Heuristic Search Techniques

Heuristic Search

- Heuristics are criteria for deciding which among several alternatives be the most effective in order to achieve some goal.
- Heuristic is a technique that
  - improves the efficiency of a search process possibly by sacrificing claims of systematicity and completeness.
  - It no longer guarantees to find the best answer but almost always finds a very good answer.

Heuristic Search – Contd..

- Using good heuristics, we can hope to get good solution to hard problems (such as travelling salesman) in less than exponential time.
- There are **general-purpose** heuristics that are useful in a wide variety of problem domains.
- We can also construct **special purpose** heuristics, which are domain specific.

General Purpose Heuristics

- A general-purpose heuristics for combinatorial problem is
  - **Nearest neighbor algorithms** which works by selecting the locally superior alternative.
  - For such algorithms, it is often possible to prove an upper bound on the error which provide reassurance that one is not paying too high a price in accuracy for speed.
- In many AI problems,
  - it is often hard to measure precisely the goodness of a particular solution.
  - But still it is important to keep performance question in mind while designing algorithm.

Contd…

- For real world problems,
  - it is often useful to introduce heuristics based on relatively unstructured knowledge.
  - It is impossible to define this knowledge in such a way that mathematical analysis can be performed.
- In AI approaches,
  - behavior of algorithms are analyzed by running them on computer as contrast to analyzing algorithm mathematically.

Contd..

- There are at least two reasons for the adhoc approaches in AI.
  - It is a lot more fun to see a program do something intelligent than to prove it.
  - AI problem domains are usually sufficiently complex, so generally not possible to produce analytical proof that a procedure will work.
  - It is even not possible to describe the range of problems well enough to make statistical analysisof program behavior meaningful.

Contd..

- One of the most important analysis of the search process is straightforward i.e.,
  - Number of nodes in a complete search tree of depth D and branching factor F is F*D .
- This simple analysis motivates to
  - look for improvements on the exhaustive search.
  - find an upper bound on the search time which can be compared with exhaustive search procedures.

Informed Search Strategies- Branch & Bound Search

- It expands the least-cost partial path. Sometimes, it is called **uniform cost search**.
- **Function g(X)** assigns some cumulative expense to the path from *Start* node to *X* by applying the sequence of operators .
  - *For example*, in salesman traveling problem, g(X) may be the actual distance from *Start* to current node *X*.
- During search process there are many incomplete paths contending for further consideration.

Contd..

- The shortest one is extended one level, creating as many new incomplete paths as there are branches.
- These new paths along with old ones are sorted on the values of function **g**.
- The shortest path is chosen and extended.
- Since the shortest path is always chosen for extension, the path first reaching to the destination is certain to be nearly optimal.

Contd..

- Termination Condition:
  - Instead of terminating when a path is found, terminate when the shortest incomplete path is longer than the shortest complete path.
- If g(X) = 1, for all operators, then it degenerates to simple Breadth-First search.
- It is as bad as depth first and breadth first, from AI point of view,.
- This can be improved if we augment it by dynamic programming i.e. delete those paths which are redundant.

Hill Climbing- (Quality Measurement turns DFS into Hill climbing (Variant of generate and test strategy)

- Search efficiency may be improved if there is some way of ordering the choices so that the most promising node is explored first.
- Moving through a tree of paths, hill climbing proceeds
  - in depth-first order but the choices are ordered according to some heuristic value (i.e, measure of remaining cost from current to goal state).

Hill Climbing- Algorithm

Generate and Test *Algorithm*

Start

- Generate a possible solution
- Test to see, if it is goal.
- If not go to start else quit

End

Example of heuristic function

- Straight line (as the crow flies) distance between two cities may be a heuristic measure of remaining distance in traveling salesman problem .

Simple Hill climbing : Algorithm

- Store initially, the root node in a OPEN list (maintained as stack) ; Found = false;
- While (OPEN $\neq$ empty and Found = false) Do

{

  - Remove the top element from OPEN list and call it NODE;
  - If NODE is the goal node, then **Found = true** else find SUCCs, of NODE, if any, and **sort SUCCs** by estimated cost from NODE to goal state and add them to the front of OPEN list.

