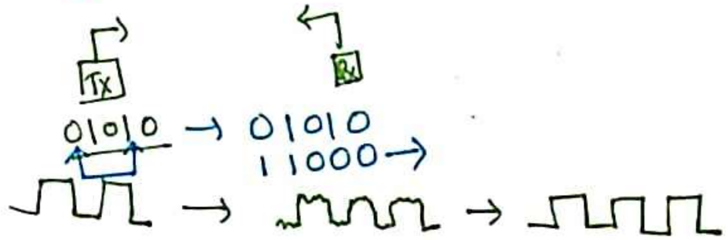
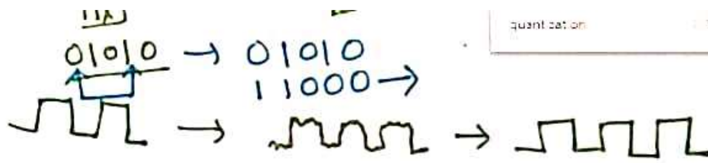
Advantages and Disadvantages of Digital Communication System

Advantages



- Storing Capability
- Inexpensive
- Use of repeaters
- Privacy and Security through the use of encryption
- Data compression, error detection & error correction is possible
- Flexible Hardware Implementation
- Easier & efficient multiplexing by TDMA & CDMA.

- Storing Capability
- Inexpensive
- Use of repeaters

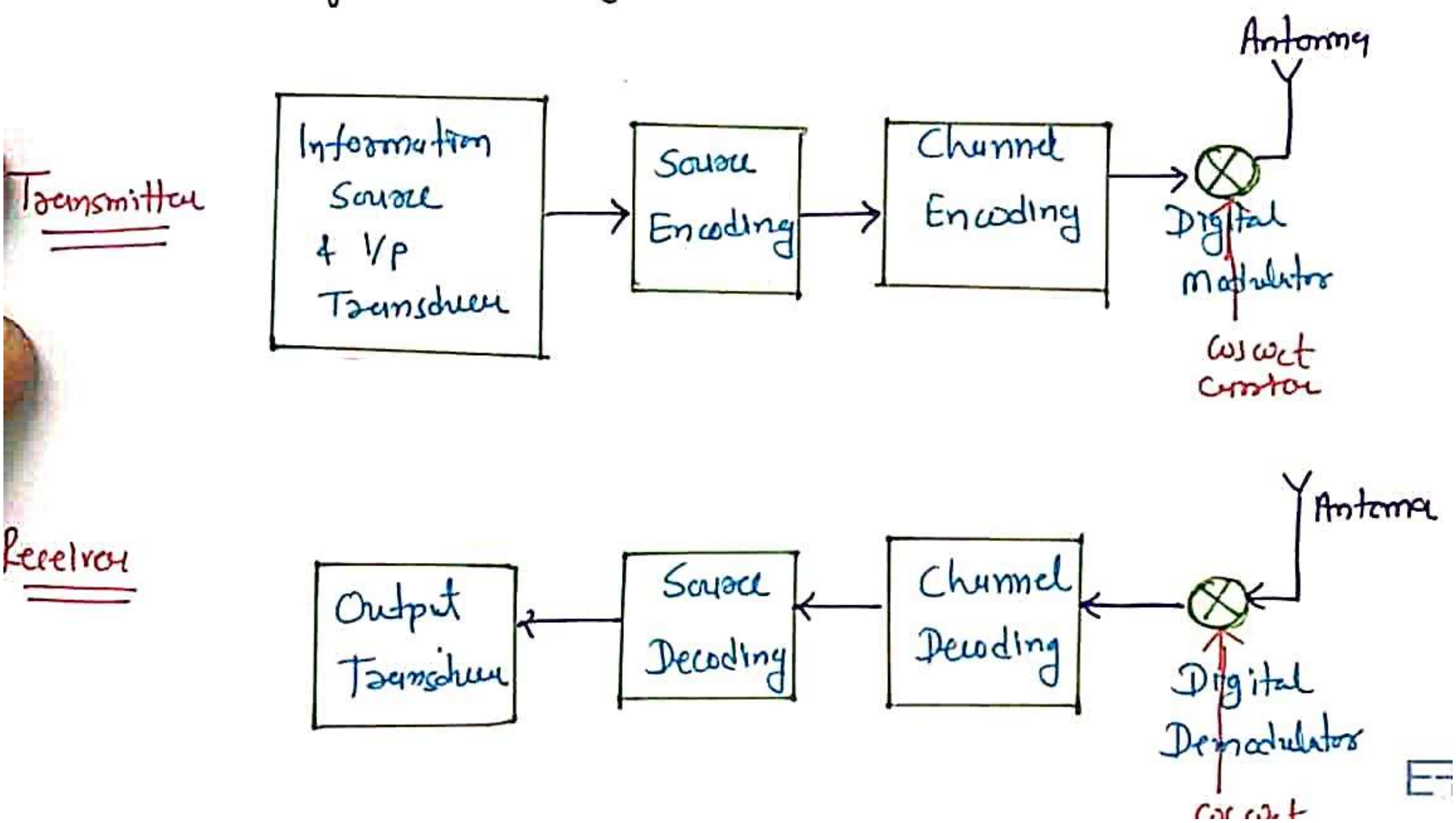


- Privacy and Security through the use of encryption
- Data compression, error detection & error correction is possible
- Flexible Hardware Implementation
- Easier & efficient multiplexing by TDMA & CDMA.

<u>Disadvantages</u>

- Bandwidth is high for channel
- Syncronization is complesory.
- High power consumption due to multiple stages.
- Complex Circuit.

Basic Blockdiagram of digital communication system



Information

- Audio, Video, image and discrete data e.g. Computer O/P
- Non electrical signal Into electrical signal

Source Encoding

- It is used to reduce redundancy
- It is utilized to use BW effecitively
- Data Compression

Huffman Coding

Shannon Fano Coding

Channel Encoding

- It is used to provide noise immunity

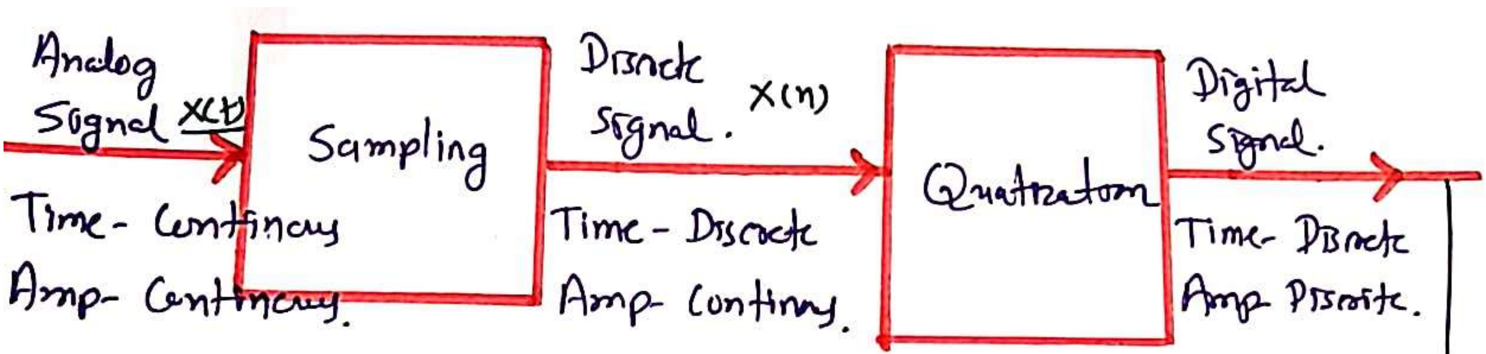
By adding redundancy we can do that

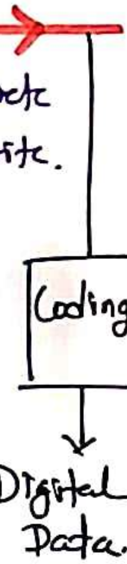
eg. - Block Code
- Cyclic Code
- Convolutional Code

Digital Modulator

- Convert Electrical signal into high freq. modulated signal.

- ASK, FSK, PSK, QPSK



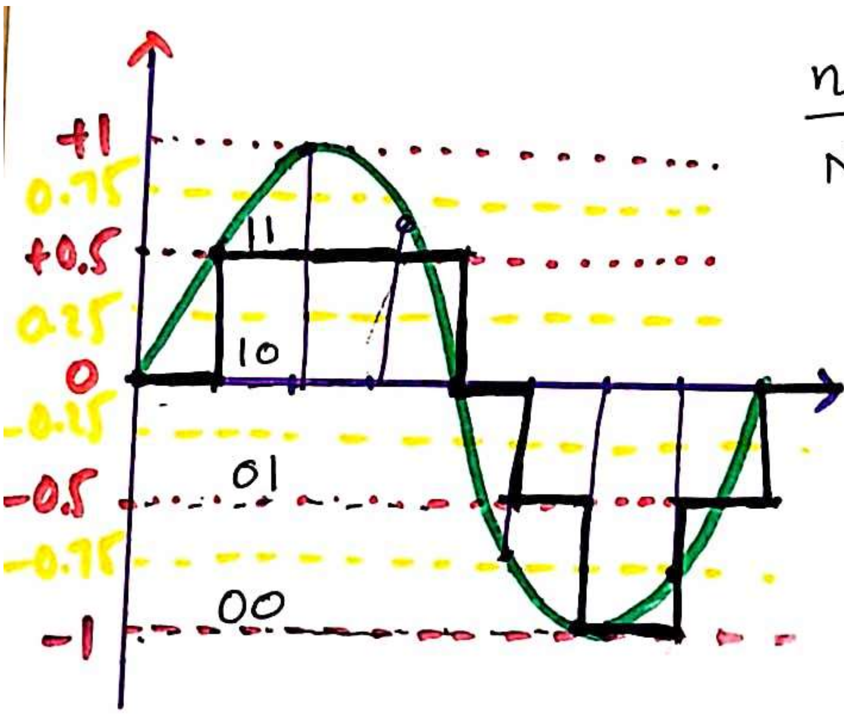


- Analog Signal - $x(t) = \sin(2\pi f t)$

  $t = 0.1, 0.11, 0.112, \ldots$

- Discrete Signal - $x(n) = \sin(2\pi f n T_s)$

  $n = 1, 2, 3, \ldots\ldots$



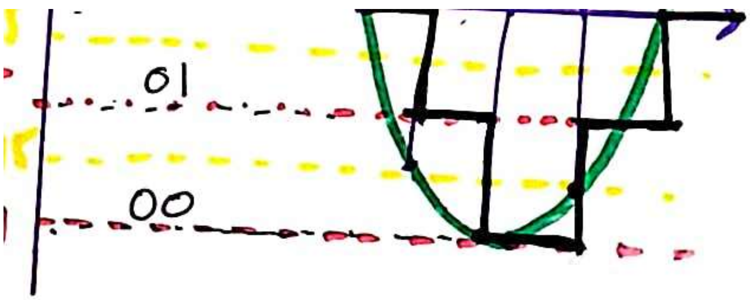
$n = 2$ bit

No of Quntize level $= 2^n$

$= 2^2$

$= 4$

$$\Delta = \frac{X_{max} - X_{min}}{L}$$



L

$$e_q = x_q - x$$

X (Sample Value)	0	0.707	1	0.707	0	-0.701	-1
X_q (Quantized Value)	0	0.5	0.5	0.5	0	-0.5	-1
Digital signal	10	11	11	11	10	01	00
Quantized error	0	0.207	0.5	0.207	0	-0.207	0

Formalities of Quantization

- mapping the continuous range of values into a finite se of values.
- Changing the infinite precision to the finite precision
- Rounding off the Samples to nearest quantization level

→ Bit depth $= n$ = no of bits

→ No of Quantized Levels

$$L = 2^n$$

→ Step - size

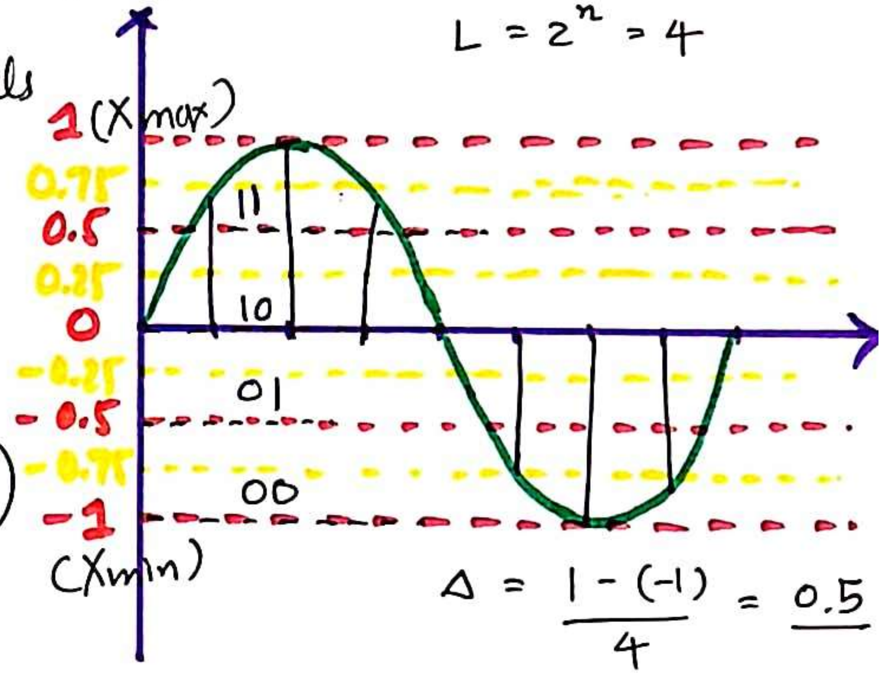
$$\Delta = \frac{X_{max} - X_{min}}{L}$$

→ $I = \text{Round}\left(\frac{X - X_{min}}{\Delta}\right)$

→ $X_q = X_{min} + I\Delta$

$n = 2$

$L = 2^n = 4$



$$\Delta = \frac{1 - (-1)}{4} = \underline{0.5}$$

$= \text{Round}\,(3.414) = 3$

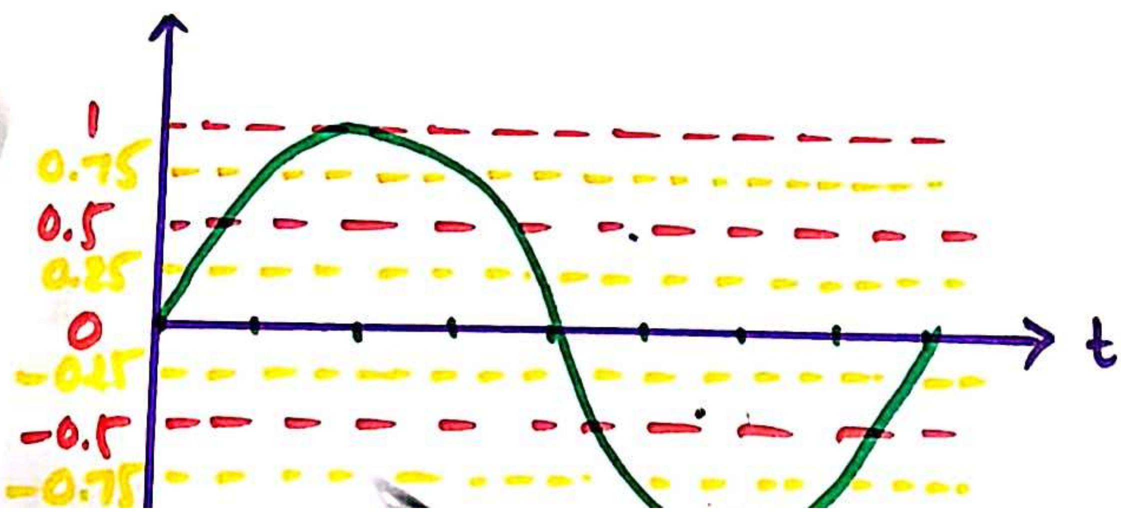
$\rightarrow \quad x_q = X_{min} + I\Delta = -1 + 3\,(0.5) = 0.5$

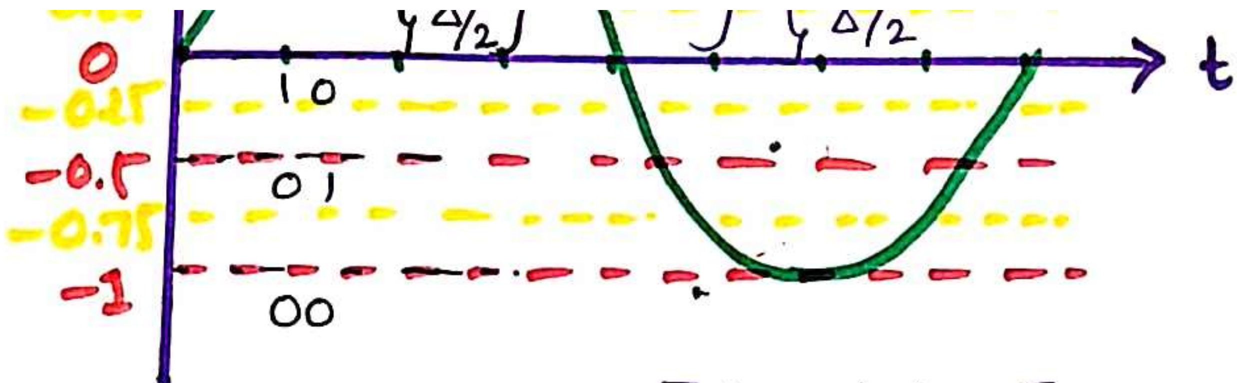
(Sample Value)	0	0.707	1	0.707	0	-0.707	-1
(Index value)	2	3	4 3	3	2	1	0
(Quantized Value)	0	0.5	1 0.5	0.5	0	-0.5	-1
	10	11	11	11	10	01	00

Dynamic Range

(Loudest) (Quietest)

- It is ratio of Largest to Smallest measurable amplitude.





→ Dynamic Range $= 20 \log \left[\dfrac{\text{Largest Amp}}{\text{Smallest Amp}} \right]$

$$= 20 \log \left[\frac{2^{n-1}}{2^{-1}} \right]$$

$$= 20 \log 2^{n} = 20 n \log 2$$

$$\boxed{\text{Dynamic Range} = 6.02\, n}$$

Dynamic range for Human	(Theoritical)	(Practial)	(Noise practial)
Threshold of hearing	0 dB	0 dB	50 dB
Threshold of ear	120 dB	85 dB	85 dB
Dynamic Range	= 120 dB	85 dB n = 16 bit	35 dB n = 16 bit

$\rightarrow$ $SNR = 20 \log \left[\dfrac{\text{Signal rms Voltage}}{\text{Noise rms Voltage}} \right]$

$$\Rightarrow \overline{q_e^2} = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2 \, dq = \frac{1}{\Delta} \left[\frac{q^3}{3} \right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{\Delta 3} \left[\frac{\Delta^3}{8} - \left(-\frac{\Delta^3}{8} \right) \right]$$

$$= \frac{\Delta^2}{12}$$

$$\Rightarrow \boxed{q_e = \frac{\Delta}{2\sqrt{3}}}$$

- full scale signal = $V_{max} - V_{min} = V_{fs}$
- Peak voltage = $V_{fs}/2$
- Rms of Peak voltage = $\frac{V_{fs}}{2\sqrt{2}}$.
- $\Delta = \frac{V_{max} - V_{min}}{2^n} = \frac{V_{fs}}{2^n} \Rightarrow V_{fs} = \Delta 2^n$
- Signal rms voltage = $\frac{2^n \Delta}{2\sqrt{2}}$

- $$\boxed{\text{Signal rms voltage} = \frac{2^n \Delta}{2\sqrt{2}}} \quad \text{— ①}$$

- $$SNR = 20 \log \left(\frac{2^n \cancel{\Delta}}{\cancel{2}\sqrt{2}} \Big/ \frac{\cancel{\Delta}}{\cancel{2}\sqrt{3}} \right)$$

$$= 20 \log \left(2^n \sqrt{\frac{3}{2}} \right)$$

$$= 20 \log 2^n + 20 \log \sqrt{3/2}$$

$$\boxed{SNR = 6.02n + 1.76} \quad \text{— Ⓐ}$$

A 12 bit ADC, with analog Input voltage ranging from -2 to 2 V. Determine the following

a. No of Quantization levels

b. Step Size

c. Quantization level, when the analog voltage is 1.33 V

d. Quantization error

e. Dynamic range.

f. SNR of quantization.

f. SNR of quantization.

