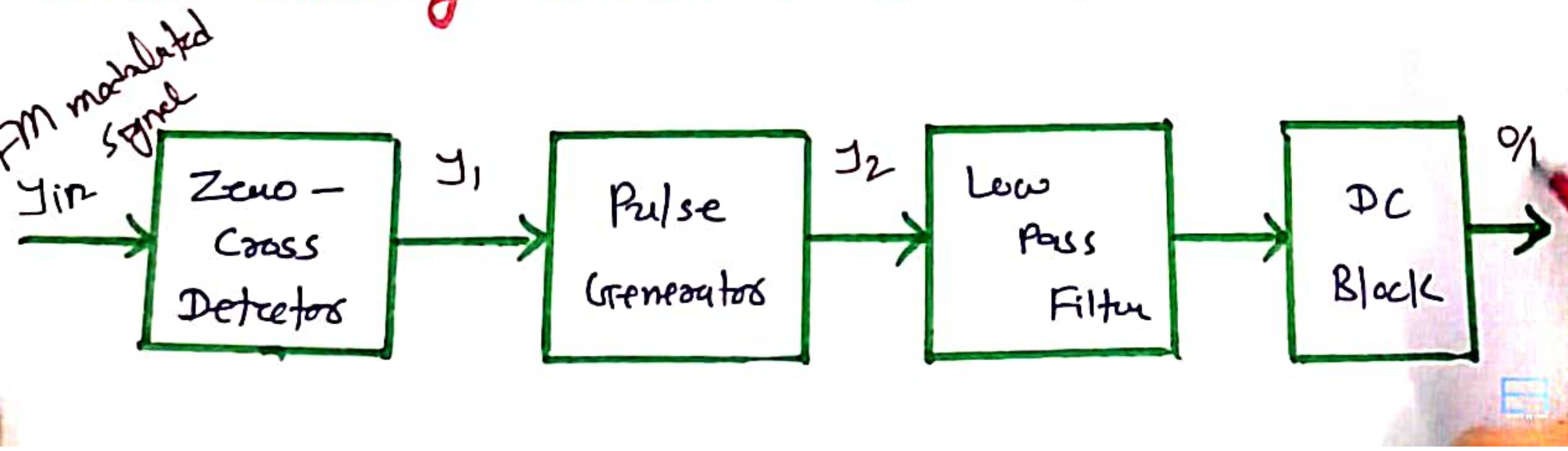
Advantages

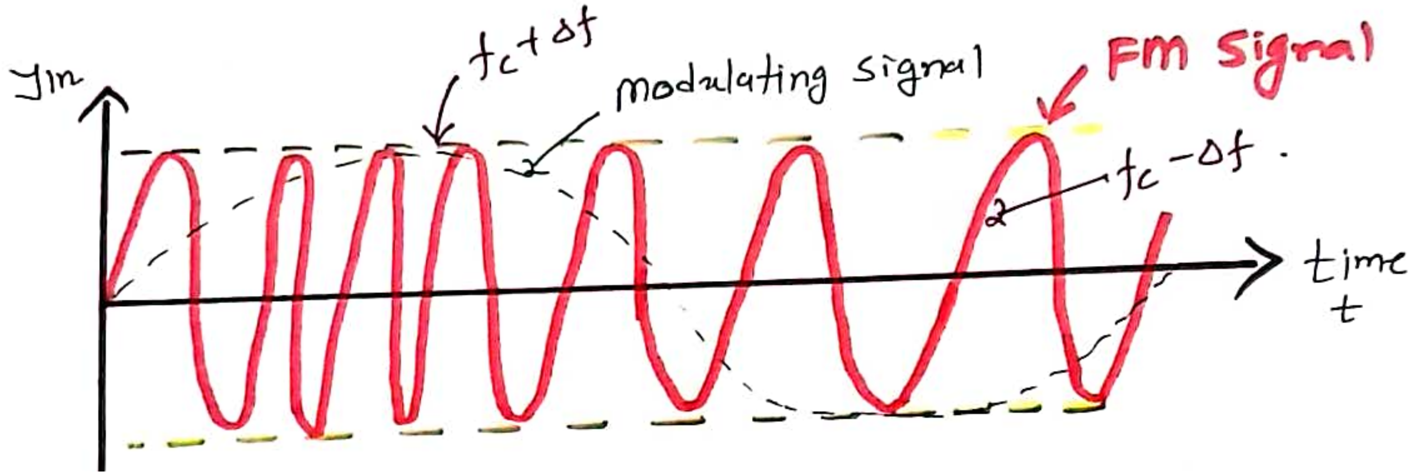
1) good performance and reasonable linearity
2) Better immunity against amplitude noise.
3) Wider Bandwidth
4) Easy to construct (L, C)