} /* end while */

- If **Found = true** then return **Yes** otherwise return **No**
- Stop

Problems in hill climbing

- There might be a position that is not a solution but from there no move improves situations?
- This will happen if we have reached a *Local maximum*, a *plateau* or a *ridge.*
  - **Local maximum:** It is a state that is better than all its neighbors but is not better than some other states farther away. All moves appear to be worse.
    - *Solution to this is to backtrack to some earlier state and try going in different direction.*

Contd…

- **Plateau:** It is a flat area of the search space in which, a whole set of neighboring states have the same value. It is not possible to determine the best direction.
  - *Here make a big jump to some direction and try to get to new section of the search space.*
- **Ridge:** It is an area of search space that is higher than surrounding areas, but that can not be traversed by single moves in any one direction. (Special kind of local maxima).
  - *Here apply two or more rules before doing the test i.e., moving in several directions at once.*

Beam Search

- Beam Search progresses level by level.
- It moves downward from the best **W** nodes only at each level. Other nodes are ignored.
  - W is called **width** of beam search.
- It is like a BFS where also expansion is level wise.
- Best nodes are decided on the heuristic cost associated with the node.
- If B is the **branching factor**, then there will be only **W*B** nodes under consideration at any depth but only W nodes will be selected.

Algorithm – Beam search

- Found = false;
- NODE = Root_node;
- If NODE is the goal node, then *Found* = *true* else find SUCCs of NODE, if any with its estimated cost and store in OPEN list;
- While (Found = false and not able to proceed further)

  {
  - Sort OPEN list;
  - Select top W elements from OPEN list and put it in W_OPEN list and empty OPEN list;

Algorithm – Contd…

- While (W_OPEN ≠ ϕ and Found = false)

{

- Get NODE from W_OPEN;
- If NODE = Goal state then Found = true else

{

- Find SUCCs of NODE, if any with its estimated cost
- store in OPEN list;

