

Sampling Theory

- It is a process to convert continuous time signal into discrete signal.
- Sufficient no of samples must be taken, so that the original signal is reconstructed properly.
- No of samples to be taken depends on maximum signal freq. present in the signal.

□

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- No of samples to be taken depends on maximum signal freq. present in the signal.

- Different types of sampling.

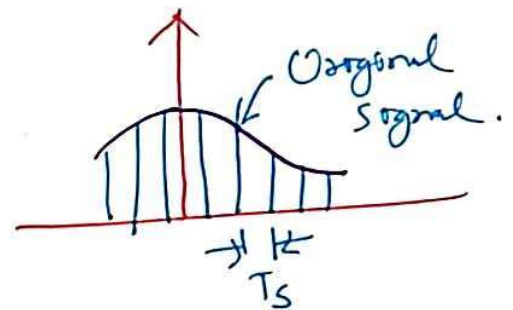
- Ideal samples.

- Natural samples.

- Flat-top samples.

Different types of Sampling.

- Ideal Samples.
- Natural Samples.
- Flat-top Samples.



Statement of Sampling theorem

i] A band limited signal of finite energy, which has no freq. component higher than f_m (Hz), is completely described by its sample values at uniform intervals less than or equal to $1/2f_m$

$$T_s \leq 1/2f_m$$

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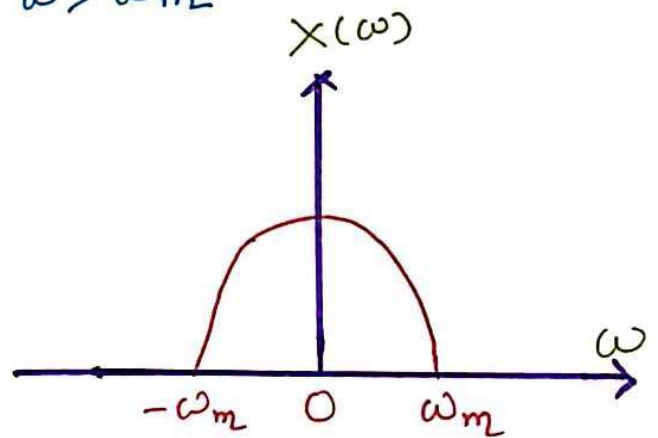
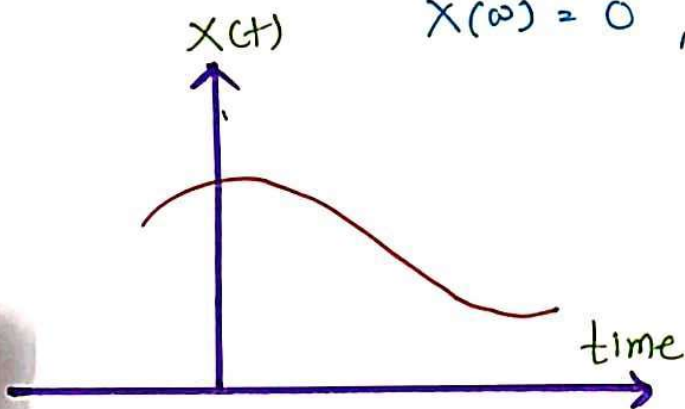
$$T_s \leq 1/2f_m$$

ii) A band limited signal of finite energy, which has no freq. components higher than f_m (Hz), may be completely recovered from the knowledge of its samples taken at the rate of $2f_m$ samples per second.

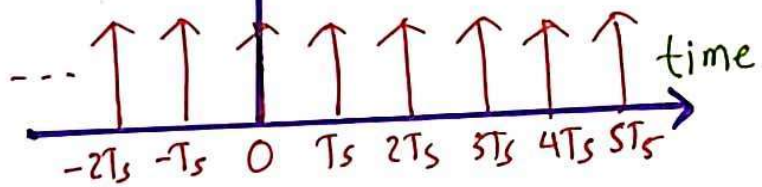
$$f_s \geq 2f_m$$

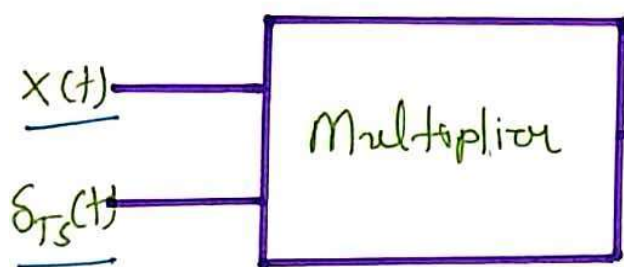
→ If signal is band limited to f_m

$$X(\omega) = 0, \quad \omega > \omega_m$$



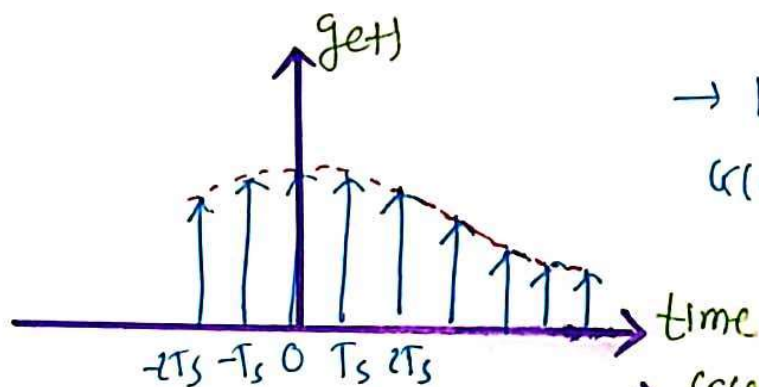
$$\delta_{T_s}(t) = \frac{1}{T_s} [1 + 2\cos \omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \dots]$$





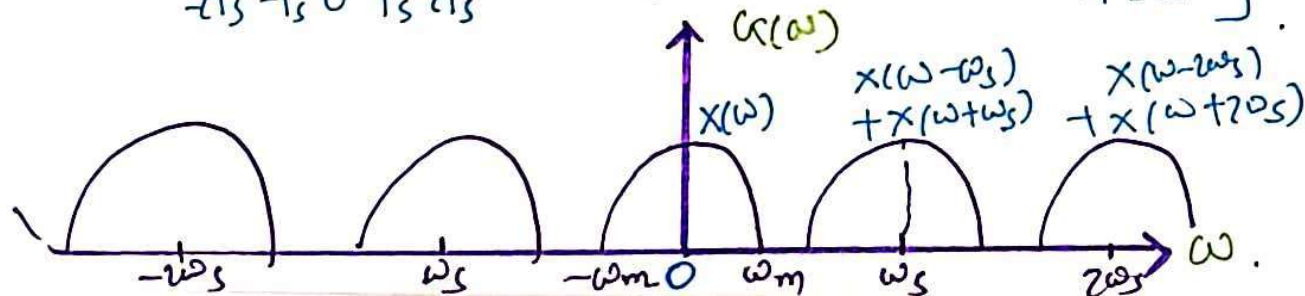
$$g(t) = X(t) \delta_{Ts}(t)$$

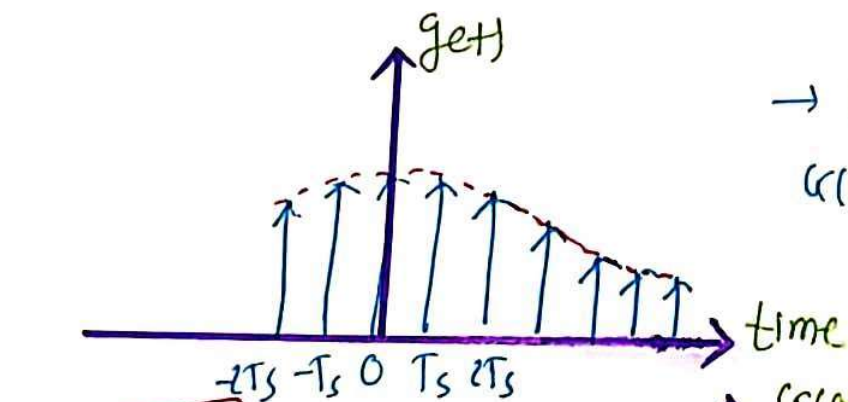
$$= X(t) \left[\frac{1}{T_s} \left(1 + 2\cos \omega_s t + \frac{2\cos 2\omega_s t}{2} + \frac{2\cos 3\omega_s t}{3} + \dots \right) \right]$$



→ In freq domain

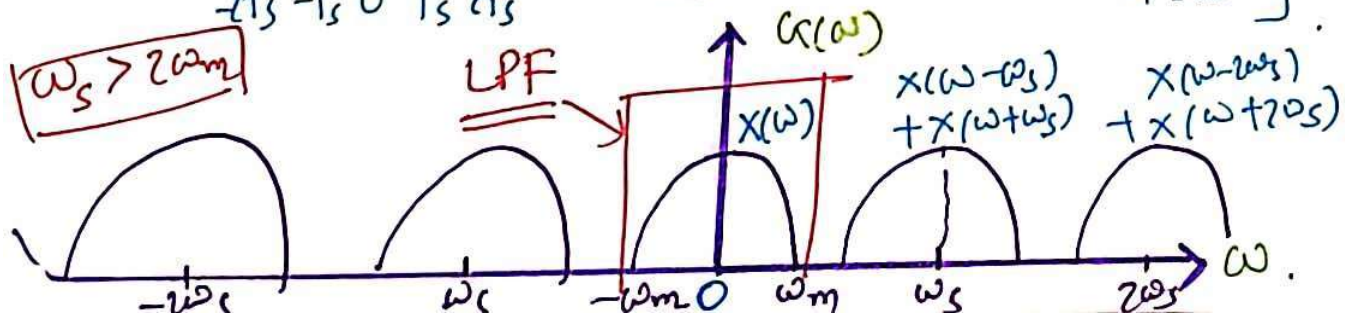
$$G(\omega) = \frac{1}{T_s} \left[\underline{X(\omega)} + \underline{X(\omega - \omega_s)} + \underline{X(\omega + \omega_s)} + \underline{X(\omega - 2\omega_s)} + \underline{X(\omega + 2\omega_s)} + \dots \right]$$

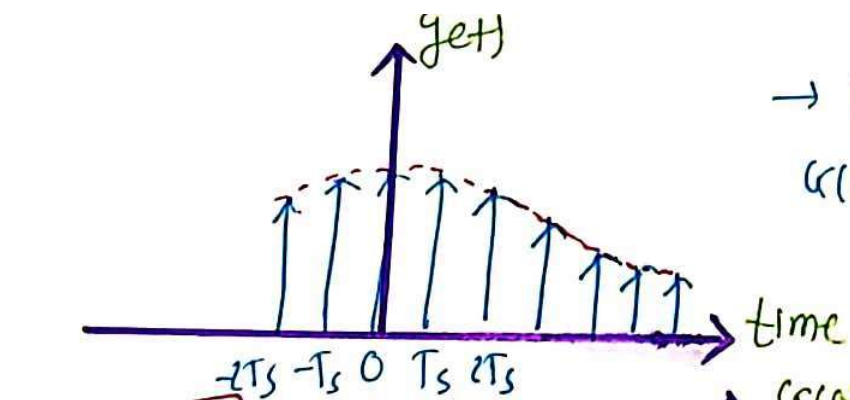




→ In frequency domain

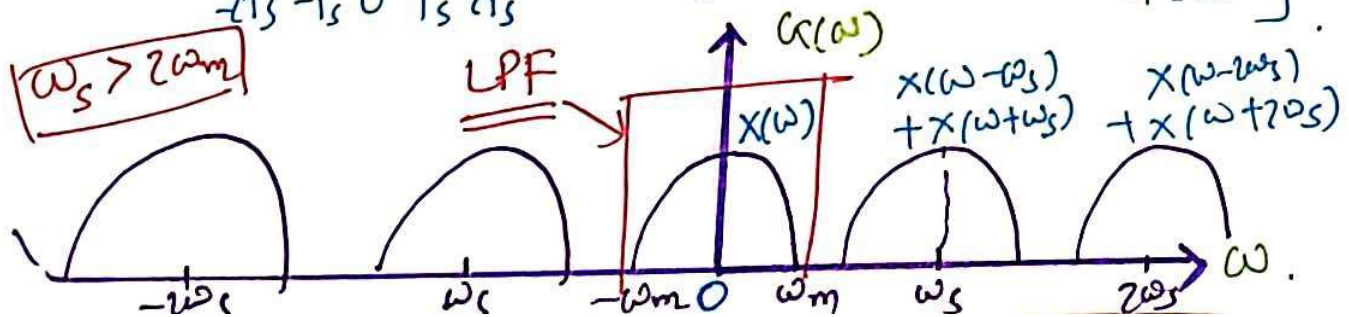
$$G(\omega) = \frac{1}{T_s} \left[\frac{X(\omega) + X(\omega - \omega_s)}{+ X(\omega + \omega_s)} + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) + \dots \right]$$






→ In frequency domain

$$Y(\omega) = \frac{1}{T_s} \left[\underline{X(\omega)} + \underline{X(\omega - \omega_s)} + \underline{X(\omega + \omega_s)} + \underline{X(\omega - 2\omega_s)} + \underline{X(\omega + 2\omega_s)} + \dots \right]$$



E-

- As long as $f_s > 2f_m$, $K(\omega)$ will repeat periodically without overlapping.
- Spectrum $K(\omega)$ extends upto ∞ freq. but our purpose is to extract original Spectrum $X(\omega)$ out of the Spectrum $K(\omega)$.
- At receiver we place LPF of freq ω_m . So we can extract original Information.
- $f_s > 2f_m$. To avoid successive ~~cycles~~ not to overlap 

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- $f_s = 2f_m$, Successive cycles just touch each other.
- $f_s < 2f_m$, Successive cycles overlap each other
- Hence, for reconstruction without distortion

... original information.

