Information Theory Basics, defination, Uncertainty & Property.

Information – It is the itelligence / Ideas / message in Information theory.

Message – Electrical signal
– Speech / voice
– Picture / Image
– Video
– text

- In communication system, Information is transmitted from source to destination from source to destination

Information source $\rightarrow$ Encoder $\rightarrow$ Tx $\rightarrow$ Channel $\rightarrow$ Rx $\rightarrow$ Decoder $\rightarrow$ Information

$\underbrace{\text{Encoder} \rightarrow \text{Tx}}_{\text{Source}}$ $\qquad$ $\underbrace{\text{Rx} \rightarrow \text{Decoder}}_{\text{destination}}$

Uncertainty

- $X = \{x_0, x_1, \dots, x_n\}$
- $P = \{P_0, P_1, \dots, P_n\}$
- total Probability $P = \sum_{i=1}^{n} P_i$

$$\left.\begin{array}{l} x_0 \rightarrow P_0 = 0 \\ \quad \rightarrow P_0 = 1 \end{array}\right\} \text{No Uncertainty}$$

$$x_0 \rightarrow P_0 = 0.5$$
$$\quad \rightarrow P_0 = 0.1$$

- As probability of message $\downarrow^{ce}$ then uncertainty $\uparrow^{ce}$

$\rightarrow P_0 = 0.1$

- As probability of message $\downarrow^{ce}$ then uncertainity $\uparrow^{ce}$

Measure of Information

- It is a Information content of a message.
- Consider an Information source emitting Independent messages $m = \{m_1, m_2, \ldots, m_n\}$ with probabilities of occurence is $P = \{P_1, P_2, \ldots, P_n\}$.
- Here, $P_1 + P_2 + \ldots + P_n = 1$.
- Amount of Information is given by,

$$I_K = \log_2\left(\frac{1}{P_K}\right) = \frac{\log(1/P_K)}{\log 2} \quad \boxed{\text{bits}}$$

Properties of Information

- More Uncertainty about Message then Information is more.

→ $P_1 = 1/4$, $P_2 = 1/2$

U_1 U_2

→ $U_1 > U_2$

→ $I_1 = \log_2\left(\frac{1}{P_1}\right)$

$= \log_2\left(\frac{1}{(1/4)}\right)$

$= \log_2 4$

$= 2\log_2 2$

$= 2$ bits

$I_2 = \log_2\left(\frac{1}{P_2}\right)$

$= \log_2\left(\frac{1}{(1/2)}\right)$

$= \log_2(2)$

$= 1$ bit

$I_1 > I_2$

- Receiver knows message being transmitted then information is zero.
- $P = 1$
- $I = \log_2\left(\frac{1}{P}\right)$

$= \log_2 1$

$= 0$ bit

$\log xy = \log x + \log y$

- I_1 is the Information of message m_1 and I_2 of m_2, then Combined Information of m_1 & m_2 $= I_1 + I_2$

→ $I_1 = \log_2\left(\frac{1}{P_1}\right)$

→ $I_2 = \log_2\left(\frac{1}{P_2}\right)$

→ Since messages m_1 & m_2 are Independent. So combined probability $= P_1 P_2$

→ $I = \log_2\left(\frac{1}{P}\right) = \log_2\left(\frac{1}{P_1 P_2}\right) = \log_2\left(\frac{1}{P_1}\right) + \log_2\left(\frac{1}{P_2}\right)$

⇒ $\boxed{I = I_1 + I_2}$

- If there are $m = 2^N$ equally likely messages, then amount of Information carried by each message will be $= N$ bits.

- probability of each message $= \frac{1}{m}$

- $I = \log_2 \left(\frac{1}{P}\right)$

$= \log_2 (m)$

$\rightarrow m = 2^N$

$\Rightarrow I = \log_2 2^N$

$= N \log_2 2$

$$\boxed{I = N \text{ bits}}$$

Examples on Information in Information Theory

① Calculate amount of Information for

a) $P_1 = 1/4$, b) $P_2 = 3/4$

→ $P_1 = 1/4$

$$I_1 = \log_2\left(\frac{1}{P_1}\right) = \log_2(4) = 2\log_2 2 = 2 \text{ bits}$$

→ $P_2 = 3/4$

$$I_2 = \log_2\left(\frac{1}{P_2}\right) = \log_2\left(\frac{4}{3}\right) = \frac{\log(4/3)}{\log 2} = 0.415 \text{ bits.}$$

2) A Card is Selected at random from a deck of playing Cards. If you have been told that It is red In Colour.

1) How much Information you have received.

2) How much more Information you needed to completely Specify the Card.

- Total Cards = 52

Total Red Cards = 26

$$P = \frac{26}{52} = \frac{1}{2}$$

- $I = \log_2\left(\frac{1}{P}\right)$

$= \log_2(2)$

$= 1$ bit

cards. If you have been told that it is red in colour.

1) How much information you have received.

2) How much more information you needed to completely specify the card.

- Total cards = 52

Total Red Cards = 26

$$p = \frac{26}{52} = \frac{1}{2}$$

- $$I = \log_2\left(\frac{1}{p}\right)$$

$$= \log_2(2)$$

$$= 1 \text{ bit}$$

→ Probability $p = \frac{1}{26}$

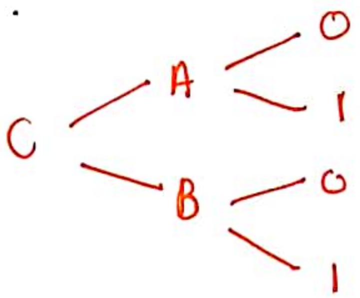
→ $$I = \log_2\left(\frac{1}{p}\right)$$

$$= \log_2(26)$$

$$= \frac{\log 26}{\log 2}$$

$$= 4.7 \text{ bits.}$$

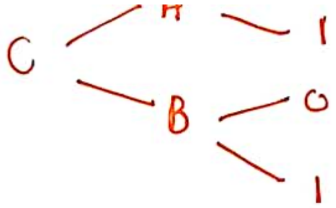
③ Consider discrete memoryless source 'C' that o/p two bits at a time. This source comprises two binary sources 'A' and 'B'. whose o/p are equally likely to occur and each source contributing one bit. Suppose that the sources within the source 'C' are Independent. What is the Information content of each o/p from source 'C'.



→ total samples of C

$C = \{00, 01, 10, 11\}$

→ $P_c = \frac{1}{4}$



→ Total samples of C

$C = \{00, 01, 10, 11\}$

→ $P_C = \frac{1}{4}$

→ $P_A = 1/2$

$P_B = 1/2$

→ Combined Probability of A & B

$$P_C = P_A P_B$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

→ Information of C

$$I_C = \log_2\left(\frac{1}{P_C}\right)$$

→ Information of c

$$I_c = \log_2\left(\frac{1}{P_c}\right)$$

$$= \log_2(4)$$

$$= 2\log_2 2$$

$$\boxed{I_c = 2 \text{ bits}}$$

Entropy basics, defination & property

Defination - It is Average Information of symbols.

- If we have $M = \{x_1, x_2, \ldots, x_n\}$ messages with probability

$$P = \{P_1, P_2, \ldots, P_n\}.$$

- then Information of each messages

$$I_1 = \log_2\left(\frac{1}{P_1}\right), \quad I_2 = \log_2\left(\frac{1}{P_2}\right), \ldots, \quad I_n = \log_2\left(\frac{1}{P_n}\right)$$

- So Entropy $H = \dfrac{\text{Total Information}}{\text{No of messages}}$

$$= \frac{I_1 + I_2 + \ldots + I_n}{n}$$

$$= \frac{I_1 + I_2 + \dots + I_n}{n}$$

$$\boxed{H = \sum_{i=1}^{n} p_i \log_2\left(\frac{1}{p_i}\right)}$$ bits/symbol

Properties

1) Entropy is zero, if the event is sure.

– $P = 0$, $P = 1$

$$H = \sum_{k=1}^{m} P_k \log_2\left(\frac{1}{P_k}\right) \qquad H = \sum_{k=1}^{m} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$\boxed{H = 0} \qquad = 1 \log_2\left(\frac{1}{1}\right)$$

$$\boxed{H = 0}$$

$$H = \sum_{k=1}^{m} p_k \log_2\left(\frac{1}{p_k}\right)$$

$$\boxed{H = 0}$$

$$H = \sum_{k=1}^{m} p_k \log_2\left(\frac{1}{p_k}\right)$$

$$= 1 \log_2\left(\frac{1}{1}\right)$$

$$\boxed{H = 0}$$

2] when $p_k = 1/m$ for all 'm' symbols, then symbols are equally. so $H = \log_2 m$

→ $p_k = 1/m$

→ $H = \sum_{k=1}^{m} p_k \log_2\left(\frac{1}{p_k}\right)$

$$= \boxed{\sum_{k=1}^{m} \left(\frac{1}{m}\right)} \log_2 m$$

→ $\boxed{H = \log_2 m} = \underline{H_{max}}$

upper bound of Entropy

Entropy Source Efficiency, Redundancy & Information rate.