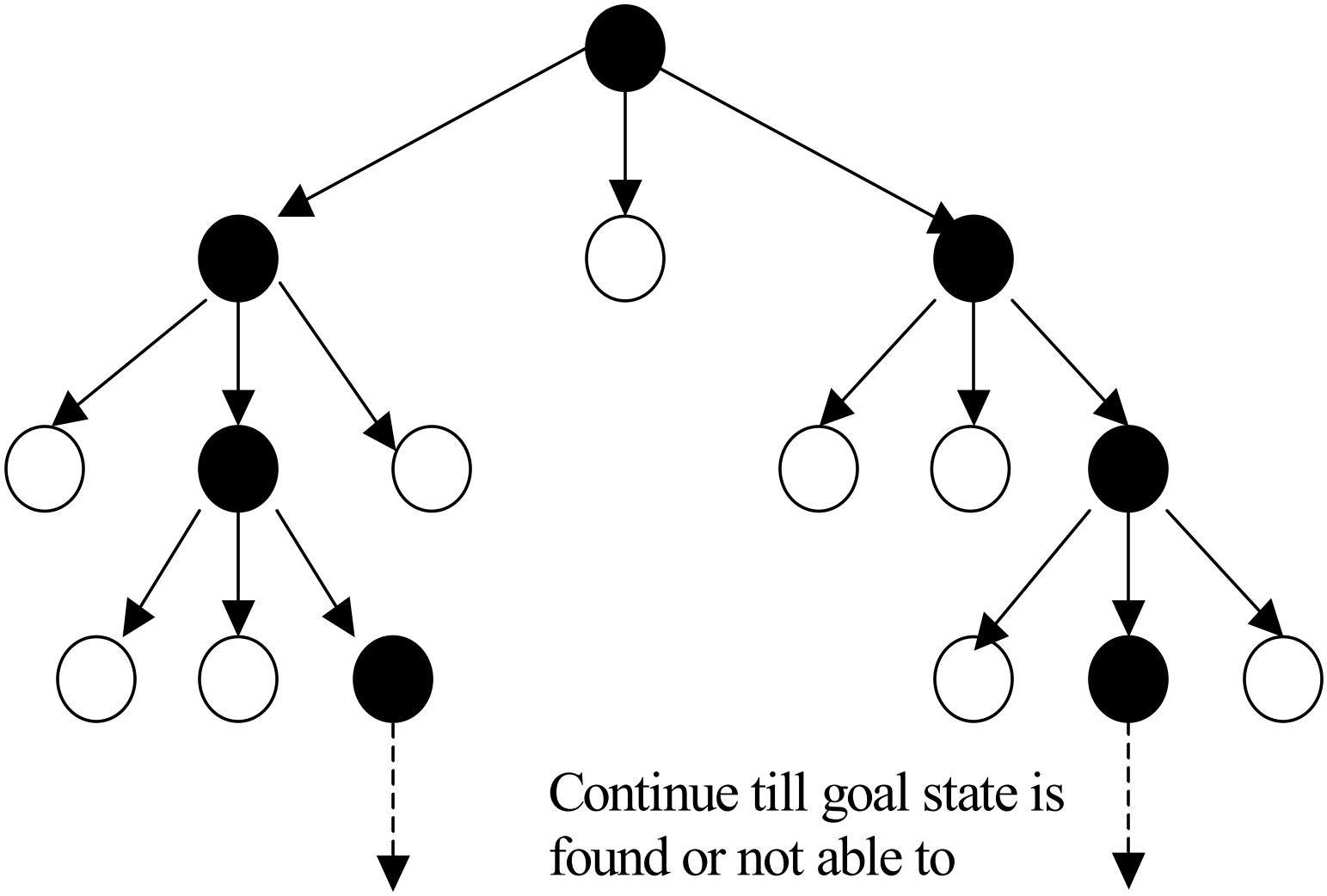
}

} // end while

} // end while

- If *Found* = *true* then return *Yes* otherwise return *No* and Stop

W = 2



Continue till goal state is found or not able to proceed further

Best First Search

- Expand the best partial path.
- Here forward motion is carried out from the best open node so far in the entire partially developed tree.

Algorithm (Best First Search)

- Initialize OPEN list by root node; CLOSED = ϕ;
- Found = false;
- While (OPEN $\neq \phi$ and Found = false) Do

  {
  - If the first element is the goal node, then **Found = true** else remove it from OPEN list and put it in CLOSED list.
  - Add its successor, if any, in OPEN list.
  - Sort the **entire list** by the value of some heuristic function that assigns to each node, the estimate to reach to the goal node

  } /* end while */
- If the **Found = true**, then announce the success else announce failure.
- Stop.

Observations

- In hill climbing, sorting is done on the successors nodes whereas in the best first search sorting is done on the entire list.
- It is not guaranteed to find an optimal solution, but normally it finds some solution faster than any other methods.
- The performance varies directly with the accuracy of the heuristic evaluation function.

Termination Condition

- Instead of terminating when a path is found, terminate when the shortest incomplete path is longer than the shortest complete path.

A* Method

- A* ("Aystar") (Hart, 1972) method is a combination of **branch & bound** and **best search**, combined with the **dynamic programming principle.**
- The heuristic function (or Evaluation function) for a node N is defined as **f(N) = g(N) + h(N)**
- The function **g** is a measure of the cost of getting from the **Start** node (initial state) to the **current** node.
  - It is sum of costs of applying the rules that were applied along the best path to the current node.
- The function **h** is an estimate of additional cost of getting from the **current** node to the **Goal** node (final state).
  - Here knowledge about the problem domain is exploited.
- A* algorithm is called OR graph / tree search algorithm.