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→ Hence, for reconstruction without distortion

$$\boxed{f_s \geq 2f_m}$$

→ $f_s = 2f_m$, Here f_s is referred as Nyquist rate.

$T_s = 1/2f_m$, T_s is Nyquist Interval.

$$\boxed{f_s \geq 2f_m}$$

→ $f_s = 2f_m$, Hence f_s is referred as Nyquist rate
 $T_s = 1/2f_m$, T_s is Nyquist Interval.

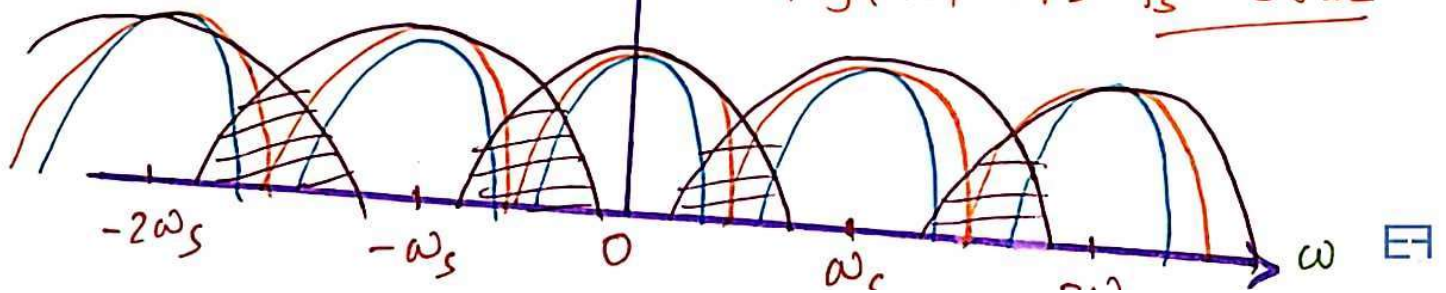
$f_s < 2f_m$ - Overlap

\uparrow $K(\omega)$

No overlap

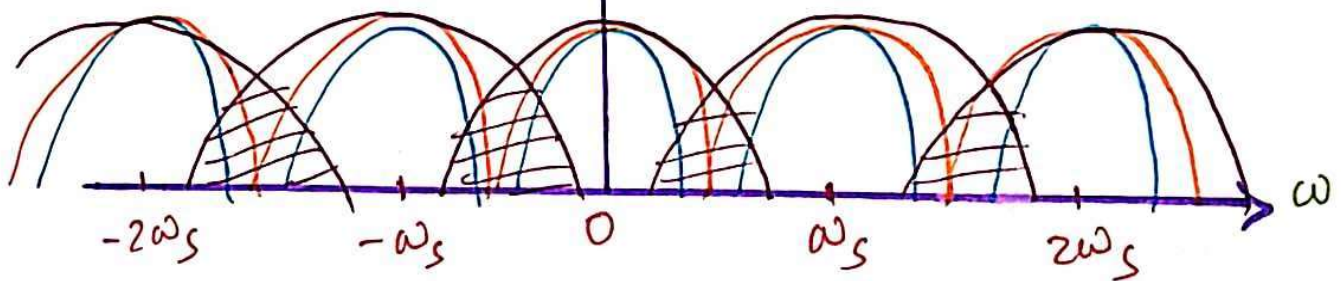
$$\boxed{f_s > 2f_m}$$

Nyquist sampling $f_s = 2f_m$



$f_s < 2f_m$ - Overlap

$\kappa(\omega)$ No overlap $f_s > 2f_m$
Nyquist sampling $f_s = 2f_m$



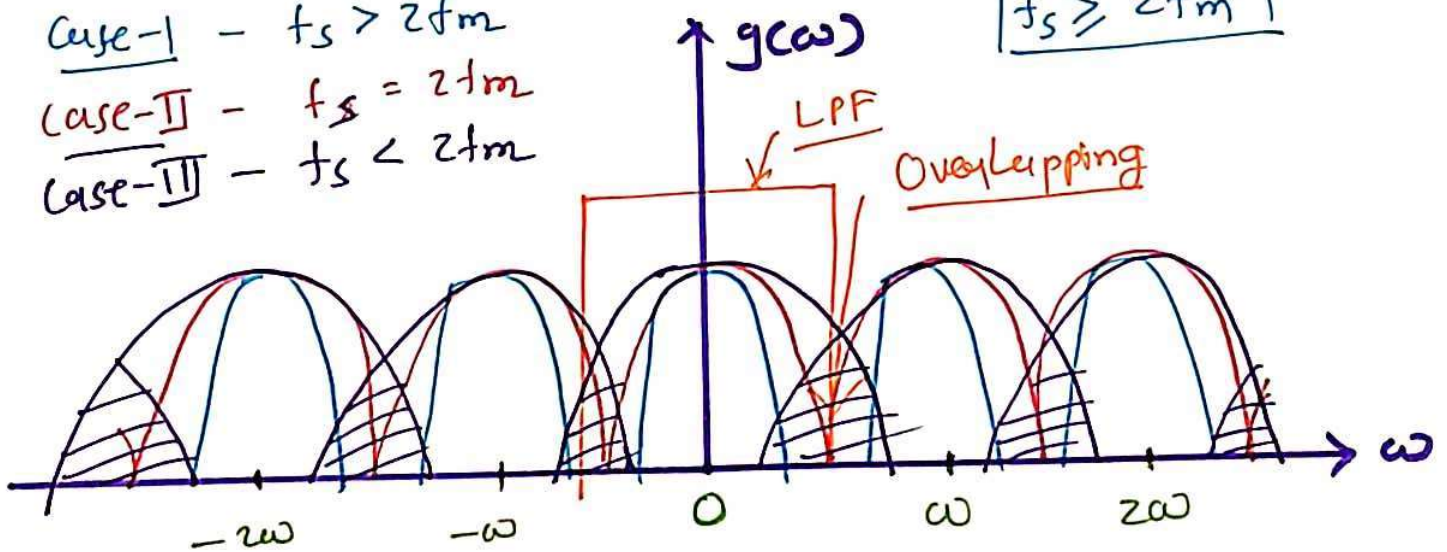
★ Effect of under sampling - Aliasing

Case-I - $f_s > 2f_m$

Case-II - $f_s = 2f_m$

Case-III - $f_s < 2f_m$

$$f_s \geq 2f_m$$



- If $t_s < 2t_m$, then successive samples (sets of $x(n)$) will overlap each other.
- Due to Aliasing effect, it is not possible to recover original signal $x(t)$ by LPF.
- Hence due to overlap of one region to other region, signal $x(t)$ is distorted.
- So before we go for sampling, we pass original signal through LPF. This is even referred as pre-alias filter, other name is band limit filter.

... due to overlap of one region to other region, signal $x(t)$ is distorted.

- So before we go for sampling, we pass original signal through LPF. This is even referred as pre-alias filter, other name is band limit filter.
- In Short, to avoid aliasing.
 - 1) Pre alias Filter can be used
 - 2) $f_s \geq 2f_m$

Examples on Sampling & Nyquist Rate

1) $X(t) = 3 \cos(50\pi t) + 10 \sin(300\pi t) - \cos(100\pi t)$

Calculate the Nyquist rate for this signal.

$$f_1 = \frac{\omega_1}{2\pi} = 25 \text{ Hz}$$

$$f_2 = \frac{\omega_2}{2\pi} = 150 \text{ Hz}$$

$$f_3 = \frac{\omega_3}{2\pi} = 50 \text{ Hz}$$

→ Max freq. $f_m = 150 \text{ Hz}$.

$$\rightarrow f_s = 2f_m$$

$$= 2 \times 150$$

$$= 300 \text{ Hz}$$

∴ 1. 2. Nyquist Interval for $X(t)$

2) Find the Nyquist rate & Nyquist Interval for the signal $x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$

$$= \frac{1}{4\pi} [2 \cos(4000\pi t) \cos(1000\pi t)]$$

$$= \frac{1}{4\pi} [\cos(3000\pi t) + \cos(5000\pi t)]$$

$$\begin{aligned} \rightarrow f_1 &= \frac{\omega_1}{2\pi} = 1500 \text{ Hz} \\ f_2 &= \frac{\omega_2}{2\pi} = 2500 \text{ Hz} \end{aligned} \quad \left. \begin{array}{l} \rightarrow \text{Max freq.} \\ f_m = 2500 \text{ Hz.} \end{array} \right\}$$

$$\begin{aligned} \rightarrow f_s &= 2f_m \\ &= 2(2500) \\ &= 5000 \text{ Hz} \end{aligned} \quad \left| \quad \begin{aligned} \rightarrow T_s &= 1/f_s \\ &= 1/5000 \\ &= 0.2 \text{ msec} \end{aligned} \right.$$

□

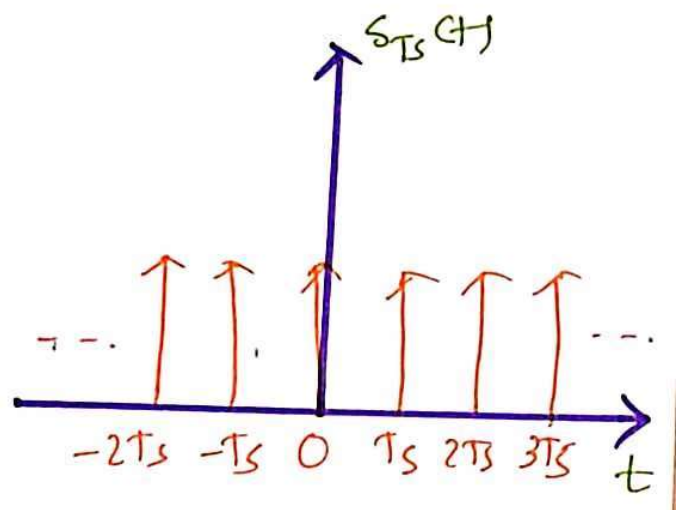
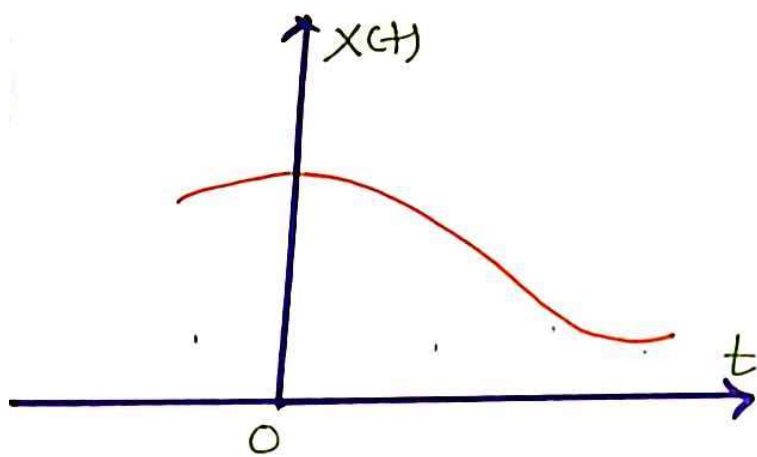
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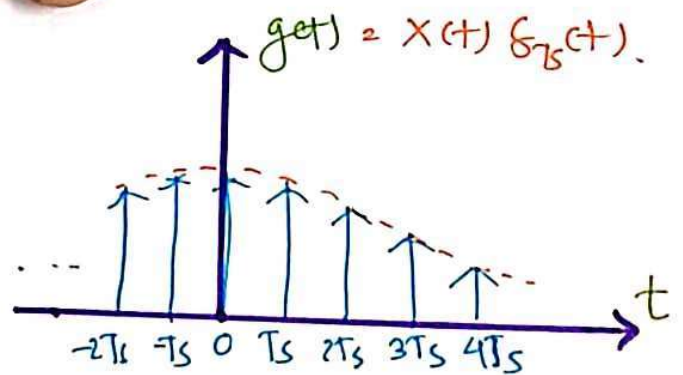
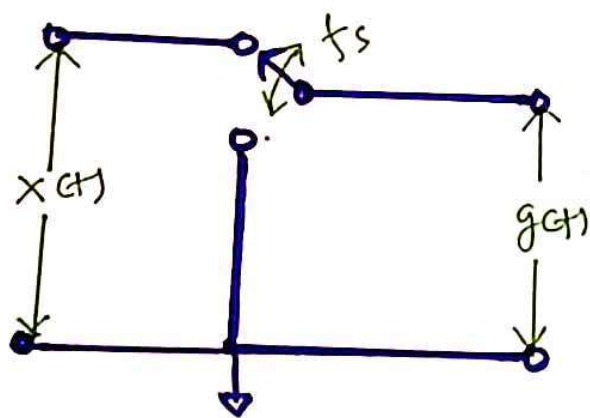
3] Determine the Nyquist rate for a continuous-time signal $x(t) = 6 \cos 500\pi t + 20 \sin 3000\pi t - 10 \cos 100\pi t$

$$\begin{aligned} \rightarrow f_1 &= \frac{\omega_1}{2\pi} = 25 \text{ Hz} \\ f_2 &= \frac{\omega_2}{2\pi} = 150 \text{ Hz} \\ f_3 &= \frac{\omega_3}{2\pi} = 50 \text{ Hz} \end{aligned} \quad \left. \vphantom{\begin{aligned} f_1 \\ f_2 \\ f_3 \end{aligned}} \right\} \begin{aligned} \rightarrow \text{max freq. } f_m &= 150 \text{ Hz} \\ \rightarrow f_s &= 2f_m \\ &= 2 \times 150 \\ &= 300 \text{ Hz} \end{aligned}$$

□

Instantaneous Sampling or Impulse Sampling or Ideal Sampling
 → It uses principle of multiplication





- To generate ideal Samples train, we use Switching Sampler.
- If we assume, closing time $t \rightarrow 0$, then it has to be Consider Ideal Impulse train.
- Impulse train

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- If we assume, closing time $t \rightarrow 0$, then it has to be considered ideal impulse train.