$\rightarrow$ $L = 2^n = 2^{12} = 4096$

$\rightarrow$ $\Delta = \dfrac{V_{max} - V_{min}}{L} = \dfrac{2-(-2)}{4096} = \dfrac{4}{4096} = 0.0009765625$

$\rightarrow$ for $x = 1.33$. find x_q

$$I = \text{round}\left(\frac{x - x_{min}}{\Delta}\right)$$

$$= \text{round}\left(\frac{1.33-(-2)}{0.0009765625}\right)$$

$$= \text{round}(3409.92)$$

$$= 3410$$

$\rightarrow$ $x_q = x_{min} + \Delta I$

$$= -2 + 3410 \times 0.000976562$$

$$= 1.330078125$$

$= 3410$

→ Quantization error

$e = x_q - x$

$= 1.330078125 - 1.33$

$= 0.000078125$

→ Dynamic Range $= 6.02n = 6.02 \times 12 = 72.24\ dB$

→ SNR $= 6.02n + 1.76$

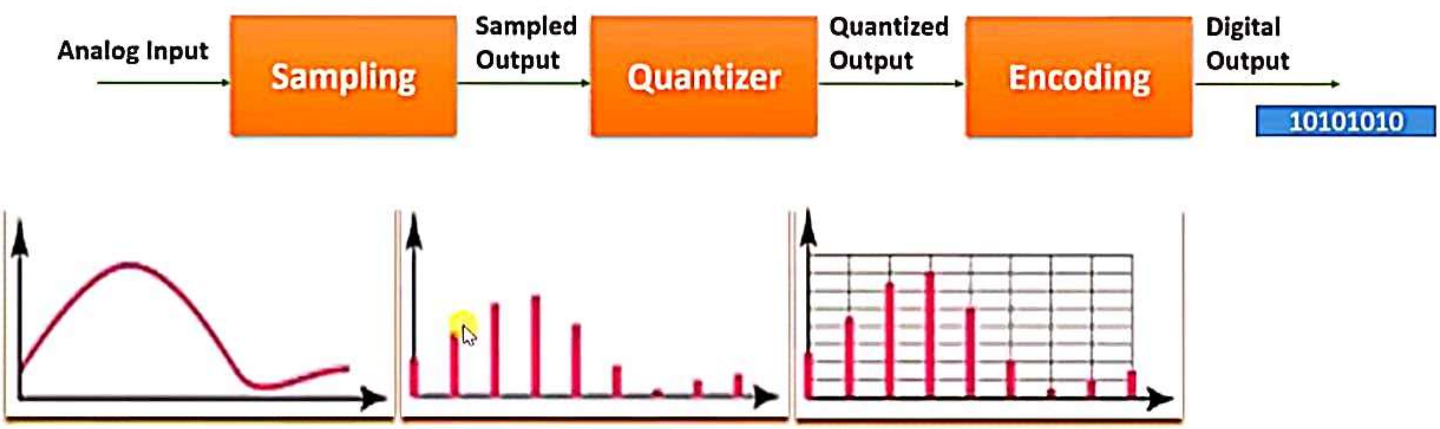
$= 72.24 + 1.76$

$= 74\ dB$

Basics of Pulse Code Modulation

- ❖ Pulse Code Modulation is used to convert Analog signal into Digital data.
- ❖ In PCM, 1st we do sampling to convert analog signal into discrete signal.
- ❖ After that, we do quantization to convert discrete signal into digital signal.
- ❖ After that, we do encoding of that digital signal.

Block Diagram of Pulse Code Modulation



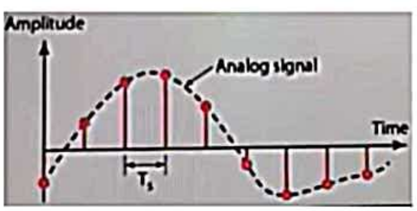
Note : If high frequency components are there at Analog input, then we should use LPF at input before we give signal to sampling

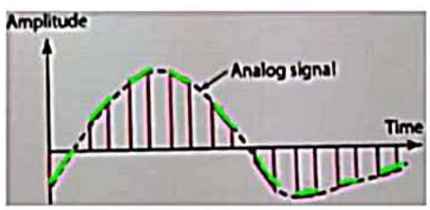
Process of Pulse Code Modulation

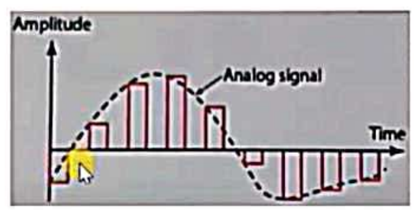
- Filtering
- Sampling
- Quantization
- Encoding

Sampling in Pulse Code Modulation

- ❖ Analog signal is sampled after every Ts interval.
- ❖ So sampling frequency is fs = 1/Ts.
- ❖ There are three sampling method
  - ❑ Ideal sampling : An impulse at each instant.
  - ❑ Natural sampling : A pulse of short width with varying amplitude.
  - ❑ Flat Top sampling : A pulse of short width with fixed amplitude.







Quantization in Pulse Code Modulation

- ❖ The process of measuring the numerical values of the samples and giving them a table value in a suitable scale.
- ❖ The finite number of amplitude intervals is called 'quantizing interval'.
- ❖ There can be two categories
  - ❑ Linear quantization
  - ❑ Non Linear quantization
- ❖ Difference between sampled output and quantized output is quantization distortion.
- ❖ To decrease this distortion, we can increase number of levels by increasing number of bits.

Standards of Pulse Code Modulation

- There are two main standards with PCM
  - The European standards
  - The American standards
- They slightly differ in details but working will remain same.
- European PCM has 30 channels.
- North American PCM has 24 channels.
- Japanese PCM has 24 channels.
- In INDIA, it follows European standards with 30 channels.

Bitrate and Bandwidth of PCM

- Bitrate of PCM can be calculated by

$$Bitrate = n \times f_s$$

- Where, n is bit per sample and fs is sampling frequency.
- Bandwidth is depending on type of encoding is used.
- Digital signal requires more bandwidth but we pay price for robustness and digital communication.

Advantages of Pulse Code Modulation

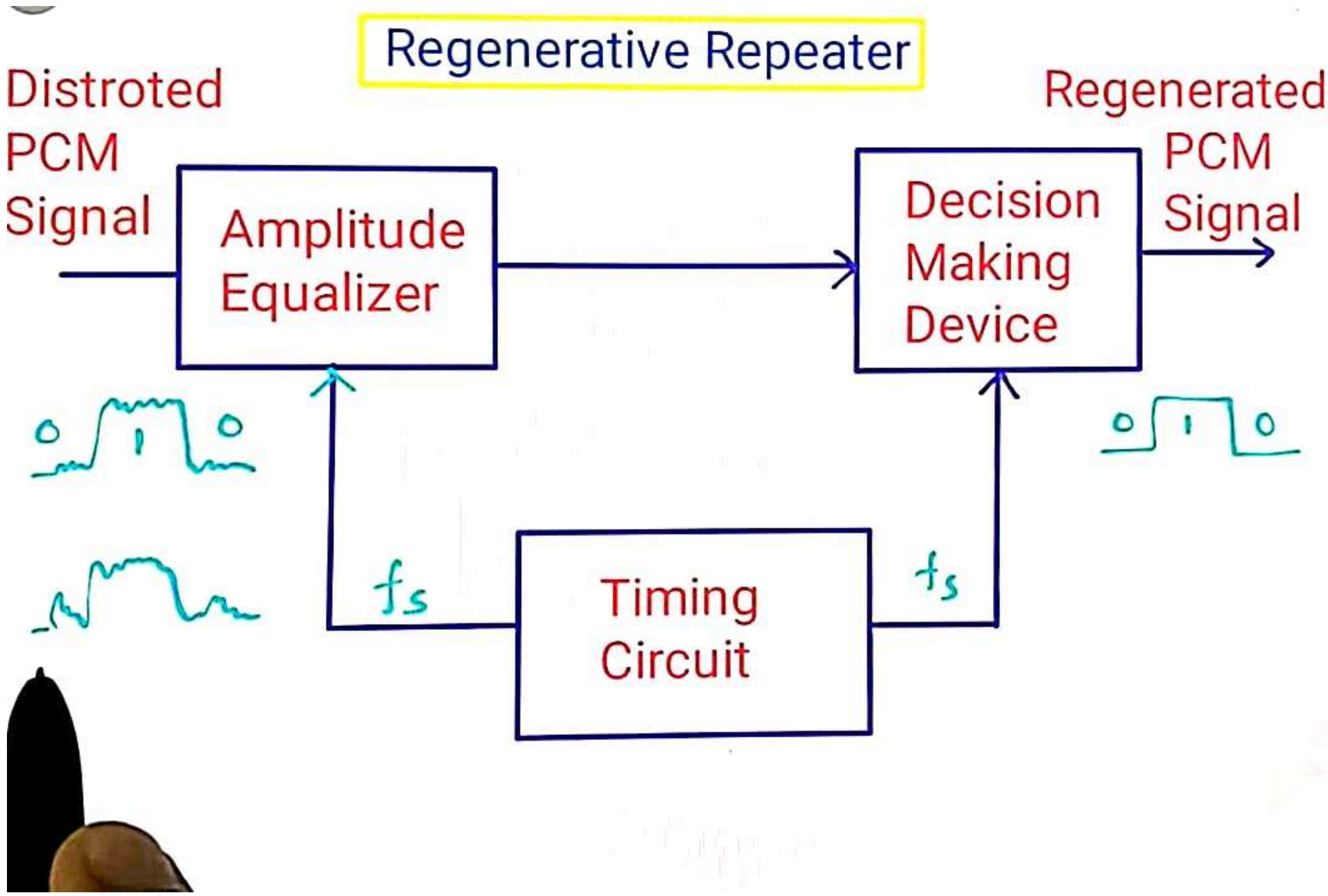
- Uniform transmission quality
- Compatibility of different class of traffic in network
- Integrated Digital Network
- Increased utilization of existing circuit.
- Good performance over poor transmission path.

Disadvantages of Pulse Code Modulation

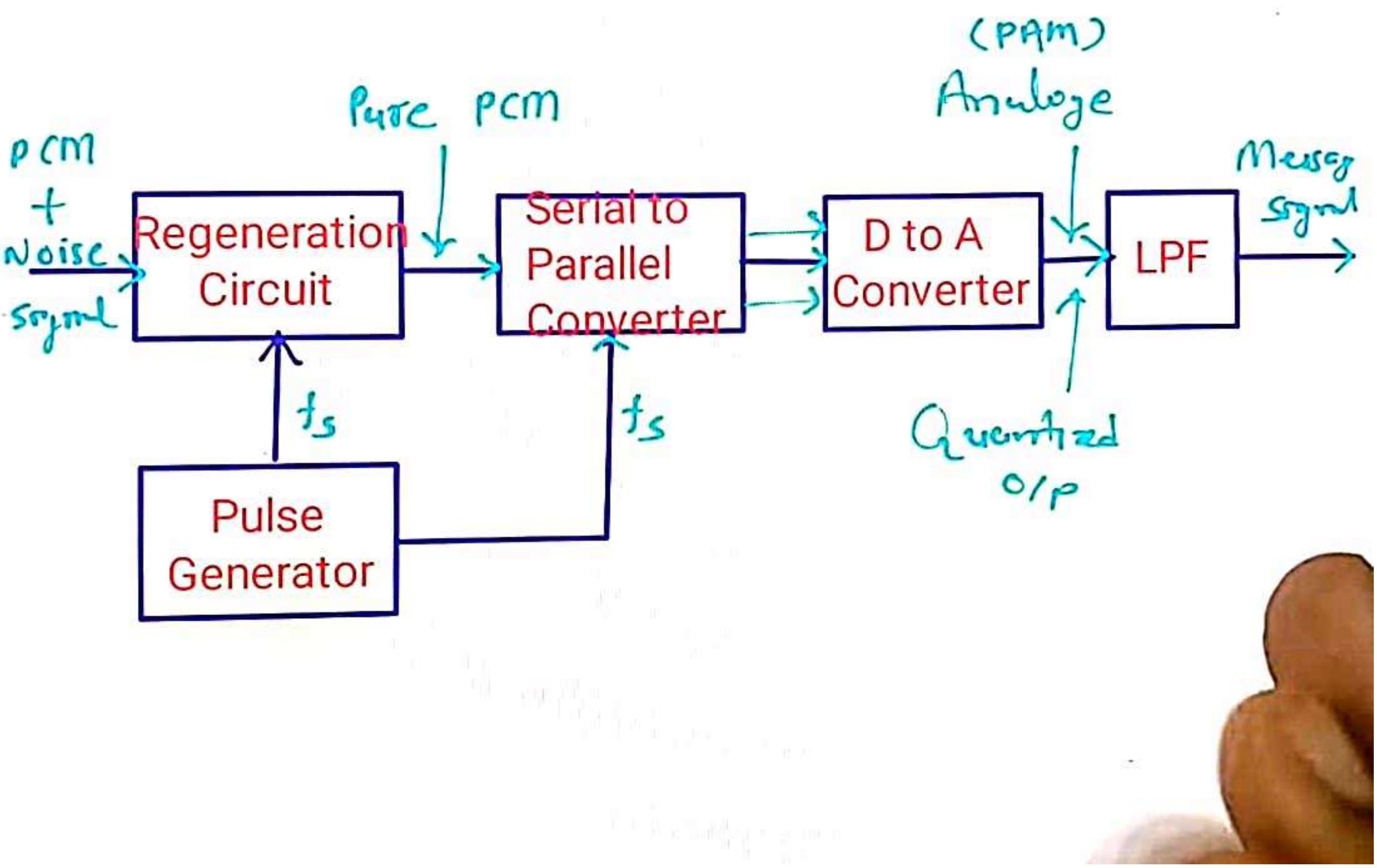
- ❖ Large Bandwidth requires for transmission
- ❖ Noise and Crosstalk leaves low but attenuation will increase

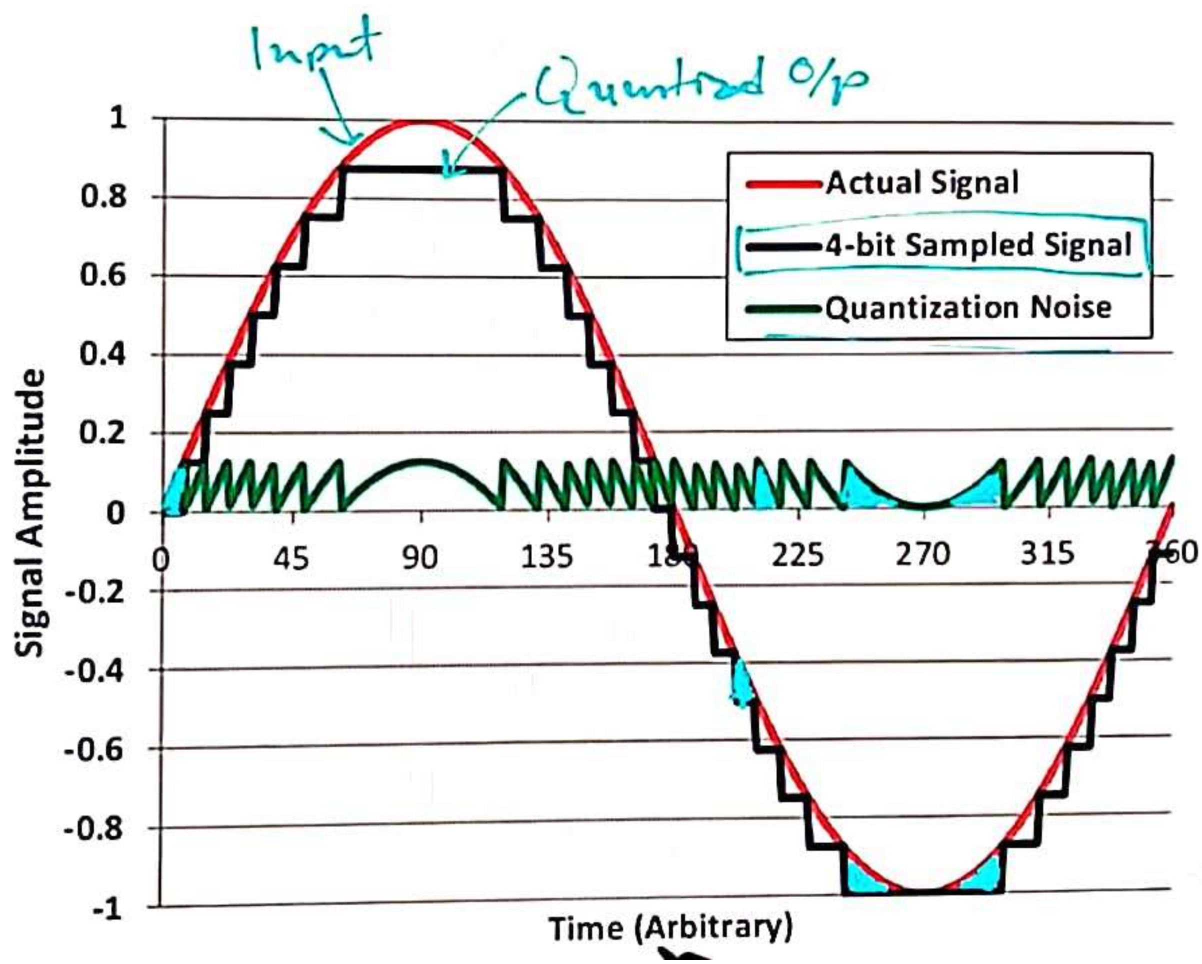
Applications of Pulse Code Modulation

- In Compact disk
- Digital Telephony
- Digital Audio applications



Regenerative Repeater
Distroted PCM Signal
Amplitude Equalizer
Decision Making Device
Regenerated PCM Signal
Timing Circuit
fs
fs
0 1 0
0 1 0


PCM
+
Noise
signal
Regeneration Circuit
Pure PCM
Serial to Parallel Converter
D to A Converter
(PAM)
Analoge
LPF
Message signal
f_s
f_s
Pulse Generator
Quantized
O/P



Input
Quantized O/p
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
45
90
135
180
225
270
315
360
Signal Amplitude
Time (Arbitrary)
Actual Signal
4-bit Sampled Signal
Quantization Noise

→ If n = no of bits per sample

No of quantization levels $L = 2^n$

Step size $\Delta = \frac{V_H - V_L}{L}$

Mean error of quantization $\boxed{e = \frac{\Delta^2}{12}}$

SNR of Quantization $= 1.76 + 6.02n$

$\approx 1.8 + 6n$

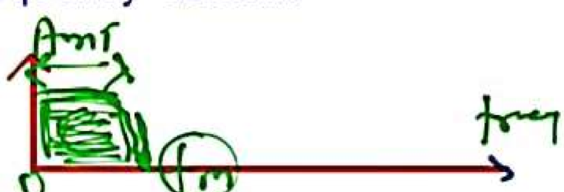
Baseband Signal

1. All sources of information,
Generates baseband signal.
E.g. audio, video, image.

2. Signals are transmitted without
Modulation.
E.g. landline

3. (0 to 20khz) audio signal
(0 to 55Mhz) video signal

4. Frequency domain



5. Unmodulated signals or
demodulated signals

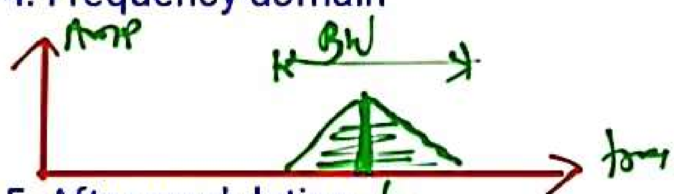
Passband Signal

1. Baseband signal transtransmitted
At high frequency modulated signal.
E.g. AM, FM, PM ....

2. It is high frequency modulated
Carrier signal.
E.g. Satalite signals

3. (550khz - 1650khz) for AM
(88Mhz - 108Mhz) for FM

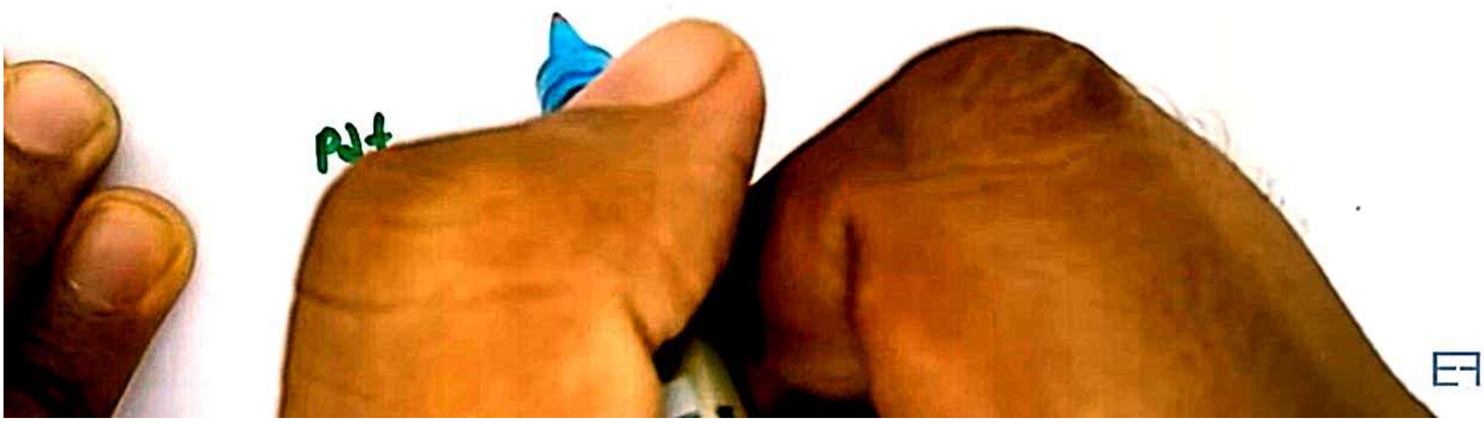
4. Frequency domain



5. After modulation f_c

Quantization Noise & SNR

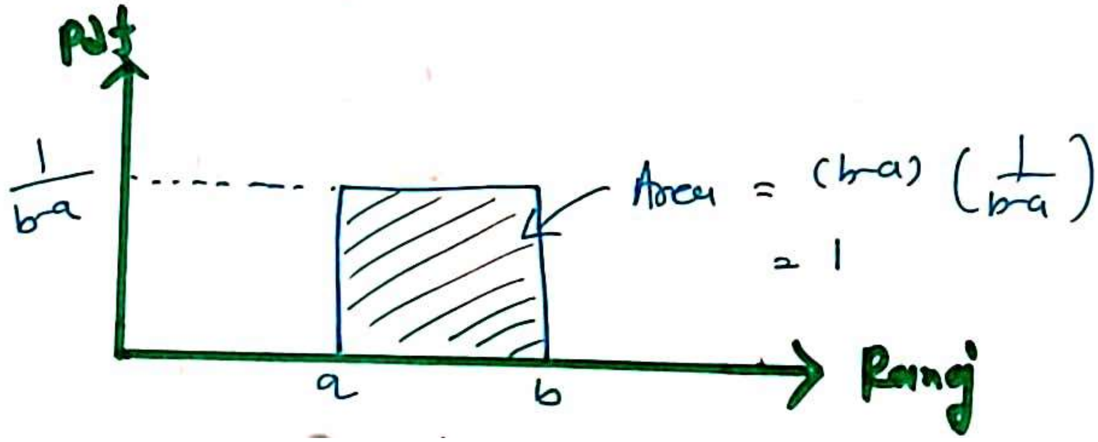
- I have Consider uniform Quantization here.
- the pdf of uniformly distributed random variable 'x' is denoted by