Algorithm (A*)

- Initialization OPEN list with initial node; CLOSED= ϕ; g = 0, f = h, **Found = false**;
- While (OPEN $\neq \phi$ and **Found = false**)

  {[1]

  - Remove the node with the lowest value of **f** from OPEN to CLOSED and call it as a **Best_Node**.
  - If Best_Node = Goal state then **Found = true** else

    {[2]

    - Generate the **Succ** of **Best_Node**
    - For each **Succ** do

      {[3]

      - Compute g(Succ) = g(Best_Node) + cost of getting from Best_Node to Succ.

A* - Contd..

- If Succ $\in$ OPEN then /* already being generated but not processed */

{4

- Call the matched node as OLD and add it in the list of Best_Node successors.
- Ignore the **Succ** node and change the parent of OLD, if required.

  - If g(Succ) < g(OLD) then make parent of OLD to be Best_Node and change the values of g and f for OLD
  - If g(Succ) >= g(OLD) then ignore

}4

A* - Contd..

- If Succ $\in$ CLOSED then /* already processed */

{5

- Call the matched node as OLD and add it in the list of Best_Node successors.
- Ignore the **Succ** node and change the parent of OLD, if required

  - If g(Succ) < g(OLD) then make parent of OLD to be Best_Node and change the values of g and f for OLD.

  - Propogate the change to OLD's children using depth first search

  - If g(Succ) >= g(OLD) then do nothing

}5

A* - Contd..

- If Succ $\notin$ OPEN or CLOSED
  {6
  - Add it to the list of Best_Node's successors
  - Compute f(Succ) = g(Succ) + h(Succ)
  - Put **Succ** on OPEN list with its f value