- Source Efficiency

$$\eta = \frac{H}{H_{max}} \qquad \Big| \qquad H_{max} = \log_2 M$$

where H = Calculated Entropy of Source

H_{max} = max. Entropy

- Redundancy of source

$$R_e = 1 - \eta$$

- Information Rate

where H = Calculated Entropy of Source

H_{max} = max. Entropy

- Redundancy of Source

$$R_e = 1 - \eta$$

- Information Rate

$$R = rH = \left[\frac{\text{Messages}}{\text{sec}}\right]\left[\frac{\text{bits}}{\text{Message}}\right] = \frac{\text{bits}}{\text{sec}}.$$

where, r = rate at which messages are generated [messages/sec]

H = Entropy $\left[\frac{\text{bits}}{\text{Messages}}\right]$

1) For a discrete memoryless source there are three symbols with $P_1 = \alpha$ and $P_2 = P_3$. Find the entropy of the source

$\rightarrow$ $P_1 = \alpha$

$P_2 = P_3$

$\therefore$ $P_1 + P_2 + P_3 = 1$

$\Rightarrow \alpha + P_2 + P_2 = 1$

$\Rightarrow \alpha + 2P_2 = 1$

$\Rightarrow P_2 = P_3 = \dfrac{1-\alpha}{2}$

$$\rightarrow H = \sum_{K=1}^{3} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$H = \alpha \log_2\left(\frac{1}{\alpha}\right) + 2\left(\frac{1-\alpha}{2}\right) \log_2\left(\frac{2}{1-\alpha}\right)$$

$$= \alpha \log_2\left(\frac{1}{\alpha}\right) + (1-\alpha) \log_2\left(\frac{2}{1-\alpha}\right)$$

2] Show that the Entropy of the source with following Probability distribution is $\left[2 - \frac{2+n}{2^n}\right] = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{n}{2^n}$ ✓

S	S_1	S_2	S_3	....	S_n
P	$1/2$	$1/4$	$1/8$	....	$1/2^n$

→ $H = \sum_{k=1}^{n} P_k \log_2\left(\frac{1}{P_k}\right)$

$= P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + P_3 \log_2\left(\frac{1}{P_3}\right) + \ldots + P_n \log_2\left(\frac{1}{P_n}\right)$

P	$1/2$	$1/4$	$1/8$	$\dots$	$1/2^n$

$$\rightarrow H = \sum_{k=1}^{n} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + P_3 \log_2\left(\frac{1}{P_3}\right) + \dots + P_n \log_2\left(\frac{1}{P_n}\right)$$

$$= \left(\frac{1}{2}\right)\log_2(2) + \left(\frac{1}{4}\right)\log_2(4) + \left(\frac{1}{8}\right)\log_2(8) + \dots + \left(\frac{1}{2^n}\right)\log_2(2^n)$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \dots + \frac{n}{2^n}$$

$$= 2 - \frac{2+n}{2^n}$$

3) The source emits three messages with probabilities $P_1 = 0.7$, $P_2 = 0.2$ and $P_3 = 0.1$

Calculate,

1) Source Entropy
2) Maximum Entropy
3) Source Efficiency
4) Redundancy

- $H = \sum_{K=1}^{3} P_K \log_2 \left(\frac{1}{P_K}\right)$

$= 0.7 \log_2 \left(\frac{1}{0.7}\right) + 0.2 \log_2 \left(\frac{1}{0.2}\right) + 0.1 \log_2 \left(\frac{1}{0.1}\right)$

$= 1.1568$ bits / messages

- $H_{max} = \log_2 m$

$= \log_2 3 = \frac{\log 3}{\log 2} = 1.585 \frac{\text{bits}}{\text{message}}$

Calculate,

1) Source Entropy

2) Maximum Entropy

3) Source Efficiency

4) Redundancy

$$- \quad H = \sum_{K=1}^{3} P_K \log_2 \left(\frac{1}{P_K}\right)$$

$$= 0.7 \log_2 \left(\frac{1}{0.7}\right) + 0.2 \log_2 \left(\frac{1}{0.2}\right) + 0.1 \log_2 \left(\frac{1}{0.1}\right)$$

$$= 1.1568 \text{ bits/messages}$$

$$- \quad H_{max} = \log_2 m$$

$$= \log_2 3 = \frac{\log 3}{\log 2} = 1.585 \ \frac{\text{bits}}{\text{message}}$$

$$- \quad \eta = \frac{H}{H_{max}} = \frac{1.1568}{1.585} = 0.73$$

$$- \quad R_e = 1 - \eta = 1 - 0.73 = 0.27$$

4) A discrete source emits one of six symbols once every msec. The symbol probability are $1/2$, $1/4$, $1/8$, $1/16$, $1/32$ & $1/32$. Find the source Entropy and Information rate.

$$R = rH$$

- $r = \frac{1}{T} = \frac{1}{10^{-3}} = 10^{3}$ messages/sec

- $H = \sum_{k=1}^{6} p_k \log_2\left(\frac{1}{p_k}\right)$

$$= \frac{1}{2}\log_2(2) + \frac{1}{4}\log_2(4) + \frac{1}{8}\log_2(8) + \frac{1}{16}\log_2(16) + 2\times\frac{1}{32}\times\log_2 32$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + 2\times\frac{1}{32}\times 5$$

$$= 1.9375 \text{ bits/sec}$$

- $r = \frac{1}{T} = \frac{1}{10^{-3}} = 10^{3}$ messages/sec $R = rH$

- $H = \sum_{K=1}^{6} P_K \log_2\left(\frac{1}{P_K}\right)$

$= \frac{1}{2}\log_2(2) + \frac{1}{4}\log_2(4) + \frac{1}{8}\log_2(8) + \frac{1}{16}\log_2(16) + 2\times\frac{1}{32}\times\log_2 32$

$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + 2\times\frac{1}{32}\times 5$

$= 1.9375$ bits/sec

- $R = rH$

$= 10^{3}\times 1.9375$

$= 1937.5$ bits/sec

Shannon Fano Encoding Algorithm

1. The messages are first written in the order of decreasing probability.
2. Then divide the messages set into two most equiprobable subset X and Y.
3. The message of 1st set X is given bit 0 and message in the 2nd subset is given bit 1.
4. The procedure is now applied for each set seprately till end.
5. Finally we get the code word for respective symbol.
6. Calculation

probability.

2. Then divide the messages set into two most equiprobable subset X and Y.
3. The message of 1st set X is given bit 0 and message in the 2nd subset is given bit 1.
4. The procedure is now applied for each set seprately till end.
5. Finally we get the code word for respective symbol.
6. Calculation

→ efficiency $(\eta) = \frac{H}{\hat{H}}$

where, H = Entropy $= \sum_{i=1}^{n} p_i \log_2\left(\frac{1}{p_i}\right)$

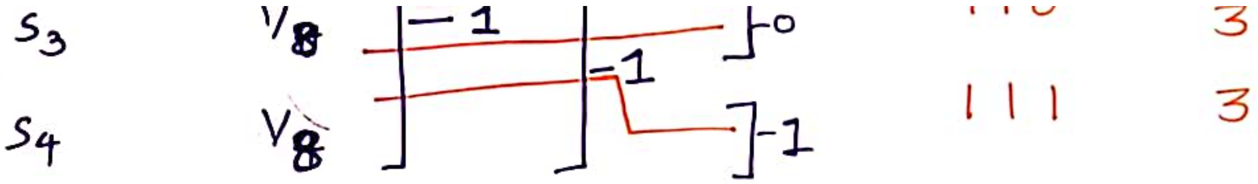
$\hat{H} = \sum_{i=1}^{n} p_i n_i$

→ Redundancy

$R_e = 1 - \eta$

Example – Find the code words occuring in the probability $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\}$ for symbols s_1, s_2, s_3 & s_4. Find efficiency and redendancy of code.

Symbol	Prob	Codeword	Length
s_1	$1/2$	0	1
s_2	$1/4$	10	2
s_3	$1/8$	110	3
s_4	$1/8$	111	3



- Efficiency $\eta = \dfrac{H}{\hat{H}}$

where H = entropy $= \sum_{i=1}^{4} p_i \log_2\left(\frac{1}{p_i}\right)$

$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \left(\frac{1}{8}\log_2 8\right) \times 2$

$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4}$

$= 1.75$ bits / symbol

$$\left(\frac{1}{8}\log_2 8\right)\times 2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4}$$

$$= 1.75 \text{ bits / symbol}$$

$$\hat{H} = \sum_{i=1}^{4} P_i n_i$$

$$= \frac{1}{2}\times 1 + 1/4 \times 2 + \frac{1}{8}\times 3 \times 2$$

$$= 1.75 \text{ bits / symbol}$$

- Redendancy $R_e = 1 - \eta$

$$= 1 - 1 = 0$$

Example Find the code word for the probability $\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}$ for symbols $s_1, s_2, \dots s_8$. Find the code efficiency and redundancy.

Symbol	Prob.
s_1	

$\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}$ for symbols $S_1, S_2, \ldots S_8$.

Find the code efficiency and redundancy.