The pdf of uniformly distributed random variable 'x' is denoted by

$$f_X(x) = \frac{1}{b-a} \quad ; \quad a < x < b$$

$$= 0 \quad ; \quad \text{else.}$$



- The pdf of quantization error is given by

$$f_Q(q) = \begin{cases} \frac{1}{\Delta} & ; \quad -\frac{\Delta}{2} < q < \frac{\Delta}{2} \\ 0 & ; \quad \text{else.} \end{cases}$$

- Mean of Qantization error

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{-\Delta/2}^{\Delta/2} q \left(\frac{1}{\Delta}\right) dq$$

$$= \frac{1}{\Delta}\left[\frac{q^2}{2}\right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{2\Delta}\left[\frac{\Delta^2}{4} - \frac{\Delta^2}{4}\right]$$

$$= 0$$

- Variance of Quantization

- Variance of Quantization : Quantization error

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} q^2 \left(\frac{1}{\Delta}\right) dq.$$

| $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

$$= \frac{1}{\Delta} \left[\frac{q^3}{3} \right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\Delta} \left[\frac{\Delta^3}{8} - \left[-\frac{\Delta^3}{8} \right] \right] = \frac{\Delta^2}{12}$$

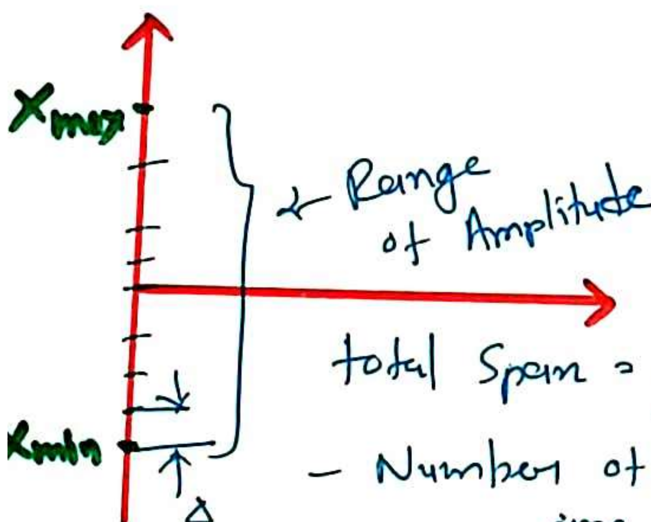
$$\sigma_Q^2 = \frac{\Delta^2}{12}$$

- If n bits are used to represent a sample then Number of quantization levels L.

$$L = 2^n \Rightarrow \log L = n \log 2$$

$$\Rightarrow \boxed{n = \log_2 L}$$

Number of Quantization Level based on Max. amplitude



total Span = $2X_{max}$

- Number of Levels are L.

$$\Rightarrow L = \frac{2|X_{max}|}{\Delta}$$

$$\Rightarrow \Delta = \frac{2|X_{max}|}{L}$$

$$\Rightarrow \boxed{\Delta = \frac{2|X_{max}|}{2^n}}$$

- Varionce

$$\sigma_Q^2 = \frac{\Delta^2}{12} = \left(\frac{2|X_{max}|}{2^n}\right)^2 / 12 = \boxed{\frac{1}{3}|X_{max}|^2 \, 2^{-2n}}$$

- SNR [for Non sinusoidal signal]

$$SNR = \frac{P}{\sigma_Q^2} = \frac{P}{\left(\frac{1}{3}|X_{max}|^2 \, 2^{-2n}\right)} = \boxed{\frac{3P}{|X_{max}|^2} \, 2^{2n}}$$

EA

SNR [for Sinusoidal signal]

- $SNR = \frac{3P}{|X_{max}|^2} 2^{2n}$

- For Sinusoidal signal

$x(t) = X_{max} \sin \omega_m t$

- Avg Power $P = X_{max}^2 / 2$

$$\Rightarrow SNR = \frac{3\left(x_{max}/2\right)}{x_{max}^2}\; 2^{2n}$$

$$\Rightarrow SNR = \frac{3}{2} \times 2^{2n}$$

$$\Rightarrow SNR\,(dB) = 10 \log\left(\frac{3}{2} \times 2^{2n}\right)$$

$$= 10 \log\left(\frac{3}{2}\right) + 10 \log 2^{2n}$$

$$= 1.76 + 20n \log 2$$

$$\boxed{SNR\,(dB) = 1.76 + 6.02\, n}$$

1. A $\boxed{\text{Sinusoidal signal}}$ is transmitted using PCM scheme. The target output SNR should be greater than $\boxed{13\ \text{dB}}$. Find the minimum number if representation levels (L) and the minimum number of bits required to represent each sample to achieve the above performance.

- $SNR\ (dB) = 13 = 1.8 + 6n$

$$\Rightarrow 6n = 11.2$$

$$\Rightarrow n = 1.86 \approx 2$$

- $L = 2^n$

$$= 2^2$$

$$= 4$$

EA

2. Consider a $\boxed{\text{sinusoidal signal}}$ given by

$$s(t) = 3\ \sin(1000\pi t)$$

i. Find the output SNR when the signal is quantized using a $\boxed{\text{9 bit}}$ PCM

ii. Also find the minimum number if bits needed to achieve the output SNR of $\boxed{\text{atleast}}$ 40 dB.

i) $n = 9$ bit

$$SNR\ (dB) = 1.8 + 6n$$
$$= 1.8 + 6 \times 9$$
$$= 55.8\ (dB)$$

ii) $SNR\ (dB) = 40\ dB$

$\Rightarrow SNR\ (dB) = 1.8 + 6n$

$\Rightarrow 40 = 1.8 + 6n$

$\Rightarrow 6n = 38.2$

$\Rightarrow n = 6.36 \approx 7$ bits

Sin Signal   B

3. A TV signal with a maximum frequency of $\boxed{42\ \text{MHz}}$ is transmitted using binary PCM. The number of quantization level is $\boxed{1024.}$ Calculate

L

i. Code word length (n)
ii. Transmission bandwidth
iii. Average output SNR
iv. Bit rate

$\rightarrow L = 1024 = 2^n$

$\Rightarrow \log 1024 = n \log 2$

$\Rightarrow n = \log 1024 / \log 2$

$\Rightarrow \boxed{n = 10 \text{ bits}}$

$\Rightarrow n = \log 1024 / \log 2$

$\Rightarrow \boxed{n = 10 \text{ bits}}$

$\rightarrow$ $W = 42$ MHz

$B = nW$

$= 10 \times 42$ MHz

$= 420$ MHz

$\rightarrow$ $SNR = 1.8 + 6n$

$= 1.8 + 6 \times 10$

$= 61.8$ dB

$\Rightarrow \boxed{n = 10 \text{ bits}}$

$\rightarrow$ $BW = 42\ MHz$

$B = nW$

$= 10 \times 42\ MHz$

$= 420\ MHz$

$SNR = 1.8 + 6n$

$= 1.8 + 6 \times 10$

$= 61.8\ dB$

$\rightarrow$ $f_s = 2W$

$= 2 \times 42\ MHz$

$= 84\ MHz$

$\rightarrow$ $R_b = n f_s$

$\left(\frac{bits}{Sample}\right)\left(\frac{Samples}{Sec}\right)$

$= 10 \times 84\ MHz$

$= 840\ mbps$

X_{max} — W

4. An input signal applied to PCM has a maximum frequency of $\boxed{4\text{ kHz}}$ & the input range varies from -4.8 V to $\boxed{+4.8\text{ V}}$. The average power of input signal is $\boxed{30\text{mW.}}$ ← P The target output SNR is $\boxed{20\text{dB}}$. Assume uniform quantization and PCM produces binary Output.

SNR

   i. Calculate the number of bits required to represent each sample.
   ii. Identify the transmission bandwidth.

$$\Rightarrow SNR = \frac{3P}{|X_{max}|^2} 2^{2n}$$

$$SNR = 20\text{ dB} = 10^2 = 100$$

$$\Rightarrow 100 = \frac{3 \times 30 \times 10^{-3}}{4.8^2} 2^{2n}$$

$$\Rightarrow 2^{2n} = \frac{100 \times 4.8^2}{3 \times 30 \times 10^{-3}}$$

$$\Rightarrow SNR = \frac{3P}{|X_{max}|^2} 2^{2n}$$

$$\Rightarrow 100 = \frac{3 \times 30 \times 10^{-3}}{4.8^2} 2^{2n}$$

$$\Rightarrow 2^{2n} = \frac{100 \times 4.8^2}{3 \times 30 \times 10^{-3}}$$

$$\Rightarrow n = 7.32 \approx 8 \text{ bits}$$

$SNR = 20 \text{ dB}$

$= 10^2$

$= 100$

$$\frac{3 \times 36 \times 10^{-3}}{4.8^2} \; 2^{2n} = 100$$

$$\Rightarrow 2^{2n} = \frac{100 \times 4.8^2}{3 \times 36 \times 10^{-3}}$$

$$\Rightarrow n = 7.32 \approx 8 \text{ bits}$$

$$\rightarrow B = nW$$

$$= 8 \times 4$$

$$= 32 \text{ KHz}$$

5. A compact disc CD records audio signal digitally by using PCM. Assume the audio signal has $\boxed{15 \text{ kHz}}$ frequency. Calculate

W

i. Nyquist rate f_s

ii. If the Nyquist samples are quantized into $\boxed{16384 \text{ levels}}$, identify the number of binary digits to encode a sample.

iii. Determine the signalling rate. (bit rate)

iv. For practical reason, the signals are sampled at a rate above Nyquist rate at 40100 samples per sec. If L = 16384, determine the number of bits per sec received to encode the signal. Also find the transmission bandwidth of the encoded signal.

— $W = 15 \text{ KHz}$

i) $f_s = 2W$

$= 2 \times 15 \text{ KHz}$

$= 30 \text{ KHz}$

of binary digits to encode a sample. (n)

iii. Determine the signalling rate. (bit rate)

iv. For practical reason, the signals are sampled at a rate above Nyquist rate at 40100 samples per sec. If L = 16384, determine the number of bits per sec received to encode the signal. Also find the transmission bandwidth of the encoded signal.

$W = 15$ KHz

$f_s = 2W$

$= 2 \times 15$ KHz

$= 30$ KHz

$L = 16384$

ii) $L = 2^n$

$\Rightarrow 16384 = 2^n$

$\Rightarrow \log 16384 = n \log 2$

$\Rightarrow n = 14$ bits

$= 2 \times 15$ KHz

$= 30$ KHz

$\Rightarrow 16384 = 2^{n}$

$\Rightarrow \log 16384 = n \log$

$\Rightarrow n = 14$ bits

iii) $R_b = n f_s$

$= 14 \times 30$

$= 420$ Kbps

iv) $f_s = 40100$ samples per sec

iv) $f_s = 40100$ samples per sec

$L = 16384$

$\Rightarrow L = 2^n$

$\Rightarrow \boxed{n = 14 \text{ bits}}$

$\Rightarrow f_s = 2W$

$\Rightarrow W = f_s/2$

$= 40100/2$

$= 20050$ Hz

— $B = nW$

$= 14 \times 20050$

≈ 280.7 KHz

W $\quad\quad$ $\Delta/2$ $\quad\quad$ %

6. Consider a PCM system with an accuracy of $\boxed{\pm 0.1\%}$. The analog signal has a maximum frequency of $\boxed{100\text{ Hz}}$ and amplitude of -5V to $\boxed{+5}$. ← X_{max}

   i. Determine the maximum required sampling rate.

   ii. Determine the maximum number of bits required to represent each PCM word.

   iii. Determine the minimum bit rate required for the PCM signal.

   iv. Determine the minimum absolute BW required for the transmission of PCM signal.

i. Determine the maximum required sampling rate. (f_s)

ii. Determine the maximum number of bits required to represent each PCM word. (n)

iii. Determine the minimum bit rate required for the PCM signal. P_b

iv. Determine the minimum absolute BW required for the transmission of PCM signal. B

$$\rightarrow \quad f_s = 2W = 2 \times 100 = 200 \text{ Hz}$$

$$\rightarrow \quad L = \frac{2|X_{max}|}{\Delta} = \frac{|X_{max}|}{(\Delta/2)} = \frac{5}{0.001}$$

$$\Rightarrow \quad L = 5000$$

$$\Rightarrow \quad 2^n = 5000$$

$$\Rightarrow \quad n = 12.28 \approx 13 \text{ bits}$$

$\Rightarrow 2^n = 5000$

$\Rightarrow n = 12.28 \approx 13$ bits

$\rightarrow R_b = n f_s$

$= 13 \times 200$

$= 2600$ bps

$\rightarrow B = nW$

$= 13 \times 100$

$= 1300$ Hz

7. The signal $x(t) = 2\sin(2000\pi t) - 4\sin(4000\pi t)$ is quantized using a 12 bit quantizer. What is the $\boxed{\text{RMS quantization error}}$ and $\boxed{\text{quantization SNR.}}$

$\sigma = \Delta/\sqrt{12}$ $\qquad$ P/σ^2

- $\Delta = \dfrac{2|X_{max}|}{L} = \dfrac{2 \times 6}{2^{12}} = \dfrac{12}{2^{12}} = 2.93 \times 10^{-3}$

- $x(t) = 2\underline{\sin(2000\pi t)} - 4\sin(4000\pi t)$

$x_{max} = 2(1) - 4(-1) = 6$

$x_{max} = 2(1) - 4(-1) = 6$

$6 = \Delta / \sqrt{12}$

$= \frac{2.93 \times 10^{-3}}{\sqrt{12}}$

$= 8.457 \times 10^{-4}$

- $x(t) = V \sin f_m t$

$P = V_m^2 / 2$

- $P = 2^2/2 + 4^2/2$

$= 10$ W.

- $SNR = P/\sigma^2$

$= 10 / 8.457 \times 10^{-4}$

$= 10 \log (\_\_\_\_) = 71.45$

8. **A PCM system uses an uniform quantizer followed by a 6 bit encoder. The bit rate of the system is 50 Mbps.**

$= 10/8.457\times10^{-4}$

$= 10\log(\_\_\_\_) = 71.45$

8. A PCM system uses an uniform quantizer followed by a $\boxed{\text{6 bit}}$ encoder. The bit rate of the system is $\boxed{\text{50 Mbps}}$ $\leftarrow R_b$

n

   i. Identify the message BW for which the system represents satisfactorily.

   ii. Determine the output SNR when a sinusoidal signal of 1MHz frequency is applied as the output.

– $n = 6$

$R_b = 50\,\text{Mbps}$

– q

of the system is $\boxed{50\ \text{Mbps}}$ ← r_b n

i. Identify the message BW for which the system represents satisfactorily.

ii. Determine the output SNR when a sinusoidal signal of 1MHz frequency is applied as the output.

- $n = 6$
  $r_b = 50\ \text{Mbps}$

$\Rightarrow f_s = 2W$

$\Rightarrow W = f_s/2$

- ~~$W\ f_s / r_b$~~

$= \dfrac{(r_b/n)}{2}$

- $\text{SNR(dB)} = 1.8 + 6n$

$= 1.8 + 6 \times 6$

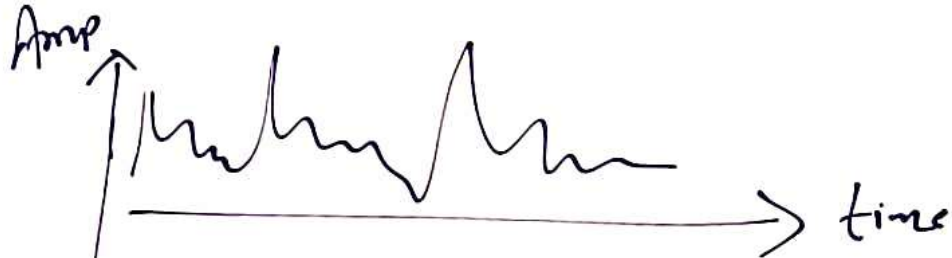
$= 37.8\ \text{dB}.$

$= \dfrac{50/6}{2} = 4.166 \times 10^{6}\ \text{H}$

Nonuniform Quantization

- if quantization characteristic is nonlinear then Step size is not constant, it means quantization is nonuniform quantization
- in non uniform quantization, step size reduce with respect to reduction in signal so quantization noise decreases.
- by companding we can achieve it.
- Nonuniform Quantization is generally used for speech and music signals

→ Crest factor = $\frac{\text{Peak Value of Signal}}{\text{Rms Value of Signal}} = \frac{X_{max}}{X_{rms}}$



→ for music & Speech signal crest factor is very high.

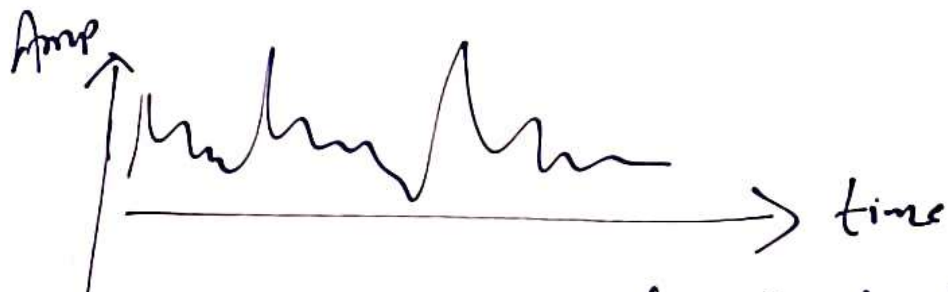
→ Signal power $P = \frac{x^2(t)}{R}$

where, $x(t)$ = mean value of signal

$R = 1$, for normalized Power

E

Rms Value of Signal $\quad x_{rms}$



→ for music & Speech signal crest factor is very high.