Drawbacks

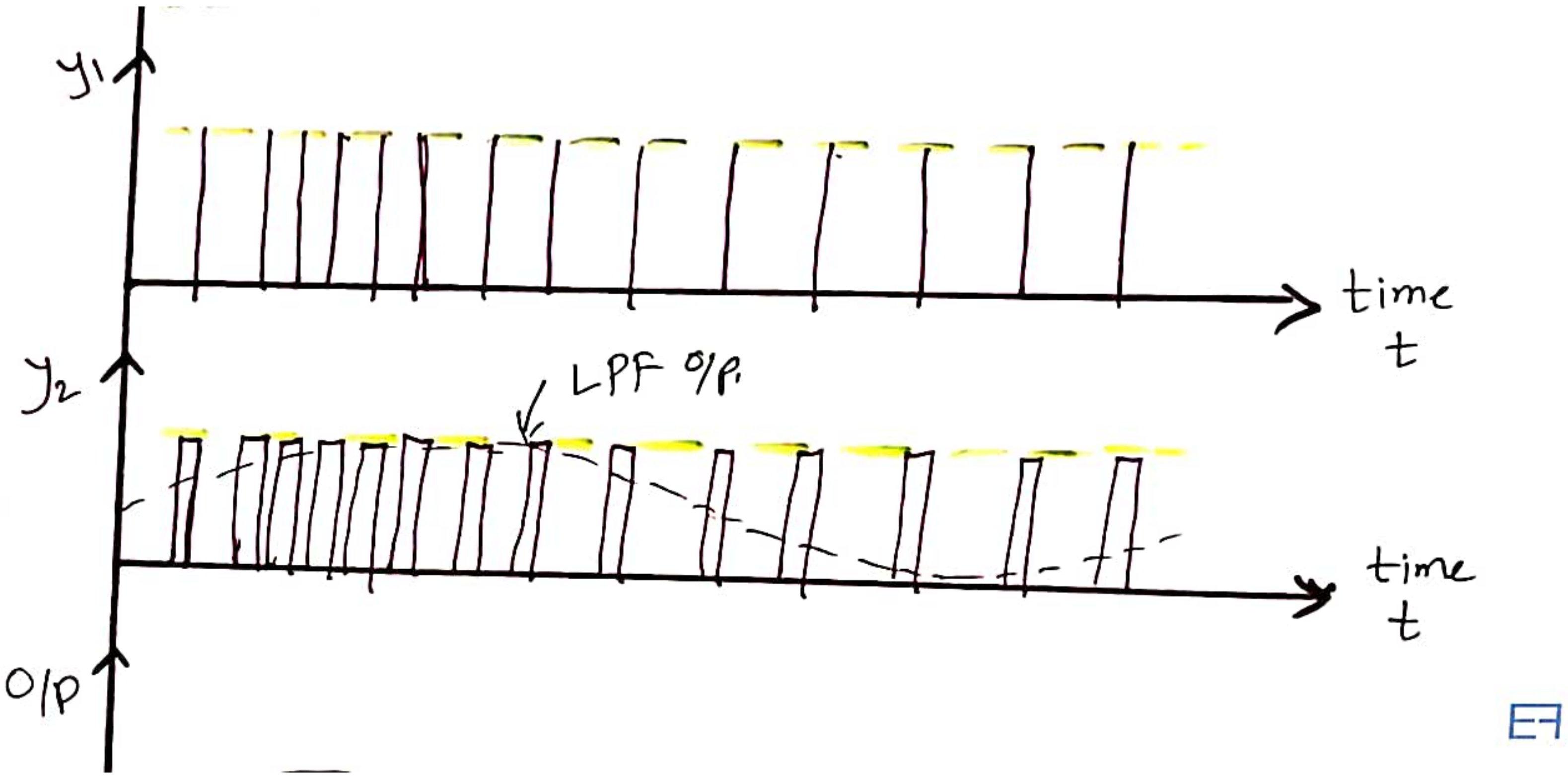
1) High cost
2) O/p efficiency is just half of foster seeley detector
3) Higher Distortion
4) Not suitable to Implement on IC Technology