Symbol	Prob.	Codeword	length
S_1	1/4	00	2
S_2	1/4	01	2
S_3	1/8	100	3
S_4	1/8	101	3
S_5	1/16	1100	4
S_6	1/16	1101	4
S_7	1/16	1110	4
S_8	1/16	1110	4

$$H = \sum p_i \log_2 \left(\frac{1}{p_i}\right)$$

$$= 2\left(\frac{1}{4}\log_2(4)\right) + 2\left(\frac{1}{8}\log_2(8)\right) + 4\left(\frac{1}{16}\log_2 16\right)$$

$$= 1 + \frac{3}{4} + 1$$

$$= 2.75 \text{ bits / symbol}$$

$$\hat{H} = \sum p_i n_i$$

$$= 2\left(\frac{1}{4}\times 2\right) + 2\left(\frac{1}{8}\times 3\right) + 4\left(\frac{1}{16}\times 4\right)$$

$$= 2.75 \text{ bits / symbol}$$

$$= 2\left(\frac{1}{4}\log_2(4)\right) + 2\left(\frac{1}{8}\log_2(8)\right) + 4\left(\frac{1}{16}\log_2 16\right)$$

$$= 1 + \frac{3}{4} + 1$$

$$= 2.75 \text{ bits/symbol}$$

- $\hat{H} = \sum p_i n_i$

$$= 2\left(\frac{1}{4}\times 2\right) + 2\left(\frac{1}{8}\times 3\right) + 4\left(\frac{1}{16}\times 4\right)$$

$$= 2.75 \text{ bits/symbol}$$

- efficiency $\eta = \frac{H}{\hat{H}} = \frac{2.75}{2.75} = 1$

- Redundancy $R_c = 1 - \eta = 1 - 1 = 0$

Example - Apply the sannon - Fano coding procedure for the message $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]$

Probability $P = [0.4, 0.2, 0.12, 0.08, 0.08, 0.08, 0.04]$

message	Prob
x_1	0.4
x_2	0.2
x_3	0.12
x_4	0.08
x_5	0.08
x_6	0.08
x_7	0.04

message	Prob	Codeword	length
x_1	0.4	0	1
x_2	0.2	100	3
x_3	0.12	101	3
x_4	0.08	1100	4
x_5	0.08	1101	4
x_6	0.08	1110	4
x_7	0.04	1111	4

- $\{x_1\}$ $\{x_2, x_3, \dots x_7\}$

  0.4   0.6

- $\{x_1, x_2\}$ $\{x_3, x_4, \dots x_7\}$

  0.6   0.4

message	Prob	codeword	length
x_1	0.4	0	1
x_2	0.2	100	3
x_3	0.12	101	3
x_4	0.08	1100	4
x_5	0.08	1101	4
x_6	0.08	1110	4
x_7	0.04	1111	4

- $\{x_1\}$ $\{x_2, x_3, \dots x_7\}$

  0.4   0.6

- $\{x_1, x_2\}$ $\{x_3, x_4, \dots x_7\}$

  0.6   0.4

$$\eta = \frac{H}{\hat{H}}$$

$$\hat{H} = \sum p_i n_i$$

$$= 0.4\times1 + 0.2\times3 + 0.12\times3 + 3\times0.08\times4 + 0.04\times4$$

$$= 2.48 \text{ bits/symbol}$$

message	Prob	codeword	length
x_1	0.4	00	2
x_2	0.2	01	2
x_3	0.12	100	3
x_4	0.08	101	3
x_5	0.08	110	3
x_6	0.08	1110	4
x_7	0.04	1111	4

$$- H = \sum p_i \log_2\left(\frac{1}{p_i}\right)$$

$$= 0.4 \log_2\left(\frac{1}{0.4}\right) + 0.2 \log_2\left(\frac{1}{0.2}\right) + 0.12 \log_2\left(\frac{1}{0.12}\right) + 3 \times 0.08 \log_2\left(\frac{1}{0.08}\right) + 0.04 \log_2\left(\frac{1}{0.04}\right)$$

$$\hat{H} = \sum p_i n_i$$

$$= 0.4\times2 + 0.2\times2 + 0.12\times3 + 0.08\times3\times2 + 0.08\times4 + 0.04\times$$

$$= 2.52 \text{ bits/symbol} \quad X$$

message	Prob	codeword	length
x_1	0.4	00	2
x_2	0.2	01	2
x_3	0.12	100	3
x_4	0.08	101	3
x_5	0.08	110	3
x_6	0.08	1110	4
x_7	0.04	1111	4

$$- H = \sum p_i \log_2\left(\frac{1}{p_i}\right)$$

$$= 0.4 \log_2\left(\frac{1}{0.4}\right) + 0.2 \log_2\left(\frac{1}{0.2}\right) +$$

$$0.12 \log_2\left(\frac{1}{0.12}\right) + 3 \times 0.08 \log_2\left(\frac{1}{0.08}\right) +$$

$$0.04 \log_2\left(\frac{1}{0.04}\right) = 2.42 \text{ bits/Symbol}$$

$$\hat{H} = \sum p_i n_i$$

$$= 0.4 \times 2 + 0.2 \times 2 + 0.12 \times 3 +$$

$$0.08 \times 3 \times 2 + 0.08 \times 4 + 0.04 \times$$

$$= 2.52 \text{ bits/symbol} \quad X$$

x_4	0.08	1100	4
x_5	0.08	1101	4
x_6	0.08	1110	4
x_7	0.04	1111	4

- $\{x_1\}\ \{x_2, x_3, \dots x_7\}$
  0.4   0.6
- $\{x_1, x_2\}\ \{x_3, x_4, \dots x_7\}$
  0.6   0.4

$$\boxed{\eta = \frac{H}{\hat{H}}}$$

$$= \frac{2.42}{2.48}$$

$$= \boxed{97.6\%}$$

$$\hat{H} = \sum p_i n_i$$

$$= 0.4 \times 1 + 0.2 \times 3 + 0.12 \times 3 + 3 \times 0.08 \times 4 + 0.04 \times 4$$

$$= 2.48 \text{ bits/symbol}$$

message	Prob	Codeword	length
x_1	0.4	00	2
x_2	0.2	01	2
x_3	0.12	100	3

Huffman Coding

1. The source symbols are arranged in order of decreasing prob. Then the two of lowest probability are assigned bit 0 and 1.
2. Then combine last two symbols and move the combined Symbol as high as possible.
3. Repeat the above step untill end.
4. Code for each symbol is found by moving backward.
5. Calculation

efficiency $\eta = \dfrac{H}{L \log_2 r}$

L = Avg Codeword $= \sum_{i=1}^{n} p_i n_i$

Then the two of lowest probability are assigned bit 0 and 1.

2. Then combine last two symbols and move the combined Symbol as high as possible.
3. Repeat the above step untill end.
4. Code for each symbol is found by moving backward.
5. Calculation

efficiency $\eta = \dfrac{H}{L \log_2 r}$

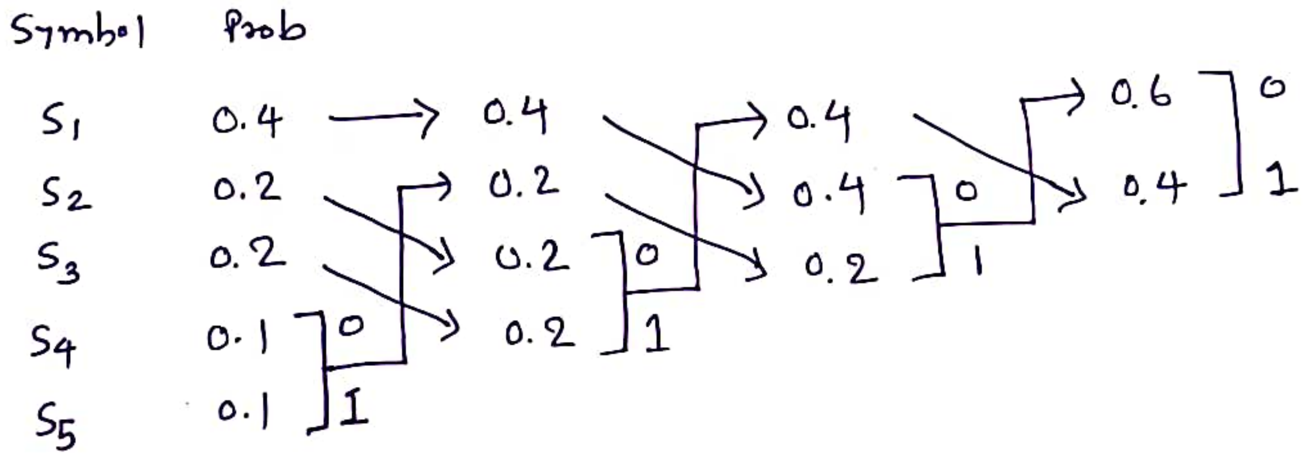
binary $r = 2$ $(0, 1)$
ternary $r = 3$ $(0, 1, 2)$
Quarternary $r = 4$ $(0, 1, 2, 3)$

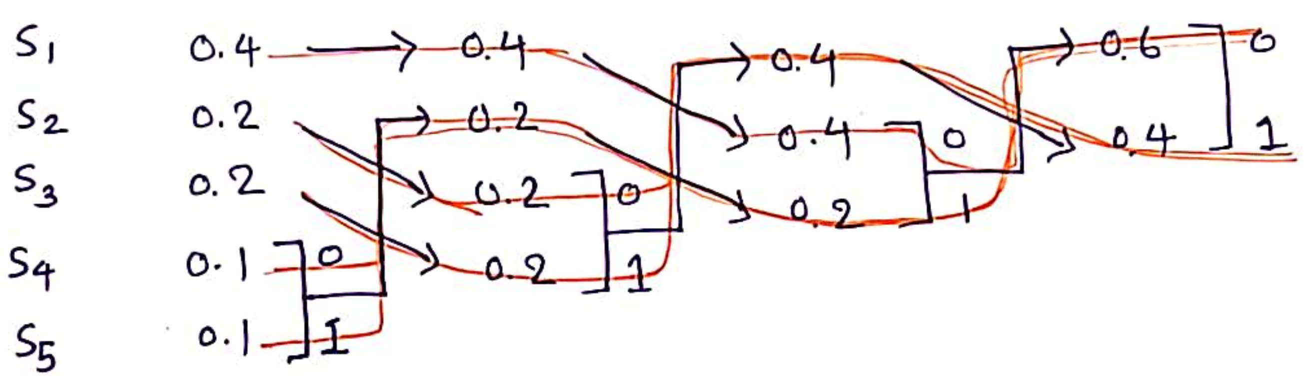
L = Avg Codeword $= \sum_{i=1}^{n} P_i n_i$