- Impulse train

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Practically not possible
High noise interference.

- Output

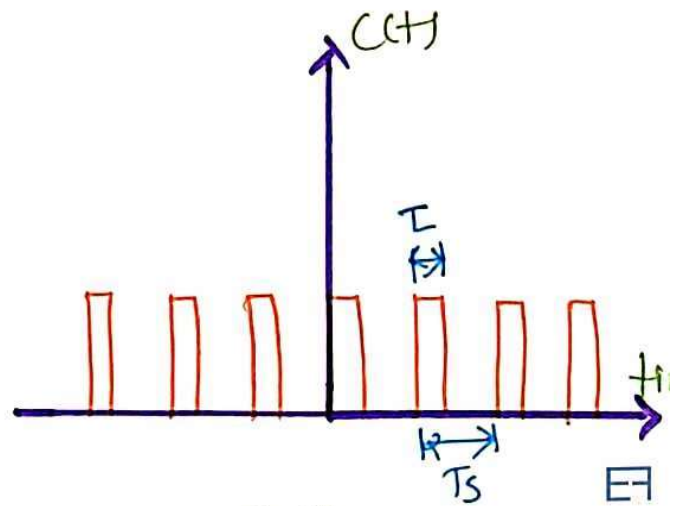
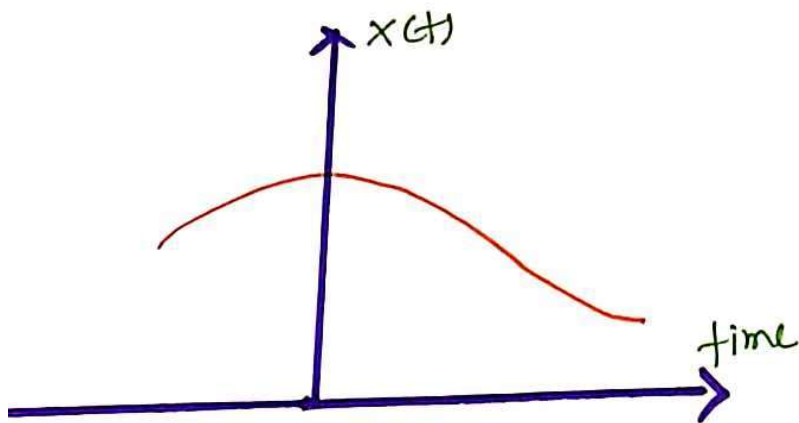
$$\begin{aligned} y(t) &= x(t) \delta_{T_s}(t) \\ &= x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \end{aligned}$$

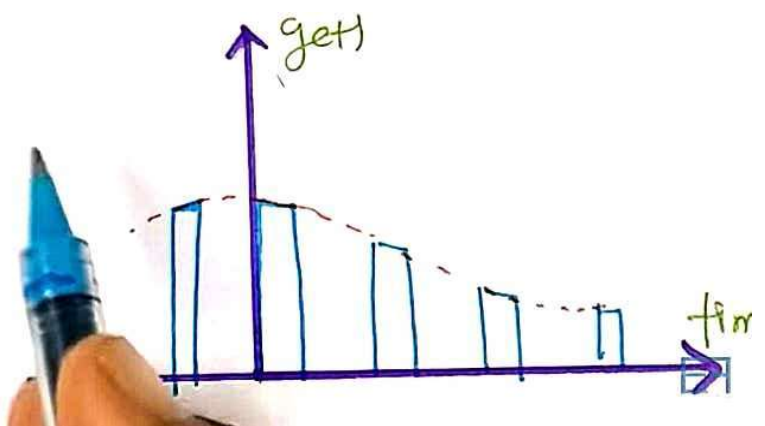
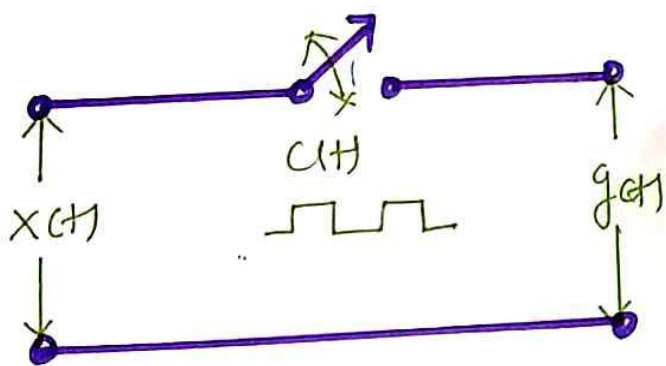
- In Frequency Domain,

$$Y(\omega) = T_s \sum_{n=-\infty}^{\infty} X(t - nT_s)$$

Natural Sampling

- It uses chopping principle





$$- \quad g(t) = x(t) \quad , \quad c(t) = A$$

$$g(t) = 0 \quad , \quad c(t) = 0$$

- So mathematically

$$g(t) = x(t) c(t)$$

$$= \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t) \sin(f_n T) e^{j 2 \pi f_n t}$$

• freq. domain

$$X(\omega) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \sin(n f_s T) X(f - n f_s)$$

$$g(t) = 0 \quad , \quad c(t) = 0$$

- So mathematically

$$g(t) = x(t) c(t) \\ = \frac{TA}{T_s} \sum_{n=-\infty}^{\infty} x(t) \sin(f_n T) e^{j2\pi f_n t}$$

- freq. domain

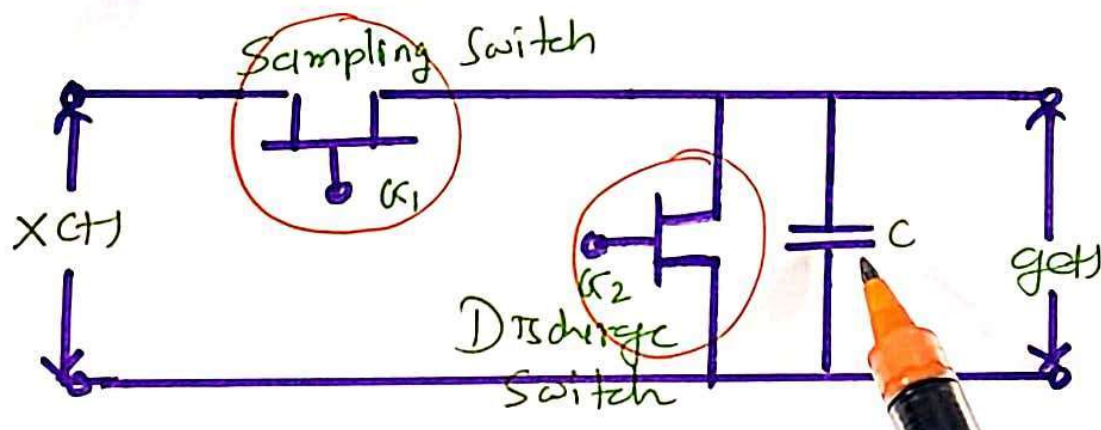
$$X(\omega) = \frac{TA}{T_s} \sum_{n=-\infty}^{\infty} \sin(n f_s T) X(f - n f_s)$$

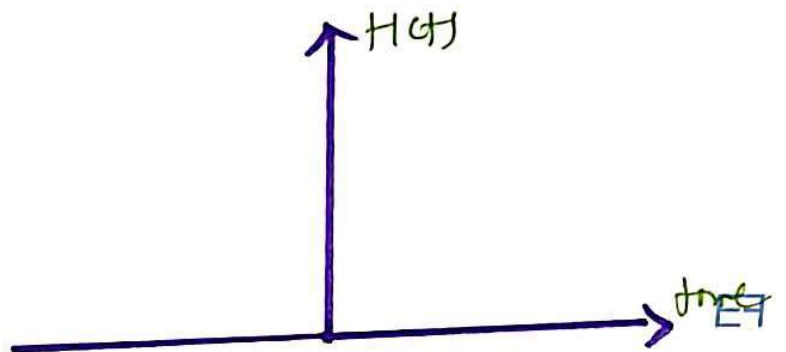
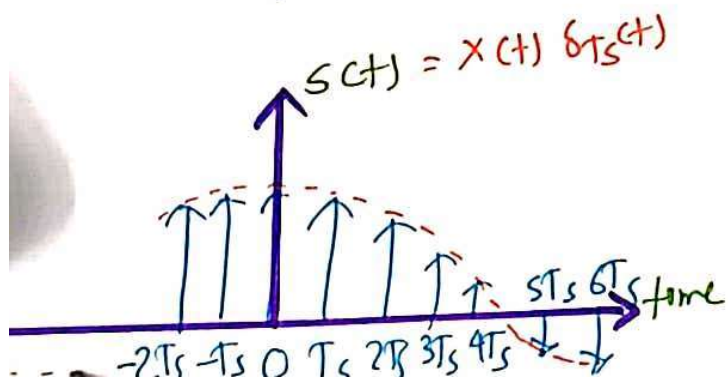
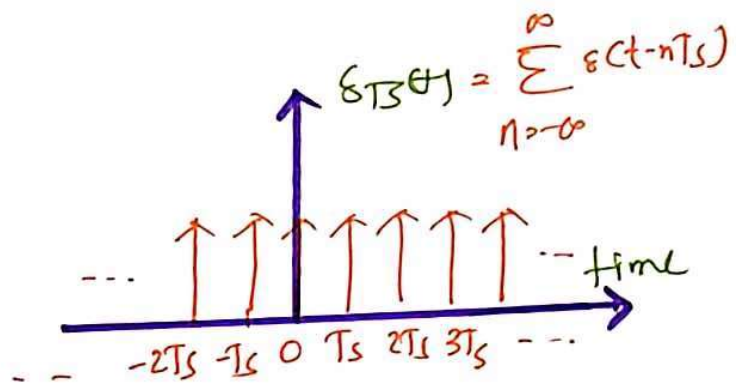
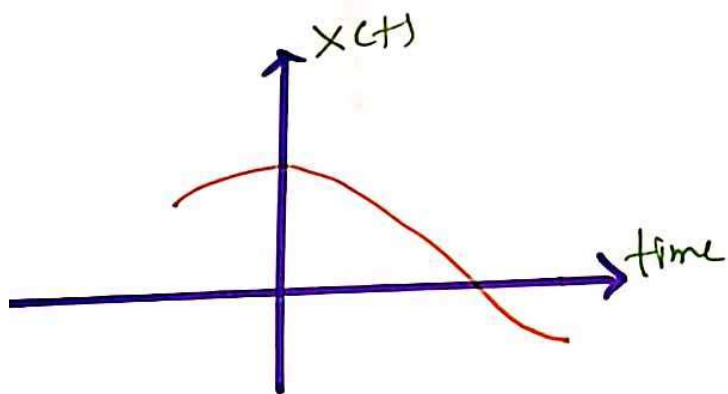
- This method is used practically.

