Classification of Systems

1. Linear and Non Linear System
2. Time Variant and Time Invariant Systems
3. Linear Time Variant and Linear Time Invariant Systems
4. Static and Dynamic Systems
5. Causal and Non Causal Systems
6. Invertible and Non invertible System
7. Stable and Unstable System

1. Linear and Non Linear System

A system is said to be linear if it satisfies the superposition principle. Consider a system with input $x_1(t)$, $x_2(t)$ and output $y_1(t)$ and $y_2(t)$.

For Linearity

$$T[a_1 x_1(t) + a_2 x_2(t)] = a_1 T[x_1(t)] + a_2 T[x_2(t)]$$

2. Time Variant and Time Invariant System

A system is said to be time variant if its input, output characteristics changes with time otherwise called time invariant.

Condition for time invariance is $y(n,k) = y(n-k)$

where $y(n,k) = T[x(n-k)]$

3. Linear Time Variant and Linear Time Invariant System

* A system is called linear time variant if it satisfies both linearity and time variance.
* A system is called linear time invariant if it satisfies both linearity and time invariance.

eg. $y(n) = n x^2(n)$

For linearity

$$LHS = T[a_1 x_1(n) + a_2 x_2(n)] = n[a_1 x_1(n) + a_2 x_2(n)]^2$$

$$RHS = a_1 y_1(n) + a_2 y_2(n) = a_1 n x_1^2(n) + a_2 n x_2^2(n)$$

$LHS \neq RHS$ Non linear

For time invariance

LHS = $y(n,k) = T[x(n-k)] = n\,x^2(n-k)$

RHS = $y(n-k) = (n-k)\,x^2(n-k)$

LHS ≠ RHS

So Time Variant

4. Static and Dynamic System

* Static system is memoryless system whereas dynamic system is memory system
* In static system ~~response is~~ output depends only on present input. In dynamic system output depends on present, past and future inputs.

eg. $y(n) = x(n)$

$y(0) = x(0)$ Static System

$y(n) = x(n) + x(n-1)$

$y(1) = x(1) + x(0)$ Dynamic System

5. Causal and Non Causal System

* A system is said to be causal, if its response is dependent on present and past input and doesn't depend on future input.
* For a non causal system the output depends on future input also.

eg. $y(n) = x(n) + \frac{1}{x(n-1)}$

is causal

eg. $y(t) = 2x(t) + \frac{1}{x(t+1)}$

is Non Causal

* All non causal system are dynamic, but all dynamic system are not non causal.
* All static system are causal, but all causal system are not static.

6. Invertible and Non Invertible System

* A system is called invertible if input of system appears at output i.e. $y(t)=x(t)$ else called non invertible system.

7. Stable and Unstable System

* A system is said to be stable when it produces bounded output for a bounded input
* Bounded signal means, we can estimate the value of signal. Unbounded means we can't estimate it's value.
* We use $u(n)$ to test stability.

eg. $y(n)= x^2(n)$

let $x(n)= u(n)$

$y(n) = [u(n)]^2 = u(n)$ (bounded)

eg. $y(t)= 2x^2(t)$

let $x(t)= u(t)$

$y(t)= 2u^2(t)= 2u(t)$ (bounded)

eg. $y(t) = \int x(t)\, dt$

let $x(t)= u(t)$

$y(t) = \int u(t)\, dt = r(t)$

as we can't estimate the value of $r(t)$

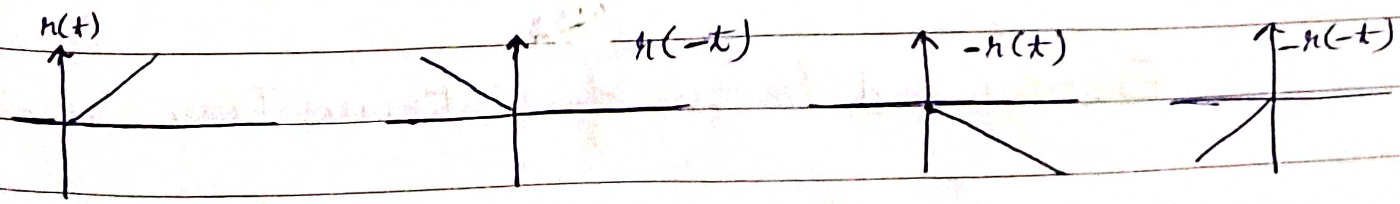
(value is infinite at $t=\infty$)