$H = \sum_{i=1}^{n} P_i \log_2 \left(\frac{1}{P_i}\right)$

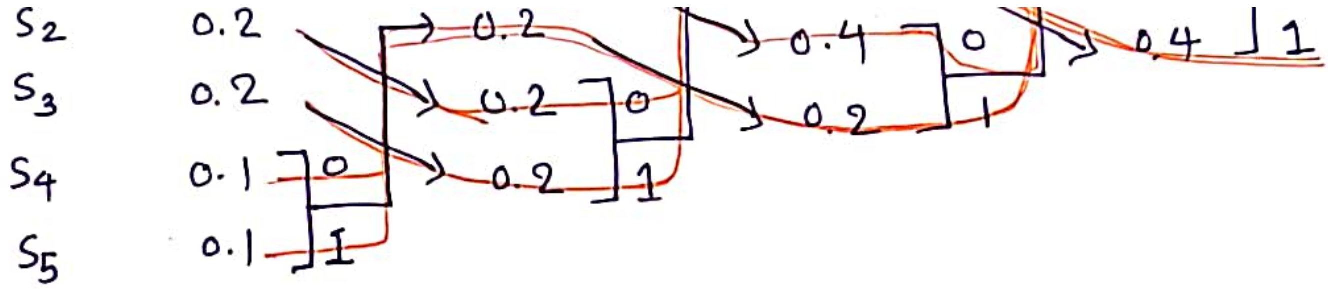
Variance $\sigma^2 = \sum_{i=1}^{n} P_i (n_i - L)^2$

<u>Example</u> : Alphabet with prob = {0.4, 0.2, 0.2, 0.1, 0.1} For Symbols $\{s_1, s_2, \ldots, s_5\}$. Find Huffman codes and also Find efficiency & variance





Symbol	Codeword	length
s_1	00	2
s_2	10	2
s_3	11	2
s_4	010	3
s_5	011	3



Symbol	Codeword	length
S_1	00	2
S_2	10	2
S_3	11	2
S_4	010	3
S_5	011	3

- Entropy

$$H = \sum p_i \log_2\left(\frac{1}{p_i}\right)$$

$$= 0.4 \log_2\left(\frac{1}{0.4}\right) + 2 \times 0.2 \log_2\left(\frac{1}{0.2}\right) + 2 \times 0.1 \log_2\left(\frac{1}{0.1}\right)$$

$$= 2.1216 \text{ bits / symbol}$$

S_5	011	3

$2 \times 0.2 \log_2\left(\frac{1}{0.2}\right) +$

$2 \times 0.1 \log_2\left(\frac{1}{0.1}\right)$

$= 2.1216$ bits / symbol

- $L = \sum p_i n_i$

  $= 2 \times 0.4 + 2(2 \times 0.2) + 3 \times 0.1 \times 2$

  $= 2.2$ bits / symbol

- $\eta = \dfrac{H}{L \log_2 r} = \dfrac{2.1216}{2.2 \times \log_2 2} = 96.4\,\%$.

- $\sigma^2 = \sum p_i (n_i - L)^2$

  $= 0.4\,(2-2.2)^2 + 2 \times 0.2\,(2-2.2)^2 + 2 \times 0.1\,(3-2.2)^2$

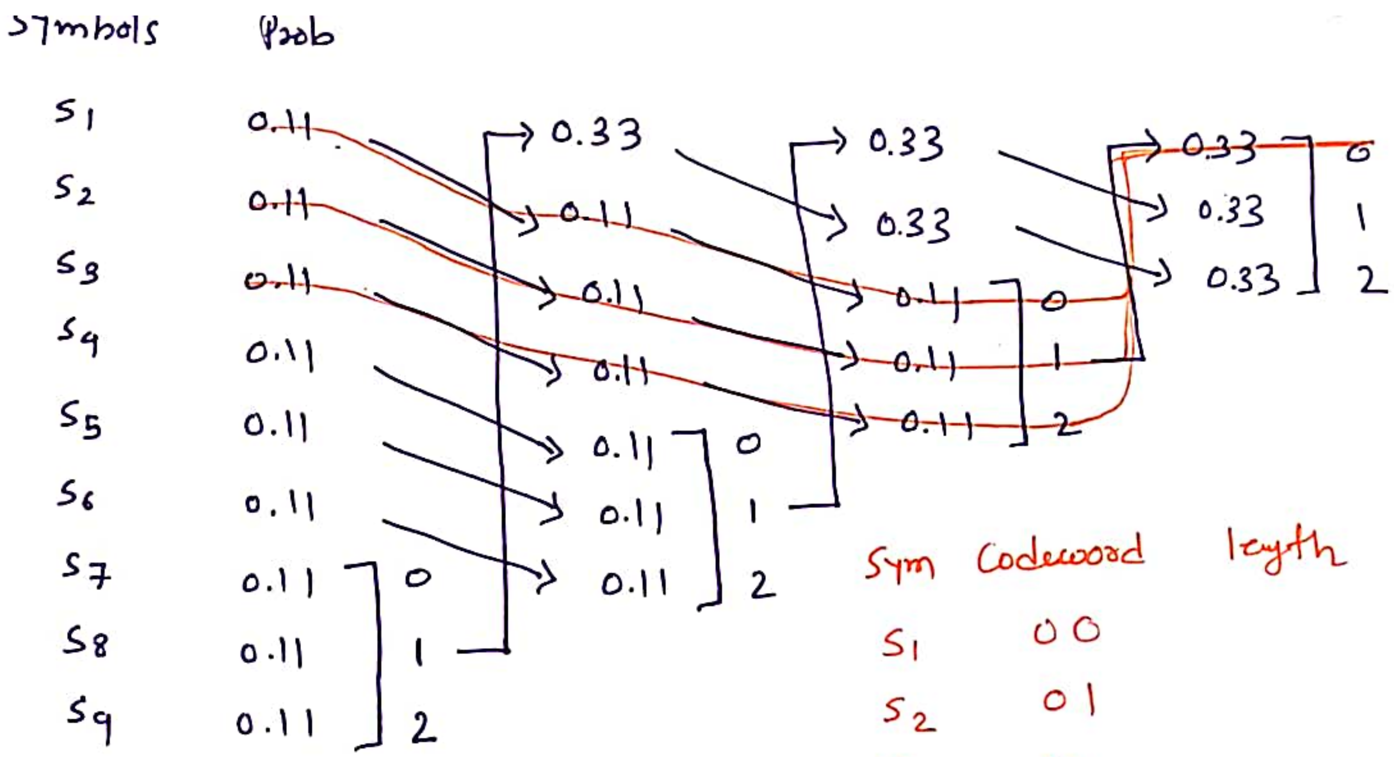
  $= 0.16$

  ↑ As low as possible.

Huffman coding procedure and example [ternary code].

- A source produces 9 symbols ($s_1, s_2, \ldots, s_9$). Construct bi ternary & Quaternary Huffman coding by moving symbols as high as possible. Also find efficiency & variance of the coding. Probability of symbols is given by

$$P_i = \{0.11, 0.11, 0.11, 0.11, 0.11, 0.11, 0.11, 0.11, 0.11\}$$



Sym	Codeword	length
S_1	00	
S_2	01	
S_3	0	
S_4		

s_7 0.11 0 → 0.11 2

s_8 0.11 1

s_9 0.11 2

Sym	Codeword	length
s_1	00	2
s_2	01	2
s_3	02	2
s_4	10	2
s_5	11	2
s_6	12	2
s_7	20	2
s_8	21	2
s_9	22	2

$$H = \sum p_i \log_2\left(\frac{1}{p_i}\right)$$

$$= 9\left(0.11 \log_2\left(\frac{1}{0.11}\right)\right)$$

$$= 3.1525 \text{ bits / symbol}$$

$$L = \sum p_i n_i$$

$$= 9(0.11 \times 2)$$

$$= 1.98 \text{ bits / symbol}$$

$$H = \sum P_i \log_2\left(\frac{1}{P_i}\right)$$

$$= 9\left(0.11 \log_2\left(\frac{1}{0.11}\right)\right)$$

$$= 3.1525 \text{ bits/symbol}$$

$$L = \sum P_i n_i$$

$$= 9(0.11 \times 2)$$

$$= 1.98 \text{ bits/symbol}$$

$$\eta = \frac{H}{L \lg_2 r}$$

$$= \frac{3.1525}{1.98 \times \log_2 3} = \frac{3.1525 \times \lg 2}{1.98 \times \lg 3} = 100\%$$