Zero Crossing Detection of FM, FM Demodulation

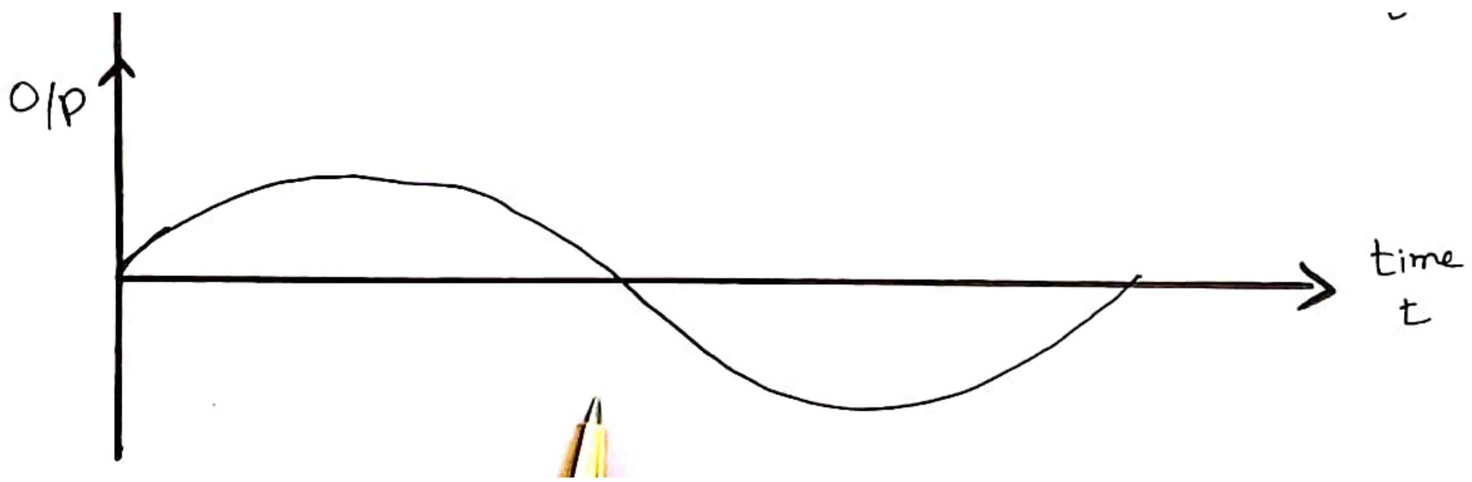




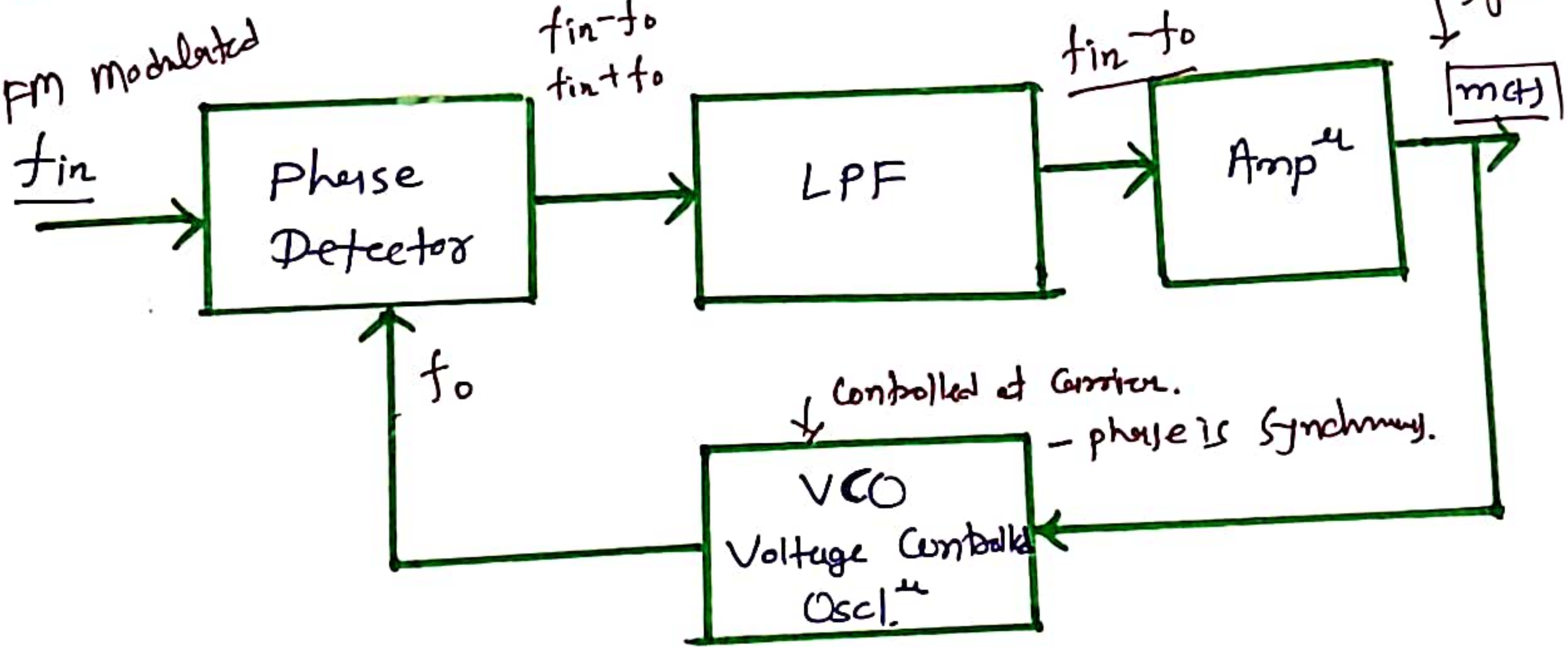
yin
fc+Δf
modulating signal
FM Signal
fc-Δf
time
t



y1
time
t
y2
LPF o/p
time
t
o/p


O/p
time
t

PLL FM Demodulator



→ The O/P freq. of VCO is equal to the freq. of unmodulated carrier.

→ The phase detector generates voltage proportional to difference between FM signal & VCO O/P.

→ Then O/P of phase detector passes through LPF & amp.^{r}

→ Here freq. correction is not req.rd at VCO as it is done at Tx.

Advantages

- No need of tuned circuit
- Simple circuit that can be implemented in integrated circuit.