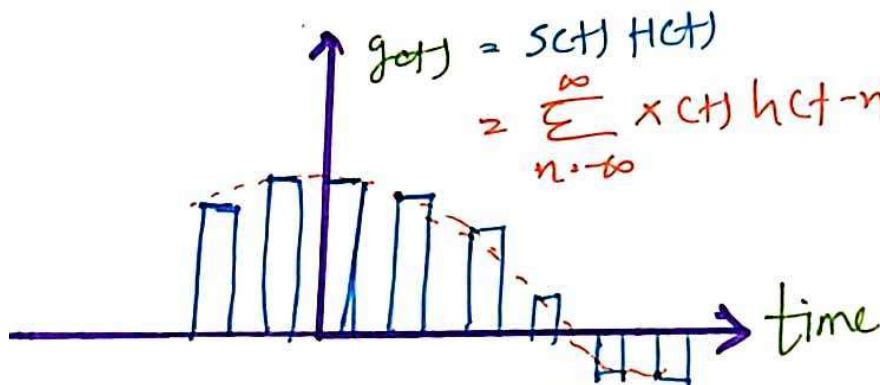
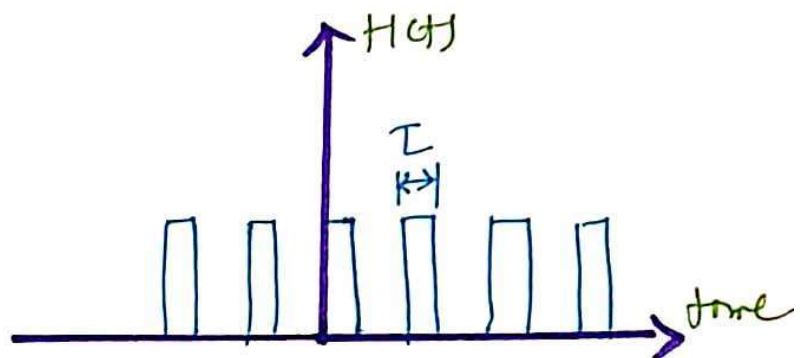
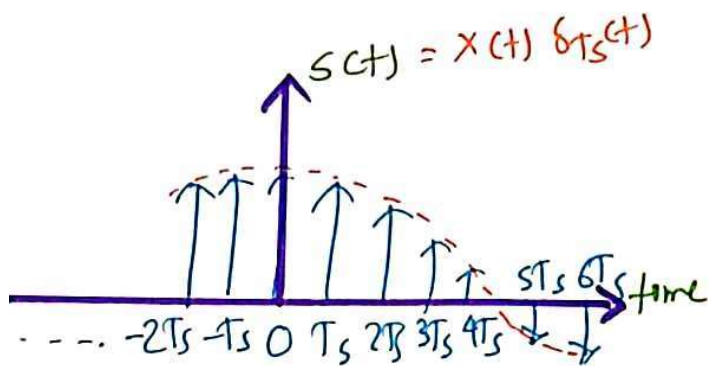
- Noise interference is less.

Flat top sampling [PAM]

- It uses Sample and hold circuit
- It is practically possible like natural sampling but Flat top sampling is easier compared to natural sampling
- It has very high noise interference.



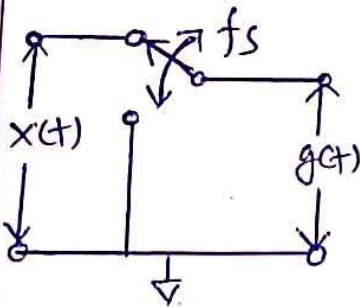
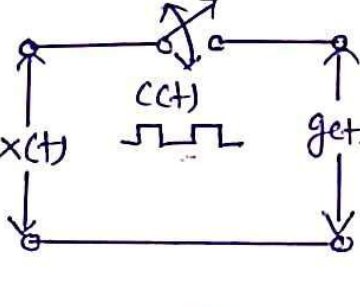
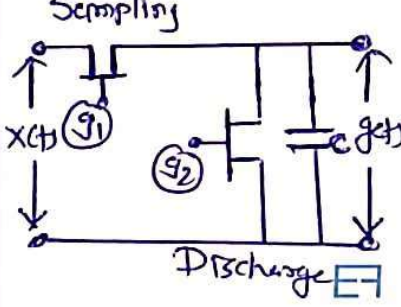


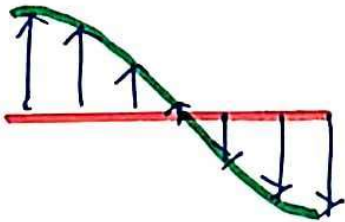
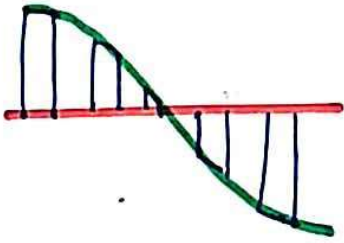
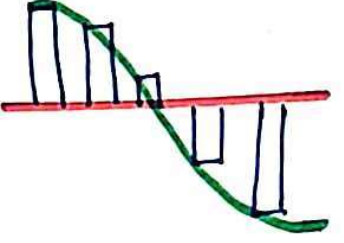


→ freq. domain

$$G(\omega) = f_s \sum_{n=-\infty}^{\infty} X(t - nT_s) H(t)$$

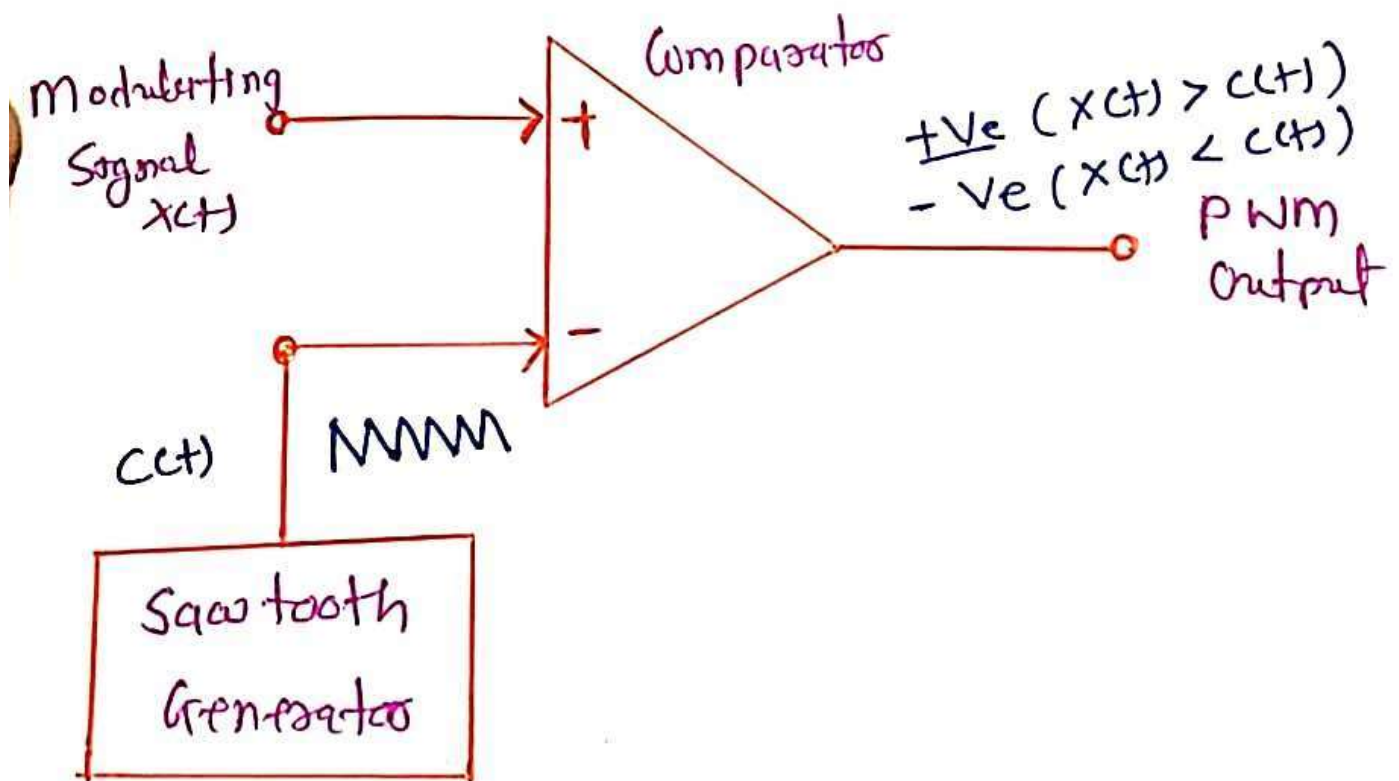
Performance Comparison of Sampling Techniques.

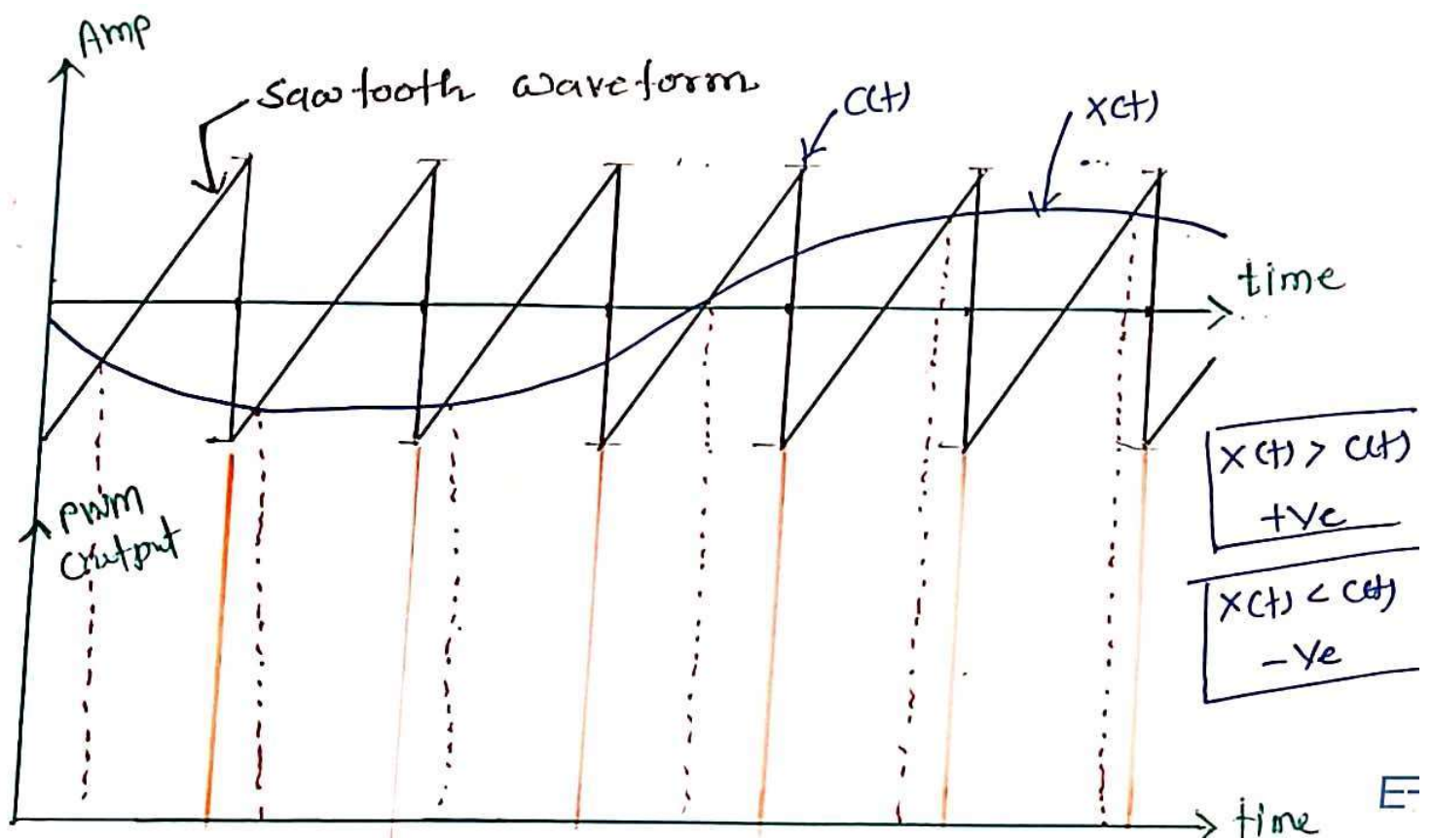
Performance Parameter	Ideal Sampling	Natural Sampling	Flat top Sampling
Sampling Principle	Multiplication	Chopping	Sample & Hold Circuit
Generation Circuit			