  }6

}3 /* for loop*/

}2 /* else if */

}1 /* End while */

- If **Found = true** then report the best path else report failure
- Stop

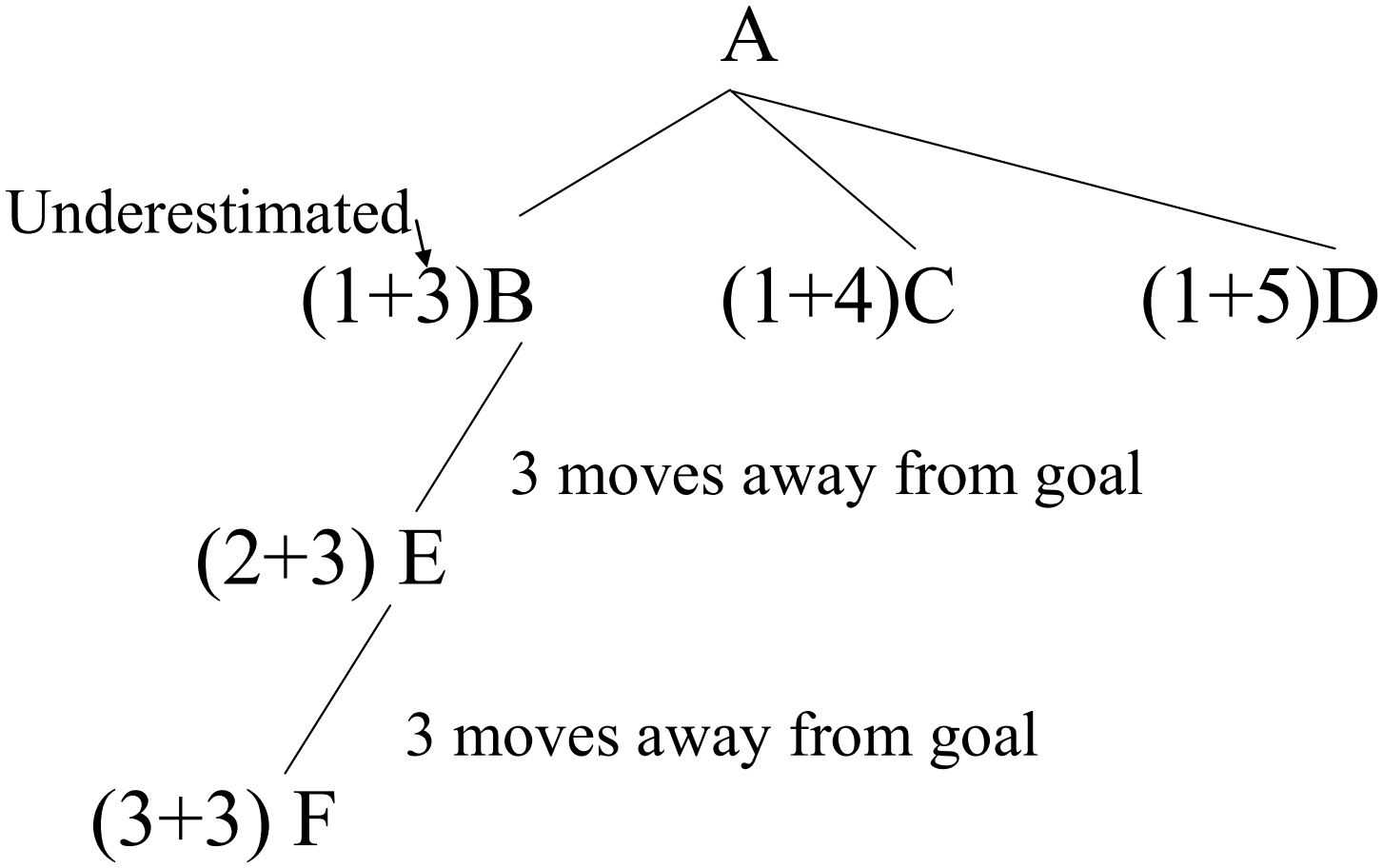
Behavior of A* Algorithm

Underestimation

- If we can guarantee that **h** never over estimates actual value from current to goal, then A* algorithm is guaranteed to find an optimal path to a goal, if one exists.