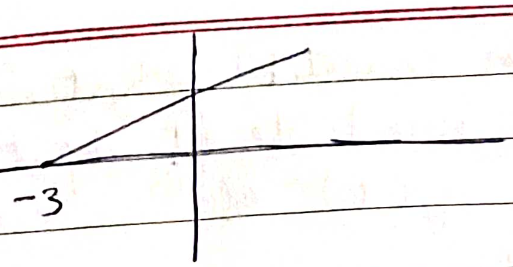
so unstable

$r(t)$ and $\delta(t)$ are unbounded signals

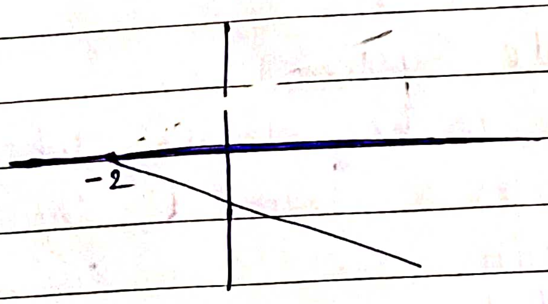
Ramp Signal Problem

$y(t)= r(t+3) - r(t+2) - r(t-2) + r(t-3)$

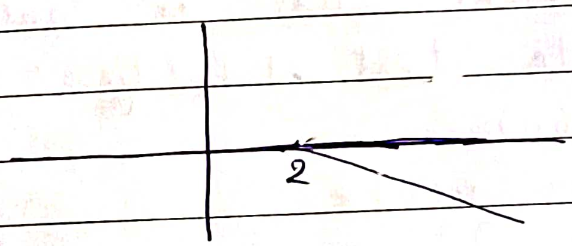




$r(t+3)$

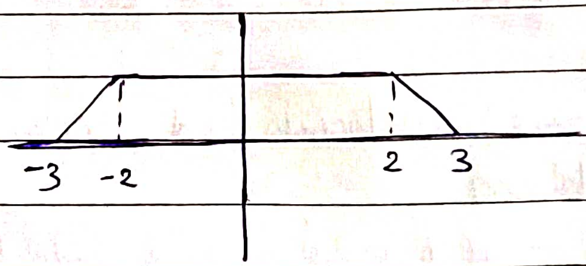


$-r(t+2)$



$-r(t-2)$

$r(t-3)$



Energy and Power of Continuous Time Signals

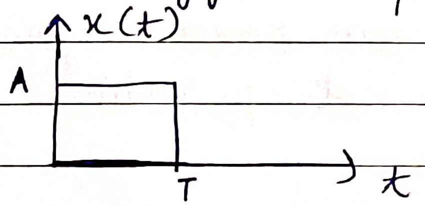
$$E = \int_{-T}^{T} x^2(t)\,dt$$ [Finite Duration]

$$E = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)\, dt \quad \text{[Infinite Duration]}$$

$$P = \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt \quad \text{[Finite Duration]}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt \quad \text{[Infinite Duration]}$$

eg. Find energy and power of given signal



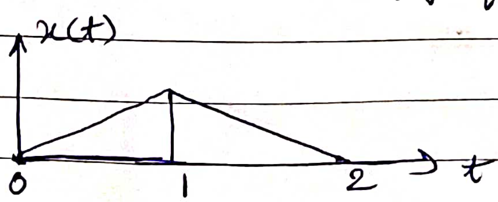
$$x(t) = \begin{cases} A & 0 \leq t \leq T \\ 0 & \text{else} \end{cases}$$

$$E = \int_{-T}^{T} x^2(t)\, dt = \int_{0}^{T} A^2\, dt = A^2 T \text{ (finite)}$$

so energy signal, hence power $= 0$

NOTE: If energy of $x(t) = E$
then energy of $x(at) = E/a$

eg. Determine total energy of the signal shown in figure



$$E = \int_{-T}^{T} x^2(t)\, dt = \int_{0}^{1} x^2(t)\, dt + \int_{1}^{2} x(2-t)\, dt$$

$$= \int_{0}^{1} t^2 dt + \int_{1}^{2} (2-t)^2\, dt$$

$$= \frac{1}{3} + \frac{-1}{3}[0-1] = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

NOTE: Ramp signal is neither energy nor power signal

Energy and Power of Discrete Signal

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$$

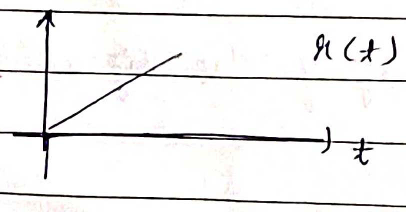
$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

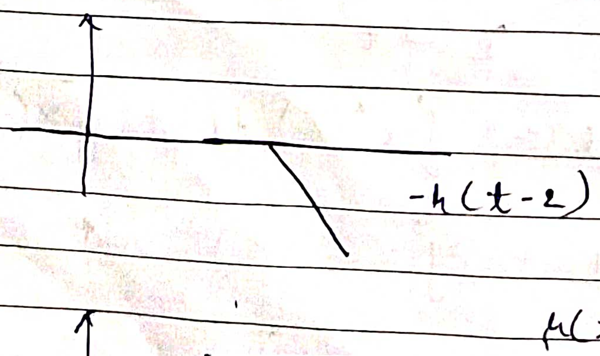
* All finite duration signals of finite amplitude are energy signals
* All periodic signals are power signals but all power signals are not periodic.
  $P_{av} = \frac{A^2}{2}$