Determine the percentage modulation for an FM wave with a frequency deviation of 10 kHz if the maximum deviation allowed is 25 kHz.

→ $\Delta f = 10 \text{ kHz}$

$\Delta f_{max} = 25 \text{ kHz}$

→ $\% \text{ Modulation} = \dfrac{\text{Actual deviation}}{\text{Max. allowed deviation}} \times 100$

$= \dfrac{10}{25} \times 100$

$= 40\%$

In an FM system, if the maximum value of deviation ~~is 75 kHz~~ and the maximum modulating frequency is 10 kHz, calculate the deviation ratio and bandwidth of the system using Carsons rule.

→ $f_{m(max)} = 10$ kHz

→ $\Delta f_{max} = 75$ kHz

→ Deviation ratio $= \dfrac{\Delta f_{max}}{f_{m(max)}} = \dfrac{75}{10} = 7.5$

→ System bandwidth $= 2(f_{m(max)} + \Delta f)$

$= 2(10 + 75)$

$= 2(85)$

$= 170$ kHz

The equation of an angle modulated voltage is

$$e = 10\sin(10^8 t + 3\sin 10^3 t)$$

What form of angle modulation is this? Calculate the carrier and modulating frequencies, the modulation index and deviation and the power dissipated in 100Ω resistor.

→ $e = 10 \sin(10^8 t + 3 \sin 10^3 t)$

→ Standard form of FM

→ $e = E_c \sin(\omega_c t + m_f \sin \omega_m t)$

→ $E_c = 10$ volt

$\omega_c = 10^8 \Rightarrow f_c = \frac{10^8}{2\pi} = 15.91$ MHz

$m_f = 3$

$\omega_m = 10^3 \Rightarrow f_m = \frac{10^3}{2\pi} = 159$ Hz

→ $m_f = \frac{\Delta f}{f_m} \Rightarrow \Delta f = m_f f_m$

$= 3 \times 159$

$= 477$ Hz

→ $P = \frac{E_c^2}{2R}$

$= \frac{10^2}{2 \times 100} = 0.5$ W

Use Carson's rule to compare the bandwidth that would be required to transmit a baseband signal with a frequency range from 300 Hz to 3 kHz using

a. NBFM with maximum deviation of 5 kHz.
b. WBFM with maximum deviation of 75 kHz

→ For NBFM

$$\Delta f_{max} = 5 \text{ kHz}$$

→ $BW = 2((f_m)_{max} + \Delta f_{max})$

$= 2(3+5)$

$= 16 \text{ kHz}$

→ For WBFM, $\Delta f_{max} = 75 \text{ kHz}$

→ $BW = 2((f_m)_{max} + \Delta f_{max})$

$= 2(3+75)$

$= 156 \text{ kHz}$

The carrier frequency of an FM broadcast transmitter is 100 MHz and maximum frequency deviation is 75 kHz. If the highest audio frequency modulating carrier is 15kHz. What is the approximate bandwidth of the signal?

→ $f_c = 100\ MHz$

$\Delta f_{max} = 75\ KHz$

$(f_m)_{max} = 15\ KHz$

→ $BW = 2((f_m)_{max} + \Delta f_{max})$

$= 2(15 + 75)$

$= 180\ KHz$

A single tone FM signal is given by,

$$V = 10\sin(16\pi \times 10^8 t \;+ 3\sin 2\pi \times 10^3 t)\ Volt$$

Find the modulation index, modulating frequency, deviation, carrier frequency and power of the FM signal.

→ $V = 10\ \sin(\underline{16\pi \times 10^8} t + \underline{3}\ \sin\ \underline{2\pi \times 10^3} t)$ Volt

→ Standard eq.ⁿ for FM

→ $V = E_c \sin(\underline{\omega_c} t + \underline{m_f} \sin \underline{\omega_m} t)$

→ $E_c = 10$ Volt

$\omega_c = 16\pi \times 10^8$

$m_f = \underline{3}$

$\omega_m = 2\pi \times 10^3$

→ $f_c = \dfrac{\omega_c}{2\pi}$

$= 8 \times 10^8$ Hz

→ $f_m = \dfrac{\omega_m}{2\pi}$

$= 10^3$ Hz

→ $m_f = \dfrac{\Delta f}{f_m}$

⇒ $\Delta f = m_f f_m$

⇒ $\Delta f = 3 \times 10^3$

→ $P = \dfrac{E_c^2}{2R} = \dfrac{10^2}{2R} = \dfrac{50}{R}$

A FM wave is represented by the following equation

$$V = 10\sin(5 \times 10^8 t + 4\sin 1250t)$$

Find,

a. Carrier and modulating frequencies
b. Modulation index and maximum deviation
c. The power dissipated by this FM wave in 5Ω resistor.