Example – Underestimation – f=g+h

Here h is underestimated

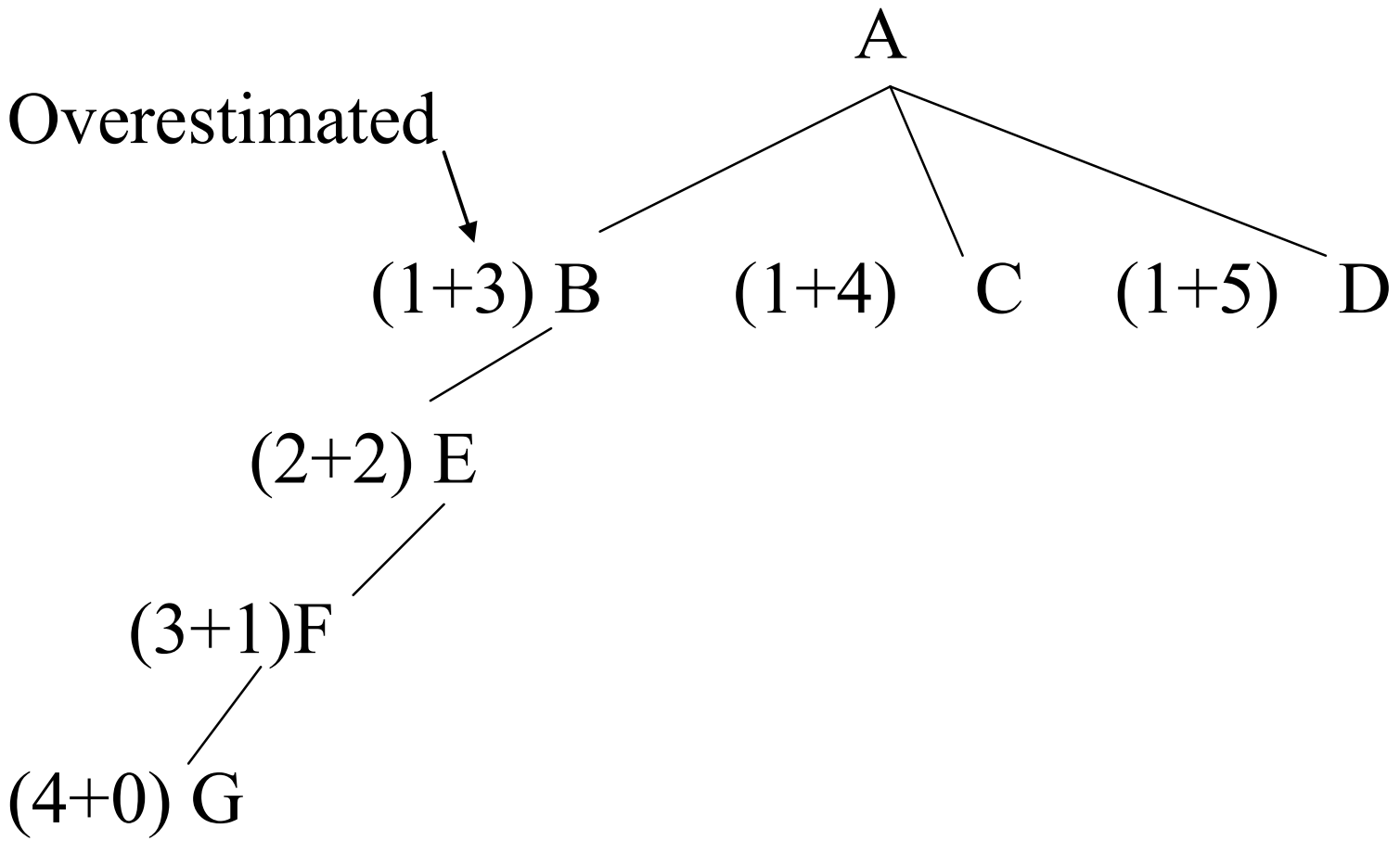


Explanation -Example of Underestimation

- Assume the cost of all arcs to be 1. A is expanded to B, C and D.
- ‘f’ values for each node is computed.
- B is chosen to be expanded to E.
- We notice that f(E) = f(C) = 5
- Suppose we resolve in favor of E, the path currently we are expanding. E is expanded to F.
- Clearly expansion of a node F is stopped as f(F)=6 and so we will now expand C.
- Thus we see that by underestimating h(B), we have wasted some effort but eventually discovered that B was farther away than we thought.
- Then we go back and try another path, and will find optimal path.

Example – Overestimation

Here h is overestimated



Explanation –Example of Overestimation

- A is expanded to B, C and D.
- Now B is expanded to E, E to F and F to G for a solution path of length 4.
- Consider a scenario when there a direct path from D to G with a solution giving a path of length 2.
- We will never find it because of overestimating h(D).
- Thus, we may find some other worse solution without ever expanding D.
- So by overestimating h, we can not be guaranteed to find the cheaper path solution.

Admissibility of A*

- A search algorithm is **admissible,** if
  - for any graph, it always terminates in an optimal path from initial state to goal state, if path exists.
- If heuristic function **h** is **underestimate** of actual value from current state to goal state, then the it is called **admissible function**.
- Alternatively we can say that A* always terminates with the optimal path in case
  - h(x) is an **admissible heuristic function**.

Monotonicity

- A heuristic function **h** is monotone if
  - ∀ states Xi and Xj such that Xj is successor of Xi

    $$h(Xi) - h(Xj) \leq \text{cost}(Xi, Xj)$$

    where, cost (Xi, Xj) actual cost of going from Xi to Xj
  - h (goal) = 0

- In this case, heuristic is locally admissible i.e., consistently finds the minimal path to each state they encounter in the search.

• The monotone property in other words is that search space which is every where locally consistent with heuristic function employed i.e., reaching each state along the shortest path from its ancestors.

• With monotonic heuristic, if a state is rediscovered, it is not necessary to check whether the new path is shorter.

• Each monotonic heuristic is admissible

• A **cost function** f(n) is monotone if $f(n) \leq f(succ(n)), \forall n$.

• For any admissible cost function f, we can construct a monotone admissible function.

Contd..