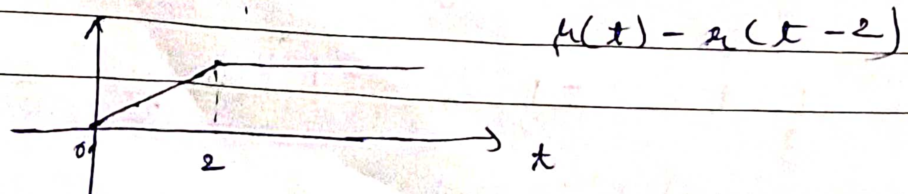
* Power signal + Energy Signal = Power Signal
* Continuous impulse is neither energy nor power signal.

eg. Calculate energy and power of $x(t)$ and $\frac{dx(t)}{dt}$ where $x(t) = r(t) - r(t-2)$

Ans)





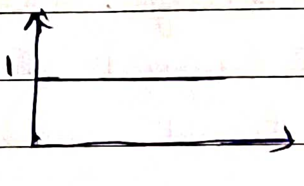


From 0 to 2 $\quad E_1 = \int_0^2 t^2\,dt = \frac{8}{3}$ Joules

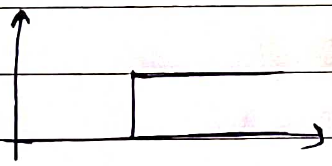
From 2 to ∞ $\quad E_2 = \int_2^\infty 4\,dt = \infty$

$E = E_1 + E_2 = \infty$ (Power signal)

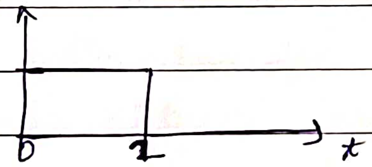
$\frac{d}{dt}x(t) = u(t) - u(t-2)$



$u(t)$



$u(t-2)$

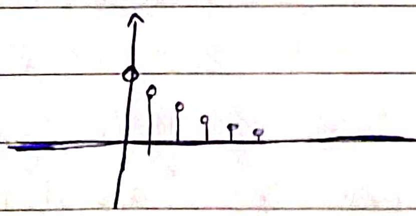


$u(t) - u(t-2)$

$E = \int_0^2 1^2\,dt = 2$ Joules (Energy Signal)

eg. $x(n) = a^n u(n)$

If $a < 1$ $\quad x(n) = 0$ at $n = \infty$

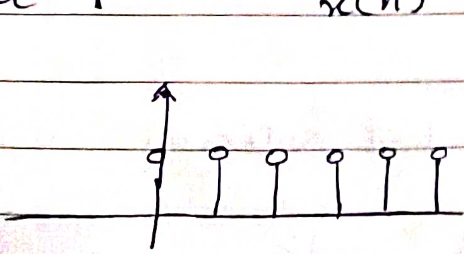


$E = \sum_{n=0}^{\infty} |x(n)|^2$

$E = x^2(0) + x^2(1) + x^2(2) + \dots + 0$ (finite value)

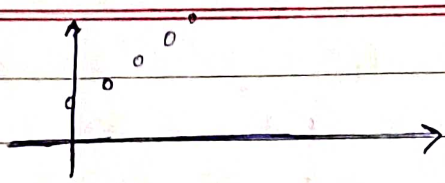
energy signal

If $a = 1$ $\quad x(n) = u(n)$



Since it is periodic hence power signal

If $a > 1$



$$E = \sum_{n=0}^{\infty} |x^2(n)| = \infty$$

Power Signal

Periodicity of Continuous Time Signals

Consider two signals $x_1(t)$ and $x_2(t)$ with periods T_1 and T_2 respectively. When they are summed, the resultant signal is said to be periodic when $\frac{T_1}{T_2}$ = rational number

eg. $\cos 4t + \sin \pi t$

$\omega_1 = 4$ $\qquad\qquad$ $\omega_2 = \pi$

$\frac{2\pi}{T_1} = 4$ $\qquad\qquad$ $\frac{2\pi}{T_2} = \pi$

$\frac{T_1}{T_2} = \frac{\pi}{4} \neq$ Rational

So not periodic

Periodicity of Discrete Time Signals

* A discrete signal $x(n)$ is said to be periodic when $x(n) = x(n+N)$
* If a discrete signal is periodic then the ratio of $\frac{\omega_0}{2\pi}$ must be rational number $\left[\frac{M}{N}\right]$

M = No. of full cycles
N = No. of samples

eg. $x(n) = \sin\left(\frac{n}{2} - \frac{\pi}{2}\right)$

$\omega = \frac{1}{2}$

$\frac{\omega}{2\pi} = \frac{1}{4\pi}$ Not Rational

Not Periodic

eg. $x(n) = e^{j\frac{\pi}{2}n}$ $\qquad \omega = \frac{\pi}{2}$

$$\frac{\omega}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4} \quad \text{(Rational)} \qquad \text{Periodic}$$

eg. $x(n) = \sin\left(\frac{\pi n}{2}\right) + \cos\left(\frac{\pi n}{4}\right)$

$$\frac{\omega_1}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4} = \frac{M_1}{N_1} \qquad \frac{\omega_2}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8} = \frac{M_2}{N_2}$$

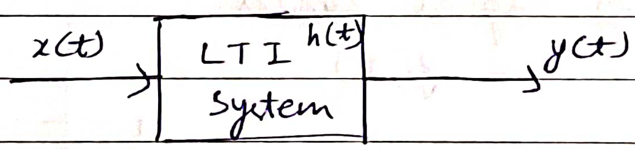
$N_1 = 4$ $\qquad$ $N_2 = 8$

$N = LCM(N_1, N_2) = 8$, $\qquad$ Periodic

eg. ~~x(n) = u(n) + u(-n)~~

Convolution

It is a mathematical operation which expresses input and output relation of an LTI system



★ By using convolution we can find zero state response of the system.