→ Signal power $P = \frac{x^2(t)}{R}$

where, $x(t)$ = mean value of signal

$R = 1$, for normalized Power

$$\boxed{P = x^2(t)}$$

Crest factor $CF = \frac{X_{max}}{X_{rms}} = \frac{X_{max}}{\sqrt{\overline{X^2(t)}}} = \frac{X_{max}}{\sqrt{P}}$

→ for normalized signal $X_{max} = 1$

$$CF = \frac{1}{\sqrt{P}} \Rightarrow P = \frac{1}{CF^2}$$

→ for non-sinusoidal signal

$$SNR = 3 \times 2^N \times P$$

→ for voice & speech

$CF >> 1$, $P << 1$, $SNR \rightarrow$ Poor

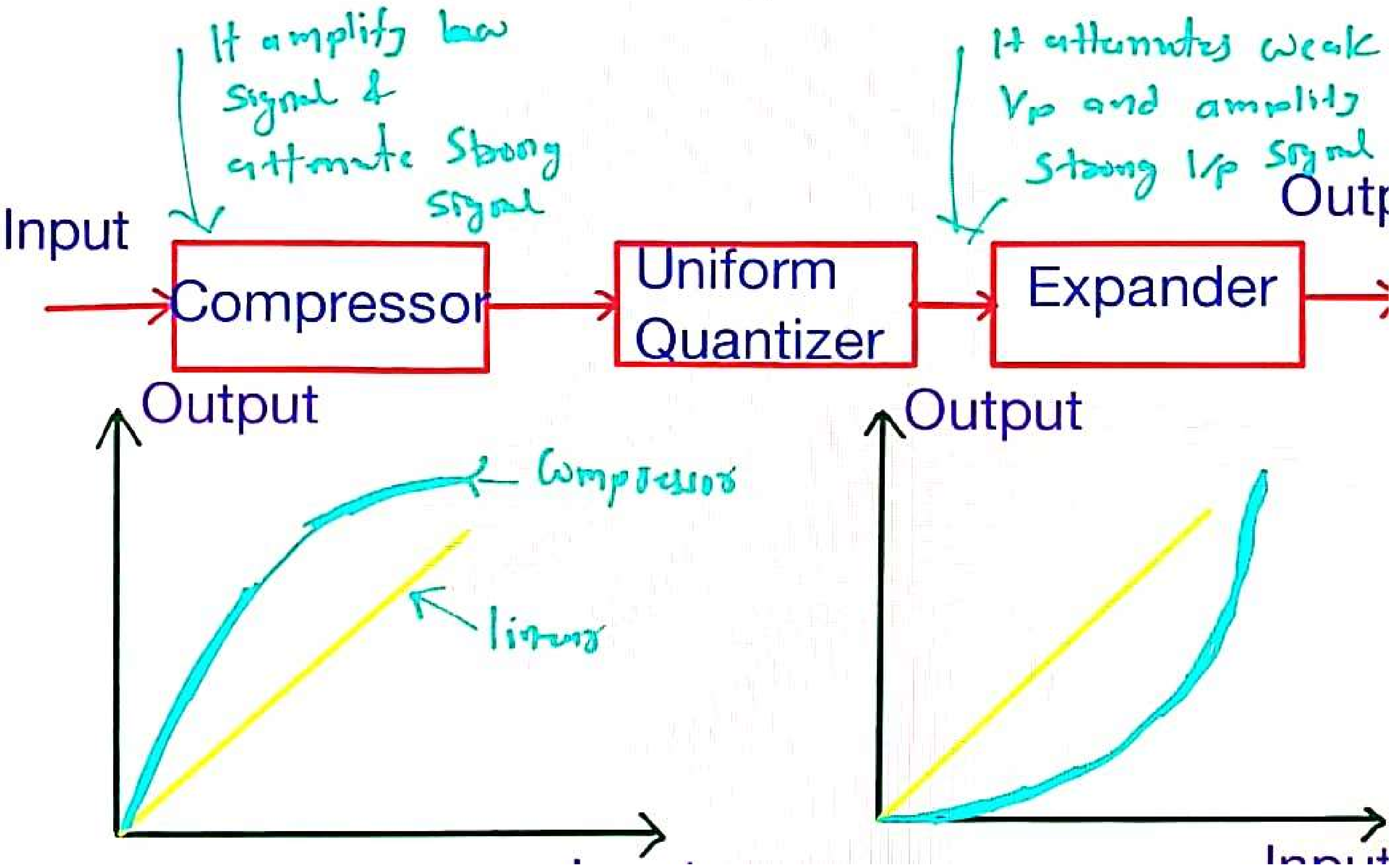
Companding

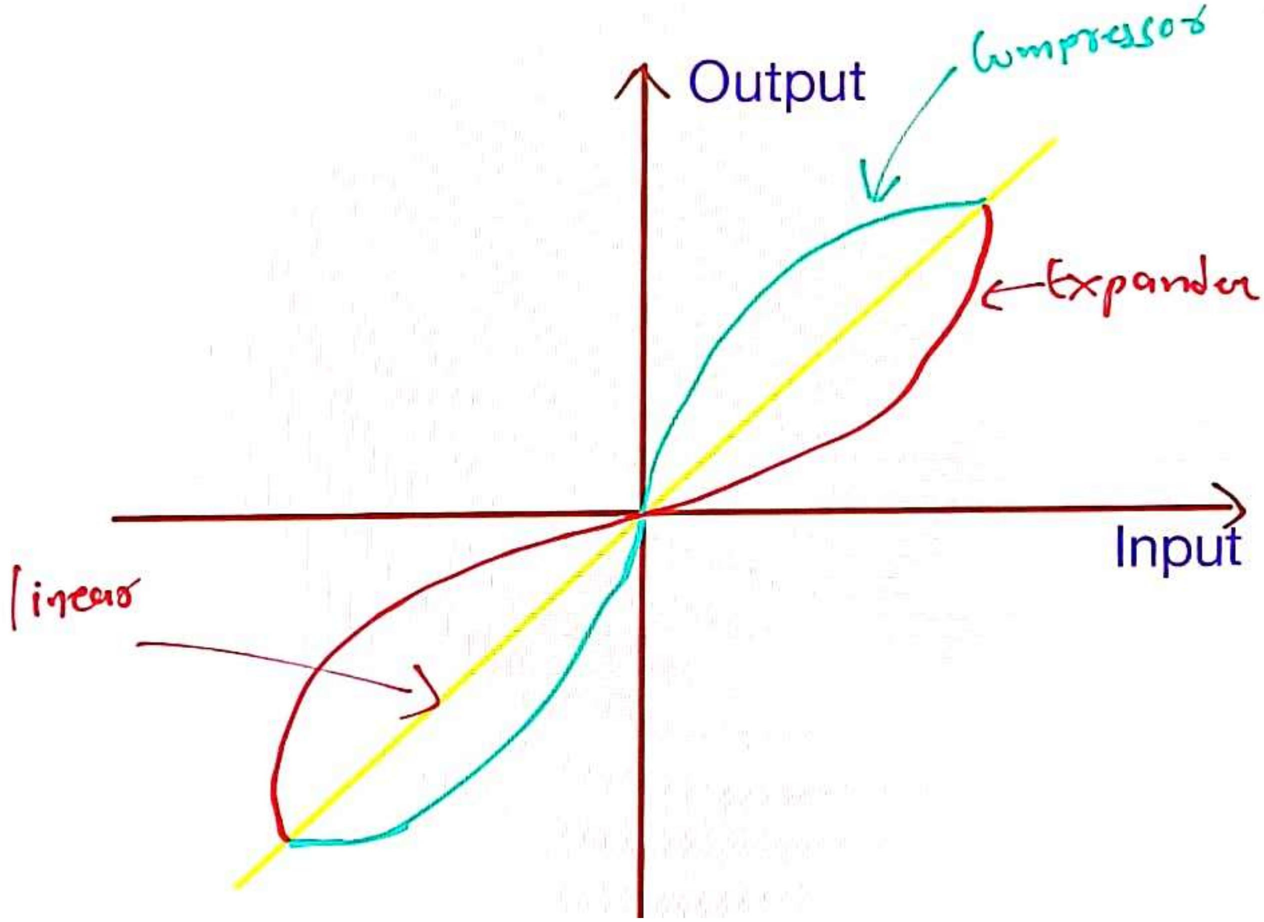
- Companding is Nonuniform Quantization
- It is required to be implemented to improve SNR of weak signal
- Quantization noise is given by

$$\boxed{N_q = \frac{\Delta^2}{12}}$$

- For weak signal noise is constant.
- Companding is derived from two words
  1. Compression
  2. Expansion

Companding (2/3)




Output
Compressor
Expander
Input
linear

* μ - Law Companding, Non - uniform Quantization

- Very popular in USA & Japan
- I/p → O/p relationship is given by

$$\frac{|y|}{x_{max}} = \frac{\ln\left[1 + \mu \frac{|x|}{x_{max}}\right]}{\ln[1+\mu]}$$

where,

x = Amp. of I/p signal at a particular Instant

y = Compressed O/p

μ = Unitless parameter used to define the amount of compression.

- Very popular in USA & Japan

- I/p → O/p relationship is given by

$$\frac{|y|}{x_{max}} = \frac{\ln\left[1 + \mu \frac{|x|}{x_{max}}\right]}{\ln[1+\mu]}$$

→ for $\mu = 0$

$$\frac{|y|}{x_{max}} = \frac{\ln[1+0]}{\ln[1+0]} = 1$$

→ there is No Compression

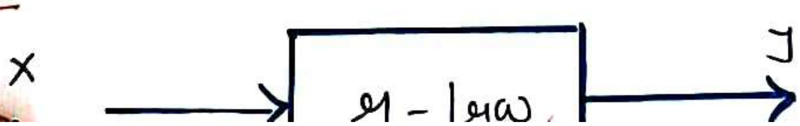
where,

x = Amp. of I/p signal at a particular Instant

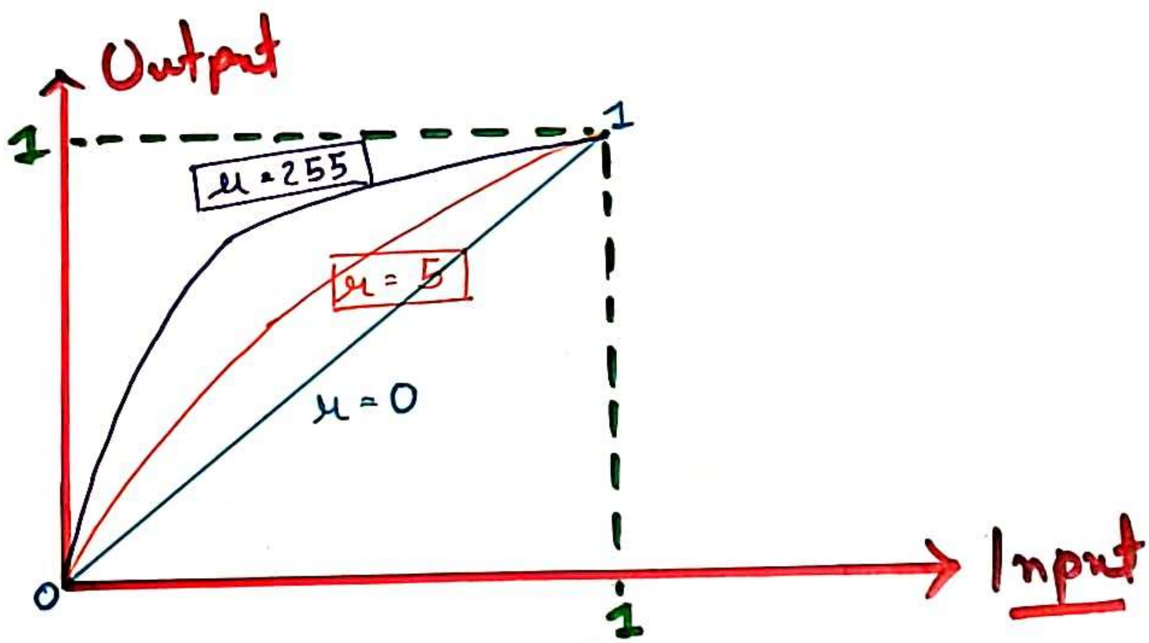
y = Compressed O/p

μ = Unitless parameter used to define the amount of compression.

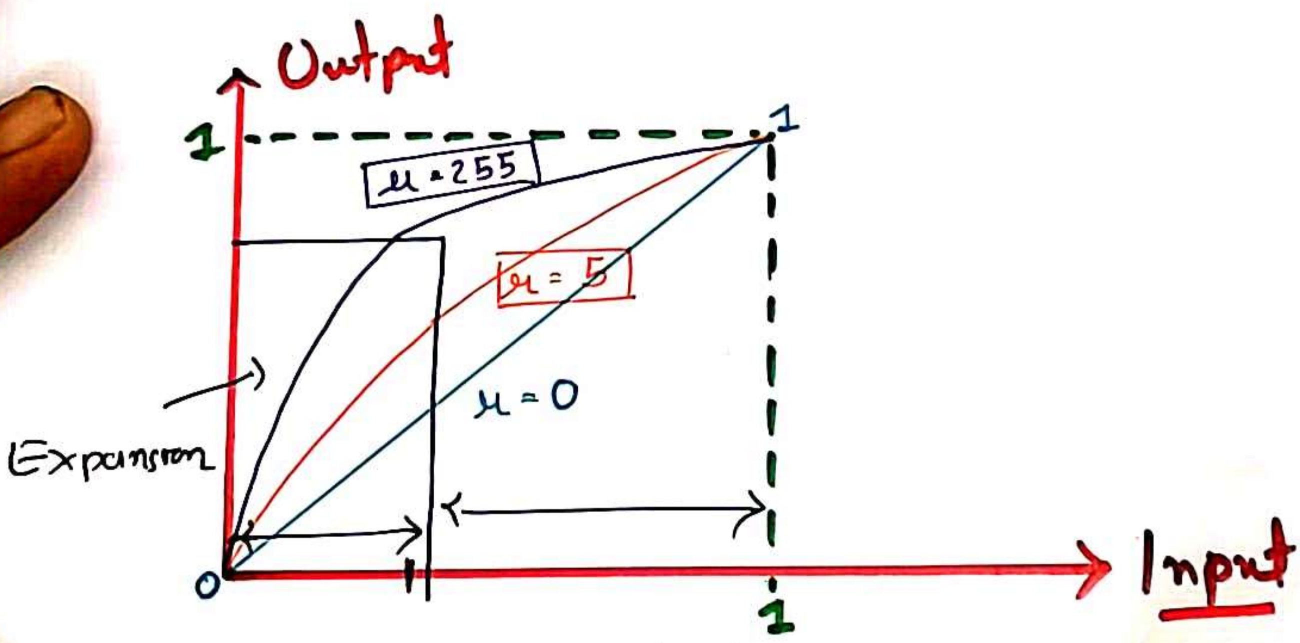
5 to 5

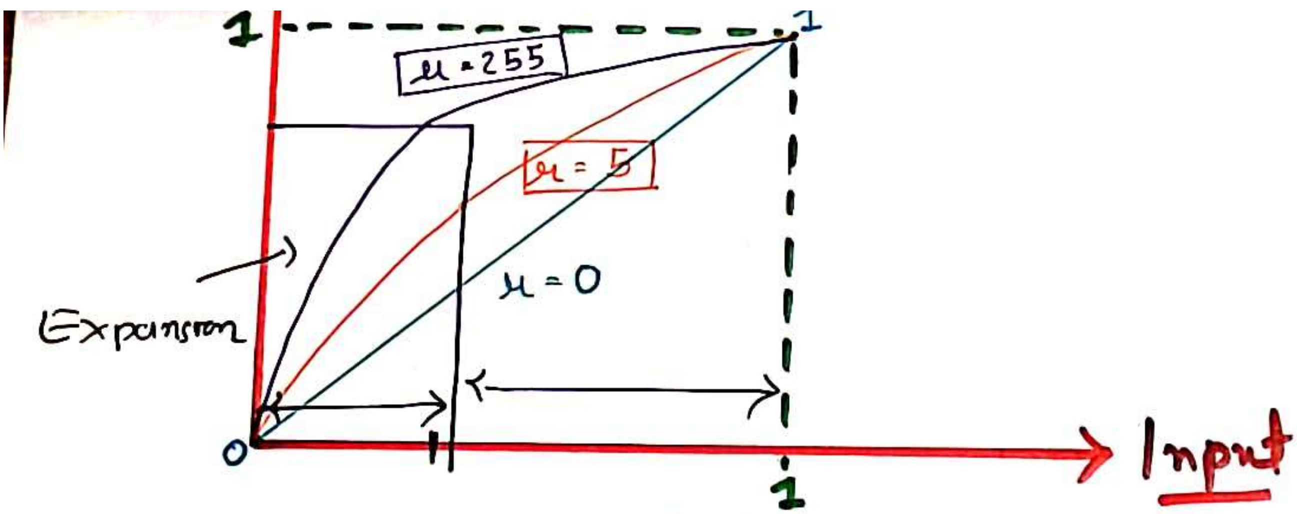


$\left(\frac{|x|}{x_{max}}\right) \to$ (0 to 1) $\longrightarrow$ $\boxed{\mu\text{-law}}$ $\longrightarrow$



$\left(\frac{|x|}{x_{max}}\right) \to$ (0 to 1) $\to$ μ - law $\to$





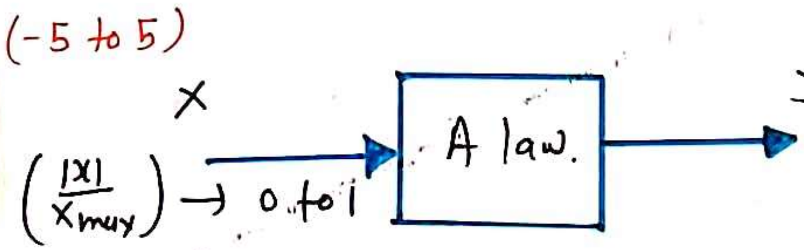
- Larger the value of μ, results into larger compression o/p → i/p with higher amplitude.
- Early Bell Lab Systems used 7 bit PCM with μ = 100
- Recent digital transmission uses 8 bit PCM with μ = 255

… Law …panding, Non-Uniform Quantization

- …y popular in India & in many European countries.
- It has slightly flatter O/p characteristics than μ-law.
- I/p $\rightarrow$ O/p relationship is given by

$$
\frac{|y|}{x_{max}} = \begin{cases} \dfrac{\dfrac{A|x|}{x_{max}}}{1+\ln A} & ; \quad 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\[2em] \dfrac{1+\ln\left[\dfrac{A|x|}{x_{max}}\right]}{1+\ln A} & ; \quad \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}
$$

$$\frac{1 + \ln\left[\frac{A|x|}{x_{max}}\right]}{1 + \ln A} \quad ; \quad \frac{1}{A} \leq \frac{|x|}{x_{max}} \leq 1$$

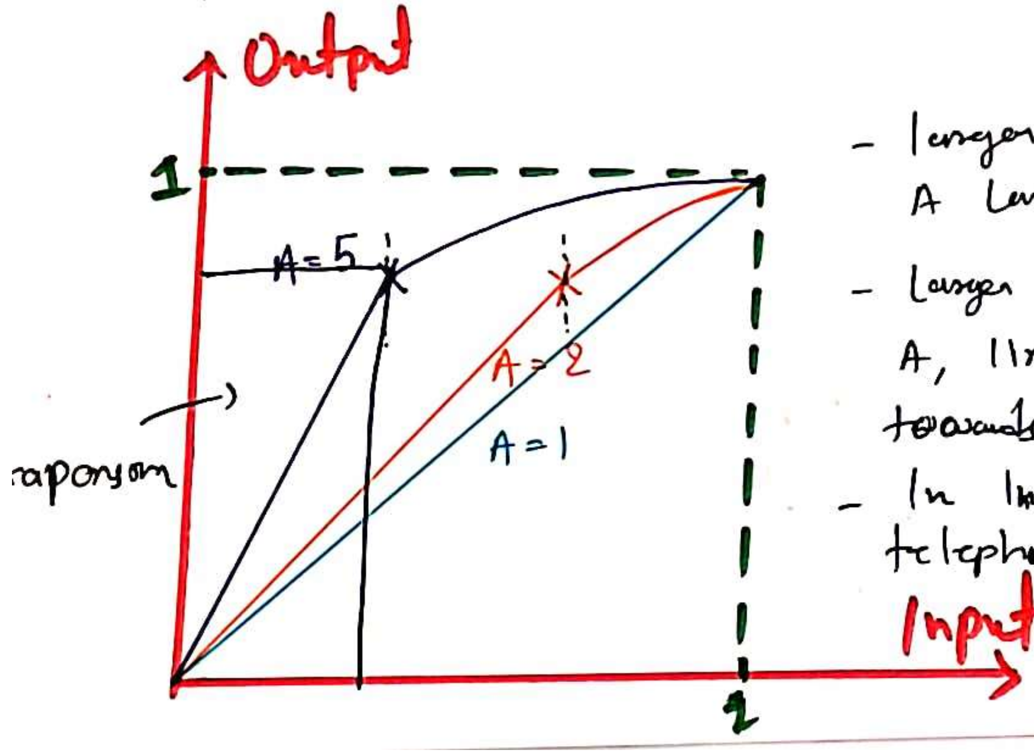


- If $A = 1$

$$\frac{|y|}{x_{max}} = \frac{A|x|/x_{max}}{1 + \ln A} = \frac{1\,|x|/x_{max}}{1 + \ln A} = \frac{|x|}{x_{max}}$$

- So at $A = 1$, there is no compression as I/p = O/p

So at $A = 1$, there is no compression as I/p = O/p



- larger the value of A larger the compression
- Larger the value of A, linear char. shift towards Left.
- In India, for digital telephone system $A = 87.6$

Data rate of PCM

- PCM is the first and popular voice codec developed for speech signal.
- Speech signal

$$300\ Hz - \underbrace{3.4\ KHz}_{\text{Maximum freq}}$$

Maximum freq (BW = W = 3.4 KHz)

- Nyquist rate $= 2W$

$$= 2 \times 3.4 \times 10^3$$

$$= 6.8 \times 10^3 \text{ samples / sec}$$

$= 6.8 \times 10^3$ Samples / sec

- Sampling rate $f_s = 8 \times 10^3$ Samples / sec
- good Quality by FM (Analog) → SNR = 30 dB.
- PCM (Digital) - SNR = $1.8 + 6n$ (I/p Sinusoidal)
- for 8 bit PCM $n = 8$ bits
  - SNR = $1.8 + 6 \times 8 = 49.8$ dB.

$(SNR)_{PCM} > (SNR)_{FM}$.

- SNR = 1.8 + 6×8 = 49.8 dB.

$(SNR)_{PCM} > (SNR)_{FM}$.

- No of Quantization Levels = $L = 2^n = 2^8 = 256$ Levels
- Data rate $R_b = n f_s \left[\frac{\text{bits}}{\text{sample}}\right]\left[\frac{\text{Samples}}{\text{sec}}\right]$

$= 8 \times 8 \times 10^3$

$= \boxed{64 \text{ Kbps}}$ ← 8 bit Audio PCM

- For video signal $B = 4\,MHz \rightarrow f_s = 2B = 8\,MHz$

$R_b = n f_s$

$= 8 \times 8 \times 10^6$

$= \boxed{64 \text{ Mbps}}$ ← 8 bit Video PCM

m is the first and popular voice codec developed
r Speech Signal.