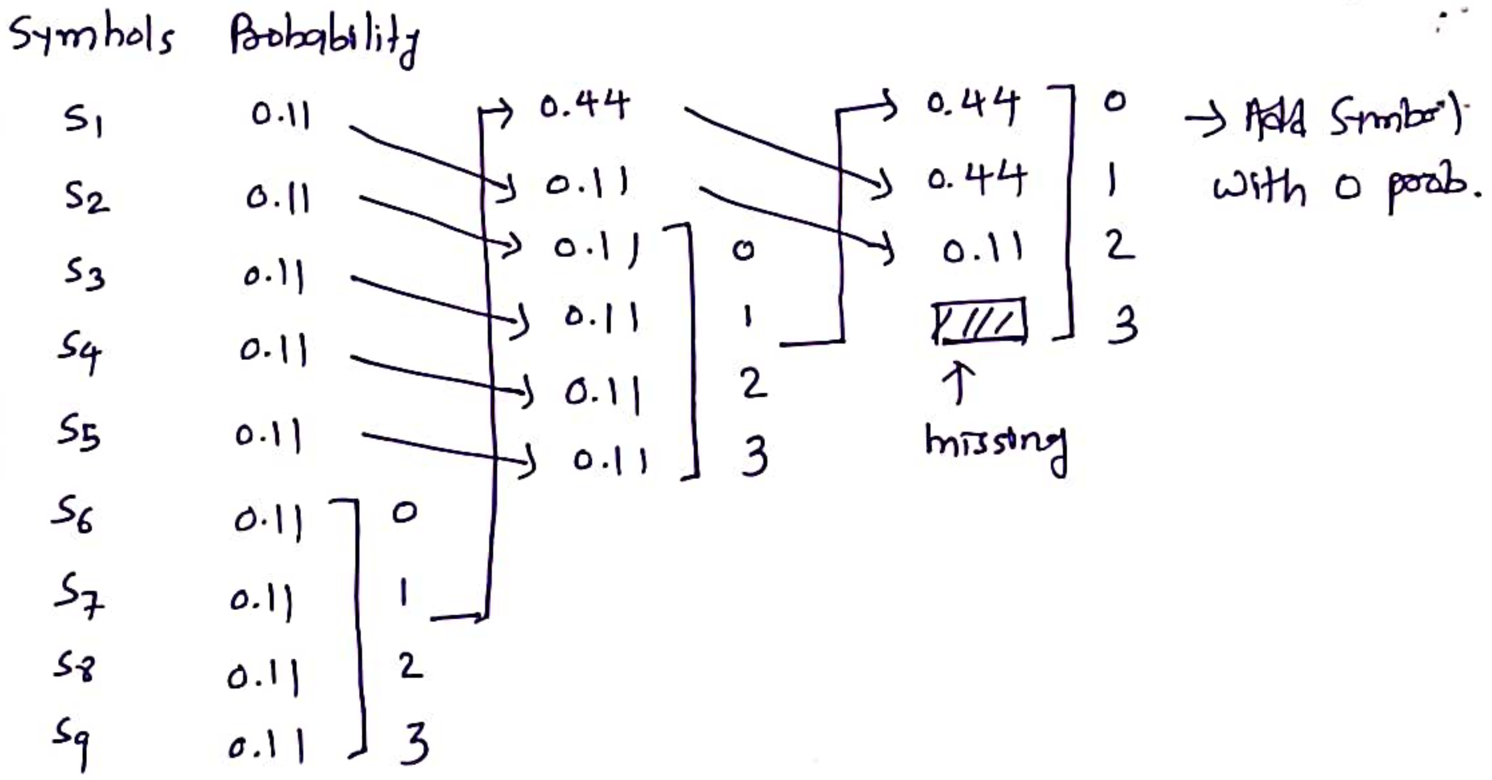
S_3	02	2
S_4	10	2
S_5	11	2
S_6	12	2
S_7	20	2
S_8	21	2
S_9	22	2

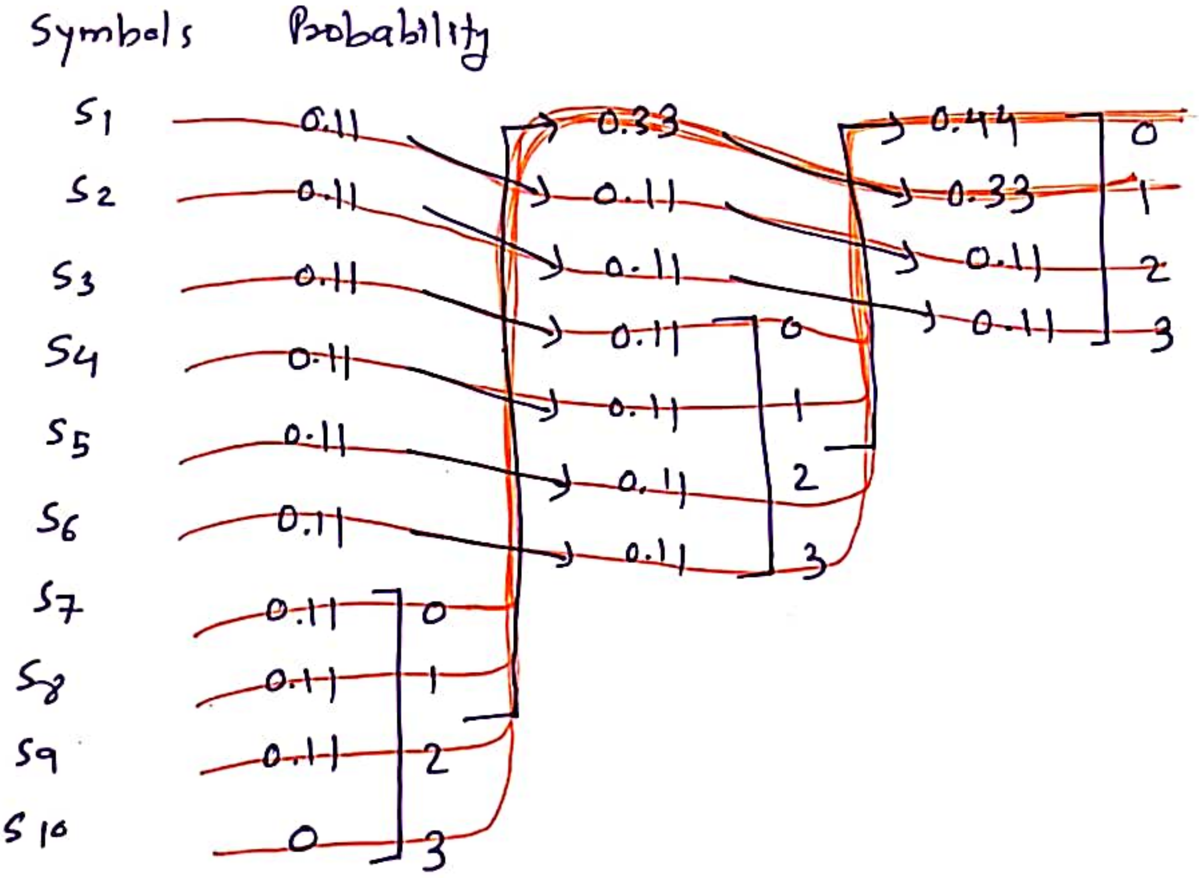
$$\sigma^2 = \sum P_i (n_i - L)^2$$

$$= 9 \times 0.11 \times (1.98 - 2)^2$$

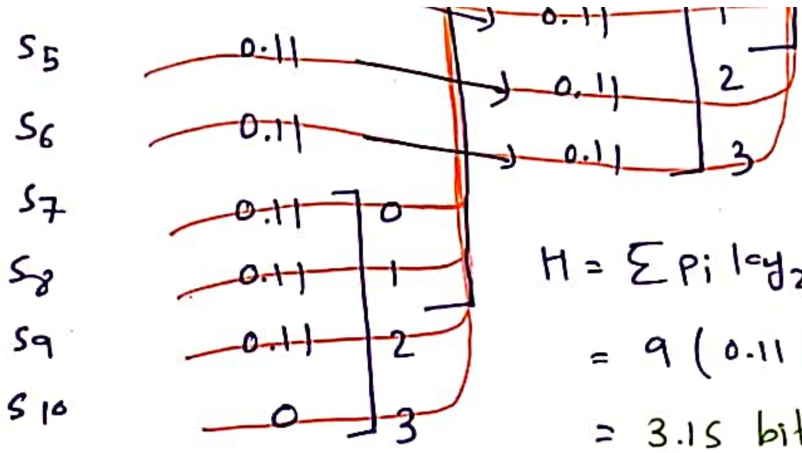
$$=$$

Huffman Coding procedure and example [Quatornary Code]





$\boxed{C}$	$\boxed{n_i}$
2	1
3	1
0 0	2
0 1	2
0 2	2
0 3	2
1 0	2
1 1	2
1 2	2
1 3	2



$$H = \sum p_i \log_2\left(\frac{1}{p_i}\right)$$
$$= 9\left(0.11 \log_2\left(\frac{1}{0.11}\right)\right)$$
$$= 3.15 \text{ bits / symbol.}$$

0 2	2
0 3	2
1 0	2
1 1	2
1 2	2
1 3	2

$\rightarrow L = \sum p_i n_i$
$$= 2(0.11 \times 1) + 0.11 \times 2 \times 7$$
$$= 1.76 \text{ bits / symbol}$$

$\rightarrow \eta = \dfrac{H}{L \log_2 r} = \dfrac{3.15}{1.76 \log_2 4} = 0.8948$

$\sigma^2 = \sum p_i (n_i - L)^2$

$=$ ______

Sources of noise

1. Natural noise sources
2. Man made noise sources
3. Fundamental sources (internal sources)

Natural source of noise

the natural phenomena that give rise to noise are Electronic storms, solar flares and radiation in space

the noise originating from the Sun and the outer space , Is known as extraterrestrial noise.

the extra Terrestrial noise is subdivided into two groups

1. Solar noise
2. Cosmic noise

our sun radiates lots of noise.

the temperature changes follow cycle 11 years

cosmic noise comes from stars

it is identical to noise by sun

it is uniformly distributed over entire Sky

we also receives noise from centre of our Galaxy and From other galaxies such as Quasars and Pulsars

Man made noise

it is generated because of make and break process in a current carrying circuit.