For Continuous Time Signals

$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

For Discrete Time Signals

$$y(n) = x(n) * h(n)$$
$$= \sum_{K=-\infty}^{\infty} x(n)\, h(n-K) = \sum_{K=-\infty}^{\infty} h(n)\, x(n-K)$$

$x(t) = t^2 + 2t + 1$

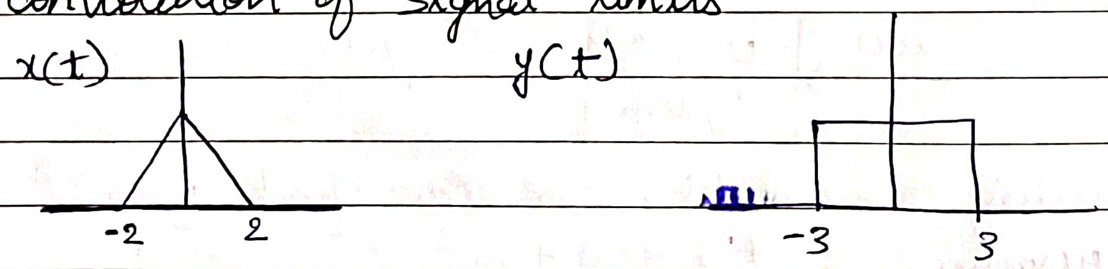
$h(t) = t^2 + 3t + 4$

	$t^2 + 2t + 1$	
$4 \quad 3t \quad t^2$		
$4 \quad 3t$	t^2	$= t^4$
4	$3t \quad t^2$	$= 3t^3 + 2t^3 = 5t^3$
	$4 \quad 3t \quad t^2$	$= t^2 + 6t^2 + 4t^2 = 11t^2$
	$4 \quad 3t$	$= 8t + 3t = 11t$
	4	$= 4$

$y(t) = x(t) * h(t)$

$y(t) = t^4 + 5t^3 + 11t^2 + 11t + 4$

Convolution of signal limits



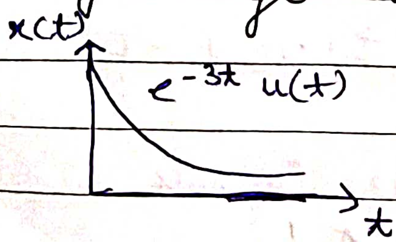
Convoluted signal limits are

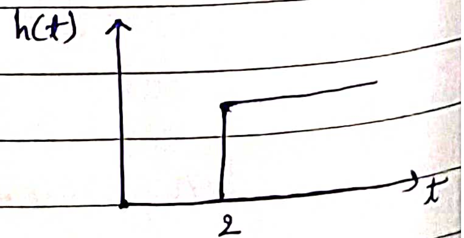
Sum of lower limits $< t <$ Sum of Upper Limits

$-2 - 3 < t < 2 + 3$

$-5 < t < 5$

Convolution by image method

$y(t) =$ 

$\circledast$ 

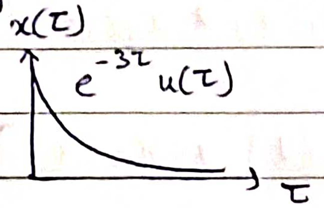
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

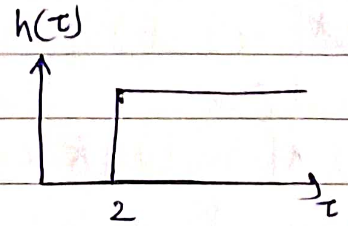
(i) Limits of $y(t) = x(t) \circledast h(t)$