$A = 87.56$ 8 bit PCM } — India
$\mu = 255$ 8 bit PCM } — USA.

peech Signal

$$300\ Hz - \underbrace{3.4\ KHz}_{\text{Maximum freq}}$$

Maximum freq ($BW = W = 3.4\ KHz$)

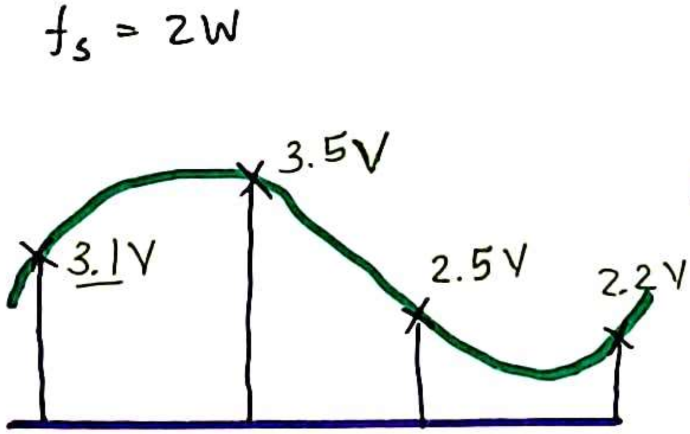
Nyquist rate $= 2W$

$= 2 \times 3.4 \times 10^3$

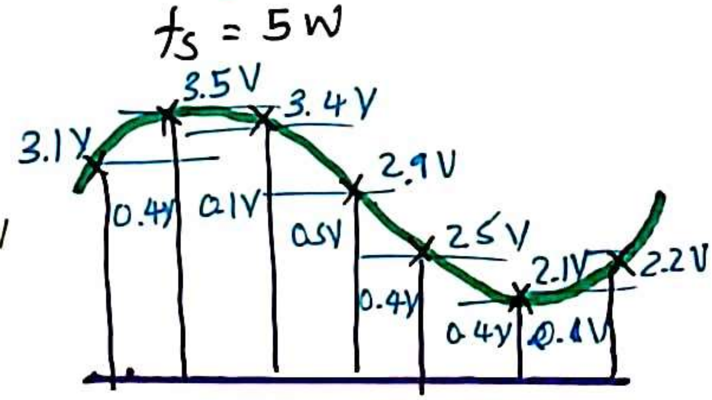
$= 6.8 \times 10^3$ Samples / sec

npling rate $f_s = 8 \times 10^3$ Samp[illegible]s / sec

Differential Pulse Code Modulation [DPCM]



PCM

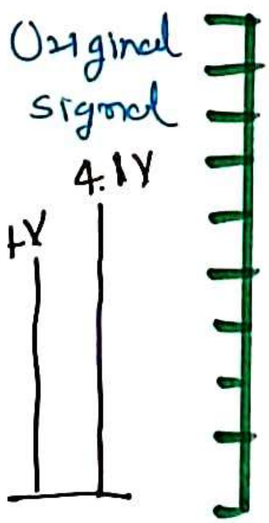


DPCM

- When the Input analog signal is sampled at a rate higher than the Nyquist rate, the successive samples become more correlated.
- Their exist a very little difference between the amplitudes of successive samples.

- Their exist a very little difference between the amplitudes of successive samples.
- When all these samples are quantized and encoded there exist more redundant information in the transmitted signal.

In DPCM, to reduce the redundant information & to achieve more compression, only the difference between the successive samples are transmitted.

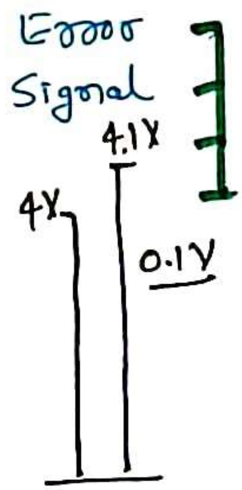


More Levels

- More number of bits per level (samples)

$$L = \frac{2|X_{max}|}{\Delta}$$

$L\uparrow \rightarrow n\uparrow$

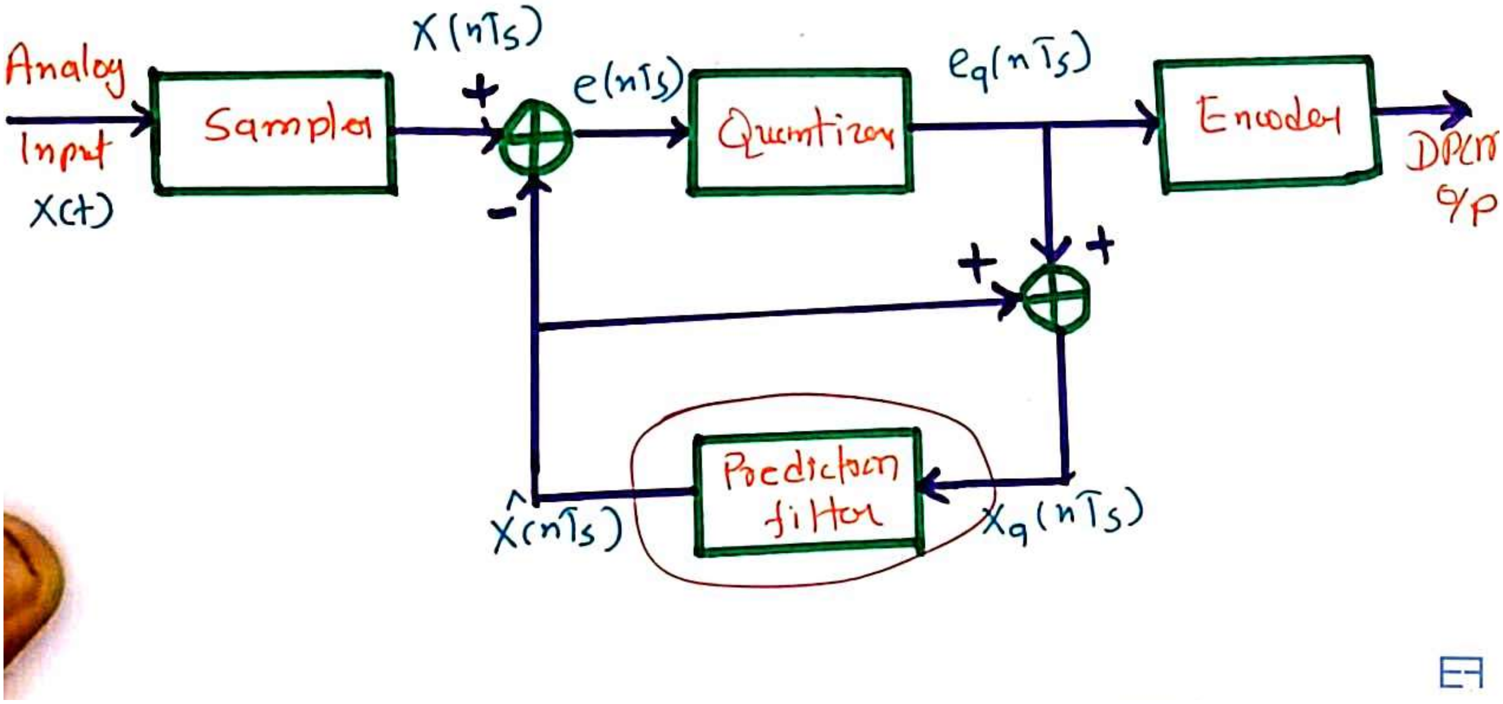


Less Levels

- Less no of bits per Levels (Sampl

$L\downarrow \rightarrow n\downarrow$

DPCM Encoder



- Error signal

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

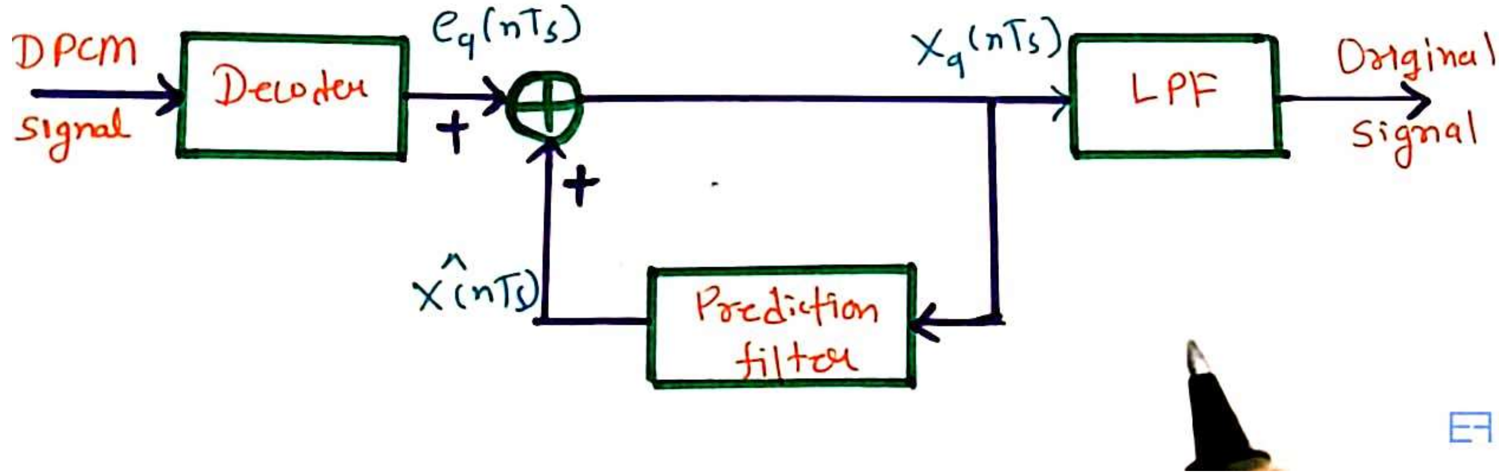
- Quntized error signal

$$e_q(nT_s) = e(nT_s) + q_e(nT_s)$$

- Input to prediction filter.

$$\begin{aligned} x_q(nT_s) &= e_q(nT_s) + \hat{x}(nT_s) \\ &= e(nT_s) + q_e(nT_s) + \hat{x}(nT_s) \\ &= x(nT_s) - \cancel{\hat{x}(nT_s)} + q_e(nT_s) + \cancel{\hat{x}(nT_s)} \\ &= x(nT_s) + q_e(nT_s) \end{aligned}$$

DPCM Decoder



Example of DPCM

Consider the Input samples $x(n) = \{2.1, 2.2, 2.3, 2.6, 2.7, 2.8\}$

Explain how encoding and decoding is done in DPCM,

Assume first order prediction filter $\hat{x}(n) = x_q(n-1)$

Encoder

$x(n)$	$\hat{x}(n) = x_q(n-1)$	$e(n) = x(n) - \hat{x}(n)$	$e_q(n)$	$x_q(n) = \hat{x}(n) + e_q(n)$
2.1	0	$2.1 - 0 = 2.1$	2	$0 + 2 = 2$
2.2	2	$2.2 - 2 = 0.2$	0	$2 + 0 = 2$
2.3	2	$2.3 - 2 = 0.3$	0	$2 + 0 = 2$
2.6	2	$2.6 - 2 = 0.6$	1	$2 + 1 = 3$
2.7	3	$2.7 - 3 = -0.3$	0	$3 + 0 = 3$
2.8	3	$2.8 - 3 = -0.2$	0	$3 + 0 = 3$

2.1	0	2.1 − 0 = 2.1	2	0 + 2 = 2
2.2	2	2.2 − 2 = 0.2	0	2 + 0 = 2
2.3	2	2.3 − 2 = 0.3	0	2 + 0 = 2
2.6	2	2.6 − 2 = 0.6	1	2 + 1 = 3
2.7	3	2.7 − 3 = −0.3	0	3 + 0 = 3
2.8	3	2.8 − 3 = −0.2	0	3 + 0 = 3

Transmitted Sequence

2 0 0 1 0 0 DPCM

010 000 000 001 000 000 Digital Data

coder

1010 000 000 001 000 0001 Digital Data

Decoder

$e_q(n)$	$\hat{x}(n) = x_q(n-1)$	$x_q(n) = \hat{x}(n) + e_q(n)$
2	0 Initially	$2+0 = 2$
0	2	$0+2 = 2$
0	2	$0+2 = 2$
1	2	$1+2 = 3$
0	3	$0+3 = 3$
0	3	$0+3 = 3$

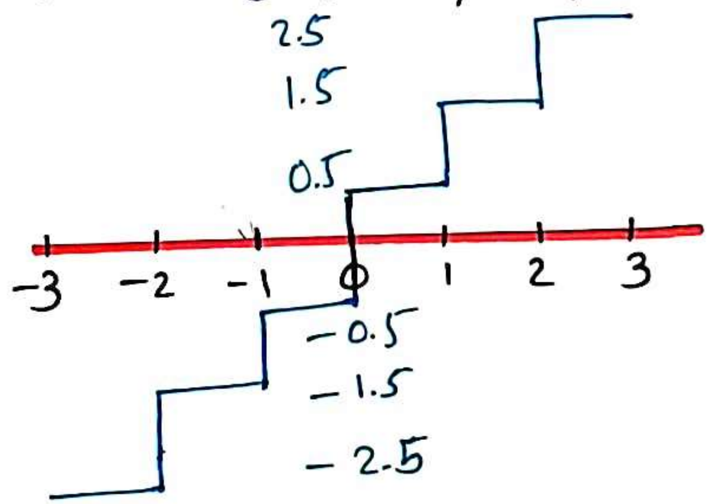
2	0 Initially	$2+0=2$
0	2	$0+2=2$
0	2	$0+2=2$
1	2	$1+2=3$
0	3	$0+3=3$
0	3	$0+3=3$.

Received Signal = $\{2, 2, 2, 3, 3, 3\}$.

Transmitted signal = $\{2.1, 2.2, 2.3, 2.6, 2.7, 2.8\}$.

Problem based on DPCM with Mid-riser Quantizer

* Using mid riser Quantizer, find DPCM output for the given input squence { 0, 0.3, 1.5, 0.7, 1, 2.3 }



-2.5

— Assume 1st Order Prediction filter

$x^{\wedge}(n) = x_q(n-1)$

Encoder

$x(n)$	$x^{\wedge}(n) = x_q(n-1)$	$e(n) = x(n) - \hat{x}(n)$	$e_q(n)$	$x_q(n) = \hat{x}(n) + e_q(n)$
0	0 Initially	0	0.5	0 + 0.5 = 0.5
0.3	0.5	0.3 - 0.5 = -0.2	-0.5	0.5 + (-0.5)
1.5	0	1.5 - 0 = 1.5	1.5	0 + 1.5
0.7	1.5	0.7 - 1.5 = -0.8	-0.5	
1				
2.3				

x(n)	$\hat{x}(n) = x_q(n-1)$	$e(n) = x(n) - \hat{x}(n)$	$e_q(n)$	$x_q(n) = \hat{x}(n) + e_q(n)$
0	0 Intially	0	0.5	0+0.5 = 0.5
0.3	0.5	0.3-0.5 = -0.2	-0.5	0.5+(-0.5) = 0.
1.5	0	1.5-0 = 1.5	1.5	0 + 1.5 = 1.5
0.7	1.5	0.7-1.5 = -0.8	-0.5	1.5+(-0.5) = 1
1	1	1-1 = 0	0.5	1 + 0.5 = 1.5
2.3	1.5	2.3-1.5 = 0.8	0.5	1.5 + 0.5 = 2.

Transmitted Seq = {0.5, -0.5, 1.5, -0.5, 0.5, 0.5}.

Decoder

$e_q(n)$	$\hat{x}(n) = x_q(n-1)$	$x_q(n)$.

Decoder

$e_q(n)$	$\hat{x}(n) = x_q(n-1)$	$x_q(n) = e_q(n) + \hat{x}(n)$.
0.5	0 // Initially	$0.5+0 = \underline{0.5}$
−0.5	0.5	$-0.5+0.5 = \underline{0}$
1.5	0	$1.5+0 = \underline{1.5}$
−0.5	1.5	$-0.5+1.5 = \underline{1}$
0.5	1	$0.5+1 = \underline{1.5}$
0.5	1.5	$0.5+1.5 = 2$

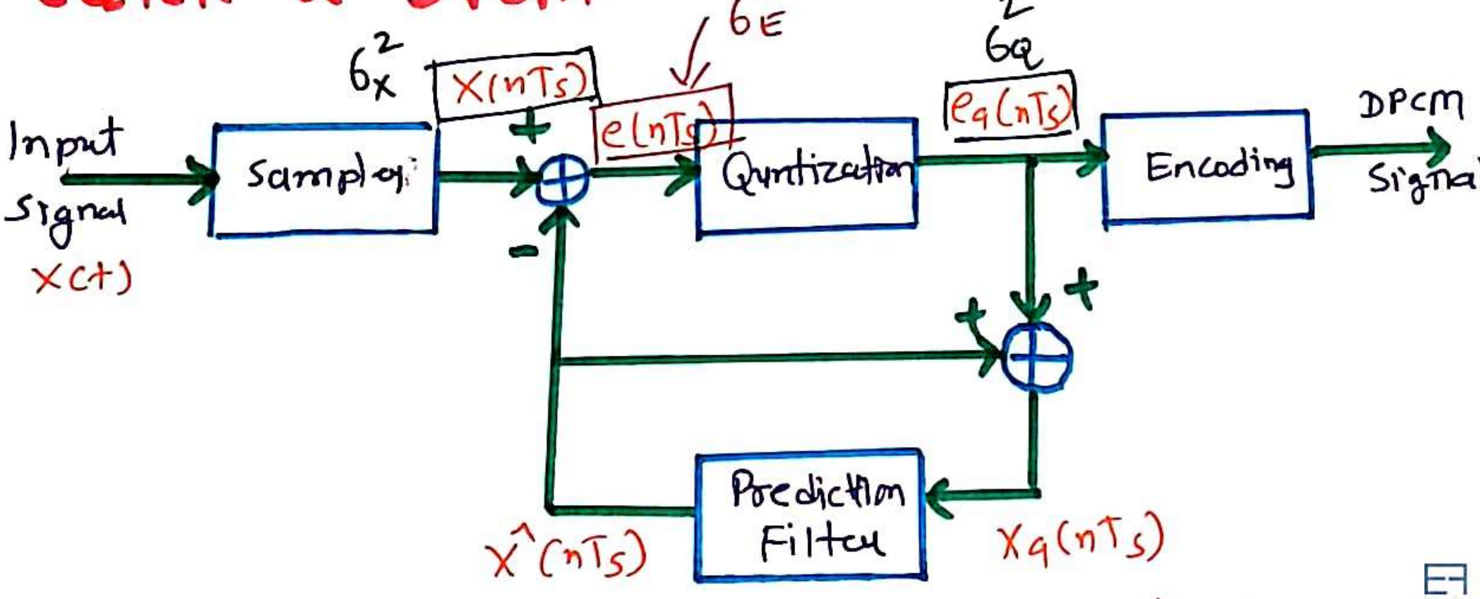
Tx Seq.

{0 , 0.3 , 1.5
0.7 , 1 , 2.3}

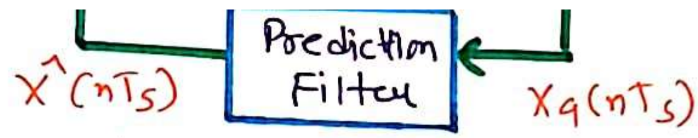
Received Seq.

{0.5 , 0 , 1.5
1 , 1.5 , 2}

SQNR of DPCM



σ_x^2
X(nTs)
σ_E^2
e(nTs)
σ_Q^2
eq(nTs)
Input Signal
x(t)
Sampling
+
-
Quntization
Encoding
DPCM Signal
+
+
Prediction Filter
$\hat{x}(nT_s)$
$x_q(nT_s)$



- Signal to noise ratio at o/p of Quantizer

$$(SNR)_o = \frac{\sigma_x^2}{\sigma_Q^2}$$

σ_x^2 = Variance of Original i/p $x(nT_s)$

σ_Q^2 = Variance of Quantization error $e_q(nT_s)$

σ_E^2 = Variance of Prediction error.

$$(SNR)_o = \underbrace{\boxed{\frac{\sigma_x^2}{\sigma_E^2}}}_{G_p} \underbrace{\boxed{\frac{\sigma_E^2}{\sigma_Q^2}}}_{(SNR)_p}$$

σ_x^2 ... produced by differential

G_p $(SNR)_p$

$G_p = \frac{\sigma_X^2}{\sigma_E^2}$ = Prediction gain produced by differential Quantization

$(SNR)_p = \frac{\sigma_E^2}{\sigma_Q^2}$ = Prediction error to quantization noise ratio.

$$(SNR)_o = G_p \, (SNR)_p$$

→ $G_p > 1 \Rightarrow \frac{\sigma_X^2}{\sigma_E^2} > 1 \Rightarrow \sigma_X^2 > \sigma_E^2$

→ G_p is maximized by minimizing the σ_E^2 of prediction error.

→ Our objective should be to design prediction filter so as to minimize σ_E^2.