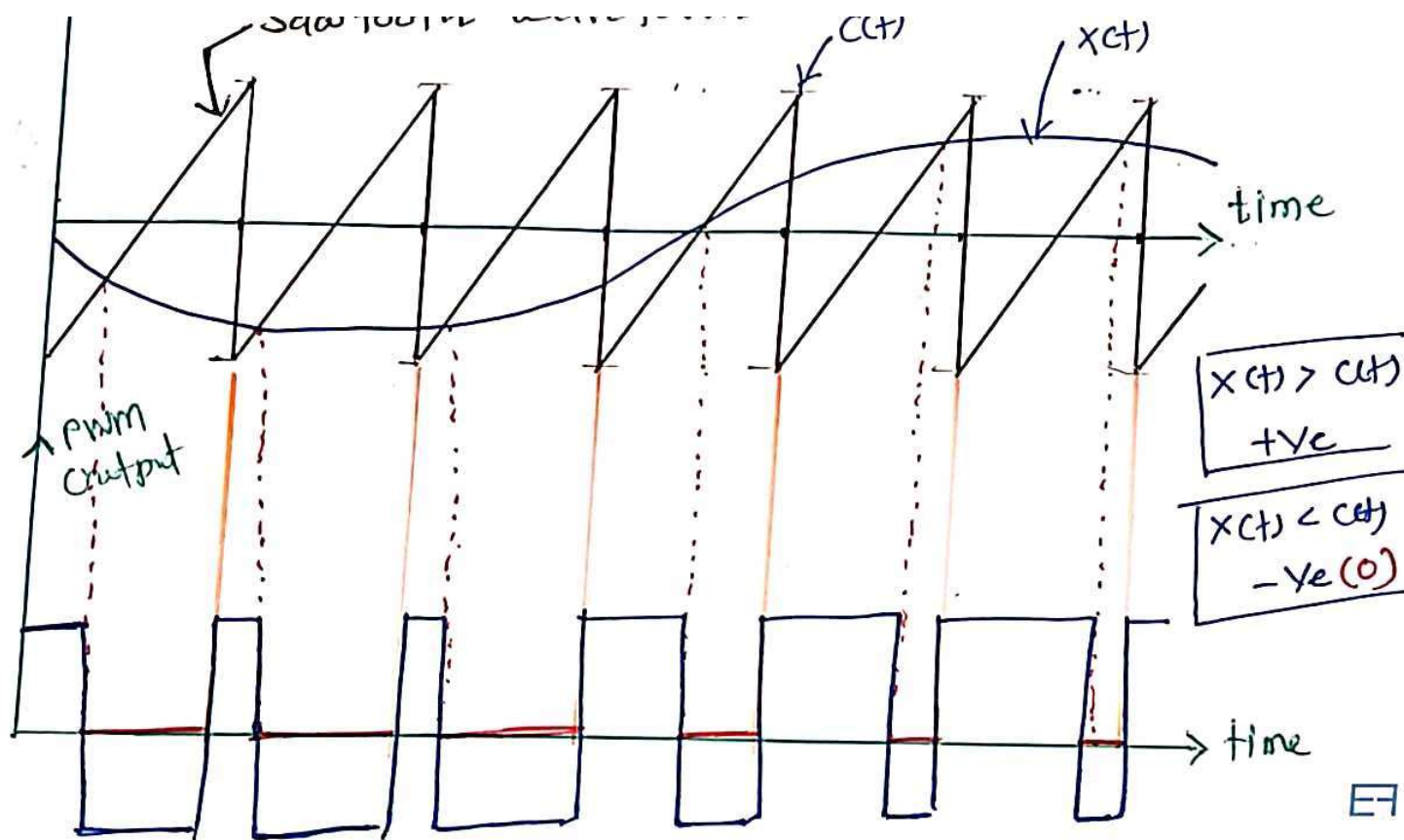
	↓		~
Waveforms			
Feasibility	Practically not possible	used practically	used practically.
Noise Interference	Very high	Less	high
			□

Feasibility	Practically not possible	used practically	used practically.
Noise Interference	Very high	Less	high
Time domain Representation	$g(t) = \sum_{n=-\infty}^{\infty} x(t) \delta(t - nT_s)$	$g(t) = \frac{TA}{T_s} \sum_{n=-\infty}^{\infty} x(t) \text{Sinc}(nt_s T) e^{j2\pi n f_s t}$	$g(t) = \sum_{n=-\infty}^{\infty} x(t) h(t - nT_s)$
Freq. domain Representation	$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s)$	$G(f) = \frac{TA}{T_s} \sum_{n=-\infty}^{\infty} \text{Sinc}(nt_s T) X(f - n f_s)$	$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s) H(f)$

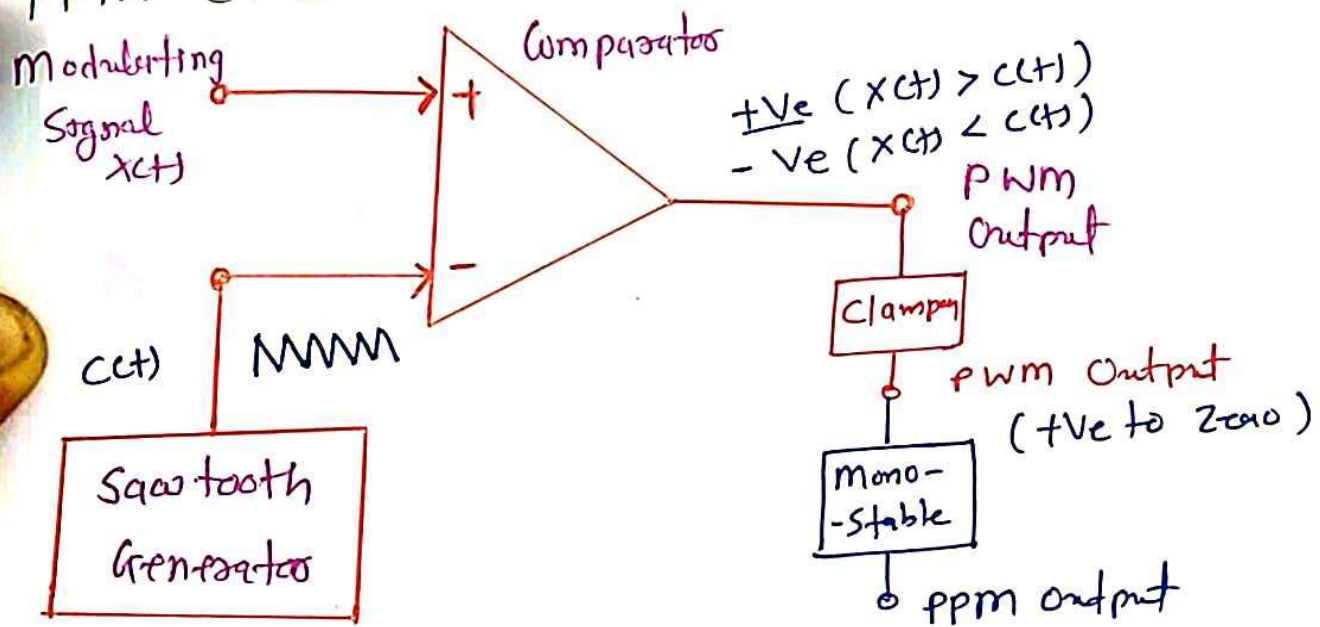
PWM (Pulse Width Modulation)

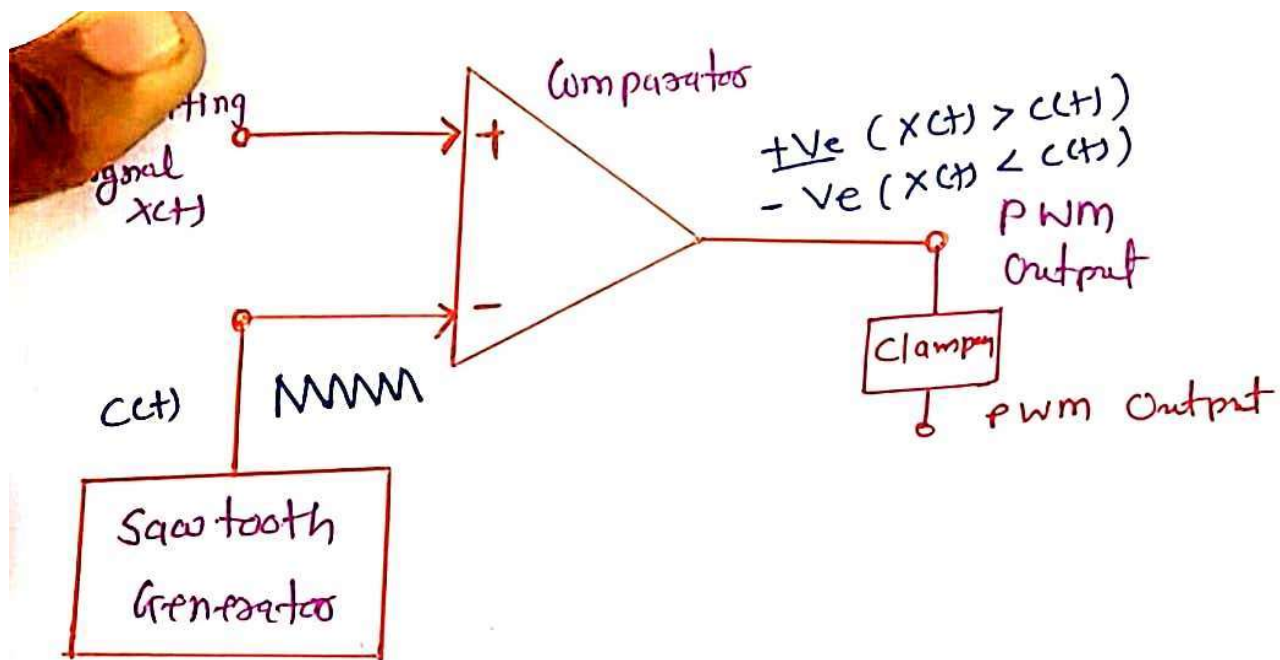






PWM (Pulse Width Modulation)
PPM (Pulse Position Modulation)