→ $V = 10\sin(5 \times 10^8 t + 4\sin 1250t)$

→ Compare given eqn with standard form

→ $V = E_c \sin(\omega_c t + m_f \sin \omega_m t)$

→ $E_c = 10$

$\omega_c = 5 \times 10^8$

$m_f = 4$

$\omega_m = 1250$

→ $f_c = \frac{\omega_c}{2\pi} = \frac{5 \times 10^8}{2\pi} = 79.57 \text{ MHz}$

→ $f_m = \frac{\omega_m}{2\pi} = \frac{1250}{2\pi} = 199 \text{ Hz}$

→ $m_f = \frac{\Delta f}{f_m}$ → $\Delta f = m_f f_m = 4 \times 199 = 796 \text{ Hz}$

→ $P = \frac{E_c^2}{2R} = \frac{10^2}{2 \times 5} = 10 \text{ W}$

In a FM system, the modulating frequency f_m = 1 kHz, the modulating voltage E_m=2volt and the deviation is 6 kHz. If the modulating voltage is raised to 4 volt than what is the new deviation? If the modulating voltage is further increased to 8 volt and modulating frequency is reduced to 500 Hz what will be deviation?

→ $f_m = 1$ KHz

$E_m = 2$ Volt

$\Delta f = 6$ KHz

→ $\Delta f = K_f E_m$

$\Rightarrow K_f = \Delta f / E_m$

$= \frac{6 \text{ KHz}}{2 \text{ Volt}} = 3 \frac{\text{KHz}}{\text{Volt}}$

→ Now If $E_m = 4$ Volt

→ $\Delta f = K_f E_m$

$= \left(3 \frac{\text{KHz}}{\text{Volt}}\right) 4$

$= 12$ KHz

→ Now If $E_m = 8$ Volt

$f_m = 500$ Hz

→ $\Delta f = K_f E_m$

$= \left(3 \frac{\text{KHz}}{\text{Volt}}\right) 8$

$= 24$ KHz

What is the bandwidth required for a FM signal if the modulating frequency is 1 kHz and the maximum deviation is 10 kHz? What is BW required for a DSBFC (AM) transmission?

→ $f_m = 1\ KHz$

$\Delta f = 10\ KHz$

→ $BW = 2(f_m + \Delta f)$

$= 2(1+10)$

$= 22\ KHz$

→ BW fo AM transmission

$BW = 2f_m$

$= 2 \times 1\ KHz$

$= 2\ KHz$

$(BW)_{FM} >> (BW)_{AM}$

A 20 MHz carrier is modulated by a 400 Hz modulating signal. The carrier voltage is 5V and maximum deviation is 10 kHz. Write down the mathematical expression for the FM waves.

→ $f_c = 20\ MHz$

$f_m = 400\ Hz$

$E_c = 5V$

$(\Delta f)_{max} = 10\ kHz$

→ For FM

$y_{FM} = \underline{E_c} \sin(\omega_c t + \underline{m_f} \sin \omega_m t)$

→ Modulating Index

$m_f = \frac{\delta f}{f_m} = \frac{10}{0.4} = 25$

→ FM signal

$y_{FM} = 5 \sin(2\pi \times 20 \times 10^6 t + 25 \sin 800\pi t)$

A carrier wave of amplitude 10V and frequency 100MHz is frequency modulated by a sinusoidal voltage. The modulating voltage has an amplitude of 5V and frequency f_m=20 kHz. The frequency deviation constant is 2 kHz/Volt. Draw the frequency spectrum of FM wave.