Delta Modulation

PCM $\longrightarrow$ DPCM $\longrightarrow$ DM

(all samples are encoded)

(difference of samples are encoded)

(saving in BW)

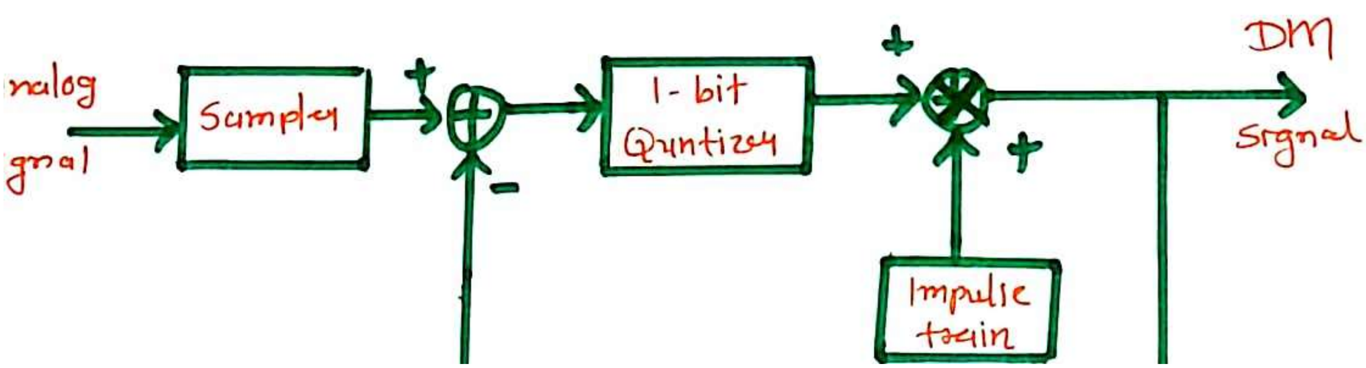
(encode error signal)

$$x(t) \overset{\Delta}{\underset{-\Delta}{\gtrless}} \hat{x}(t)$$

$+\Delta$ and $-\Delta$

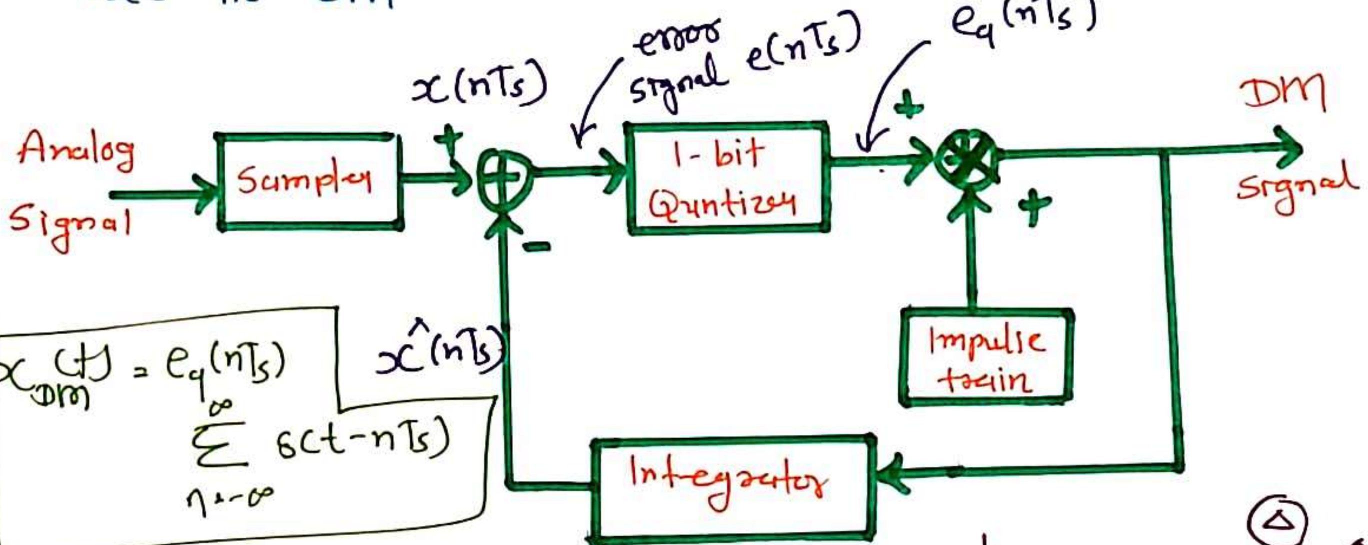
[1 bit requirement].

- In DPCM, the error signal is quantized & coded. based on quantized O/P, the length of the encoded bits for each sample will differ.
- To reduce the transmitted bits, 1-bit quantizer is used in DM



E

used in DM

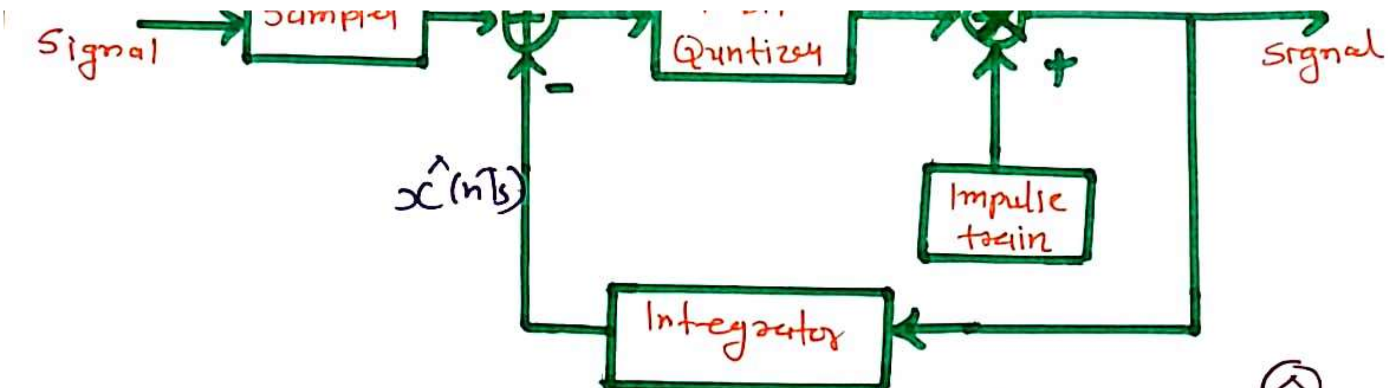


$$\boxed{x_{DM}(t) = e_q(nT_s) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)}$$

- error signal

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$e_q(nT_s) = \begin{cases} \Delta & x(nT_s) > \hat{x}(nT_s) \\ -\Delta & x(nT_s) < \hat{x}(nT_s) \end{cases}$$



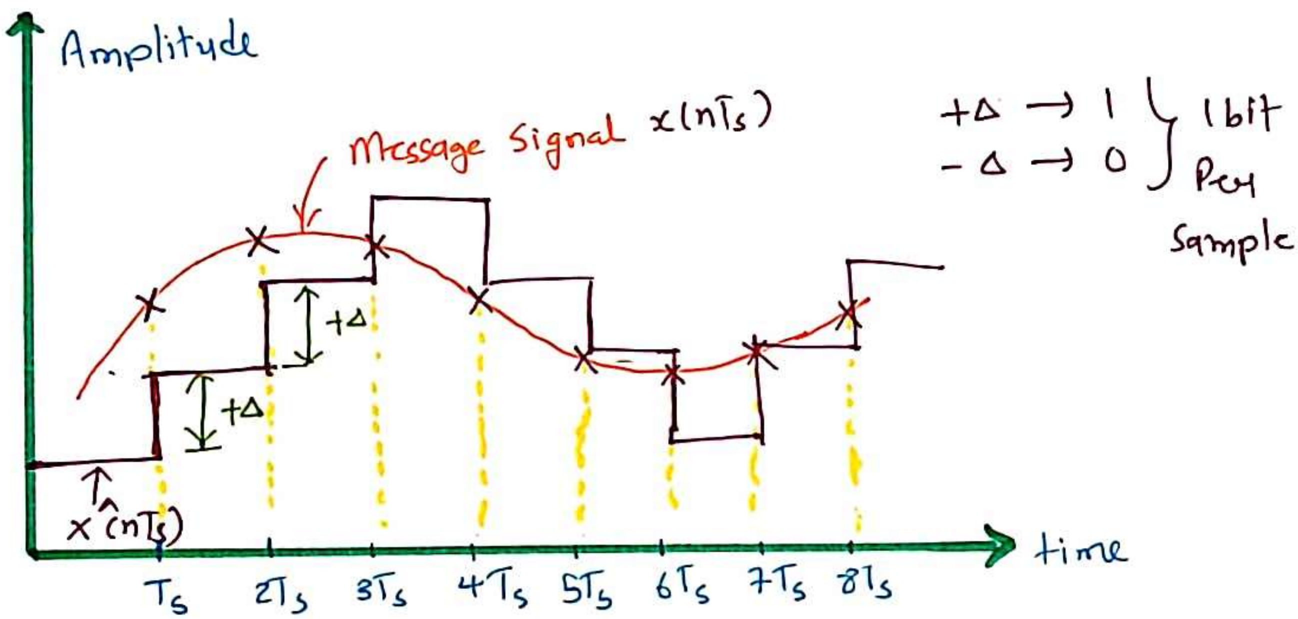
- error signal

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$e_q(nT_s) = x(nT_s) \gtrless \hat{x}(nT_s) \quad \begin{matrix} \Delta \\ -\Delta \end{matrix}$$

- quantized O/P.

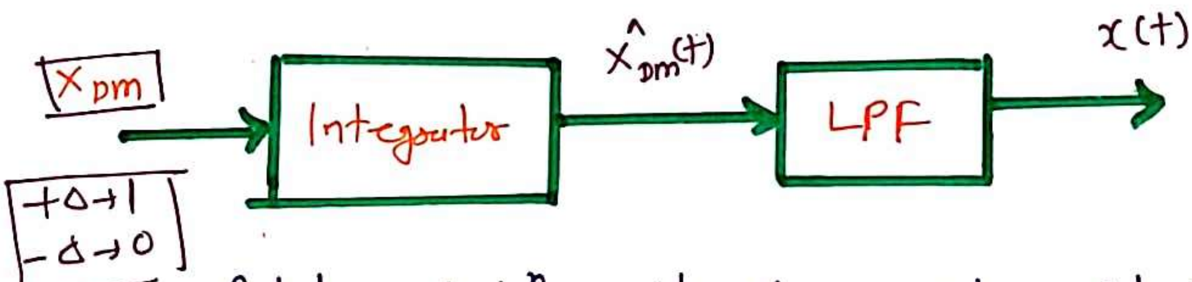
$$e_q(nT_s) = \begin{cases} \Delta & (x(nT_s) > \hat{x}(nT_s)) \\ -\Delta & (x(nT_s) < \hat{x}(nT_s)) \end{cases}$$



Amplitude
Message Signal $x(nT_s)$
$+\Delta \rightarrow 1$
$-\Delta \rightarrow 0$
1bit Per sample.
$+\Delta$
$+\Delta$
$\hat{x}(nT_s)$
time
T_s
$2T_s$
$3T_s$
$4T_s$
$5T_s$
$6T_s$
$7T_s$
$8T_s$

DM Receiver

- To get the original signal at the receiver, the received signal is passed through Integrator followed by a LPF.



- Relation bet.^{n} unit step and unit Impulse.

$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt$$

$$u(t) = \int_{-\infty} \delta(t)\, dt$$

- The O/p of Integrator

$$\hat{x}_{pm}(t) = \int_{-\infty}^{t} x_{pm}(t)\, dt$$

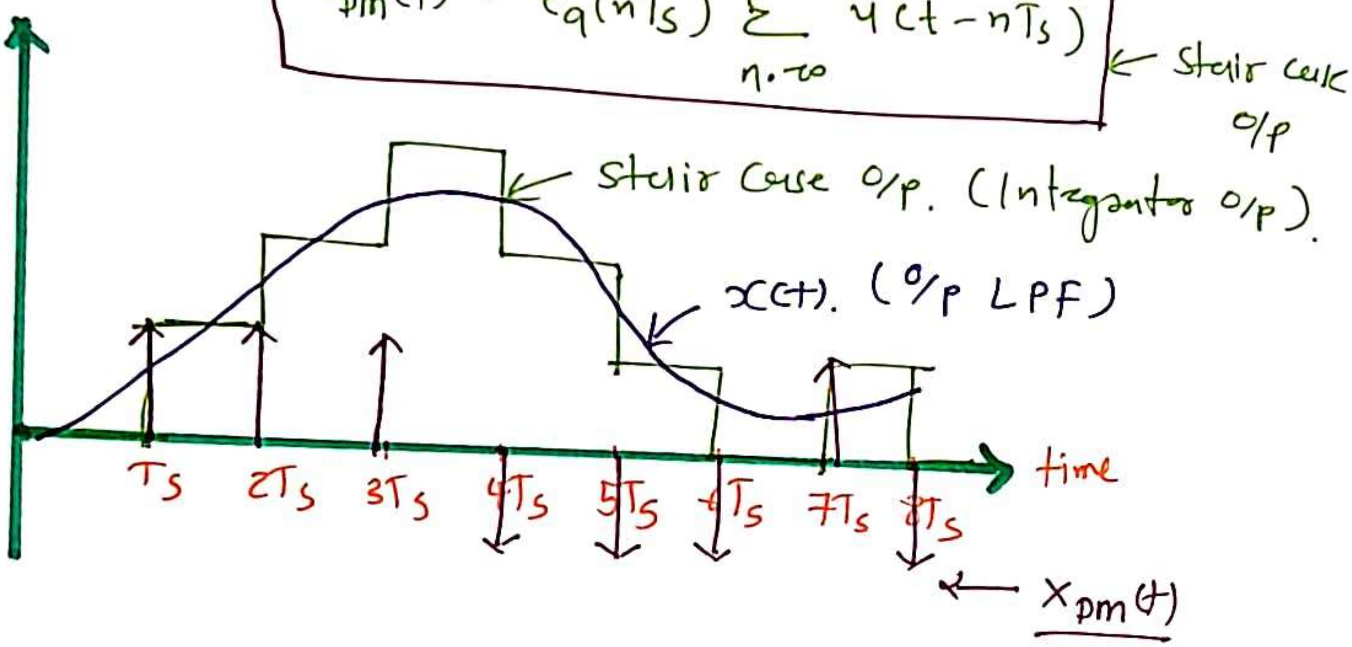
$$= \int_{-\infty}^{t} e_q(nT_s) \sum_{n=-\infty}^{\infty} \delta(t-nT_s)\, dt$$

$$= e_q(nT_s) \int_{-\infty}^{t} \sum_{n=-\infty}^{\infty} \delta(t-nT_s)\, dt$$

$$\boxed{\hat{x}_{pm}(t) = e_q(nT_s) \sum_{n=-\infty}^{\infty} u(t-nT_s)}$$

$$x_{pm}(t) = e_q(nT_s) \sum_{n=0}^{\infty} u(t - nT_s)$$

← Stair case o/p



Problem with Delta Modulation

- To make DM more efficient, we have a choice of two parameters.

  1) Step size (Δ)

  2) sampling rate (f_s)

- To account for the fastest change in the signal, both f_s and Δ must be increased.
- But increasing the sampling rate will account for larger bandwidth.

with T_s and Δ must be increased.

- But increasing the sampling rate will account for larger bandwidth.

PCM $\longrightarrow$ DPCM $\longrightarrow$ DM.

(encode all levels) with more bits

(encode diff. betⁿ level with bits)

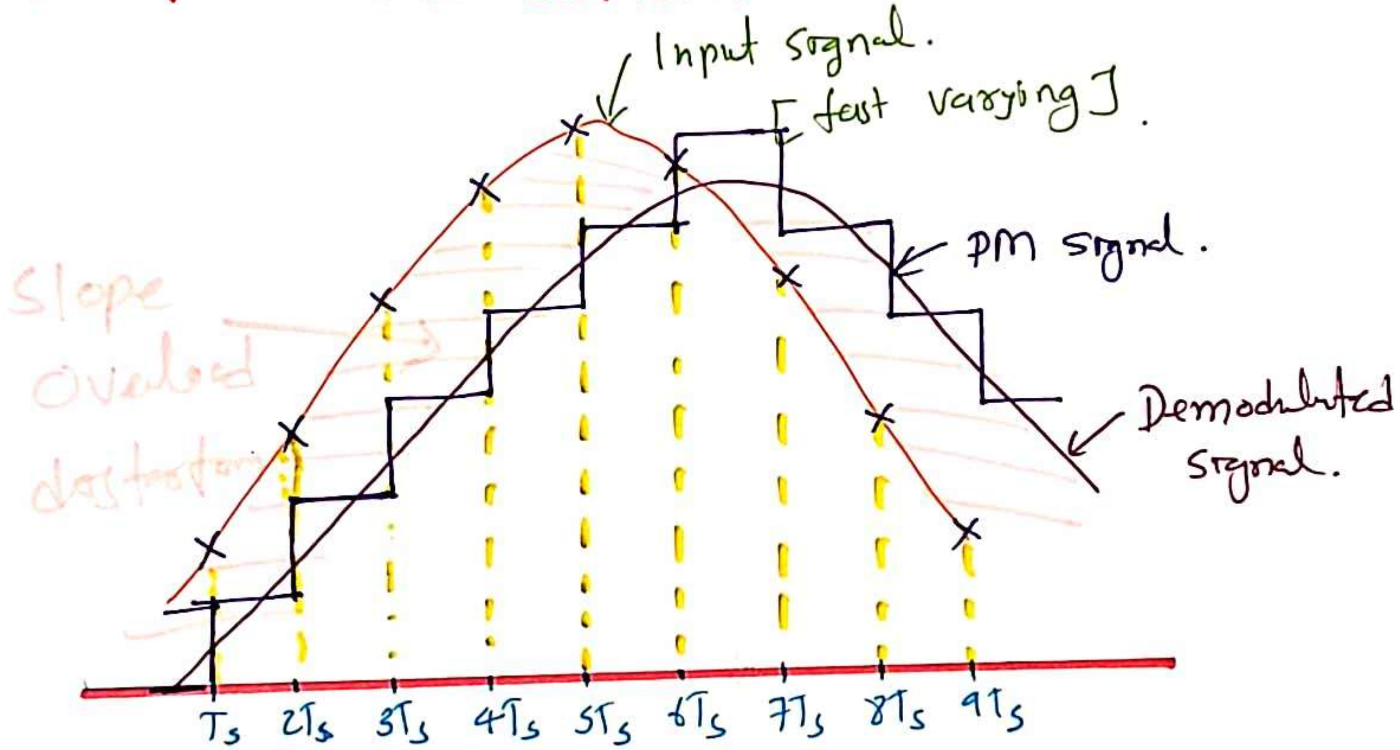
(encode one bit)

$+\Delta$ $-\Delta$

[error signal is quantized and encoded].

- Increasing step size account for increase in quantization error.
- Increasing f_s account for increasing in Bandwidth.
- The wrong choise of step size will introduce two problems like
  1. Slope overload distortion
  2. Granular noise.

*** Slope Overload distortion**

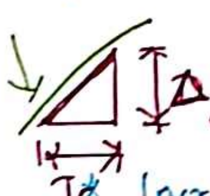


- When step size is small, slop overload distortion occurs. The envelope of staircase approximated signal will be far behind envelope of original signal.

- To avoid the quantization error due to slope overload distortion,

  * choose larger step size to satisfy the cond.^{n}

$\frac{dx(t)}{dt}$



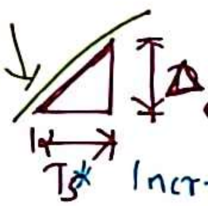
$$\frac{\Delta}{T_s} \geq \max \left| \frac{dx(t)}{dt} \right|$$

Increase f_s or decrease T_s

distortion,

* choose larger step size to satisfy the cond.^{n}

$\frac{dx(t)}{dt}$



$$\frac{\Delta}{T_s} \geq \max\left|\frac{dx(t)}{dt}\right|$$

* Increase f_s or decrease T_s

- To avoide slope overload distortion, we should have larger slope of staircase compared to max slope of orginal message signal.

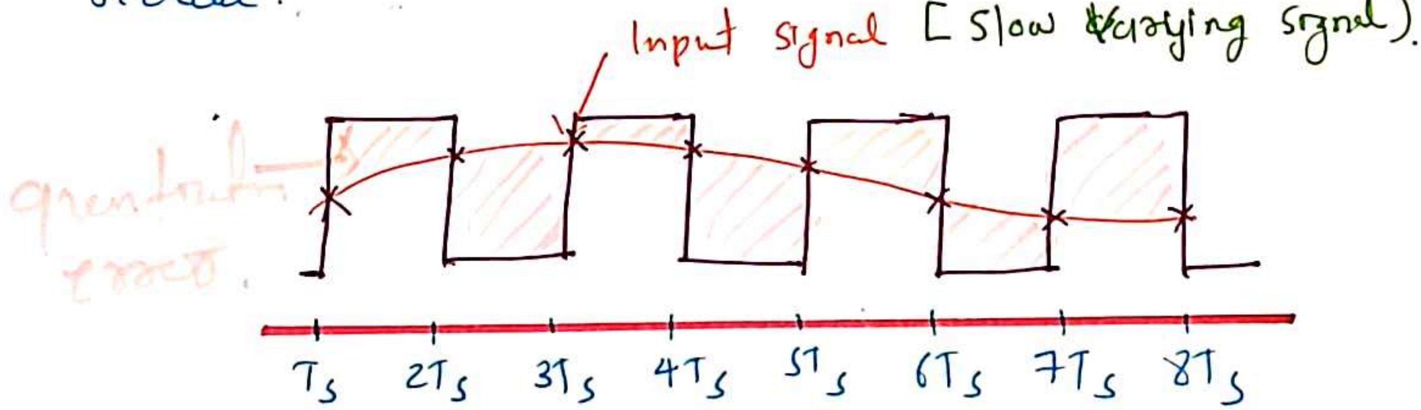
* Granular noise

- When the input signal is very slow varying and step size is very large, it introduces an error called granular noise.