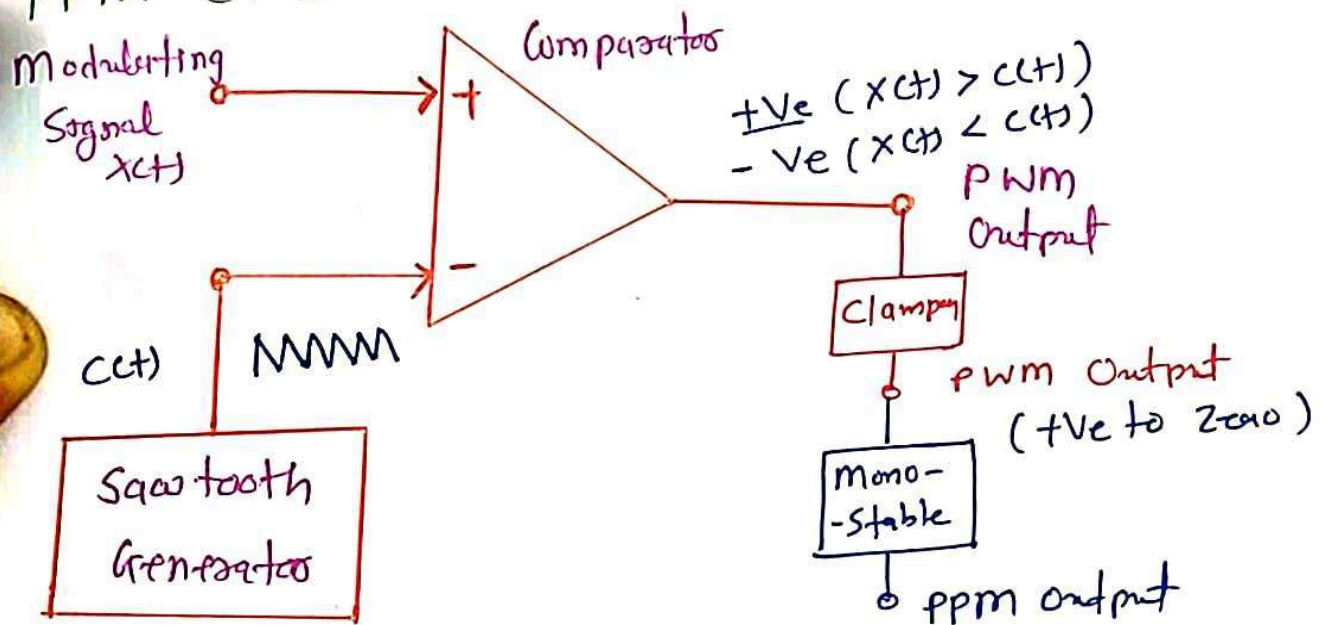


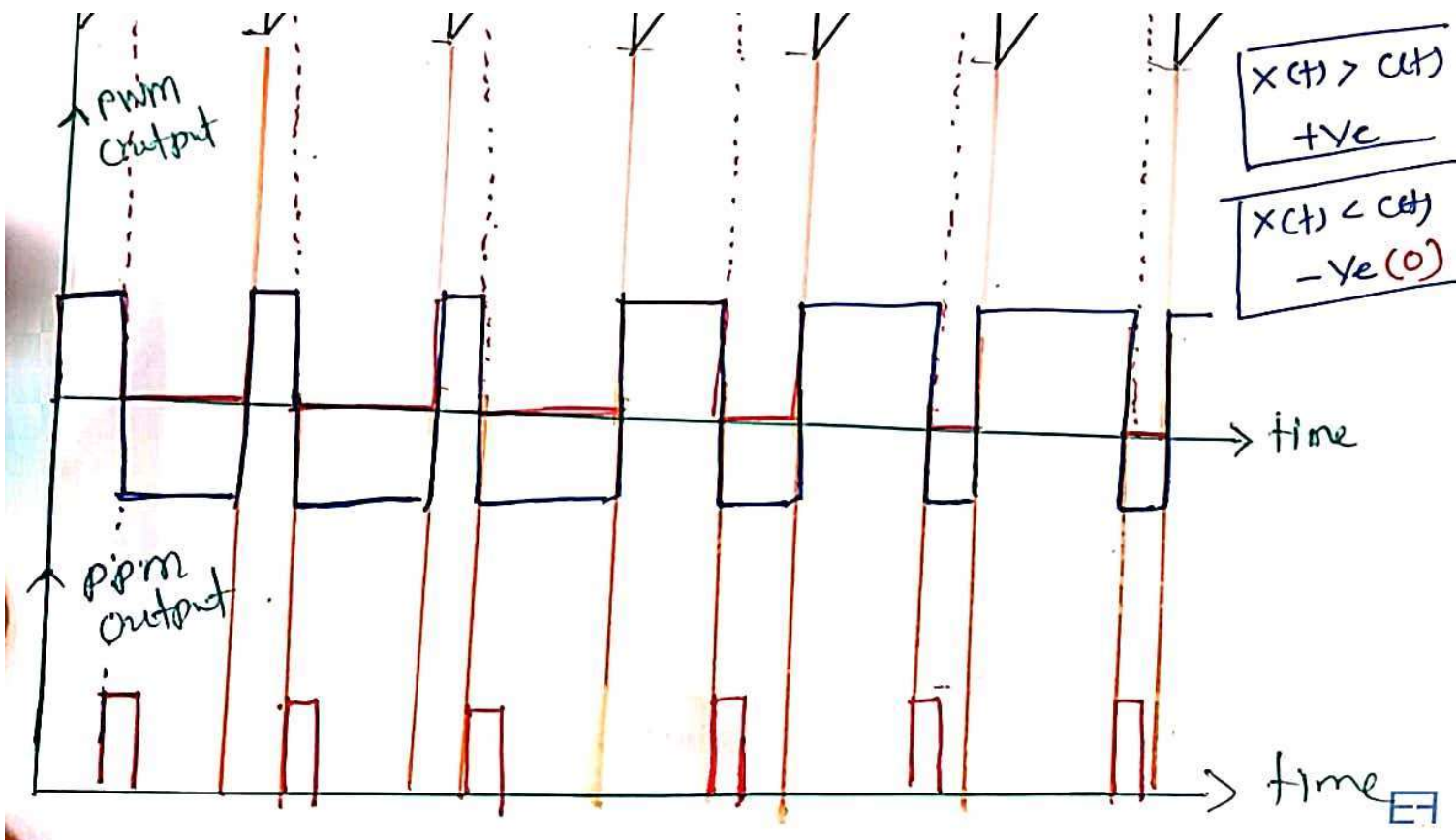


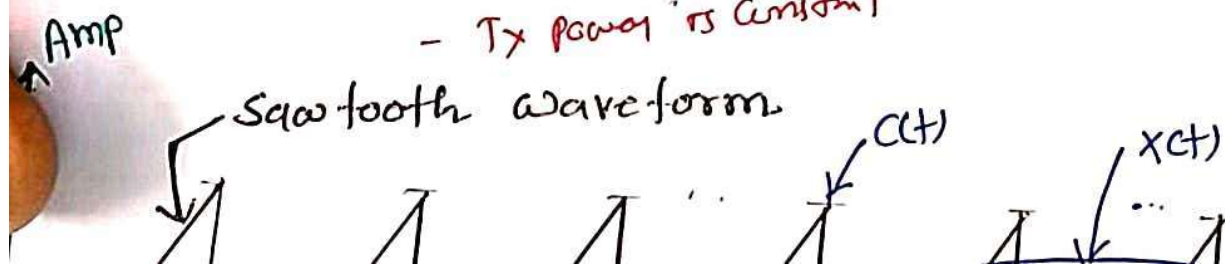
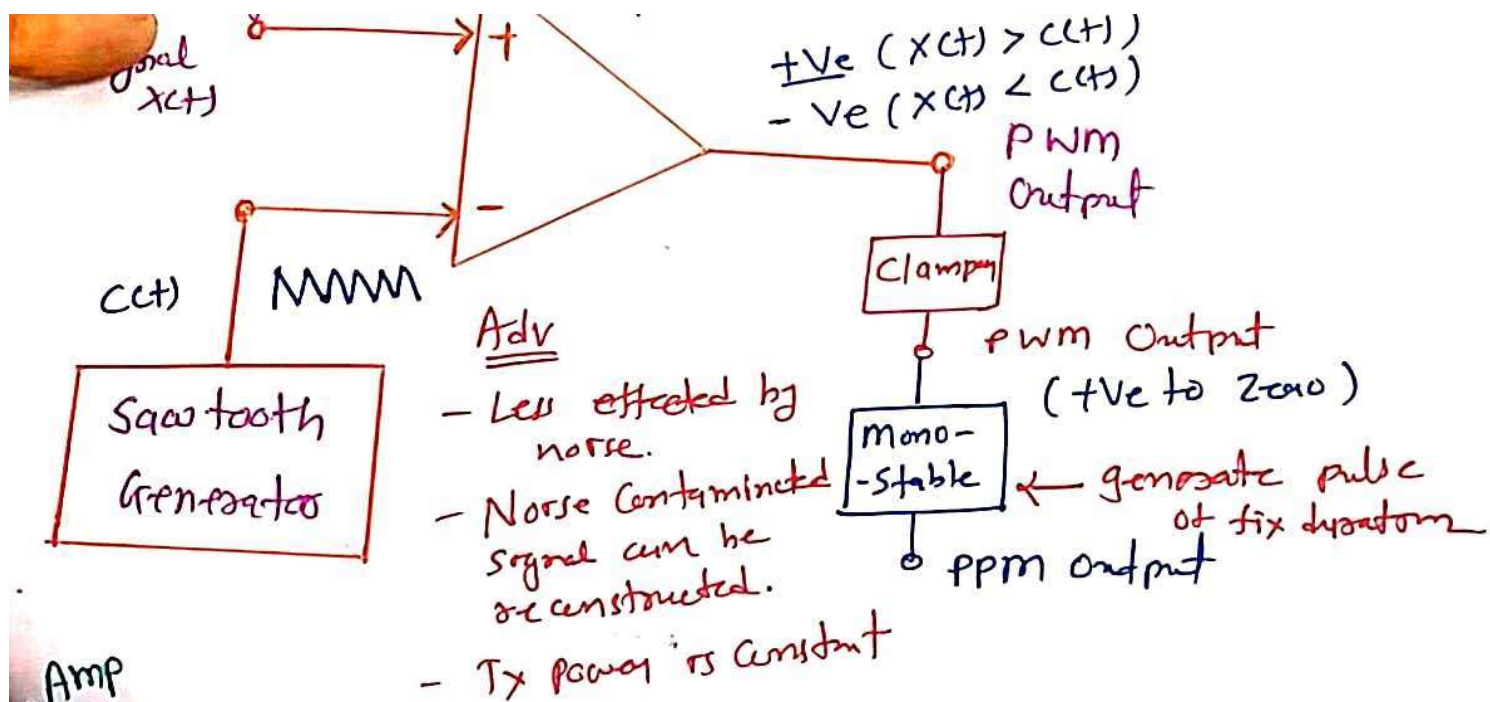
↑ Amp

— Sawtooth waveform, $c(t)$, $x(t)$

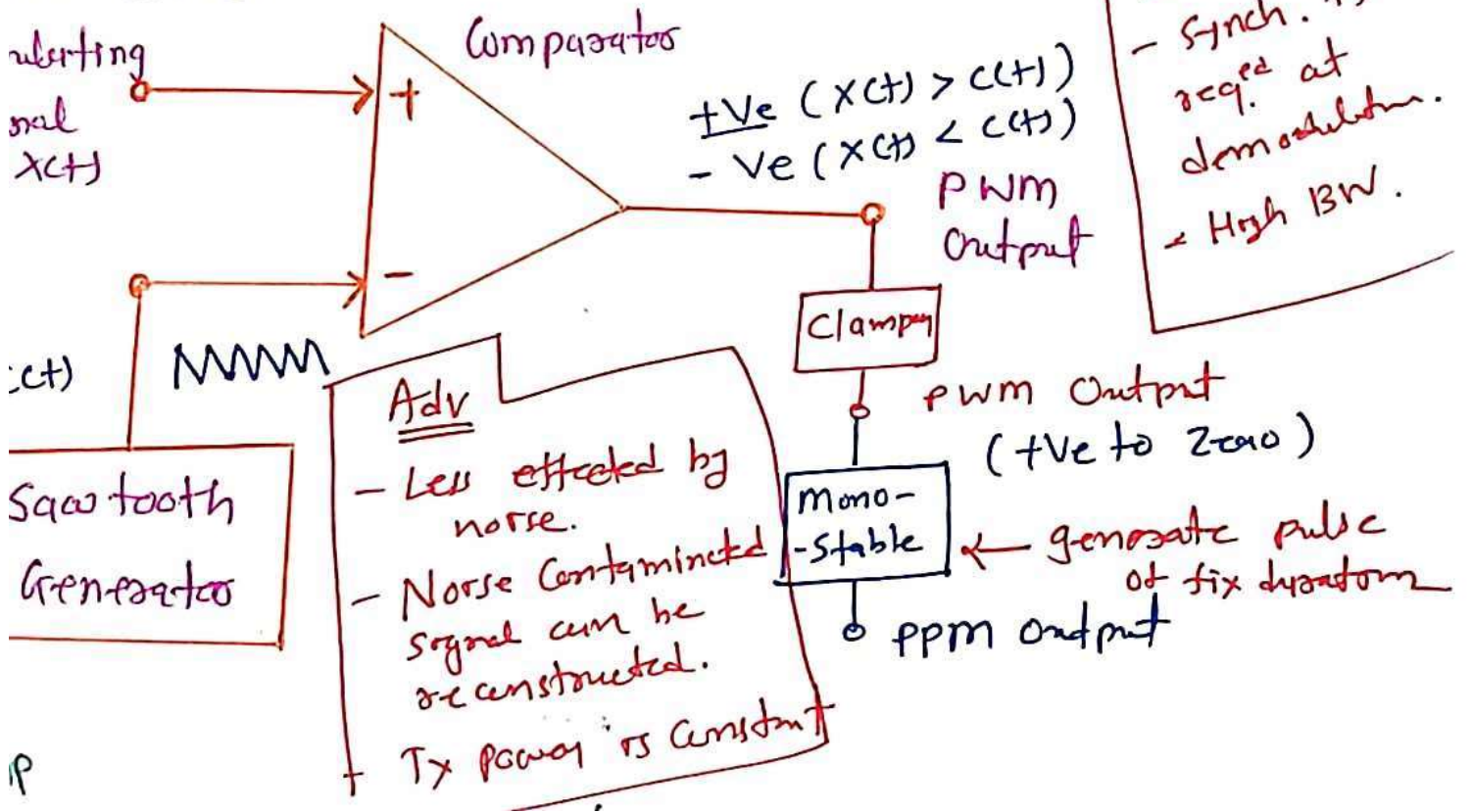
PWM (Pulse Width Modulation)
PPM (Pulse Position Modulation)



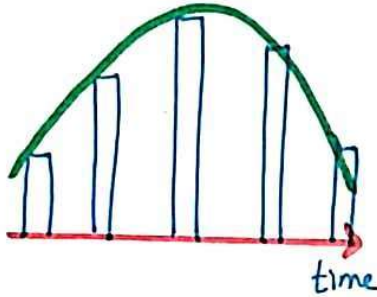
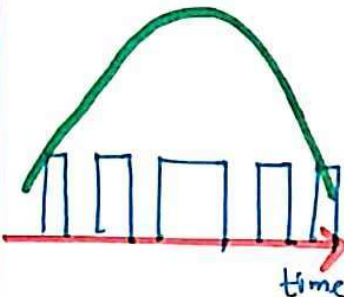
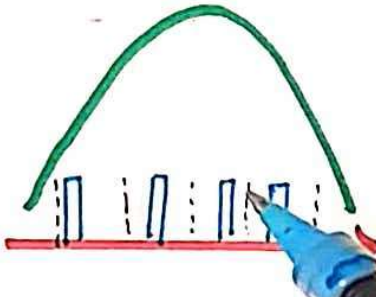




18) (Pulse Width Modulation)
 19) (Pulse Position Modulation)