→ $E_c = 10\ V$

$f_m = 100\ MHz$

$E_m = 5\ V$

$f_m = 20\ kHz$

$K_f = 2\ kHz/Volt$

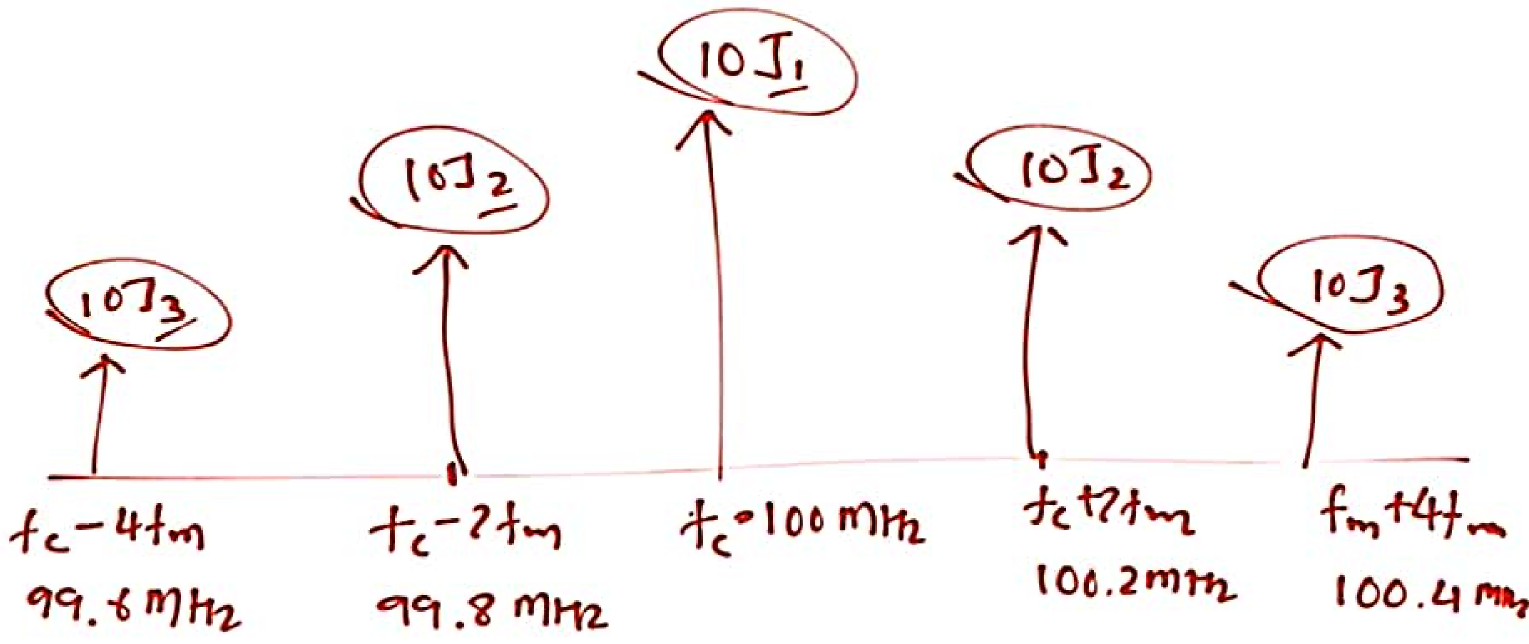
→ $\Delta f = K_f E_m$

$= (2\ kHz/Volt)\ 5$

$= 10\ kHz$

→ $m_f = \frac{\Delta f}{f_m} = \frac{10}{20} = 0.5$

→ From that calculate bessels parameter J_1, J_2 & J_3 by table.


10J3
10J2
10J1
10J2
10J3
fc-4fm
99.6 MHz
fc-2fm
99.8 MHz
fc=100 MHz
fc+2fm
100.2 MHz
fm+4fm
100.4

When the modulating frequency in an FM system is 400 Hz and the modulating voltage is 2.4 V, the modulation index is 60. Calculate the maximum deviation. What is the modulation index when modulating frequency is reduced to 250 Hz and the modulating voltage is simultaneously raised to 3.2 V.

→ $f_m = 400\ Hz$

$E_m = 2.4\ Volt$

$m_f = 60$

⇒ $m_f = \frac{\Delta f}{f_m}$

⇒ $\Delta f = m_f f_m$

$= 60 \times 400$

$= 24\ KHz$

⇒ $\Delta f = K_f E_m$

⇒ $K_f = \frac{\Delta f}{E_m}$

$= \frac{24\ KHz}{2.4\ Volt}$

$= 10\ \frac{KHz}{Volt}$

→ Now New data

$f_m = 250\ Hz$

$E_m = 3.2\ Volt$

→ $\Delta f = K_f E_m$

$= 10 \left(\frac{KHz}{Volt}\right) 3.2\ Volt$

$= 32\ KHz$

→ $m_f = \frac{\Delta f}{f_m}$

$= \frac{32\ KHz}{250\ Hz}$

$= 128$

In an FM system, the audio frequency is 1 kHz and audio voltage is 2 volts. The deviation is 4 kHz. If the AF voltage is now increased to 8 volts and its frequency dropped to 500 Hz, find the modulating index in each case and the corresponding bandwidth using Carson's rule.

$\rightarrow f_{m_1} = 1\ \text{KHz}$

$E_{m_1} = 2\ \text{volt}$

$\Delta f_1 = 4\ \text{KHz}$

$\Rightarrow \Delta f_1 = K_f E_{m_1}$

$\Rightarrow K_f = \Delta f_1 / E_{m_1}$

$= 4\ \text{KHz} / 2\ \text{volt}$

$= 2\ \text{KHz/volt}$

$\rightarrow m_{f_1} = \dfrac{\Delta f_1}{f_{m_1}} = \dfrac{4\ \text{KHz}}{1\ \text{KHz}} = 4$

$\rightarrow BW_1 = 2(\Delta f_1 + f_{m_1})$

$= 2(4+1)$

$= 10\ \text{KHz}$

$\rightarrow E_{m_2} = 8\ \text{volt}$

$f_{m_2} = 500\ \text{Hz}$

$\rightarrow \Delta f_2 = K_f E_{m2}$

$= \left(2\ \dfrac{\text{KHz}}{\text{Volt}}\right) 8$

$= 16\ \text{KHz}$

$\rightarrow m_{f2} = \dfrac{\Delta f_2}{f_{m2}} = \dfrac{16}{0.5} = 32$

$\rightarrow BW_2 = 2(\Delta f_2 + f_{m2})$

$= 2(16 + 0.5)$

[redacted] A message signal $m(t) = A_m \sin(2\pi f_m t)$ is used to modulate the phase of a carrier $A_c \cos(2\pi f_c t)$ to get the modulated signal $y(t) = A_c \cos(2\pi f_c t + m(t))$. The bandwidth of $y(t)$