- Granular noise gives altering $+\Delta$ and $-\Delta$ outputs.

- To reduce the granular noise, step size has to be reduced.

reduced.



Input signal [Slow varying signal).
quantization error.
T_s $2T_s$ $3T_s$ $4T_s$ $5T_s$ $6T_s$ $7T_s$ $8T_s$

E

- Solution is decrease Δ size.

Note

- Slope overload distortion occurs for the Input signal with large variations in the slope.
- Granular noise occurs for the Input signal with slow variation.
- Adaptive Delta Modulation [Sol.^{n}].

$\boxed{\Delta \text{ is not fix}}$

Signal with large variations in the slope.

- Granular noise occurs for the input signal with slow variation.

- Adaptive Delta Modulation [Sol.^{n}]

$\boxed{\Delta \text{ is not fix}}$

- logic $(-\Delta + \Delta - \Delta + \Delta)$ (granular noise)

decrease Δ size.

$(-\Delta - \Delta - \Delta)$ or $(+\Delta + \Delta + \Delta)$ (Slope overload)

Increase Δ size.

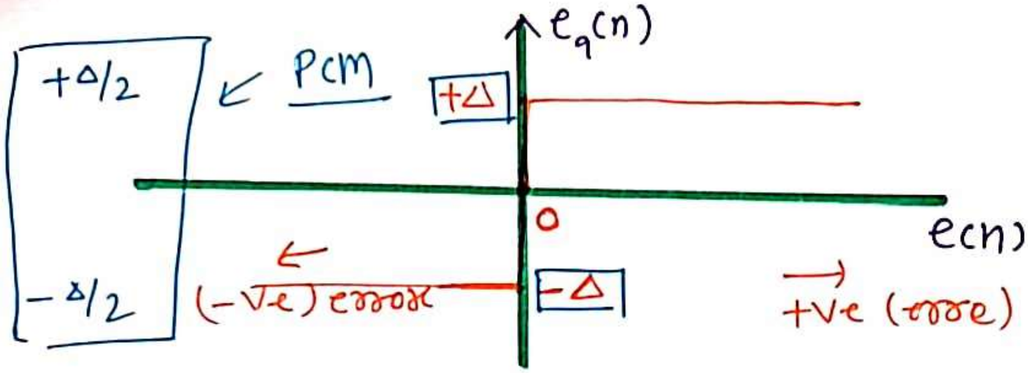
* SQNR of DM System

- Original signal $x(n)$
  Staircase signal $\hat{x}(n)$

$\rightarrow$ $x(n) \underset{-\Delta}{\overset{\Delta}{\gtrless}} \hat{x}(n)$

$\rightarrow$ $e(n) = x(n) - \hat{x}(n)$

- $\rightarrow$ +ve $\rightarrow$ Δ
- $\rightarrow$ -ve $\rightarrow$ $-\Delta$



- error will take min value $-\Delta$ and max $+\Delta$
- In PCM, it was $-\Delta/2$ to $+\Delta/2$
- Here error is uniformly distributed over $-\Delta$ to $+\Delta$
- In PCM

$$\sigma_Q^2 = \frac{\Delta^2}{12}$$

- error will take min value $-\Delta$ and max $+\Delta$
- In PCM, it was $-\Delta/2$ to $+\Delta/2$
- Here error is uniformly distributed over $-\Delta$ to $+\Delta$
- In PCM

$$\sigma_Q^2 = \frac{\Delta^2}{12}$$

- So, in DM

$$\sigma_Q^2 = \frac{(2\Delta)^2}{12} = \boxed{\frac{\Delta^2}{3}}$$

- In DM, issues of granular noise and slope overload problem does not allows direct calculation of SNR.

- To avoid slope overload distortion Cond.^{n}

$$\frac{\Delta}{T_s} \geq \max \left| \frac{dx(t)}{dt} \right|$$

→ If Input signal is sinusoidal

$$x(t) = A_m \sin(2\pi f_m t).$$

→ By diff. x(t) w.r.t time t.

$$\Rightarrow \frac{dx(t)}{dt} = A_m (2\pi f_m) \cos(2\pi f_m t)$$

$$\Rightarrow \max \left| \frac{dx(t)}{dt} \right| = 2\pi f_m A_m$$

→ So slope overload Cond.^{n}

Δ

→ If Input signal is sinusoidal

$$x(t) = A_m \sin(2\pi f_m t)$$

→ By diff. $x(t)$ w.r.t time t.

$$\Rightarrow \frac{dx(t)}{dt} = A_m(2\pi f_m)\cos(2\pi f_m t)$$

$$\Rightarrow \max\left|\frac{dx(t)}{dt}\right| = 2\pi f_m A_m$$

→ So slope overload cond.^{n}

$$\Rightarrow \frac{\Delta}{T_s} \geq 2\pi f_m A_m$$

$$\Rightarrow \boxed{A_m \leq \frac{\Delta}{2\pi f_m T_s}}$$

→ Power of signal

$$P = \frac{A_m^2}{2}$$

$$\boxed{P = \frac{\Delta^2}{8\pi^2 f_m^2 T_s^2}}$$

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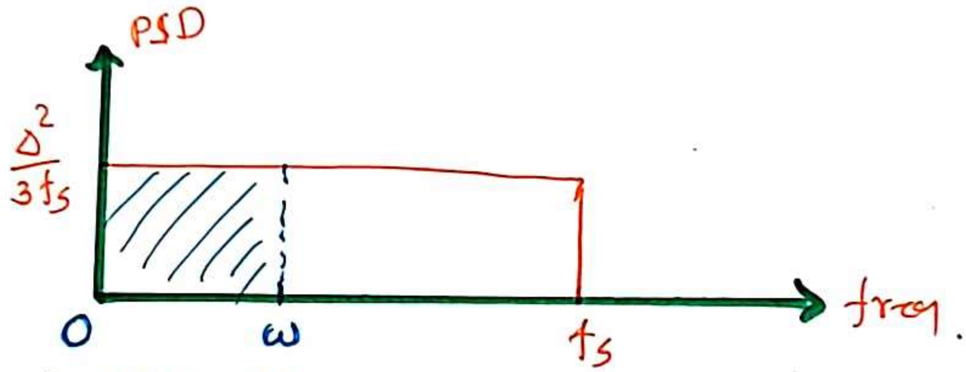
- For noise power



- In band Noise power

$$\sigma^2 = \int_0^{\omega} \frac{\Delta^2}{3f_s}\, df$$

In band Noise power

$$\sigma^2 = \int_0^{\omega} \frac{\Delta^2}{3f_s}\, df$$

$$= \frac{\Delta^2}{3f_s}\,\omega$$

$$= \frac{\Delta^2\omega}{3f_s} = \frac{\Delta^2\omega T_s}{3}$$

$=$

→ Signal to noise ratio.

$$SNR = \frac{P}{\sigma^2}$$

$$= \frac{\cancel{\Delta^2}}{8\pi^2 f_m^2 T_s^2} \times \frac{3}{\cancel{\Delta^2}\omega T_s}$$

$$= \frac{3}{8\pi^2 f_m^2 T_s^3\,\omega}$$

$$\boxed{SNR = \frac{3}{8\pi^2 f_m^3 T_s^3}}$$

(If $\omega = f_m$)

1) A DM system is designed to operate at sampling freq. of 6 kHz and step size of 350 mV.

i) Determine the maximum amplitude of a 1 kHz i/p sinusoidal signal for which the DM does not show the slope overload.

ii) Determine the o/p SNR.

– $f_s = 6\ kHz$

$\Delta = 350\ mV$

$f_m = 1\ kHz$

$f_s = 6$ KHz

$\Delta = 350$ mV

$f_m = 1$ KHz

- To avoide slope overload

$$A_m \leq \frac{\Delta}{2\pi f_m T_s}$$

$$\leq \frac{350 \times 10^{-3} \times 6 \times 10^{3}}{2\pi \times 10^{3} \times 1}$$

$$\leq 0.3357 \text{ Volt}$$

- O/p SNR of PM

$$SNR = \frac{3}{8\pi^2 f_m^2 T_s^3 W}$$

$$= \frac{3}{8\pi^2 (10^3)^2 \left(\frac{1}{6\times10^3}\right)^3 (3\times10^3)}$$

$$= \frac{3 \times 6^3 \times 10^9}{8\pi^2 \times 10^6 \times 3\times10^3}$$

$$= 2.77$$

$$SNR = \frac{3}{8\pi^2 f_m^2 T_s^3 W}$$

$$= \frac{3}{8\pi^2 (10^3)^2 \left(\frac{1}{6\times10^3}\right)^3 (3\times10^3)}$$

$$= \frac{3 \times 6^3 \times 10^9}{8\pi^2 \times 10^6 \times 3\times10^3}$$

$$= \boxed{2.77}$$

$$= 10 \log (2.77)$$

$$= \boxed{4.42 \text{ dB}}$$

2) Consider an analog I/p signal $x(t) = 0.1 \sin(2\pi \times 10^4 t)$ For a DM system, the signal is sampled at a rate of 2×10^4 Hz. Find out whether the slope overload distortion occurs for the following step size

i) $\Delta = 4\,mV$

ii) $\Delta = 60\,mV$

– $x(t) = 0.1 \sin(2\pi \times 10^4 t)$

$f_s = 2 \times 10^4$ Hz

- To avoid slope overload distortion

$$\frac{\Delta}{T_s} \text{ or } \Delta f_s \geq \left| \frac{dx(t)}{dt} \right|_{max}$$

- RHS $= \left| \frac{d(x(t))}{dt} \right|_{max}$

$$= \left[\frac{d}{dt} (0.1 \sin (2\pi \times 10^4 t) \right]_{max}$$

$$= \left[0.1 \times 2\pi \times 10^4 \cos (\underline{2\pi \times 10^4 t}) \right]_{max}$$

$$= 2\pi \times 10^3$$

- for $\Delta_1 = 4\,mV$

$$LHS = \Delta_1 f_s$$
$$= 4 \times 10^{-3} \times 2 \times 10^{4}$$
$$= 80 < 2\pi \times 10^{3}$$

- Slope over load will occure with $\Delta_1 = 4mV$

→ for $\Delta_2 = 60\,mV$

$$LHS = \Delta_2 f_s$$
$$= 60 \times 10^{-3} \times 2 \times 10^{4}$$
$$= 1200 < 2\pi \times 10^{3}$$

- Slope over load will occure with $\Delta_2 = 60mV$

3) A linear DM is designed to Operate on Speech Signa which is limited to 3.2 KHz. The DM was following specifications:

Sampling rate = 10 Nyquist rate.

Step size = 100 mV

The modulator is tested with a 1 kHz Sinusoidal signal. Determine the maximum amplitude of the test signal required to avoid the slope overload distortion

- $W = 3.2\ KHz$

$f_s = 10$ (Nyquist rate)

$= 10\ (2W)$

$= 20 \times 3.2$

$= 64\ KHz.$

$\Delta = 100\ mV$

$f_m = 1\ KHz$

- To avoid slop overload

$$A_m \leq \frac{\Delta}{2\pi f_m T_s}$$

$f_m = 1\ KHz$

- To avoid slop overload

$$\Rightarrow A_m \leq \frac{\Delta}{2\pi f_m T_s}$$

$$\Rightarrow A_m \leq \frac{\Delta f_s}{2\pi f_m}$$

$$\leq \frac{100 \times 10^{-3} \times 64 \times 10^{3}}{2\pi \times 10^{3}}$$

$$\Rightarrow \boxed{A_m \leq 1.02\ \text{Volt}}$$

Find the min sampling freq. $(f_s)_{min}$ to avoid slope overload when $x(t) = \cos(2\pi \times 800 t)$ and step size $\Delta = 0.1$

- $\Delta = 0.1$

  $x(t) = \cos(2\pi \times 800 t)$

- To avoid slope overload

$$\Rightarrow A_m \leq \frac{\Delta f_s}{2\pi f_m}$$

$$\Rightarrow f_s \geq \frac{2\pi f_m A_m}{\Delta}$$

$$\Rightarrow f_s \geq \frac{2\pi \times 800 \times 1}{0.1}$$

- $\Delta = 0.1$

$x(t) = \cos(2\pi \times 800 t)$

- To avoid slope overload

$\Rightarrow A_m \leq \dfrac{\Delta f_s}{2\pi f_m}$

$\Rightarrow f_s \geq \dfrac{2\pi f_m A_m}{\Delta}$

$\Rightarrow f_s \geq \dfrac{2\pi \times 800 \times 1}{0.1}$

$\Rightarrow \boxed{f_s \geq 16000\pi}$ Hz

- Nyquist rate $= 2f_m$

$= 2 \times 800$

$= 1600$ Hz

- Sampling freq. is 10π times of Nyquist rate.

x(t) be the Input to a DM

* Let a message signal $x(t)$ be the Input to a DM, where, $x(t) = 6 \sin(2\pi \times 10^3 t) + 4 \sin(4\pi \times 10^3 t)$ Volt. Determine the minimum sampling rate that will prevent slop over load, If the step size is 0.314 Volt.

$x(t) = 6 \sin(2\pi \times 10^3 t) + 4 \sin(4\pi \times 10^3 t)$ Volt

$\Delta = 0.314$

To avoid slop over load

$$\frac{\Delta}{T_s} \geq \left| \frac{dx(t)}{dt} \right|_{max}$$

$$LHS = \frac{\Delta}{T_s} = \Delta f_s = 0.314 f_s$$

$$RHS = \left| \frac{d(x(t))}{dt} \right|_{max}$$

$$= \left| \frac{d}{dt}(6 \sin(2\pi \times 10^3 t) + 4 \sin(4\pi \times 10^3 t) \right|_{max}$$

$$= \left| 6 \times 2\pi \times 10^3 \cos(2\pi \times 10^3 t) + 4(4\pi \times 10^3) \cos(4\pi \times 10^3 t) \right|_{max}$$

$$= 12\pi \times 10^{3} + 16\pi \times 10^{3}$$

$$= 28\pi \times 10^{3}$$

$$\Rightarrow \quad 0.314\, f_s \geq 28\pi \times 10^{3}$$

$$\Rightarrow \quad f_s \geq \frac{28\cancel{\pi} \times 10^{3}}{\cancel{0.314} \times 0.1}$$

$$\Rightarrow \quad f_s \geq 280 \times 10^{3}$$

$$\Rightarrow \quad \boxed{f_s \geq 280\ kHz}$$

EF

The Input to a linear Delta modulator is a Sinusoidal signal whose frequency can vary from 200 Hz to 4000 Hz. The Input is sampled at eight times the Nyquist rate. The peak amplitude of the sinusoidal signal is 1 V.

a) Determine the value of the step size in order to avoid slope overload when the input signal freq is 800 Hz.

b) What is the peak amplitude of the input signal to just the overload the modulator, when the Input signal freq is 200 Hz.

c) Is the modulator over loaded when the Input

c) Is the modulator over loaded when the Input signal freq. is 4 KHz.

$f_m = 200$ Hz to 4000 Hz

$W = 4000$ Hz

Nyquist rate $= 2W = 8000$ Hz

$f_s = 8$ (Nyquist rate)

$= 8(8000) = 64000$ Hz

$A_m = 1$ Volt.

→ Case - a ($f_m = 800$ Hz)

- To avoid Slop overload

$$\Rightarrow \Delta \geq \frac{2\pi A_m f_m}{f_s}$$

$$\geq \frac{2\pi \times 1 \times 800}{64000}$$

$$\geq \boxed{\pi/40}$$

$= 8\,(8000) = 64000\ Hz$

$A_m = 1$ Volt.

$\geq \dfrac{2\pi \times 1 \times 800}{64000}$

$\geq \boxed{\pi/40}$

- Case-b ($f_m = 200\ Hz$)

To occure slop overload

$\Rightarrow \Delta < \dfrac{2\pi A_m f_m}{f_s}$

$\Rightarrow A_m > \dfrac{\Delta f_s}{2\pi f_m}$

$\Rightarrow A_m > \dfrac{(\pi/40)\,64000}{2\pi \times 200}$

$\Rightarrow \boxed{A_m > 4\ V}$

Case-b (fm = ... Hz)

To occure Slop overload

$\Rightarrow \Delta < \frac{2\pi A_m f_m}{f_s}$

$\Rightarrow A_m > \frac{\Delta f_s}{2\pi f_m}$

$\Rightarrow A_m > \frac{(\pi/40)\,64000}{2\pi \times 200}$

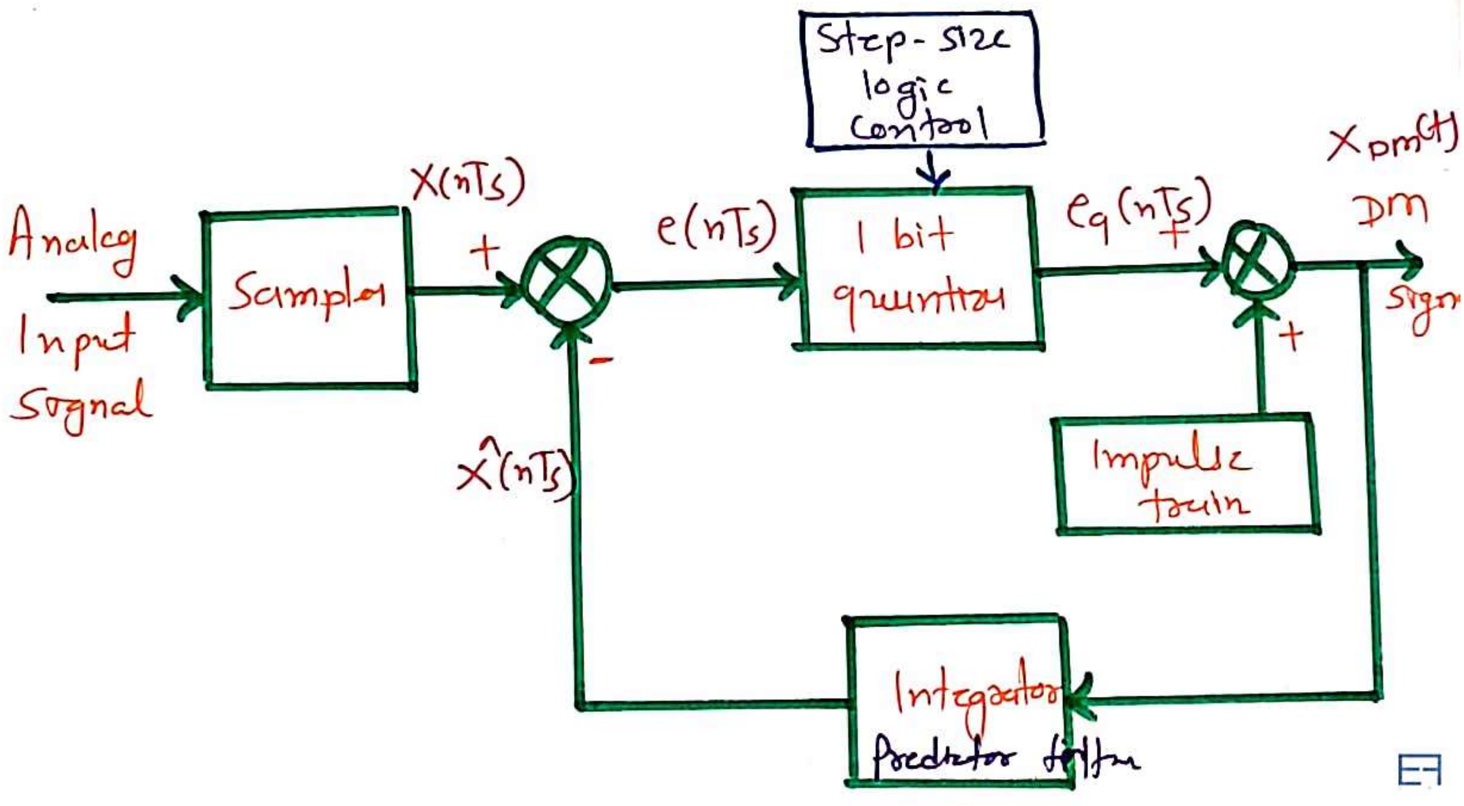
$\Rightarrow \boxed{A_m > 4\,V}$

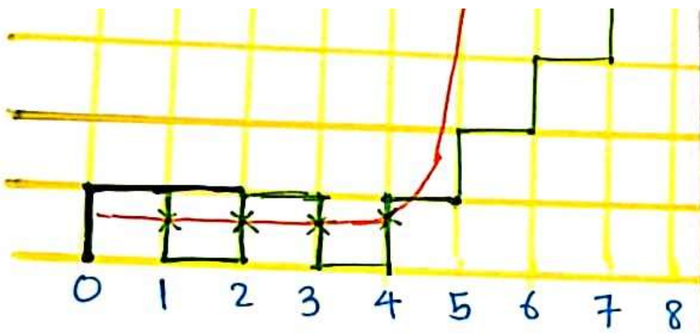
Case-c ($f_m = 4$ KHz)

$\frac{\Delta f_s}{2\pi f_m} = \frac{(\pi/40)\,64000}{2\pi \times 4 \times 1000}$

$= 0.2 < 4$

- There is no slope overload at $f_m = 4$ KHz


Step-size logic control
X(nTs)
Analog Input Signal
Sampler
+
-
$\hat{X}(nT_s)$
e(nTs)
1 bit quantizer
$e_q(nT_s)$
+
+
$X_{DM}(t)$ DM signal
Impulse train
Integrator
Predictor filter