- Alternatively, the monotone property:
  - that search space which is every where locally consistent with heuristic function employed i.e., reaching each state along the shortest path from its ancestors.
- With monotonic heuristic, if a state is rediscovered, it is not necessary to check whether the new path is shorter.
- Each monotonic heuristic is admissible
  - A **cost function** f(n) is monotone. if $f(n) \leq f(succ(n)), \forall n$.
- For any admissible cost function f, we can construct a monotone admissible function.

Example: Solve Eight puzzle problem using A* algorithm

Start state

3	**7**	**6**
5	**1**	**2**
4	□	**8**

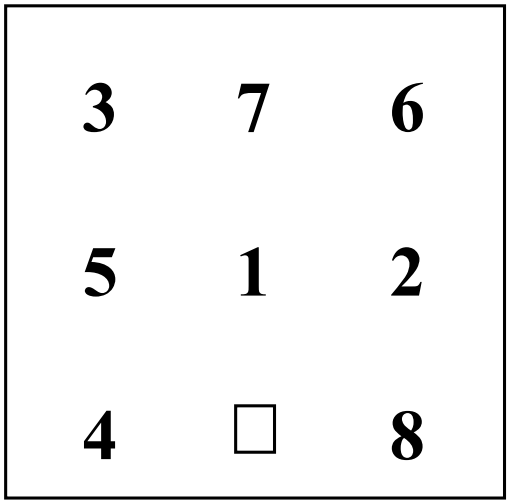
Goal state

5	**3**	**6**
7	□	**2**
4	**1**	**8**

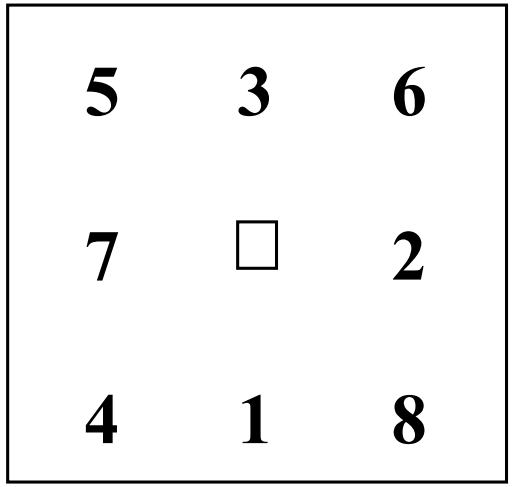
- Evaluation function f (X) = g (X) + h(X)
  h (X) = the number of tiles not in their goal position in a given state X
  g(X) = depth of node X in the search tree
- Initial node has **f(initial_node)** = 4
- Apply A* algorithm to solve it.
- The choice of evaluation function critically determines search results.

Example: Eight puzzle problem (EPP)