$$0+2 < t < \infty + \infty$$

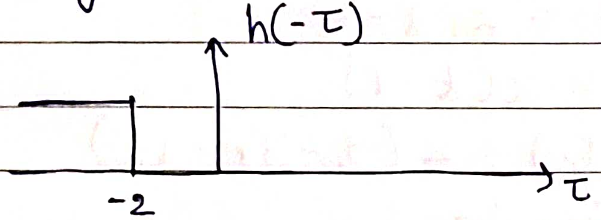
$$2 < t < \infty$$

(ii) Change Limits

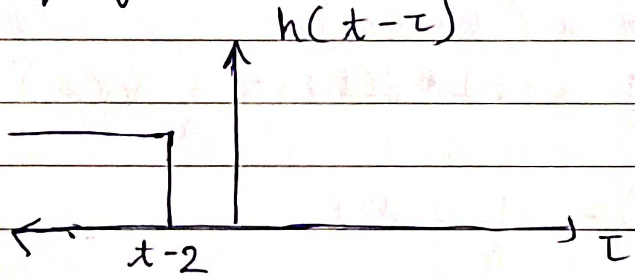




(iii) Folding $h(-\tau)$

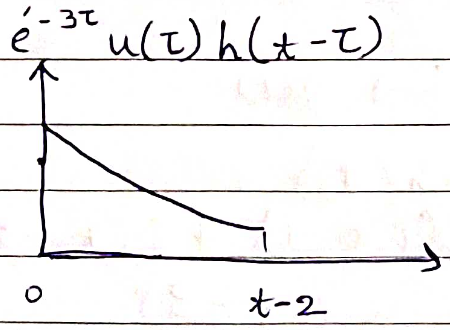


(iv) Shifting $h(t-\tau)$



(v) Multiplication

if $t-2 > 0$



$$y(t) = \int_0^{t-2} e^{-3\tau} \, d\tau$$

$$y(t) = \left[\frac{e^{-3\tau}}{-3}\right]_0^{t-2}$$

$$y(t) = \frac{1}{3}\left[1 - e^{(-3t+6)}\right]$$

if $t-2 < 0$

then $y(t) = 0$

Properties of Convolution

1) Commutative Property

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

2) Distributive Property

$$x_1(t) * [x_2(t) + x_3(t)] = [x_1(t) * x_2(t)] + [x_1(t) * x_3(t)]$$

3) Associative

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

4) Shifting Property

if $x_1(t) * x_2(t) = z(t)$

$$x_1(t) * x_2(t-T) = z(t-T)$$

$$x_1(t-T) * x_2(t) = z(t-T)$$

$$x_1(t-T_1) * x_2(t-T_2) = z(t-T_1-T_2)$$

5) Convolution with impulse

$$x(t) * \delta(t) = x(t)$$

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

$$x(t) * \delta(at) = \frac{1}{a}\, x(t) * \delta(t) = \frac{1}{a}\, x(t)$$

because $\delta(at) = \frac{1}{a}\delta(t)$

eg. The impulse response of continuous system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of step response at $t=2$ is

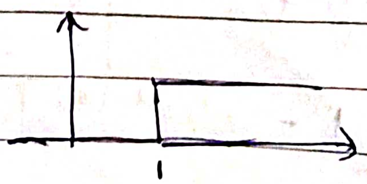
Ans)

$$u(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

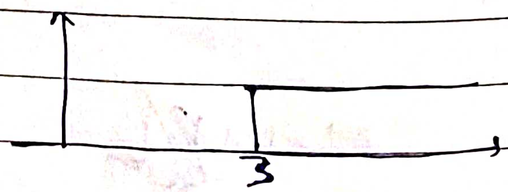
$$y(t) = x(t) * h(t)$$

$$= u(t) * [\delta(t-1) + \delta(t-3)]$$

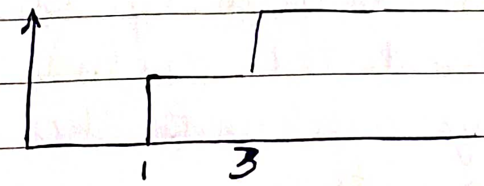
$$y(t) = u(t-1) + u(t-3)$$



u(t-1)



u(t-3)



eg Given $z(t) = \int_{-\infty}^{\infty} x(-\tau + b)\, h(t+\tau)\, d\tau$ Express $z(t)$

in terms of $y(t) = x(t) * h(t)$

Ans) $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau =$

$\int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$

Let $t + \tau = \lambda$

$z(t) = \int_{-\infty}^{\infty} x(t - \lambda + b)\, h(\lambda)\, d\lambda = y(t+b)$

eg. If $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t} u(t)$.

Find $\frac{dx(t)}{dt} * h(t)$

Ans) $\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$

$\frac{dx(t)}{dt} * h(t) = [\delta(t-3) - \delta(t-5)] * h(t)$

$= h(t-3) - h(t-5)$

$= e^{-3(t-3)} u(t-3) - e^{-3(t-5)} u(t-5)$

Other Properties of Convolution

1) $\frac{dy(t)}{dt} = \frac{d}{dt} x(t) * h(t)$ OR $x(t) * \frac{d}{dt} h(t)$

2) $u(t) * u(t) = r(t)$

3) $u(t-T_1) * u(t-T_2) = r(t - T_1 - T_2)$

4) $y(-t) = x(-t) * h(-t)$

5) If $x(t) * h(t) = y(t)$

if input and impulse response differentiated

m times and n times respectively then output will be differentiated (m+n) times.
If inputs are delayed by a, b units then output is delayed by a+b units.

