Name : Yash Avlani

ID : 1001670008

NetID : yba0008

Assignment · 8

Task - 1

(a) P of Sensor in Maine $P(M) = 0.05$

P of Sensor in Sahara $P(S) = 0.95$

P of $(T/M) := 0.20$

P of $(T/S) := 0.90$

Here : $P(M/T) = \dfrac{P(M)\, P(T/M)}{P(T)}$

$$= \frac{P(M) \cdot P(T/M)}{P(M)\, P(T/M) + P(S) \cdot P(T/S)}$$

$$= \frac{0.05 \times 0.2}{(0.05 \times 0.2) + (0.95 \times 0.9)}$$

$$= 0.01156$$

(b) $Te_1 \Rightarrow$ email 1 , $Te_2 \Rightarrow$ email2

$$P(Te_2 / Te_1) = \frac{P(Te_2 \wedge Te_1)}{P(Te_1)}$$

$$P(Te_1) = P(Te_1 / M) \cdot P(M) + P(Te_2 / S)\, P(S)$$

$P(Te_2 \wedge Te_1)$

$$= P(Te_2 \wedge Te_1 / M) \cdot P(M) + P(Te_2 \wedge Te_1 / S) \cdot P(S)$$

$$= 0.20 \times 0.20 \times 0.05 + 0.90 \times 0.90 \times 0.95$$

$$= 0.7715$$

(c) $T_{e1} \Rightarrow$ email 1, $T_{e2} \Rightarrow$ email 2, T_{e3} = email 3

$$P(T_{e1}/m) \cdot P(T_{e2}/m) \cdot P(T_{e3}/m) \cdot P(m) +$$
$$P(T_{e1}/s)\ P(T_{e2}/s) \cdot P(T_{e3}/s)$$

$$\Rightarrow 0.20 \times 0.20 \times 0.20 \times 0.05 \quad +$$
$$0.90 \times 0.90 \times 0.90 \times 0.95$$

$$\Rightarrow 0.69295$$

Task - 2

(a) Total 11 variables

for A $\Rightarrow$ 5 values
for $B_1, B_2, B_3, \ldots B_{10} \Rightarrow$ 7 values possible

So, total $A \times B_1 \times B_2 \times \ldots \times B_{10}$

Hence $5 \times \underbrace{7 \times 7 \times 7 \times \ldots \times 7}_{10 \text{ times}}$

$\Rightarrow 5 \times 7^{10}$ values of $5 \times 7^{10} - 1$ numbers

So, this can be done in $5 \times 7^{10} - 1$ numbers

(b) Conditional Probability

$P(A, B_1, B_2, B_3, \ldots B_{10})$

$= P(B_1/A)\, P(B_2/A) \ldots$

Hence, $P(B_i/A)$ needs $(7-1) \times 5 = 30$ values

$P(A)$ needs $5 - 1 = 4$ values

So In total : $\cancel{30 \times 4 + 7}$ $30 \times 10 + 4$
$\cancel{= 120 + 7}$ $= 304$ values

~~So, total 307 values~~
So total 304 values

Task-3

(a) $P(A|B=T) = \alpha [< P(A=T, B=T, C=T)$
$P(A=F, B=T, C=T) > +$
$< P(A=T, B=T, C=F)$
$P(A=F, B=T, C=F) >]$

$= \alpha [< 0.048, 0.012 > + < 0.196, 0.294 >]$

$\alpha = \frac{1}{0.56} = 1.818$

$= \alpha [< 0.244, 0.306 >]$

$= < 0.444, 0.556 >$

$P(A|B=F) = \alpha [< P(A=T, B=F, C=T)$
$P(A=F, B=F, C=T) > +$
$< P(A=T, B=F, C=F)$
$P(A=F, B=F, C=F) >]$

$= \alpha [< 0.192, 0.048 > + < 0.084, 0.126 >]$

$= \alpha [< 0.276 \quad 0.174 >]$

Here $\alpha = \frac{1}{0.45} = 1.818$

$= < 0.613, 0.387 >$

Given B is True $< 0.444, 0.556 >$

Given B is False $< 0.613 \quad 0.387 >$

(b) ~~the~~ $P(A \mid B, C)$

$P(A \mid B=T, C=T) = \alpha[\langle P(A=T, B=T, C=T), P(A=F, B=T, C=T)\rangle]$

$= \alpha \langle 0.048, 0.012 \rangle$

Here $\alpha = \frac{1}{0.06} = 16.66$

$= \langle 0.8, 0.2 \rangle$

$P(A \mid B=F, C=F) = \alpha \langle P(A=T, B=F, C=F), P(A=F, B=F, C=F)$

$= \alpha \langle 0.084, 0.126 \rangle$

Here $\alpha = \frac{1}{0.21} = 4.761$

$= \langle 0.4, 0.6 \rangle$

$P(A \mid B=T\ C=F) = \alpha \langle P(A=T, B=T, C=F), P(A=F, B=T, C=F)\rangle$

$= \alpha \langle 0.196\ 0.294 \rangle$

Here $\alpha = \frac{1}{0.49} = 2.04$

$= \langle 0.4\ 0.6 \rangle$

$P(A \mid B=F\ C=T) = \alpha \langle P(A=T, B=F, C=T), P(A=F, B=F, C=T)\rangle$

$= \alpha \langle 0.192\ 0.048 \rangle$

Here $\alpha = \frac{1}{0.24} = 4.166$

$= \langle 0.8\ 0.2 \rangle$

So, the $P(A \mid B, C)$

$= \langle 0.8 \;\; 0.2 \rangle$ when $B = T$ $C = T$

$= \langle 0.4 \;\; 0.6 \rangle$ when $B = F$ $C = F$

$= \langle 0.4 \;\; 0.6 \rangle$ when $B = T$ $C = F$

$= \langle 0.8 \;\; 0.2 \rangle$ when $B = F$ $C = T$

(c) $P(A, C \mid B)$

$\Rightarrow P(A, C \mid B = T) = \alpha[\langle P(A=T, C=T, B=T), P(A=F, C=T, B=T),$
$P(A=T, C=F, B=T), P(A=F, C=F, B=T)\rangle]$

$= \alpha \langle 0.048, 0.012, 0.196, 0.294 \rangle$

there $\alpha = \frac{1}{0.55} = 1.818$

$= \langle 0.087, 0.021, 0.356, 0.534 \rangle$

$\Rightarrow P(A, C \mid B = F) = \alpha[\langle P(A=T, C=T, B=F), P(A=F, C=T, B=F)$
$P(A=T, C=F, B=F), P(A=F, C=F, B=F)\rangle]$

$= \alpha \langle 0.192, 0.048, 0.084, 0.126 \rangle$

$\alpha = \frac{1}{0.45} = 2.22$

$= \langle 0.426, 0.106 \;\; 0.186, 0.279 \rangle$

So value for $P(A, C \mid B)$

$= \langle 0.087 \;\; 0.021 \;\; 0.356 \;\; \cancel{0.279}\ 0.534 \rangle$
when B is True

$= \langle 0.426 \;\; 0.106 \;\; 0.186 \;\; 0.279 \rangle$
when B is False

(d) Given B of n is conditionally independent of C then

$$P(A|B,C) = P(A|B)$$

As per the value calculated in previous part it can be derived that

$$P(A|B,C) \neq P(A|B)$$

Hence A is not conditionally independent of C