Examples

- × electrical Motors,
- × welding machines,
- × ignition system of automobiles,
- × fluorescent lights,
- × switching gears

Fundamental sources

it is within the electronic equipment
they are called fundamental sources because they are integral part of material and electrical component.
this type of noise follows certain rules.
hence it can be eliminated by designing properties

Classifications of noise

× thermal noise
× partition noise
× shot noise
× flicker noise or low frequency noise
× transient noise or high frequency noise

Shot Noise

it is produced due to shot effect
it is produced in amplifying devices rather than in all.
it is produced because of random variation of electrons or holes (charge carrier)
it has uniform power spectral density like thermal noise
the exact formula for shot noise only can be calculated for diodes.

$$\underbrace{I_n^2}_{\text{Shot noise current}} = 2(\underbrace{I}_{\text{Direct current}} + 2\underbrace{I_0}_{\text{Reverse sat. current}})\underbrace{q}_{\text{Charge}}\underbrace{B}_{\text{BW}}$$

For amplifying devices, shot noise is
× directly proportional to output current
× inversely proportional to transconductance of device

Partition noise

partition noise is generated when current gets divided into two or more parts.
show partition noise is higher in transistor then the diode.
the device is like gallium arsenide FET draw almost zero gate bias current and therefore, keeping the partition noise to its minimum value.

Low frequency of flicker noise

it will appear at frequency is below few kilohertz
it is also referred as 1/f noise
it is generated because of fluctuation in current density
this will change conductivity of material
this will produce fluctuation in voltage and current
the mean square value of flicker noise is directly proportional to the square of direct current flowing Throgh the device

Transient noise or high frequency noise

if the time taken by electron from emitter to collector becomes comparible to period of signal then the transit time effect takes place.
this effect can be observed at very high frequency.
due to this some carriers may diffuse back to emitter.
this gives rise to input impedance.
so minut change at input generates random fluctuations at output.
once the nice appears, it goes on increasing with frequency at a rate of 6dB per octave

Thermal noise/ Johson noise/ white noise/ resistor noise

it arises due to random motio of free charge particles.
mainly electrons in conducting media.
the intensity of random motion is proportional to thermal (heat energy) supplied. E.g. tempreture.
the net motion of the electrons gives rise to an electric current to flow through the resistor cousing the noise.
calculation

$$\boxed{\begin{aligned} &\text{Power } P = KTB \\ &\text{Voltage } V = \sqrt{4KTBR} \\ &\text{Current } I = \sqrt{4GKTB} \end{aligned}}$$

Signal to Noise ratio (SNR)

- It is defined as a ratio of signal power to noise power.

$$\frac{S}{N} = \frac{P_S}{P_N}$$

where, P_S = Signal Power

P_N = Noise Power.

- In dB,

$$\left(\frac{S}{N}\right)_{dB} = 10 \log\left(\frac{P_S}{P_N}\right)$$

- In dB,

$$\boxed{\left(\frac{S}{N}\right)_{dB} = 10 \log\left(\frac{P_S}{P_N}\right)}$$

- Power in terms of voltage.

$$P_S = V_S^2/R \quad , \quad P_N = V_n^2/R$$

- So SNR will be.

$$\left(\frac{S}{N}\right)_{dB} = 10 \log\left(\frac{V_S^2/\cancel{R}}{V_N^2/\cancel{R}}\right)$$

$$= 10 \log\left(\frac{V_S}{V_N}\right)^2$$

$$\boxed{\left(\frac{S}{N}\right)_{dB} = 20 \log\left(\frac{V_S}{V_N}\right)}$$

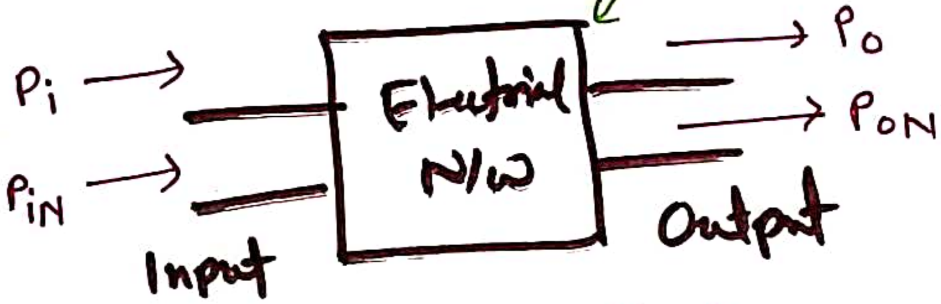
- high SNR is good for T_X and R_X

Noise Figure

- It is a ratio of SNR at Input to SNR at O/P

$$F = \frac{SNR \text{ at } I/P}{SNR \text{ at } O/P}$$

Gain $G = \frac{P_o}{P_i}$



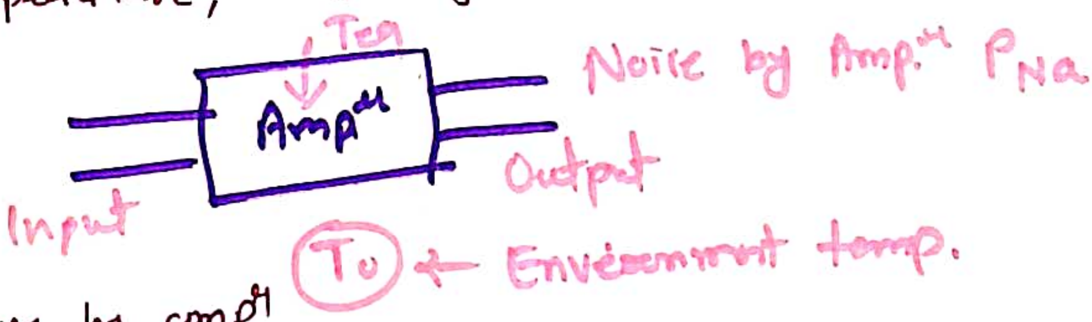
$(SNR)_{Input} = \frac{P_i}{P_{iN}}$

$(SNR)_{output} = \frac{P_o}{P_{oN}}$

$$F = \frac{P_i / P_{iN}}{P_o / P_{oN}} = \frac{P_i \; P_{oN}}{P_o \; P_{iN}} = \frac{P_{oN}}{G P_{iN}}$$

Noise Temperature

- It is temperature, which generates noise power in system.



- Noise power by ampl.

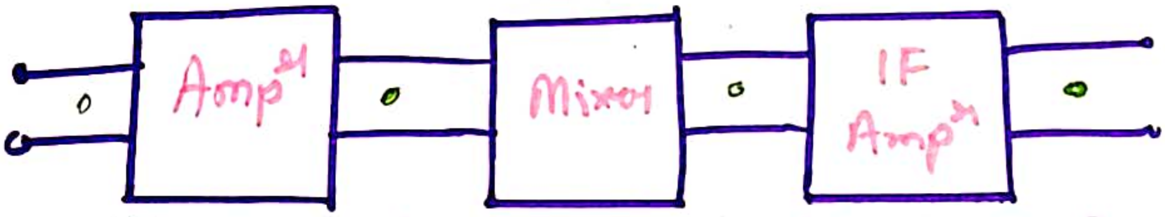
$$\Rightarrow P_{Na} = (F-1)kT_0B$$

- Noise power power in terms of noise temp T_{eq}.

$$\Rightarrow kT_{eq}B = (F-1)kT_0B$$

$$\Rightarrow \boxed{T_{eq} = (F-1)T_0}$$

Equivalent Noise Temp. and Noise Figure in cascade commⁿ system



(F_1, G_1, T_{eq_1}) (F_2, G_2, T_{eq_2}) (F_3, G_3, T_{eq_3})

→ Equivalent Noise Figure.

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

→ Do not take F_1, F_2 & F_3 in terms of dB.

(F_1, G_1, T_{eq_1}) (F_2, G_2, T_{eq_2}) (F_3, G_3, T_{eq_3})

→ Equivalent Noise Figure.

$$\boxed{F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}}$$

→ Do not take F_1, F_2 & F_3 in terms of dB.

→ Noise temp. $T_{eq} = T_0 (F - 1)$

→ $$\boxed{T_{eq} = T_{eq_1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2}}$$

Two resistors 20k and 50k are at room temperature 290K. Determine for the bendwidth of 100kHz, the thermal noise for the conditions:

I. For each resistor

II. For two resistors in series

III. For two resistors in parallel

→ $R_1 = 20\ k\Omega$

$R_2 = 50\ k\Omega$

$T = 290\ K$

$B = 100\ KHz$

→ $R_s = R_1 + R_2 = \underline{70\ k\Omega}$

→ $R_p = \dfrac{R_1 R_2}{R_1 + R_2} = 14.28\ k\Omega$

1) For $20\ k\Omega$

$$V_n = \sqrt{4KTBR_1}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 20 \times 10^{3} \times 100 \times 10^{3}}$$

$$= 5.66\ \mu V$$

→ For $50\ k\Omega$

$$V_n = \sqrt{4KTBR_2} = \underline{?}$$

→ For series $70\ k\Omega$

$$V_n = \sqrt{4KTBR_s} = \underline{?}$$

→ For Parallel $14.28\ k\Omega$

Calculate the thermal noise power available from any resistor at room temperature 290K for a bendwidth 2 MHz. Also calculate the corresponding noise voltage given that R=100.