Area of Convoluted Signal

If area under input signal is A_x and area under impulse signal is A_h. Then the area under output signal is $\boxed{A_y = A_x \cdot A_h}$

Scaling in Convolution

$$x(\alpha t) * h(\alpha t) = \frac{1}{\alpha} y(\alpha t)$$

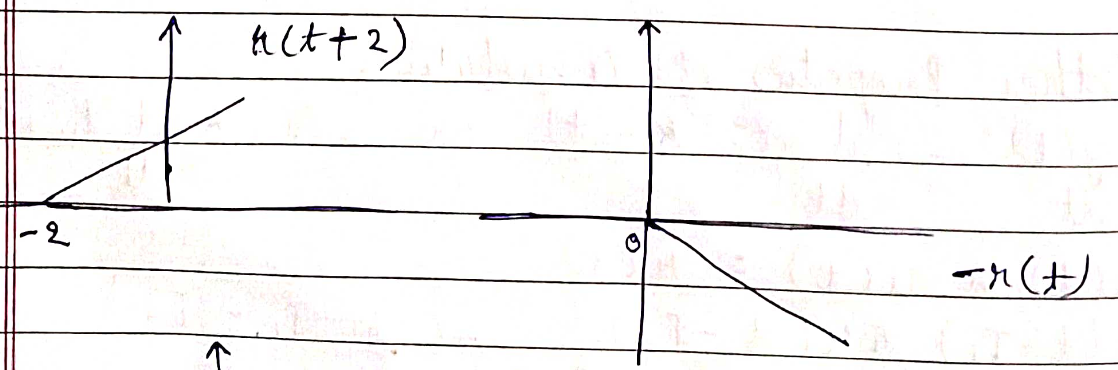
eg Find the area of the signal
$[u(t+1) - u(t-3)] * [u(t+1) - u(t-1)]$

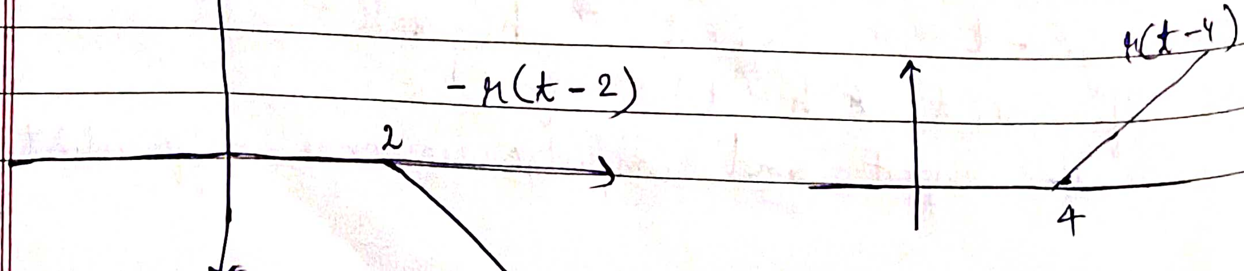
Ans) Using Distributive Property

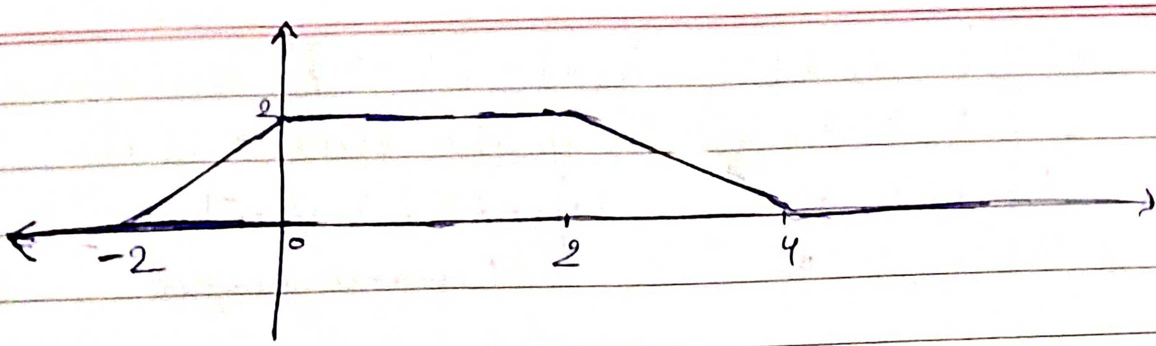
$u(t+1) * u(t+1) - u(t+1) * u(t-1)$
$- u(t-3) * u(t+1) + u(t-3) * u(t-1)$