Performance Comparison of Pulse analog Modulation

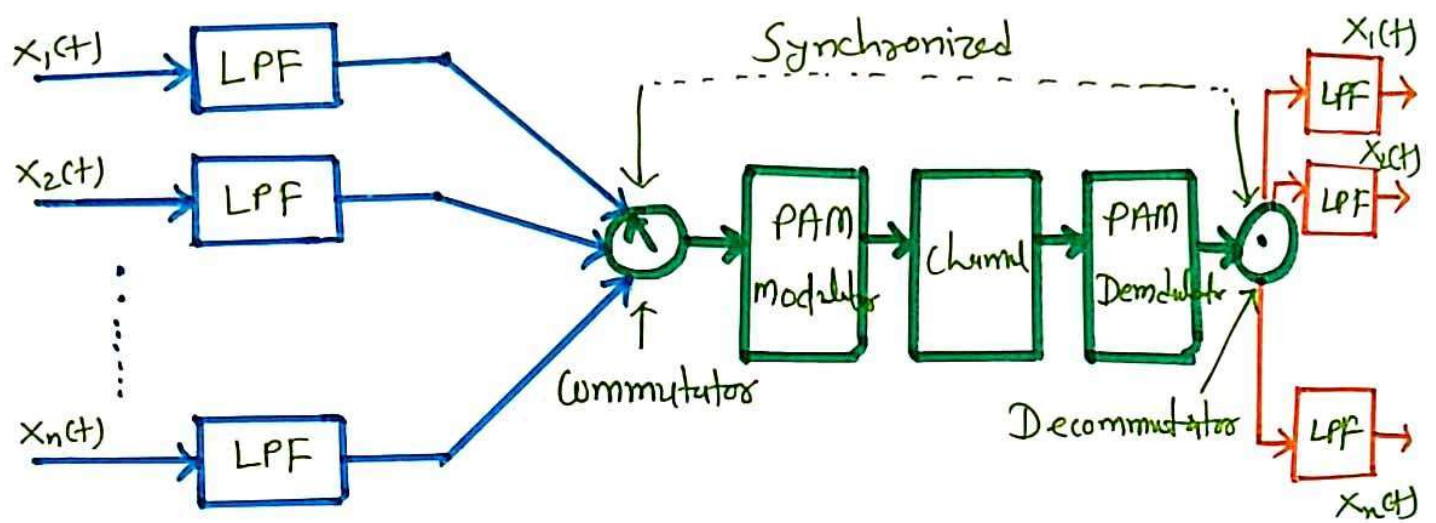
Performance Parameter	PAM	PWM	PPM
Waveform	 A diagram showing a green sinusoidal envelope curve. Inside the curve, there are five vertical rectangular pulses of varying heights. The pulses are drawn with blue outlines. The horizontal axis is a red line with an arrow pointing right, labeled 'time' in blue.	 A diagram showing a green sinusoidal envelope curve. Inside the curve, there are five rectangular pulses of equal height but varying widths. The pulses are drawn with blue outlines. The horizontal axis is a red line with an arrow pointing right, labeled 'time' in blue.	 A diagram showing a green sinusoidal envelope curve. Inside the curve, there are five narrow rectangular pulses of equal width but varying positions. The pulses are drawn with blue outlines. The horizontal axis is a red line with an arrow pointing right, labeled 'time' in blue. A blue pen is visible on the right side of the diagram.

	time	time	time
Working Principle	Amplitude of Pulse is proportional to amp. of modulating signal.	- Width of Pulse is proportional to amplitude of modulating signal	- Relative Position of Pulse is proportional to amp. of modulating signal
Bandwidth	- BW is depending on width of pulse	- BW depends on rise time of pulse.	- BW depends on rise time of pulse.
Transmitted Power	- Varies w.r.t time	- Varies w.r.t time	- Constant.
			□

Bandwidth	- BW is depending on width of pulse	- BW depends on rise time of pulse.	- BW depends on rise time of pulse.
Transmitted Power	- Varies w.r.t time	- Varies w.r.t time	- Constant.
Noise Interference	- Max	- Min	- Less
-	- It is similar to AM	- It is similar to FM	- It is similar to PM. \square

Time Division Multiplexing (TDM)

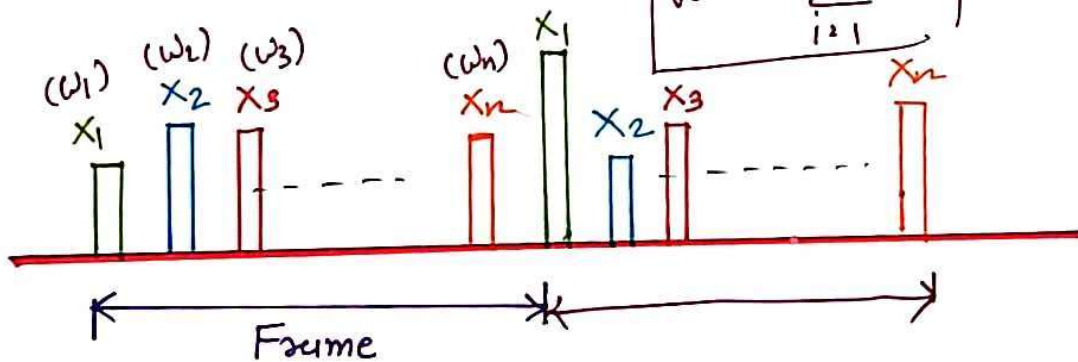
- The sampled PAM waveform is off for most of the time.
- During the off period, the channel can be used to transmit samples of other waveforms.
- The concept of interleaving samples from several signals into a single waveform is called TDM.



- The minimum bandwidth (TDM-PAM)

$$W = w_1 + w_2 + w_3 + \dots + w_n$$

$$W = \sum_{i=1}^N w_i$$



A Signal $x_1(t)$ is Band-limited to 4.2 kHz and three other signals $x_2(t)$, $x_3(t)$ and $x_4(t)$ are band limited to 1.4 kHz each. Assume that these signals are transmitted by TDM.

(a) Set-up a Scheme TDM for realizing this multiplexing requirement with each signal sampled at its Nyquist rate.

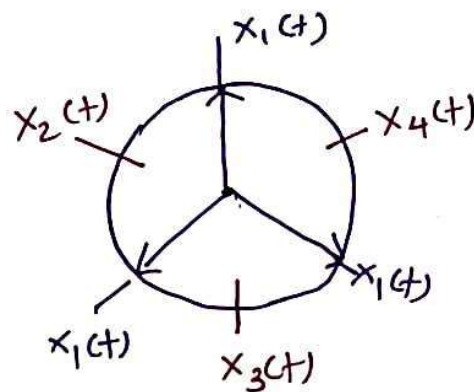
(b) Identify the required Speed of the Commutator in samples/sec.

(c) Identify the minimum Bandwidth requirement of the channel.

$$\begin{aligned}
 - x_1(t) &= 4.2 \text{ KHz} \\
 x_2(t) &= 1.4 \text{ KHz} \\
 x_3(t) &= 1.4 \text{ KHz} \\
 x_4(t) &= 1.4 \text{ KHz}
 \end{aligned}$$

(a) Nyquist plot

$x_1(t)$	$B_1 = 4.2 \text{ KHz}$	$f_{s1} = 2B_1 = 8.4 \text{ KHz}$
$x_2(t)$	$B_2 = 1.4 \text{ KHz}$	$f_{s2} = 2B_2 = 2.8 \text{ KHz}$
$x_3(t)$	$B_3 = 1.4 \text{ KHz}$	$f_{s3} = 2B_3 = 2.8 \text{ KHz}$
$x_4(t)$	$B_4 = 1.4 \text{ KHz}$	$f_{s4} = 2B_4 = 2.8 \text{ KHz}$



- If computer is rotating by 2800 rot/sec

- $x_1(t) = 4.2 \text{ KHz}$
- $x_2(t) = 1.4 \text{ KHz}$
- $x_3(t) = 1.4 \text{ KHz}$
- $x_4(t) = 1.4 \text{ KHz}$

(a) Nyquist rate

$$x_1(t) \quad B_1 = 4.2 \text{ KHz}$$

$$f_{s1} = 2B_1 = 8.4 \text{ KHz}$$

$$x_2(t) \quad B_2 = 1.4 \text{ KHz}$$

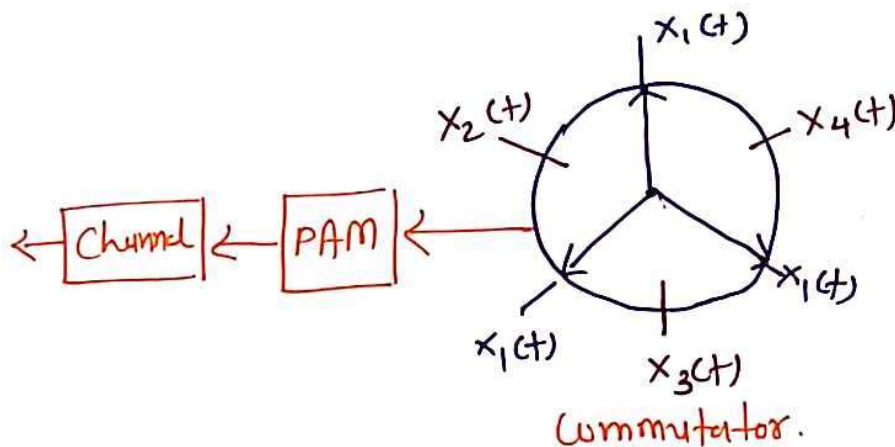
$$f_{s2} = 2B_2 = 2.8 \text{ KHz}$$

$$x_3(t) \quad B_3 = 1.4 \text{ KHz}$$

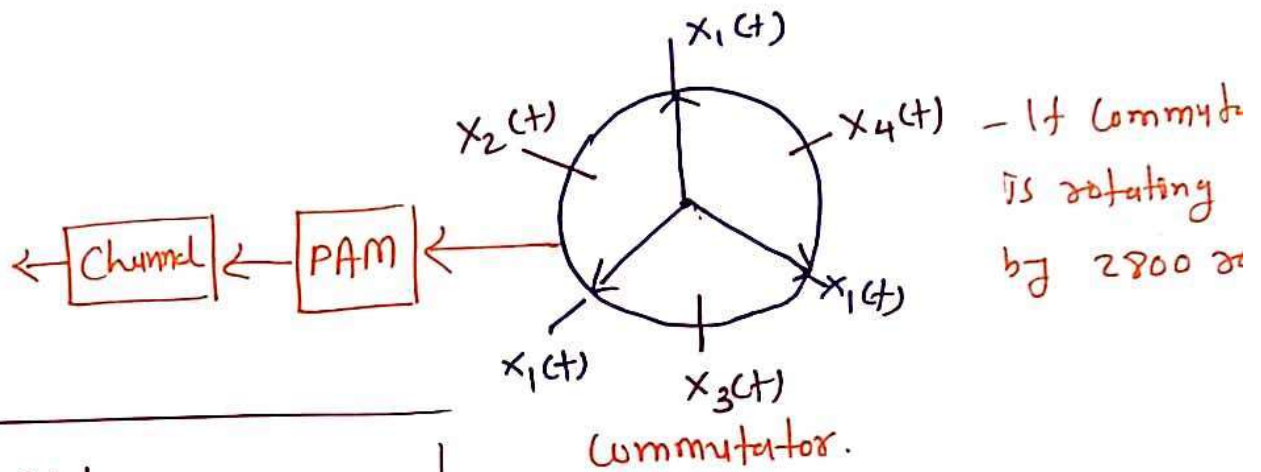
$$f_{s3} = 2B_3 = 2.8 \text{ KHz}$$

$$x_4(t) \quad B_4 = 1.4 \text{ KHz}$$

$$f_{s4} = 2B_4 = 2.8 \text{ KHz}$$



- If commutator is rotating by 2800 rot/sec



Minimum BW

$$\begin{aligned}
 B &= B_1 + B_2 + B_3 + B_4 \\
 &= 4.2 + 1.4 + 1.4 + 1.4 \\
 &= 8.4 \text{ KHz}
 \end{aligned}$$

Signals	Samples/sec
$x_1(t)$	8400 samples/sec
$x_2(t)$	2800
$x_3(t)$	2800
$x_4(t)$	2800
	<hr/> 16800 samples/sec

② Assume that the commutator o/p is quantized with $L = 1024$ and the result is binary coded. what is the o/p bit rate? [take ref. of example 1]

$$- L = 1024 = 2^m$$

$$\begin{aligned} \Rightarrow m &= \log_2 L \\ &= \log_2 1024 \\ &= 10 \text{ bits/sample.} \end{aligned}$$

→ Output bitrate

$$\begin{aligned} R_b &= m \times f_s \\ &= \left(\frac{\text{bits}}{\text{sample}} \right) \left(\frac{\text{Samples}}{\text{sec}} \right) \\ &= 10 \times 16.8 \text{ KHz} \\ &= \underline{168 \text{ Kbps}} \end{aligned}$$

Two analog signals $x_1(t)$ & $x_2(t)$ are to be transmitted over a common channel by means of TDM.

The highest freq. of $x_1(t)$ is 3 kHz and that of $x_2(t)$ is 4.5 kHz. What is the minimum permissible sampling rate?

- $x_1(t)$ — $B_1 = 3 \text{ kHz}$
- $x_2(t)$ — $B_2 = 4.5 \text{ kHz}$

The highest freq. of $x_1(t)$ is 3 kHz and that of $x_2(t)$ is 4.5 kHz. What is the minimum permissible sampling rate?

- $x_1(t)$ — $B_1 = 3 \text{ kHz}$
- $x_2(t)$ — $B_2 = 4.5 \text{ kHz}$
- Composite signal of $x_1(t)$ and $x_2(t)$ is $x_1(t) + x_2(t)$.
- Bandwidth of composite signal is highest of $x_1(t)$ and $x_2(t)$.
- Highest of $x_1(t)$ & $x_2(t)$ is 4.5 kHz.
- min sampling rate $f_s = 2W$
 - 2×4.5
 - 9 kSamples/sec.