→ $T = 290\,K$

$B = 2 \times 10^6\,Hz$

$R = 100\,\Omega$

$K = 1.38 \times 10^{-23}\,J/K$

→ $\boxed{V_{rms} = \sqrt{4KTBR}}$

→ $\boxed{P = KTB}$

→ Thermal Noise power

$\boxed{P = KTB}$

$= 1.38 \times 10^{-23} \times 290 \times 2 \times 10^6$

$= 8 \times 10^{-15}\,W$

⇒ $P = V^2/R$

→ $V = \sqrt{PR}$

$= \sqrt{8 \times 10^{-15} \times 100} = \boxed{0.894\,\mu V}$

An amplifier has a bandwidth of 4 MHz with 10k as the Input resistor. Calculate The RMS noise voltage at the Input to this amplifier if the room temperature is 25'C.

- $B = 4\ MHz$
  $R = 10\ k\Omega$
  $T = 25^\circ C = 25 + 273$
  $= 298\ K$
  $k = 1.38 \times 10^{-23}\ J/K$

$\Rightarrow V_n = \sqrt{4kTBR}$

$\Rightarrow V_n = \sqrt{4 \times 1.38 \times 10^{-23} \times 298 \times 4 \times 10^{6} \times 10 \times 10^{3}}$

$\Rightarrow V_n = 25.64\ \mu V$

The signal power and noise power measured at input of an amplifier are 150μW and 1.5μW, respectively. If the signal power at the output1.5 W and noise power is 40mW, calculate amplifier noise factor and noise figure.

$$\left.\begin{aligned} \rightarrow P_{si} &= 150\ \mu W \\ P_{ni} &= 1.5\ \mu W \end{aligned}\right\} (S/N)_i = \frac{P_{si}}{P_{Ni}} = \frac{150\ \mu W}{1.5\ \mu W} = 100$$

$$\left.\begin{aligned} P_{so} &= 1.5\ W \\ P_{no} &= 40\ mW \end{aligned}\right\} (S/N)_o = \frac{P_{so}}{P_{No}} = \frac{1.5\ W}{40 \times 10^{-3}\ W} = 37.5$$

$$\rightarrow F = \frac{(S/N)_i}{(S/N)_o} = \frac{100}{37.5} = 2.66$$

$$\rightarrow (F)_{dB} = 10 \log F = 10 \log 2.66 = 4.26\ dB$$

The signal to noise ratio at the input of amplifier is 40dB. If the noise figure of an amplifier is 20dB, calculate the Signal to noise ratio at the amplifier output.

$\rightarrow (S/N)_i (dB) = 40\ dB$

$(F)(dB) = 20\ dB$

$\Rightarrow (F)(dB) = (S/N)_i (dB) - (S/N)_o (dB)$

$\Rightarrow (S/N)_o (dB) = (S/N)_i (dB) - (F)(dB)$

$= 40 - 20$

$= \boxed{20\ dB}$

An amplifier has noise figure of 3dB. Determine its equivalent noise temperature.

$\rightarrow (F)_{dB} = 3\,dB$

$\Rightarrow (F)_{dB} = 10 \log F$

$\Rightarrow F = \text{Antilog}\left(\frac{(F)_{dB}}{10}\right)$

$= \text{Antilog}(0.3)$

$\Rightarrow \boxed{F = 2}$

$\rightarrow \boxed{T_{eq} = (F-1)T_0}$

$\rightarrow$ Room temp 300 K.

$\Rightarrow T_{eq} = (2-1)\,300$

$\Rightarrow \boxed{T_{eq} = 300\,K}$

Radio receiver with 10kHz bandwidth has a noise figure of 30dB. Determine the signal power required at input of receiver to achieve input SNR of 30dB.

→ $B = 10\ kHz$

$F_{dB} = 30\ dB \Rightarrow F = 1000$

$(S/N)_i = 30\ dB \Rightarrow (S/N)_i = 1000$

→ $P_{Ni} = FkTB$

$= 1000 \times 1.38 \times 10^{-23} \times 300 \times 10 \times 10^{3}$

$= 4.14 \times 10^{-14}\ W$

→ $(S/N)_i = \frac{P_{si}}{P_{Ni}} \Rightarrow P_{si} = (S/N)_i\ P_{Ni}$

$= 1000 \times 4.14 \times 10^{-14}$

$= 4.14 \times 10^{-11}\ W = 41.4\ pW$

If each stage has gain of 10 dB and noise figure of 10dB. Calculate the overall noise figure of two stage amplifier.

→ $(G_1)_{dB} = 10\,dB$

⇒ $G_1 = 10$

→ $(G_2)_{dB} = 10\,dB$

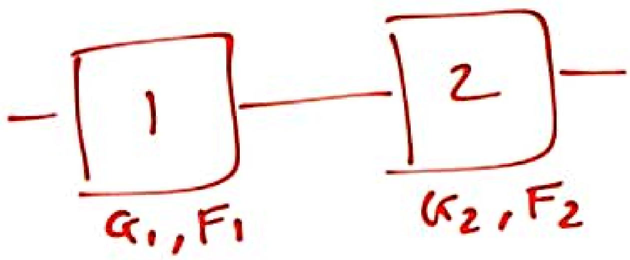
⇒ $G_2 = 10$

→ $(F_1)_{dB} = 10\,dB$

⇒ $F_1 = 10$

→ $(F_2)_{dB} = 10\,dB$

⇒ $F_2 = 10$



→ $F = F_1 + \frac{(F_2 - 1)}{G_1}$

$= 10 + \frac{(10-1)}{10}$

$= 10.9$

→ Noise Figure

$F_{dB} = 10 \log F = 10 \log 10.9$

$= 10.37\,dB$

Radio receiver with equivalent noise bandwidth of 10kHz Has a noise figure of 20dB. If input SNR to receiver is 40dB, determine the output SNR. What is the equivalent Noise temperature, if ambient temperature is 300K.

$\rightarrow B = 10\ kHz$

$F_{dB} = 20\ dB \Rightarrow F = \underline{100}$

$(S/N)_i = 40\ dB \Rightarrow (S/N)_i = 10^4$

$T_0 = 300\ K$

$\rightarrow T_{eq} = (F-1)T_0$

$= (100-1)\,300$

$= 29700\ K$

$\rightarrow F = \dfrac{(S/N)_i}{(S/N)_o} \Rightarrow (S/N)_o = \dfrac{(S/N)_i}{F}$

$\Rightarrow (S/N)_o = 10^4/100$

$\Rightarrow (S/N)_o = 100 \Rightarrow (S/N)_o^{(dB)} = 20dB$

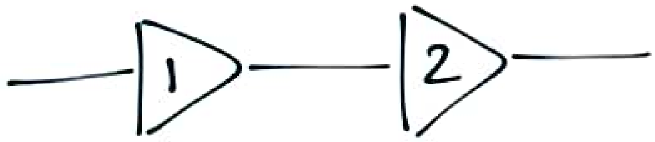
An amplifier with 10dB noise figure and 4dB power gain is cascaded with a second amplifier which has a 10dB power gain and 10dB noise figure. What is the overall noise figure and power gain?

→ $F_1 = 10\,dB = 10$

$G_1 = 4\,dB = 2.5$

$F_2 = 10\,dB = 10$

$G_2 = 10\,dB = 10$



→ $F = F_1 + \frac{(F_2 - 1)}{G_1}$

$= 10 + 9/2.5$

$= 13.6 = 11.33\,dB$

→ $G = G_1 G_2$

$= 2.5 \times 10$

$= \underline{25}$

$= \underline{13.9\,dB}$

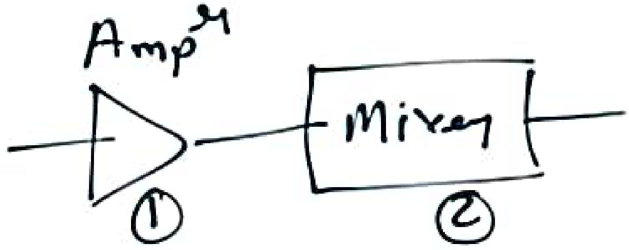
→ $T_{eq} = (F - 1) T_0$

A mixer stage has a noise figure of 20dB and it is preceded by an another amplifier with a noise figure of 9dB and an available power gain of 15dB. Calculate noise figure.

→ $F_1 = 9dB = 7.94$

$G_1 = 15\,dB = 31.62$

$F_2 = 20dB = 100$

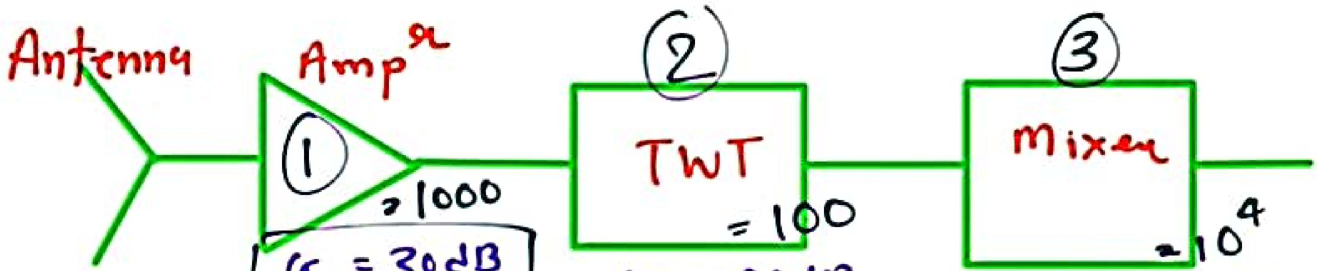


→ $F = F_1 + \frac{(F_2 - 1)}{G_1}$

$= 7.94 + \frac{99}{31.62}$

$= 11.67 = \underline{10.44\,dB}$

Calculate equivalent noise figure and noise temperature for given diagram.