We know $u(t) * u(t) = r(t)$

$\Rightarrow r(t+2) - r(t) - r(t-2) + r(t-4)$







$$A = \frac{1}{2}(2+6) \times 2 = 8$$

eg. Convolute $u(t+1) * r(t-2)$

Ans) Let $g(t) = u(t) * r(t)$

$$g'(t) = \frac{d}{dt} u(t) * r(t)$$

$$g'(t) = \delta(t) * r(t) = r(t)$$

$$g(t) = \int r(t) = \int t\,dt = \frac{t^2}{2} u(t)$$

$$g(t+1-2) = g(t-1) = \frac{(t-1)^2}{2} u(t-1)$$

eg. Find the response of the system if $h(t) = t\,u(t)$ for an input $u(t-1)$

Ans) $y(t) = u(t-1) * [t\,u(t)]$

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t \leq 0 \end{cases} = r(t)$$

$$y(t) = u(t-1) * r(t)$$

$$= \frac{(t-1)^2}{2} u(t-1)$$

Discrete Convolution

eg. $x[n] = \{1, 2, 3, 4\}$ $h[n] = \{1, 1, -1, 1\}$

	1	2	3	4
1	1	2	3	4
1	1	2	3	4
-1	-1	-2	-3	-4
1	1	2	3	4

$y(n) = x(n) * h(n)$
$= \{1, 3, 4, 6, 3, -1, 4\}$
$(m+n-1)$ samples

Now, add

eg. $x(n) = \{1, 2, 3\}$ $h(n) = \{1, -2\}$

	1	2	3
1	1	2	3
-2	-2	-4	-6

$y(n) = x(n) * h(n)$

$y(n) = \{1, 0, -1, -6\}$

$(m+n-1)$ samples

Circular Convolution

eg. $x(n) = \{1, 2, 1, 1\}$ & $h(n) = \{1, 1, -1, -1\}$

$y(n) = x(n) * h(n)$

	1	2	1	1
1	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
-1	-1	-2	-1	-1

$y(n) = \{1, 3, 2, -1, -2, -2, -1\}$

We select max (m, n)

i.e. max (4, 4) = 4

samples and add remaining to it

$\underbrace{1 \quad 3 \quad 2 \quad -1}$ $\underbrace{-2 \quad -2 \quad -1}$

Now,

1	3	2	-1
-2	-2	-1	

$\{-1, 1, 1, -1\}$ = circular convolution

eg. $x(n) = \{2, 2\}$ $h(n) = \{1, 1, -1\}$

	1	1	-1
2	2	2	-2
2	2	2	-2

$\{2, 4, 0, -2\}$ = Linear Conv

Now, we select max (2, 3) = 3

samples and add remaining to it

2,	4,	0
-2		

$\{0, 4, 0\}$ = Circular convolution

Deconvolution

eg. If $x(n) = \{1, 1, -1\}$ and $y(n) = \{1, 3, 1, -2\}$. Find $h(n)$.

Ans) ~~Let~~ $m = 3$ $\quad m+n-1 = 4 \Rightarrow n = 2$

Let $h(n) = \{a, b\}$

	1	1	-1
a	a	a	-a
b	b	b	-b

$y(n) = \{a, a+b, b-a, -b\}$

$\Rightarrow \boxed{a = 1} \quad \boxed{b = 2}$

Fourier Series Representation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad -(1)$$

Fourier Series Coefficient

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} \, dt \qquad -(2)$$

Derivation:

Multiply $e^{-jn\omega_0 t}$ on both sides

$$x(t) e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t}$$

$$x(t) e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t}$$

Integrate on both side

$$\int_0^T x(t) e^{-jn\omega_0 t} dt = \int_0^T \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t} dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\omega_0 t} dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \int_0^T \left[\cos(k-n)\omega_0 t + j\sin(k-n)\omega_0 t\right] dt$$

$$\int_0^T e^{j(k-n)\omega_0 t} = \begin{cases} T & k=n \\ 0 & k\neq n \end{cases}$$

$$\int_0^T x(t)\, e^{-jn\omega_0 t} = a_n T$$

$$a_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}$$

$$\rightarrow \quad a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}$$

Properties of Fourier Series

1) Linearity

if $x(t) \xleftrightarrow{FS} F_x$

& $y(t) \xleftrightarrow{FS} F_y$

Linearity states that

$$a x(t) + b y(t) \xleftrightarrow{FS} a F_x + b F_y$$

2) Time Shifting

if $x(t) \xleftrightarrow{FS} F_n$

then time shifting property states that

$$x(t-t_0) \xleftrightarrow{FS} e^{-jn\omega_0 t_0} F_n$$

3) Frequency Shifting

if $x(t) \xleftrightarrow{FS} F_n$

then frequency shifting states that

$$e^{jn_0\omega_0 t}\, x(t) \xleftrightarrow{FS} F(n-n_0)$$

4) Multiplication and convolution

if $x(t) \xleftrightarrow{FS} F_x$

& $y(t) \xleftrightarrow{FS} F_y$

then multiplication property states that

$$x(t) \cdot y(t) \xleftrightarrow{FS} T_0 \; F_x \circledast F_y$$

and convolution property states that

$$x(t) \circledast y(t) \xleftrightarrow{FS} T_0 \; F_x \cdot F_y$$

5) Differentiation

if $x(t) \xleftrightarrow{FS} F_n$

then differentiation property states that

$$\frac{d}{dt} x(t) \xleftrightarrow{FS} j \frac{2\pi n}{T_0} F_n$$

6) Time Scaling

if $x(t) \xleftrightarrow{FS} F_n$

$x(at) \xleftrightarrow{FS} F_n$

Time scaling changes frequency component from $1/T_0$ to a/T_0

Dirchlet's Condition for Existence of Fourier Series

1) The function should be absolutely integrable

i.e. $\int_{-\infty}^{\infty} |f(t)| \, dt < \infty$

2. There must be finite number of maxima and minima in the function

3. There must be finite number of discontinuties in the function.

Trigonometric Fourier Series

$$f(t) = a_0 + a_1 \cos w_0 t + a_2 \cos 2w_0 t + \dots$$
$$+ b_1 \sin w_0 t + b_2 \sin 2w_0 t + \dots$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + \sum_{n=1}^{\infty} b_n \sin n w_0 t$$