The T1 carrier system used in digital telephony multiplexes 24 voice channels based on 8 bit PCM. Each voice signal is put through a LPF with the cut off frequency of 3.4 KHz. The LPF o/p is sampled at 8KHz. Then a single bit is added at the end of the frame for the purpose of synchronization. Calculate

- a) Bit duration
- b) Transmission rate
- c) Nyquist Bandwidth

b) Transmission rate

c) Nyquist Bandwidth

- T₁ Carrier System

- It uses TDM-PCM

- Voice Signal

- 300 Hz - 3.4 kHz

1 m → [LPF] → [quant] →

2 m

3 m

⋮

24 m

$f_c = 3.4 \text{ kHz}$

$m = 8 \text{ bits}$

$$L = 2^m = 2^8 = 256 \text{ levels}$$

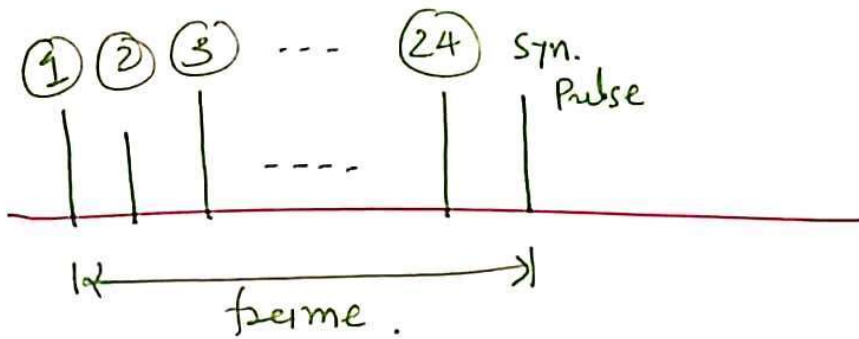
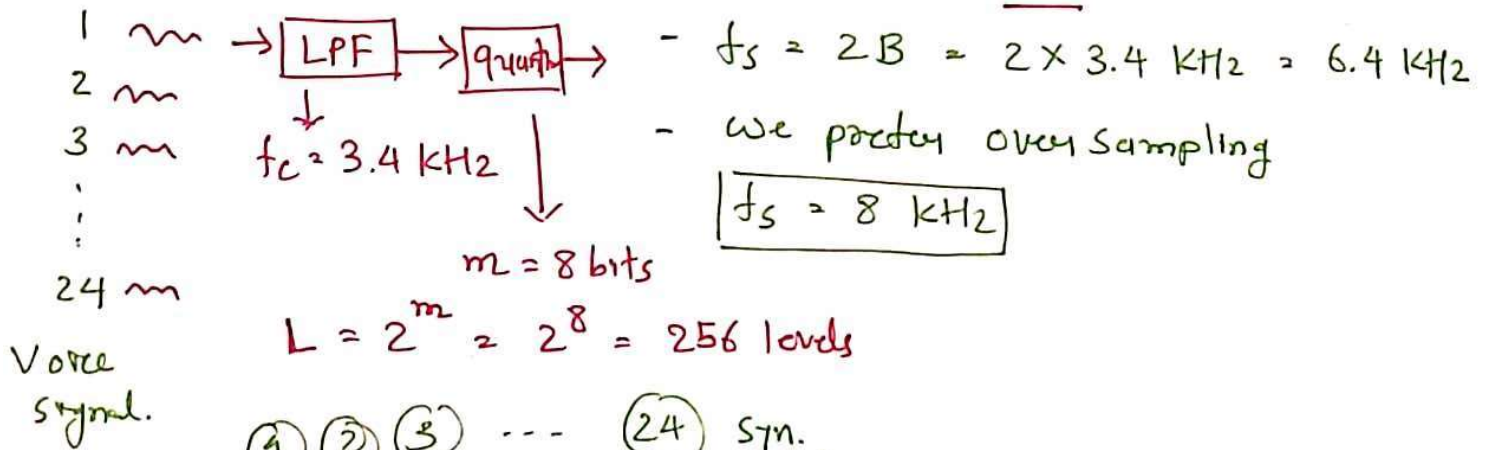
- $f_s = 2B = 2 \times 3.4 \text{ kHz} = 6.4 \text{ kHz}$

- we prefer over sampling

$$f_s = 8 \text{ kHz}$$

Voice
signal.

- It uses TDM-PCM | - 300 Hz - 3.4 kHz



- bit rate $= R_b = m f_s = 8 \times 8000 = 64 \text{ Kbps}$

- Frame period $T_f = \frac{1}{f_s} = \frac{1}{8000} = 125 \mu\text{sec}$

- bits in one frame. $= 8(24) + 1 = 193 \text{ bits}$.

(a) bit duration $= \frac{T_f}{\text{No of bits}} = \frac{125 \mu\text{sec}}{193} = 0.648 \mu\text{sec}$

(b) - Transmission rate $R_b = 1/T_b = 1/0.648 \mu\text{sec} = 1.544 \text{ Mbps}$

- bit rate $= R_b = m f_s = 8 \times 8000 = 64 \text{ kbps}$

- Frame period $T_f = \frac{1}{f_s} = \frac{1}{8000} = 125 \mu\text{sec}$

- bits in one frame. $= 8(24) + 1 = 193 \text{ bits}$.

(a) bit duration $= \frac{T_f}{\text{No of bits}} = \frac{125 \mu\text{sec}}{193} = 0.648 \mu\text{sec}$

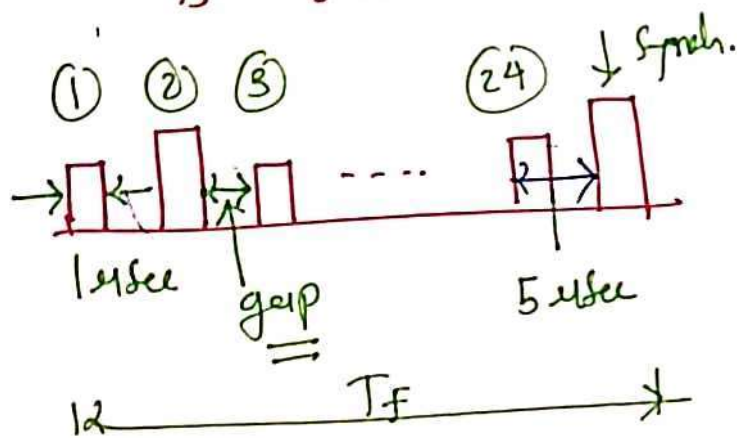
(b) - Transmission rate $R_b = 1/T_b = 1/0.648 \mu\text{sec} = 1.544 \text{ Mbps}$

(c) - $R_b = 2B \Rightarrow B = R_b/2 = 1.544/2 = 0.772 \text{ MHz or } 772 \text{ kHz}$

24 Voice Signals are uniformly sampled and then multiplexed with TDM. The sampling operation used flat Top samples with 1 μ sec duration. For synchronization one extra pulse of 1 μ sec duration is added. The highest freq. of each voice signal is 3.4 KHz.

- a) Calculate the spacing between successive pulses of multiplexed signal by assuming a sampling rate of 8 KHz
- b) Repeat for Nyquist rate of sampling.

$$- T_f = \frac{1}{f_s} = \frac{1}{8000} = 125 \text{ } \mu\text{sec}$$



- for 1 pulse time (without spacing)

$$= \frac{T_f}{\text{No of Pulses}}$$

$$= \frac{125 \text{ } \mu\text{sec}}{25}$$

$$= 5 \text{ } \mu\text{sec}$$

$$- \text{Spacing bet.}^n \text{ Pulses} = 5 \text{ } \mu\text{sec} - 1 \text{ } \mu\text{sec}$$

$$= 4 \text{ } \mu\text{sec.}$$

- Spacing bet.ⁿ Pulses = $5 \mu\text{sec} - 1 \mu\text{sec}$
= $4 \mu\text{sec}$.

- $T_f = \frac{1}{f_s} = \frac{1}{6800} = 147.06 \mu\text{sec}$

- For 1 pulse time without space

$$= \frac{T_f}{\text{No of Pulg}} = \frac{147.06}{25} = 5.88 \mu\text{sec}$$

- Spacing = $5.88 - 1 = 4.88 \mu\text{sec}$.

- Lowering sampling rate \uparrow in Spacing bet.ⁿ
Successive Pulses.

Eight message signals are sampled & multiplexed with TDM. The TDM output is passed through a LPF before transmission. Six of the V_p signals have a BW of 4 KHz and the other two are band limited to 12 KHz.

- (a) Identify the minimum sampling rate of composite signal. also find the overall sampling rate.
- (b) Design an asynchronous TDM for this application. Assume commutator speed to be 8000 rotations/sec
- (c) Compute the transmission bandwidth requirement for part (a) and (b)

$$\begin{array}{ccccccc}
 - & x_1(t) & x_2(t) & x_3(t) & x_4(t) & \dots & x_8(t) \\
 & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{4.5cm}} & & & \\
 & 12 \text{ KHz} & & 4 \text{ KHz} & & &
 \end{array}$$

- for composite signal consider highest bandwidth of signal.

$$\begin{aligned}
 f_s &= 2W \\
 &= 2(12 \text{ KHz}) \\
 &= 24 \text{ KHz}
 \end{aligned}$$

- Overall sampling rate for 8 channels

$$\begin{aligned}
 &= 8 \times f_s \\
 &= 8 \times 24 = 192000 \text{ samples/sec}
 \end{aligned}$$

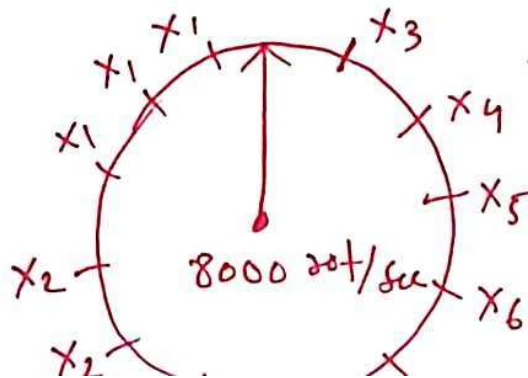
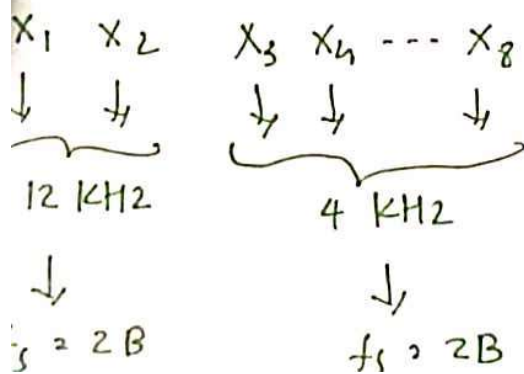
$$\begin{aligned}
 f_s &= 2W \\
 &= 2(12 \text{ KHz}) \\
 &= 24 \text{ KHz}
 \end{aligned}$$

- Overall sampling rate for 8 channels

$$\begin{aligned}
 &= 8 \times f_s \\
 &= 8 \times 24 = 192000 \text{ samples/sec}
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow f_s &= 2B \Rightarrow B = \frac{f_s}{2} \\
 \Rightarrow B &= \frac{192000}{2} \\
 \boxed{B} &= \boxed{96 \text{ KHz}}
 \end{aligned}$$

Commutator Speed 8000 rot/sec.



- Sampling rate

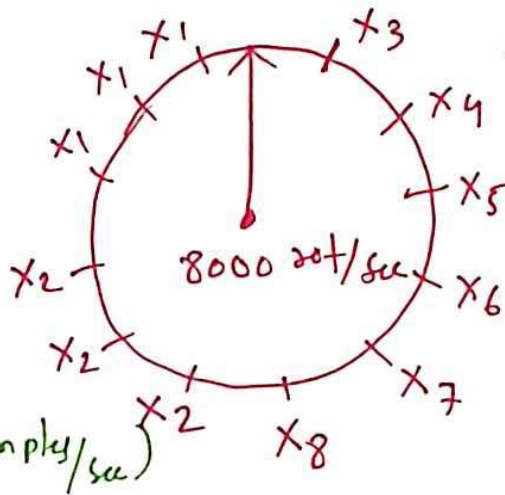
$$\begin{aligned}
 &= 12 \times 8000 \\
 &= 96000 \\
 &\text{Sample/sec}
 \end{aligned}$$

$$= 0.75 \text{ sec} = 11.25 \text{ sec}$$

$$B = 76 \text{ kHz}$$

Commutator Speed 8000 rot/sec.

$$\begin{array}{ccc} x_1 & x_2 & x_3 \quad x_4 \quad \dots \quad x_8 \\ \downarrow & \downarrow & \downarrow \quad \downarrow \quad \downarrow \\ \underbrace{\hspace{1cm}} & & \underbrace{\hspace{1cm}} \\ 12 \text{ kHz} & & 4 \text{ kHz} \\ \downarrow & & \downarrow \\ f_s = 2B & & f_s = 2B \\ = 24 \text{ kHz} & & = 8000 \text{ (sample/sec)} \\ \text{(sample/sec)} & & \end{array}$$



$$\begin{aligned} \text{Sampling rate} &= 12 \times 8000 \\ &= 96000 \text{ sample/sec} \end{aligned}$$

$$\begin{aligned} B &= \frac{f_s}{2} \\ &= \frac{96000}{2} = 48 \text{ kHz} \end{aligned}$$