- At Sampling time n, The Step size $\Delta(n)$ is given by

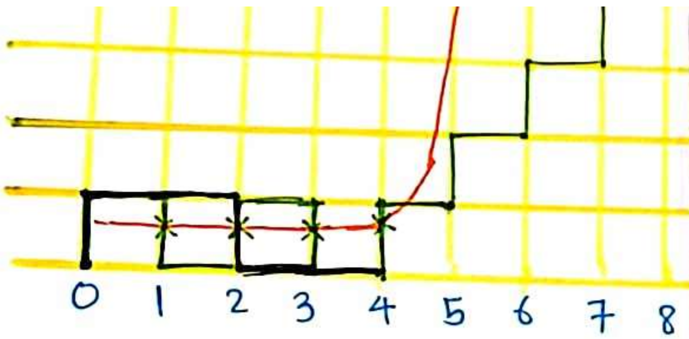
$$\Delta(n) = |\Delta(n-1)|\, e(n) + \Delta(0)\, e(n-1)$$

- for $n = 0$, $e_0 \xrightarrow{+ve} +1$

$$\Delta_0 = \Delta_0$$

- for $n = 1$, $e_1 \xrightarrow{-ve} -1$

$$\Delta(1) = |\Delta(0)|(-1) + \Delta(0)(+1) = -\Delta(0) + \Delta(0) = 0$$



- At Sampling time n, The Step size $\Delta(n)$ is given by

$$\Delta(n) = |\Delta(n-1)|\, e(n) + \Delta(0)\, e(n-1)$$

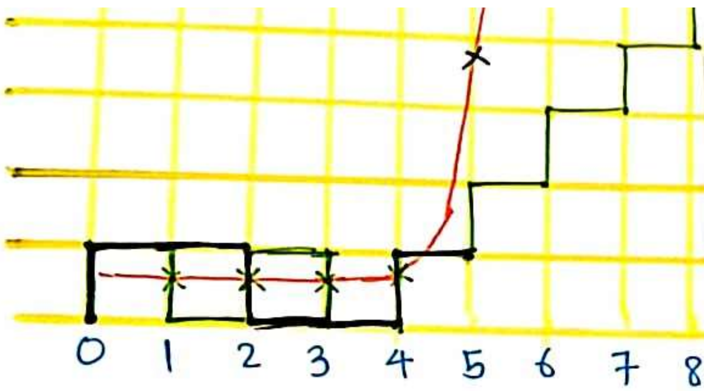
$\Delta(1) = |\Delta(0)|(-1) + \Delta(0)(+1) = -\Delta(0) + \Delta(0) = 0$

- for $n = 2$, $e_2 \xrightarrow{-ve} (-1)$

$$\Delta(2) = |\Delta(1)|(-1) + \Delta(0)(-1) = -\Delta(0).$$

- for $n = 3$, $e_3 \xrightarrow{+ve} (+1)$

$$\Delta(3) = |\Delta(2)|(+1) + \Delta(0)(-1) = \Delta(0) - \Delta(0) = 0$$



- At Sampling time n, The Step size $\Delta(n)$ is given by

$$\Delta(n) = |\Delta(n-1)|\, e(n) + \Delta(0)\, e(n-1)$$

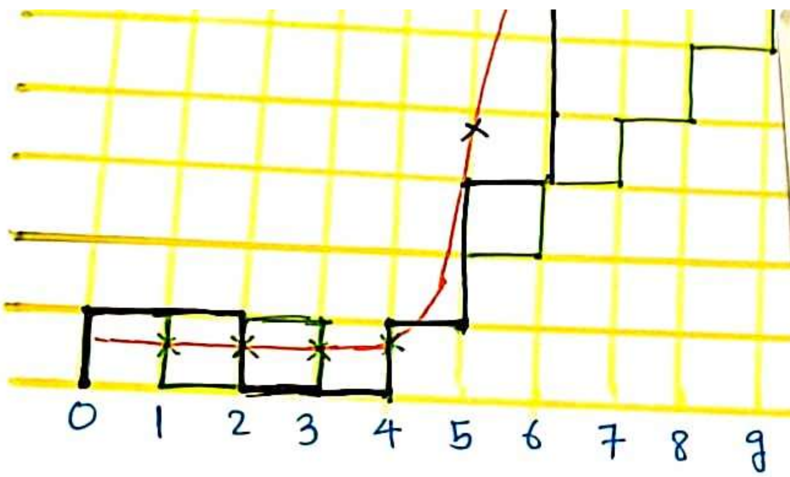
$$\Delta(3) = |\Delta(2)|(+1) + \Delta(0)(-1) = \Delta(0) - \Delta(0) = 0$$

- for $n = 4$, $e_4 \xrightarrow{+ve} (+1)$

$$\Delta(4) = |\Delta(3)|(+1) + \Delta(0)(+1) = 0 + \Delta(0) = \Delta(0)$$

for $n = 5$, $e_5 \xrightarrow{+ve} (+1)$

$$\Delta(5) = |\Delta(4)|(+1) + \Delta(0)(+1) = \Delta(0) + \Delta(0) = 2\Delta(0)$$



- At Sampling time n, The Step size $\Delta(n)$ is given by

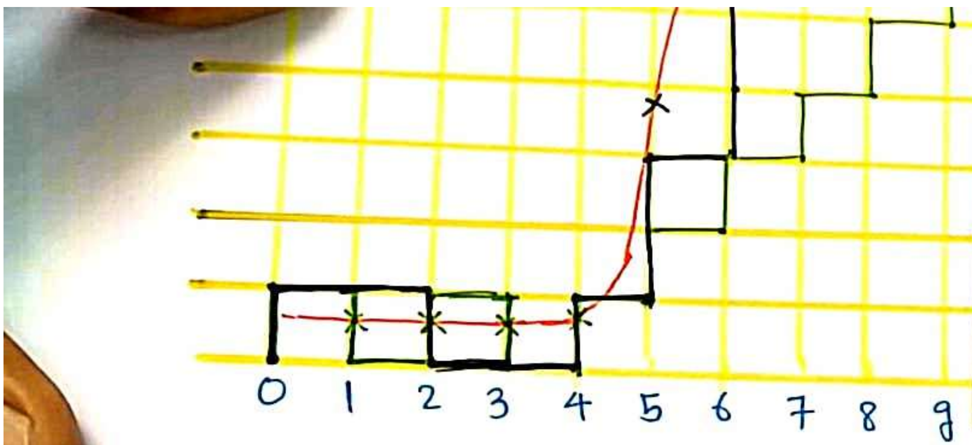
$$\Delta(n) = |\Delta(n-1)|\, e(n) + \Delta(0)\, e(n-1)$$

for $n=6$, $e_6 \xrightarrow{+ve} (+1)$

$$\Delta(6) = |\Delta(5)|(+1) + \Delta(0)(+1) = 2\Delta(0) + \Delta(0) = 3\Delta(0)$$

for $n=7$, $e_7 \xrightarrow{+ve} (+1)$

$$\Delta(7) = |\Delta(6)|(+1) + \Delta(0)(+1) = 3\Delta(0) + \Delta(0) = 4\Delta(0)$$



- At Sampling time n,
The Step size $\Delta(n)$ is given by

$$\Delta(n) = |\Delta(n-1)|\, e(n) + \Delta(0)\, e(n-1)$$

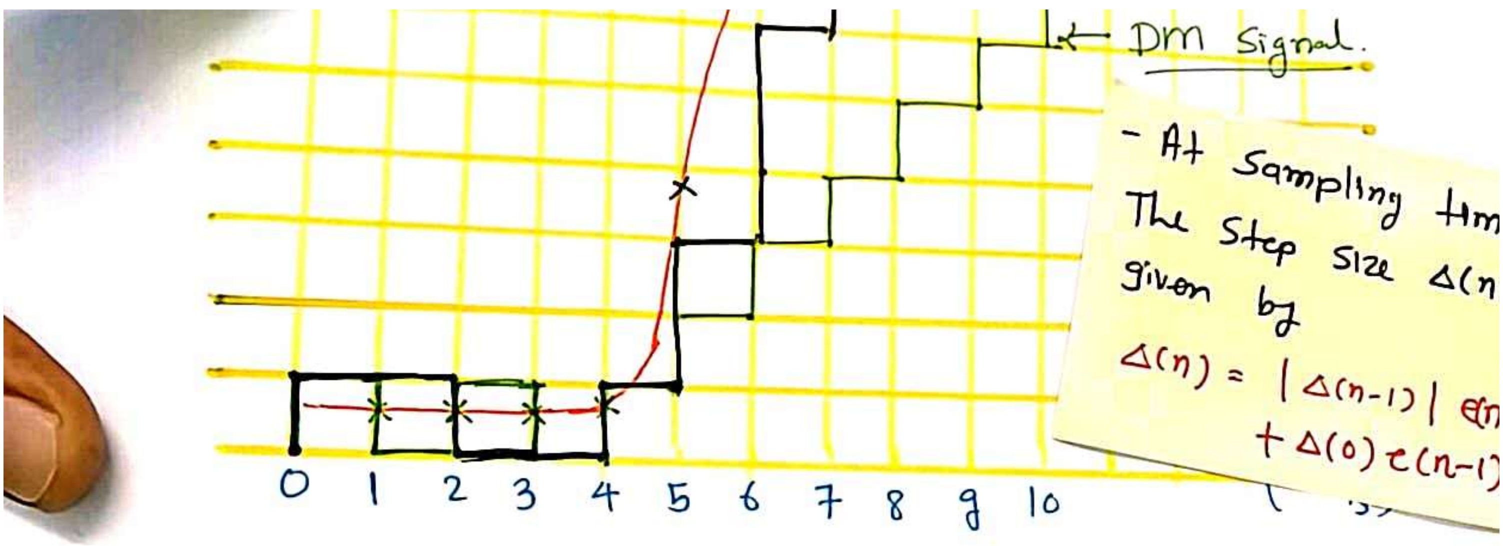
$\Delta(7) = |\Delta(6)|(+1) + \Delta(0)(+1) = 3\Delta(0) + \Delta(0) = 4\Delta(0)$

- for $n = 8$, $e_8 \xrightarrow{-ve} (-1)$

$$\Delta(8) = |\Delta(7)|(-1) + \Delta(0)(+1) = -4\Delta(0) + \Delta(0) = -3\Delta(0)$$

for $n = 9$, $e_9 \xrightarrow{+ve} (+1)$

$$\Delta(9) = |\Delta(8)|(+1) + \Delta(0)(-1) = 3\Delta(0) - \Delta(0) = +2\Delta(0)$$

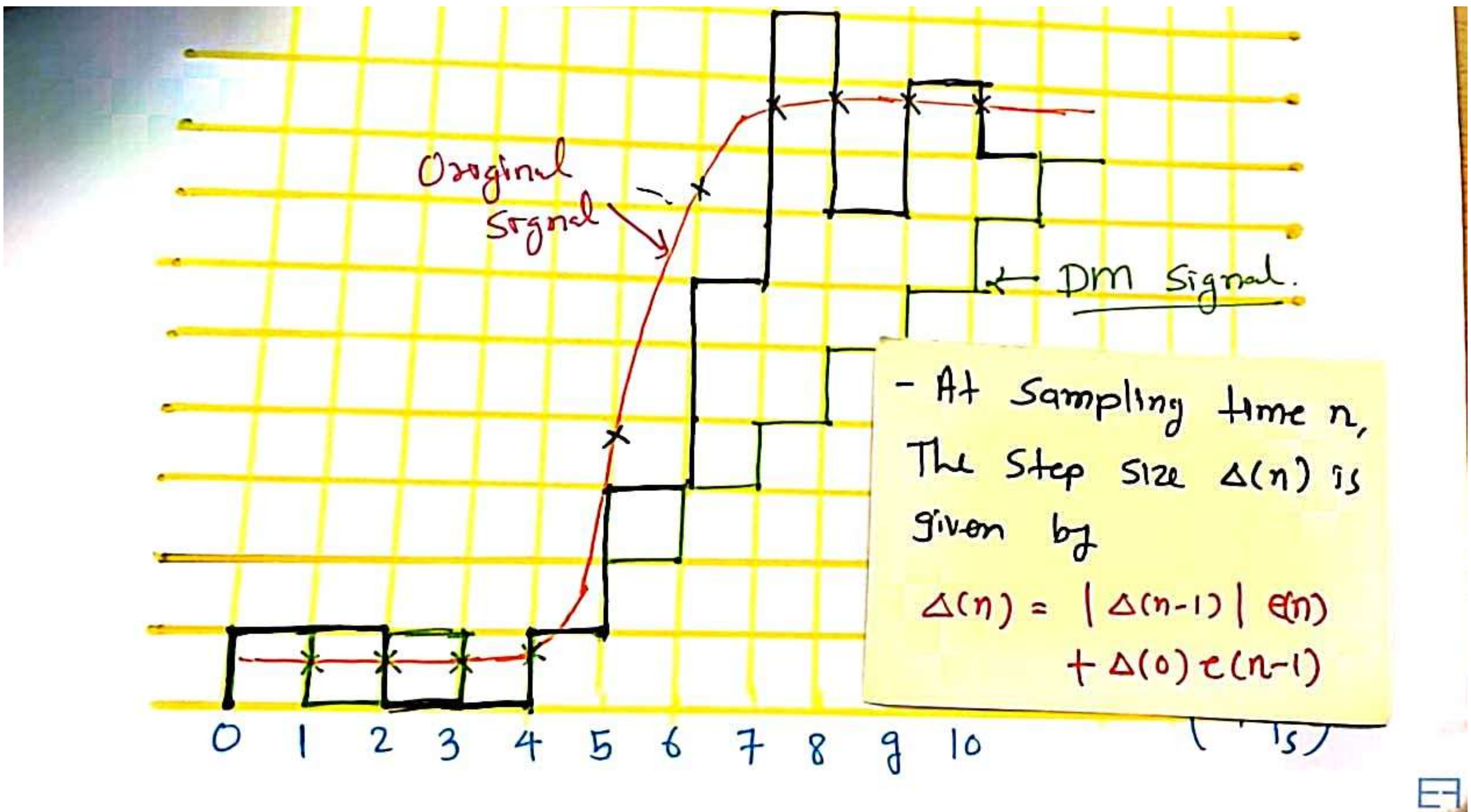


- for $n = 9$, $e_9 \xrightarrow{+ve} (+1)$

$$\Delta(9) = |\Delta(8)|(+1) + \Delta(0)(-1) = 3\Delta(0) - \Delta(0) = +2\Delta$$

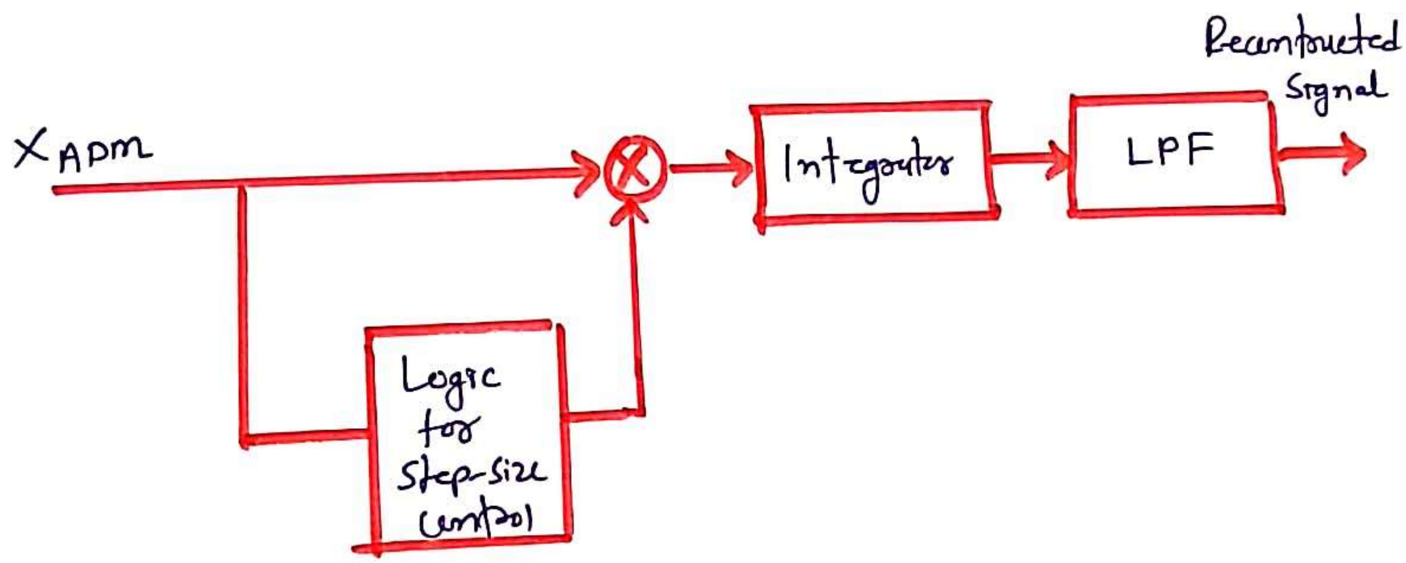
- for $n = 10$, $e_{10} \xrightarrow{-ve} (-1)$

$$\Delta(10) = |\Delta(9)|(-1) + \Delta(0)(+1) = -2\Delta(0) + \Delta(0) = -\Delta(0)$$



Original Signal
DM Signal.
0 1 2 3 4 5 6 7 8 9 10
- At Sampling time n, The Step size $\Delta(n)$ is given by
$\Delta(n) = |\Delta(n-1)|\, e(n) + \Delta(0)\, e(n-1)$

Adaptive delta Modulation Receiver



ADPCM-ADAPTIVE DPCM

THE CODING TECHNIQUE WHICH USES BOTH ADAPTIVE QUANTISATION AND ADAPTIVE PREDICTION IS CALLED AS ADPCM

Adaptive quantization is obtained by varying the step size according to the rms value of the input signal

The step size of the Adaptive Quantisation is given by $\partial(nT_s) = \phi\partial_x(nT_s)$

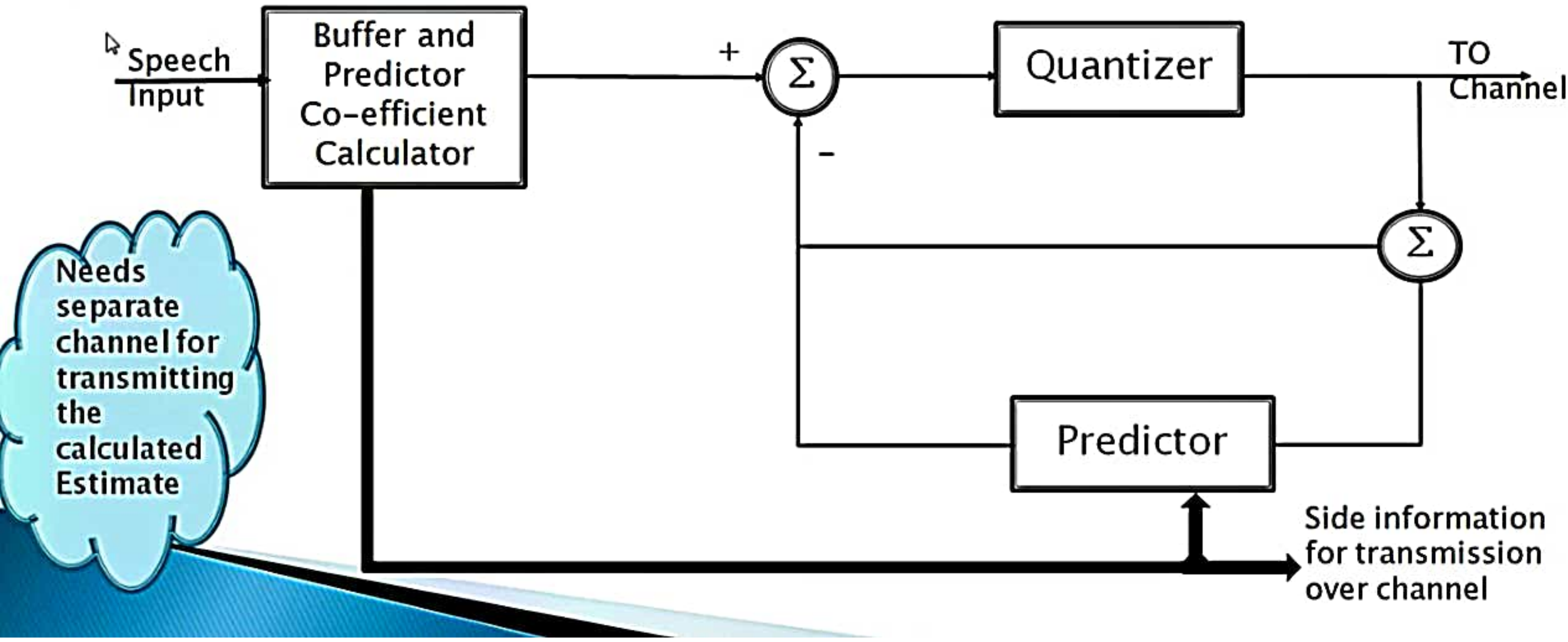
Where ϕ is the constant

$\partial_x(nT_s)$ is the estimate of the standard deviation $\sigma_x(nT_s)$

$\sigma_x(nT_s)$ can be obtained by 1.AQF or 2. AQB

APF- Adaptive Prediction with Forward estimation

- Unquantized samples of the input speech signal are buffered and the released after the computation of M predictor co efficient.



APB- Adaptive Prediction with Backward estimation