(A) depends on A_m but not on f_m

(B) depends on f_m bust not on A_m

(C) depends on both A_m and f_m

(D) does not depend on A_m or f_m

$\rightarrow$ $BW = 2(f_m + \Delta f)$

$$= 2f_m\left(1 + \frac{\Delta f}{f_m}\right)$$

$$= 2f_m(1 + m_f)$$

$$= 2f_m(1 + K_f E_m)$$

[redacted] Consider an FM signal

$$f(t) = \cos[2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t]$$

The maximum deviation of instantaneous frequency from the carrier frequency f_c is

(A) $\beta_1 f_1 + \beta_2 f_2$ (B) $\beta_1 f_2 + \beta_2 f_1$

(C) $\beta_1 + \beta_2$ (D) $f_1 + f_2$

$\Rightarrow \phi = 2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t$

$\Rightarrow \frac{d\phi}{dt} = 2\pi f_c + \beta_1 (2\pi f_1) \cos(2\pi f_1 t) + \beta_2 (2\pi f_2) \cos(2\pi f_2) t$

$\Rightarrow \frac{d\phi}{dt} = 2\pi (\underbrace{f_c}_{= \text{Constant}} + \underbrace{\beta_1 f_1 \cos(2\pi f_1 t) + \beta_2 f_2 \cos(2\pi f_2 t)}_{\text{Max. } \Delta f = \beta_1 f_1 + \beta_2 f_2})$

[redacted] Consider an angle modulated signal

$$x(t) = 6\cos[2\pi \times 10^6 t + 2\sin(800\pi t)] + 4\cos(800\pi t)$$

The average power of $x(t)$ is

(A) 10 W (B) 18 W

(C) 20 W (D) 28 W

$R = 1$

→ Avg Power $= \dfrac{E_c^2}{2R}$

$= \dfrac{6^2}{2 \times R} = \dfrac{36}{2}$

$= \boxed{18\text{ W}}$

[redacted] The **Column -1** lists the attributes and the **Column -2** lists the modulation systems. Match the attribute to the modulation system that best meets it.

Column -1	**Column -2**
P. Power efficient transmission of signals	**1.** Conventional AM
Q. Most bandwidth efficient transmission of voice signals	**2.** FM
R. Simplest receiver structure	**3.** VSB
S. Bandwidth efficient transmission of signals with significant dc component	**4.** SSB-SC

(A) P-4, Q-2, R-1, S-3

(B) P-2, Q-4, R-1, S-3

(C) P-3, Q-2, R-1, S-4

(D) P-2, Q-4, R-3, S-1

P - 2

Q - 4

R - 1

S - 3

[redacted] Consider the frequency modulated signal $10\cos[2\pi\times10^5 t + 5\sin(2\pi\times1500t) + 7.5\sin(2\pi\times1000t)]$ with carrier frequency of 10^5 Hz. The modulation index is

(A) 12.5 (B) 10

(C) 7.5 (D) 5

$\rightarrow m_f = \dfrac{\Delta f}{f_m} = \dfrac{15000}{1500} = 10$

$\Rightarrow \phi = 2\pi\times10^5 t + 5\sin(2\pi\times1500t) + 7.5\sin(2\pi\times1000t)$

$\Rightarrow \dfrac{d\phi}{dt} = 2\pi\times10^5 + 5(2\pi\times1500)\cos(2\pi\times1500t) + 7.5(2\pi\times1000)\cos(2\pi\times1000t)$

$\Rightarrow \dfrac{d\phi}{dt} = 2\pi(10^5 + \underbrace{7500\cos(2\pi\times1500t) + 7500\cos(2\pi\times1000t)}_{\downarrow})$

$\text{Max. } \Delta f = 15000$

[redacted] The signal $\cos\omega_c t + 0.5\cos\omega_m t \sin\omega_c t$ is

(A) FM only (B) AM only

(C) both AM and FM (D) neither AM nor FM

$\rightarrow$ AM $= A_c(1 + m \sin\omega_m t)\sin\omega_c t$

$\rightarrow$ NBFM $= A_c(1 + K_f g(t))\sin\omega_c t$

[redacted] Find the correct match between group 1 and group 2.

	Group 1		Group 2
P.	$\{1 + km(t) A\sin(\omega_c t)\}$	W.	Phase modulation
Q.	$km(t) A\sin(\omega_c t)$	X.	Frequency modulation
R.	$A\sin\{\omega_c t + km(t)\}$	Y.	Amplitude modulation
S.	$A\sin\left[\omega_c t + k\int_{-\infty}^{t} m(t)\,dt\right]$	Z.	DSB-SC modulation

(A) P – Z, Q – Y, R – X, S – W
(B) P – W, Q – X, R – Y, S – Z
(C) P – X, Q – W, R – Z, S – Y
(D) P – Y, Q – Z, R – W, S – X

$P \to Y$ | $R \to W$
$Q \to Z$ | $S \to X$

[redacted] An angle-modulated signal is given by $\frac{150}{2\pi}$

$s(t) = \cos 2\pi(2 \times 10^6 t + 30 \sin 150t + 40 \cos(150t))$.