Start state



Goal state

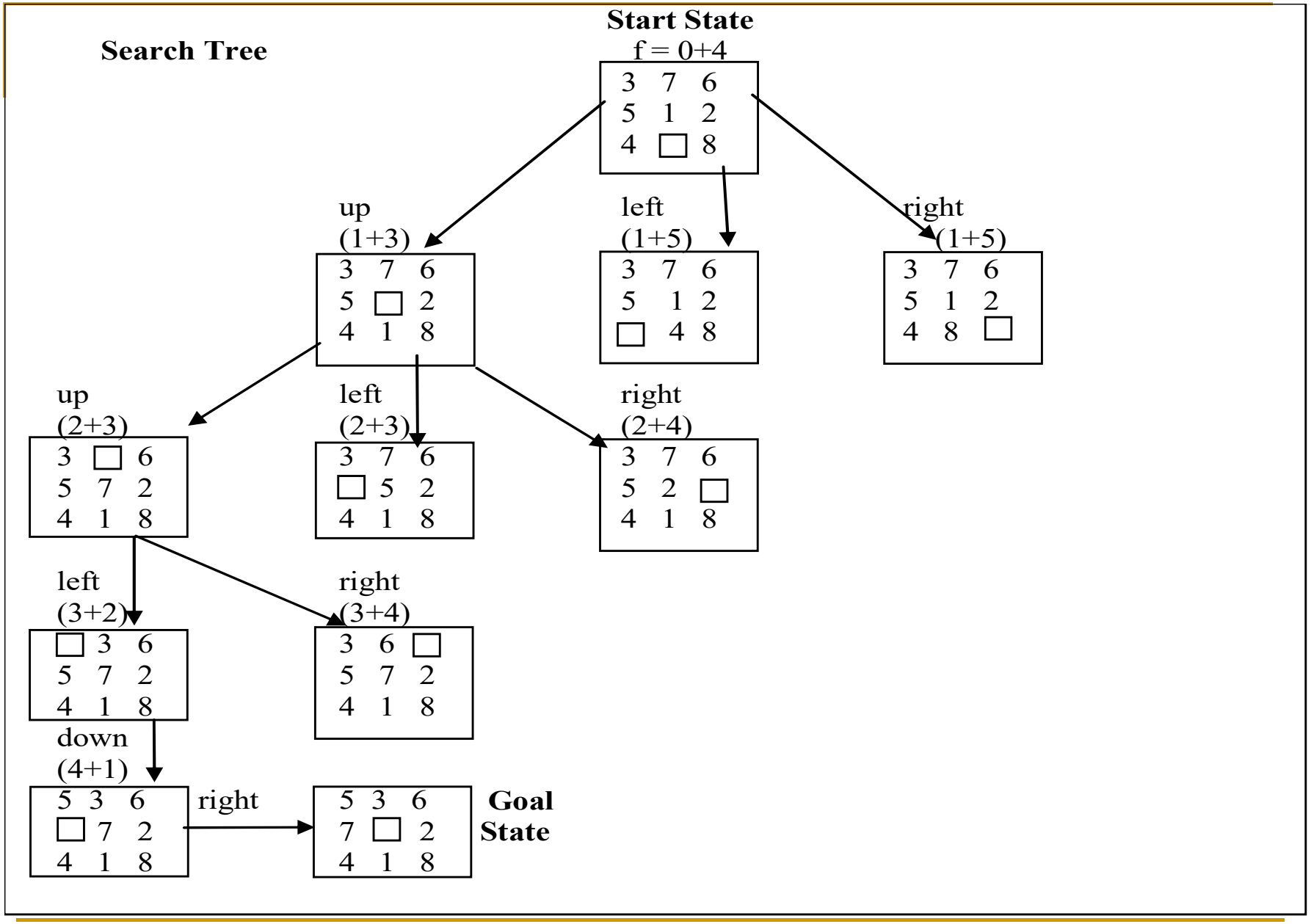


Evaluation function - f for EPP

- The choice of evaluation function critically determines search results.
- Consider Evaluation function

  f (X) = g (X) + h(X)

  - h (X) = the number of tiles not in their goal position in a given state X
  - g(X) = depth of node X in the search tree
- For Initial node
  - **f(initial_node)** = 4
- Apply A* algorithm to solve it.


Search Tree
Start State
f = 0+4
3 7 6
5 1 2
4 □ 8
up
(1+3)
3 7 6
5 □ 2
4 1 8
left
(1+5)
3 7 6
5 1 2
□ 4 8
right
(1+5)
3 7 6
5 1 2
4 8 □
up
(2+3)
3 □ 6
5 7 2
4 1 8
left
(2+3)
3 7 6
□ 5 2
4 1 8
right
(2+4)
3 7 6
5 2 □
4 1 8
left
(3+2)
□ 3 6
5 7 2
4 1 8
right
(3+4)
3 6 □
5 7 2
4 1 8
down
(4+1)
5 3 6
□ 7 2
4 1 8
right
5 3 6
7 □ 2
4 1 8
Goal
State

Harder Problem

- Harder problems (8 puzzle) can't be solved by heuristic function defined earlier.

Initial State

2	1	6
4	□	8
7	5	3

Goal State

1	2	3
8	□	4
7	6	5

- A better estimate function is to be thought.

  h (X) = the sum of the distances of the tiles from their goal position in a given state X

- Initial node has **h(initial_node)** = 1+1+2+2+1+3+0+2=12