$G_1 = 30\,dB$
$T_{e_1} = 5\,K$

$G_2 = 20\,dB$
$F_2 = 5\,dB = 4$

$G_3 = 40\,dB$
$F_3 = 12\,dB = 16$

$$\Rightarrow T_{eq_1} = (F_1 - 1)\,T_0$$

$$\Rightarrow F_1 = 1 + \frac{T_{eq_1}}{T_0} = 1 + \frac{5}{300} = 1.02$$

$$\rightarrow F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

$$= 1.02 + \frac{(4-1)}{1000} + \frac{16-1}{1000 \times 100}$$

$$= \underline{\qquad\qquad}$$

$$\rightarrow T_{eq} = (F - 1)\,T_0$$

Channel Capacity by Shannon - Hartley and It's proof

$$C = B \log_2 \left[1 + \frac{S}{N} \right]$$

where,

B = Bandwidth of channel

S = Signal Power

N = Noise power.

Proof :- Received signal = Signal Power (S) + Noise power (N) and it's mean square value is $\sqrt{S+N}$

- Noise power is N and it's mean square value is $\sqrt{N}$
- So Number of levels can be separated without error is

$$m = \frac{\sqrt{N+S}}{\sqrt{N}} = \sqrt{1+\frac{S}{N}}$$

- So digital information is

$$I = \log_2 m$$

$$= \log_2 \sqrt{1+S/N}$$

$$= \frac{1}{2} \log_2 \left(1+\frac{S}{N}\right)$$

$$m = \frac{\sqrt{N+S}}{\sqrt{N}} = \sqrt{1+\frac{S}{N}}$$

- So digital Information is

$$I = \log_2 m$$

$$= \log_2 \sqrt{1+S/N}$$

$$= \frac{1}{2}\log_2\left(1+\frac{S}{N}\right)$$

- If channel transmitter K pulses per second then channel capacity is

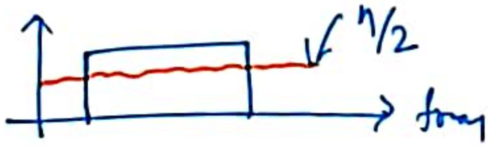
$$C = IK = \frac{K}{2}\log_2\left(1+\frac{S}{N}\right)$$

- Nyquist Bandwidth is $K = 2B$

$$\boxed{C = B\log_2\left(1+\frac{S}{N}\right)}$$

Practically If we Increase B then noise N will also Increase. So Capacity of channel can not be Infinite.

If $\eta/2$ is power density then



$$N = \eta B$$

So channel capacity is

$$C = B \log_2 \left(1 + \frac{S}{\eta B}\right)$$

$$= \frac{\eta B}{S}\left(\frac{S}{\eta}\right) \log_2 \left(1 + \frac{S}{\eta B}\right)$$

- So Channel capacity is

$$C = B \log_2 \left(1 + \frac{S}{\eta B}\right)$$

$$= \frac{\eta B}{S}\left(\frac{S}{\eta}\right) \log_2 \left(1 + \frac{S}{\eta B}\right)$$

$$= \frac{S}{\eta}\left[\frac{\log_2 \left(1 + \frac{S}{\eta B}\right)}{\left(\frac{S}{\eta B}\right)}\right] \qquad \underline{B \to \infty}$$

- For $\lim\limits_{x \to 0} \frac{\log_2 (1+x)}{(1/x)} = \log_2 e = 1.44$

$$C_{B \to \infty} = \frac{S}{\eta} \log_2 e = \boxed{1.44 \frac{S}{\eta}}$$

Examples on Channel Capacity by Shannon - Hartley

① For a typical telephone line with a signal to noise ratio of 30 dB and an audio bandwidth 3 kHz, max. data rate of ....

$\rightarrow$ SNR = 30 dB = 10^3

B = 3 kHz

C = ?

dB	
10	$\rightarrow 10$
20	$\rightarrow 10^2$
30	$\rightarrow 10^3$
40	$\rightarrow 10^4$

$\rightarrow$ $C = B \log_2 (1 + SNR)$

$$= 3\times10^3 \log_2 (1 + 10^3)$$

$$= 3\times10^3 \frac{\log 1001}{\log 2} = 3\times10^6 \text{ bps} = \boxed{3 \text{ Mbps}}$$

... signal to noise ratio

② For a satellite TV channel with a signal to noise ratio of 20 dB and a video bandwidth of 10 MHz, find max. data rate?

→ SNR = 20 dB = 10^2

B = 10 MHz = 10×10^6 Hz

C = ?

→ $C = B \log_2 (1 + SNR)$

⇒ $C = 10 \times 10^6 \log_2 (1 + 10^2)$

$= 10^7 \frac{\log (101)}{\log 2}$

$= 6.6 \times 10^7$ bps

= 66 Mbps

③ A Gaussian channel has 1 MHz bandwidth. Calculate the channel capacity. If the signal power to noise spectral density ratio S/N is 10^5 Hz. Also find the max. Information rate.

③ A Gaussian Channel has 1 MHz bandwidth. Calculate the Channel Capacity. If the signal power to noise spectral density ratio S/N is 10^5 Hz. Also find the max. Information rate.

– $B = 1\ MHz = 10^6\ Hz$

$SNR = 10^5$

$C = ?$

$\Rightarrow C = B \log_2 (1 + SNR)$

$= 10^6 \dfrac{\log (100001)}{\log 2}$

$= 16.6 \times 10^6$ bps

$$\boxed{C = 16.6 \text{ Mbps}}$$

* Calculate channel capacity with $B \to \infty$.

- Channel capacity

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

- If we have noise power $\boxed{N = \eta B}$ when $\frac{\eta}{2}$ is power density.

Power density.

$$C = B \log_2\left(1 + \frac{S}{\eta B}\right)$$

$$= \frac{\eta B}{S} \times \frac{S}{\eta} \log_2\left(1 + \frac{S}{\eta B}\right)$$

$$= \frac{S}{\eta}\left[\frac{\eta B}{S} \log_2\left(1 + \frac{S}{\eta B}\right)\right]$$

- Here $B \to \infty$.

$$C = \frac{S}{\eta} \lim_{B \to \infty}\left[\frac{\log_2\left(1 + \frac{S}{\eta B}\right)}{\left(S/\eta B\right)}\right]$$

$$= \frac{S}{\eta} \left[\frac{\eta B}{S} \log_2 \left(1 + \frac{S}{\eta B} \right) \right]$$

– Here $B \to \infty$.

$$C = \frac{S}{\eta} \lim_{B \to \infty} \left[\frac{\log_2 \left(1 + \frac{S}{\eta B} \right)}{\left(S/\eta B \right)} \right]$$

$$\Rightarrow C = \frac{S}{\eta} \log_2 e$$

$$\Rightarrow \boxed{C_{B \to \infty} = (1.44) \frac{S}{\eta}}$$

10.7 SOURCE CODING

A conversion of the output of a DMS into a sequence of binary symbols (binary code word) is called *source coding*. The device that performs this conversion is called the *source encoder* (Fig. 10-6).

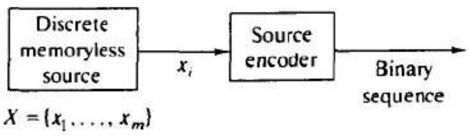


Fig. 10-6 Source coding

An objective of source coding is to minimize the average bit rate required for representation of the source by reducing the redundancy of the information source.

A. Code Length and Code Efficiency:

Let X be a DMS with finite entropy $H(X)$ and an alphabet $\{x_1, \ldots, x_m\}$ with corresponding probabilities of occurrence $P(x_i)(i = 1, \ldots, m)$. Let the binary code word assigned to symbol x_i by the encoder have length n_i, measured in bits. The length of a code word is the number of binary digits in the code word. The average code word length L, per source symbol is given by

$$L = \sum_{i=1}^{m} P(x_i)n_i \tag{10.49}$$

The parameter L represents the average number of bits per source symbol used in the source coding process.

The *code efficiency* η is defined as

$$\eta = \frac{L_{\min}}{L} \tag{10.50}$$

where $L_{\min}$ is the minimum possible value of L. When η approaches unity, the code is said to be *efficient*.

The *code redundancy* γ is defined as

$$\gamma = 1 - \eta \tag{10.51}$$

B. Source Coding Theorem:

The source coding theorem states that for a DMS X with entropy $H(X)$, the average code word length L per symbol is bounded as

$$L \geqslant H(X) \tag{10.52}$$

and further, L can be made as close to $H(X)$ as desired for some suitably chosen code.

Thus, with $L_{\min} = H(X)$, the code efficiency can be rewritten as

$$\eta = \frac{H(X)}{L} \tag{10.53}$$