The maximum frequency and phase deviations of $s(t)$ are

(A) 10.5 kHz, 140π rad

(B) 6 kHz, 80π rad

(C) 10.5 kHz, 100π rad

(D) 7.5 kHz, 100π rad

$$\phi = 2\pi(2\times10^6 t + \underbrace{30 \sin 150t + 40 \cos 150t}_{\uparrow \Delta\phi})$$

$$\Rightarrow \Delta\phi = 2\pi(30 \sin 150t + 40 \cos 150t)$$

$$= 2\pi\sqrt{30^2+40^2}\ \boxed{\sin\left(150t + \tan^{-1}\left(\frac{40}{30}\right)\right)}$$

$$\Rightarrow \boxed{\Delta\phi = 100\pi}$$

$$\rightarrow \Delta f = \Delta\phi f_m$$

$$= 100\pi\left(\frac{150}{2\pi}\right)$$

$$= 7500 = 7.5\ \text{KHz}$$

Phase Lock Loop

- A PLL is a non - Linear feedback system that tracks the phase of Input signal & minimized phase error at Local oscillator.

Why PLL ?

- In coherent demodulation PLL is used for removing phase error.

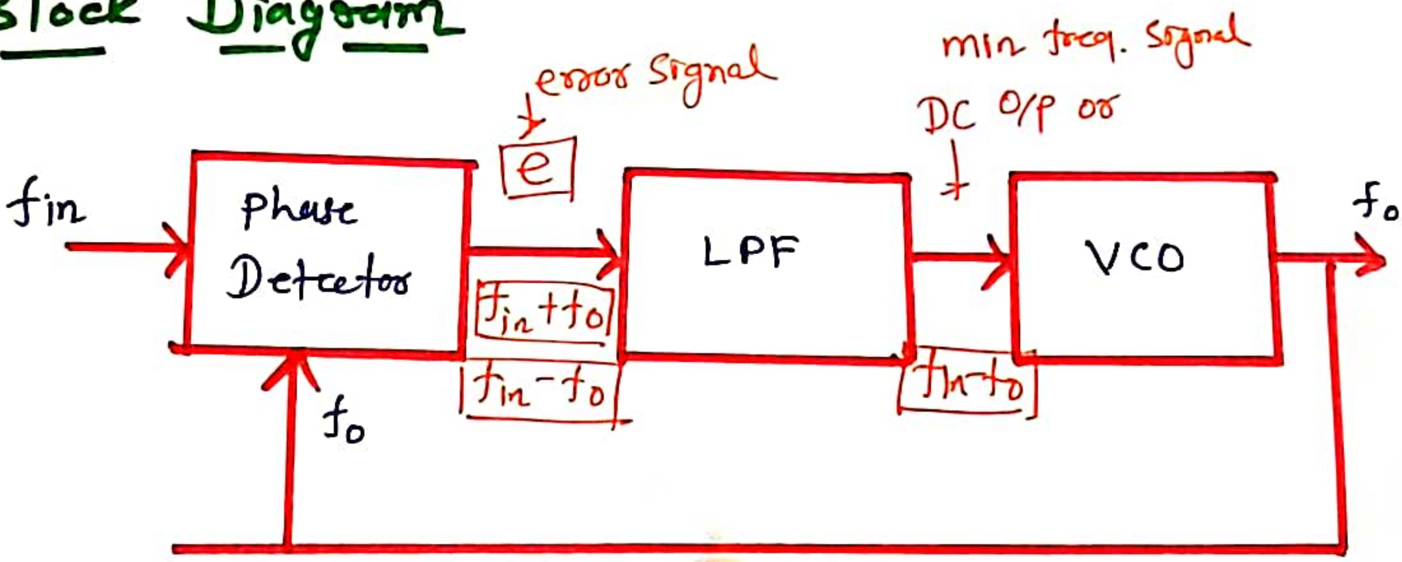
Applications

— FM coherent demodulation

— AM " "

$$c(t) = A_c \sin(\omega_c t + \underbrace{\phi}_{\text{phase error}})$$

Block Diagram



phase Detector.

- Compares f_{in} and f_o
- The O/p of the phase detector is proportional to phase difference between f_{in} and f_o
- O/p of phase detector is DC or min. freq. signal. So, It is reflected as the error voltage e.

LPF - It removes high freq. noise.
- It produce DC signal.
- we can use active low pass filter or Passive low pass filter.

VCO - It generates high freq. signal

- the Instantaneous VCO freq. is controlled by its input voltage. It is based on Vp voltage

$$f_o = f + K\omega_m$$

- The freq. of VCO is directly controlled by DC Vp voltage

Operation of PLL

- PLL Operation goes through 3 state
  1) Free running
  2) Capture
  3) phase Lock

Free Running

- If no Vp is applied, The PLL is in free running state.

Capture

- Once the Vp is applied, VCO freq. starts to change and PLL is said to be in the Capture mode.
- It even refers as "freq. Pull In".

Phase lock

- The lock range is defined as the range of freq. over which the changes the Input freq fin and Fo is zero, then it refer as phase lock. $\boxed{F_{in} - F_o = 0}$