$$w_0 = \frac{2\pi}{T}$$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos n w_0 t \, dt$$

$$b_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \sin n w_0 t \, dt$$

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) \, dt$$ (also called DC component)

Exponential Fourier Series

$$f(t) = F_0 + F_1 e^{j w_0 t} + F_2 e^{j 2 w_0 t} + \dots +$$
$$F_{-1} e^{-j w_0 t} + F_{-2} e^{-j 2 w_0 t}$$

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{j n w_0 t}$$

$$F_n = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) e^{-j n w_0 t} \, dt$$

Relation between Trigonometric and Exponential Fourier Series

$F_0 = a_0$

$a_n = F_n + F_{-n}$

$b_n = j(F_n - F_{-n})$

Fourier Transform (Dirchlet's Conditions)

(i) $f(t)$ should be absolutely integrable i.e. $\int_{-\infty}^{\infty} |f(t)| \, dt$

(ii) The function must have finite number of maximas and minimas

(iii) The function must have finite number of discontinuties.

Formula

Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \, e^{-j\omega t} \, dt$$

Inverse Fourier Transform

$$x(t) = \frac{1}{2a} \int_{-\infty}^{\infty} X(\omega) \, e^{j\omega t} \, d\omega$$

Properties

1) Linearity

If $x_1(t) \xleftrightarrow{F.T.} X_1(\omega)$

and $x_2(t) \xleftrightarrow{F.T.} X_2(\omega)$

then $a_1 x_1(t) + a_2 x_2(t) \xleftrightarrow{F.T.} a_1 X_1(\omega) + a_2 X_2(\omega)$

2. Time Shifting Property

$x(t) \xleftrightarrow{F.T.} X(\omega)$

then $x(t - t_0) \xleftrightarrow{F.T.} e^{-j\omega t_0} X(\omega)$

3. Frequency Shifting Property

If $x(t) \xleftrightarrow{FT} X(\omega)$

then $x(t)\, e^{j\omega_0 t} \longleftrightarrow F(\omega-\omega_0)$

4. Time Differentiation and Integration Properties

If $x(t) \xleftrightarrow{FT} X(\omega)$

then $\dfrac{d\,x(t)}{dt} \xleftrightarrow{F.T.} j\omega\, X(\omega)$

$$\int x(t)\,dt \xleftrightarrow{F.T.} \frac{1}{j\omega} X(\omega)$$

$$\frac{d^n x(t)}{dt^n} \xleftrightarrow{F.T.} (j\omega)^n X(\omega)$$

$$\iiint_n x(t)\,dt \xleftrightarrow{F.T.} \frac{1}{(j\omega)^n} X(\omega)$$

5. Duality or Similarity

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

then $X(t) \xleftrightarrow{F.T.} 2\pi\, x(-\omega)$

6. Time Scaling Property

If $x(t) \xleftrightarrow{F.T} X(\omega)$

then $x(at) \xleftrightarrow{F.T.} \dfrac{1}{|a|} X\left(\dfrac{\omega}{a}\right)$

7. Multiplication and Convolution Property

If $x_1(t) \xleftrightarrow{F.T.} X_1(\omega)$

and $x_2(t) \xleftrightarrow{F.T.} X_2(\omega)$

then Multiplication property is

$$x_1(t) \cdot x_2(t) \xleftrightarrow{F.T.} X_1(\omega) \circledast X_2(\omega)$$

Convolution property is

$$x_1(t) \circledast x_2(t) \xleftrightarrow{F.T.} X_1(\omega) \cdot X_2(\omega)$$

Basic Fourier Transform Pairs.

1) $e^{-at}u(t) \xleftrightarrow{F.T.} \dfrac{1}{a+j\omega}$

2) $e^{at}u(-t) \xleftrightarrow{F.T.} \dfrac{1}{a-j\omega}$

3) $t^n e^{-at}u(t) \xleftrightarrow{F.T.} \dfrac{1}{(a+j\omega)^{n+1}}$

4) $e^{-a|t|} \xleftrightarrow{F.T.} \dfrac{2a}{a^2+\omega^2}$

5) $e^{-at^2} \xleftrightarrow{F.T.} e^{-\omega^2/4a}$

6) $u(t) \xleftrightarrow{F.T.} \pi\delta(\omega) + \dfrac{1}{j\omega}$

7) $\delta(t) \xleftrightarrow{F.T.} 1$