Orthogonal Matching Pursuit

Linear Algebra:

matrix equation → $\underline{A}x = \underline{b}$

$$\begin{array}{l} \text{equation \#1} \rightarrow \\ \text{equation \#2} \rightarrow \\ \text{equation \#3} \rightarrow \end{array} \begin{bmatrix} 1 & 5 & 3 \\ 2 & 8 & 6 \\ 4 & 9 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 6 \end{bmatrix}$$

$$\text{3 equations} \begin{cases} 1x_1 + 5x_2 + 3x_3 = 10 \\ 2x_1 + 8x_2 + 6x_3 = 12 \\ 4x_1 + 9x_2 + 7x_3 = 6 \end{cases}$$

$\Rightarrow$

(# of equations) = (# of unknowns)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ -2 \end{bmatrix}$$

x = np.linalg.solve(A, b)

What if

(# of equations) $\neq$ (# of unknowns)

A+ is pseudoinverse $\leftarrow$ $A^{+} x \approx b$

$$\begin{bmatrix} 1 & 5 & 3 \\ 2 & 8 & 6 \\ 4 & 9 & 7 \\ 3 & 2 & 4 \\ 10 & 5 & 7 \\ 5 & 1 & 1 \\ 6 & 3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 6 \\ 4 \\ 24 \\ 8 \\ 20 \end{bmatrix}$$

Overdetermined System

→ Jaise yaha 7 eqn. hongi magar unknown 3 hi hai isey kahege overdetermined system.

x = np.linalg.lstsq(A, b) # solve using least squares

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$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.46919876 \\ -0.16811539 \\ 1.19239733 \end{bmatrix}$$

→ orthogonal matching pursuit is used on systems which is overdetermined or underdetermined

$$\boxed{\begin{array}{c} A^{+}x \approx b \\ y \approx A^{+}x \end{array}}$$

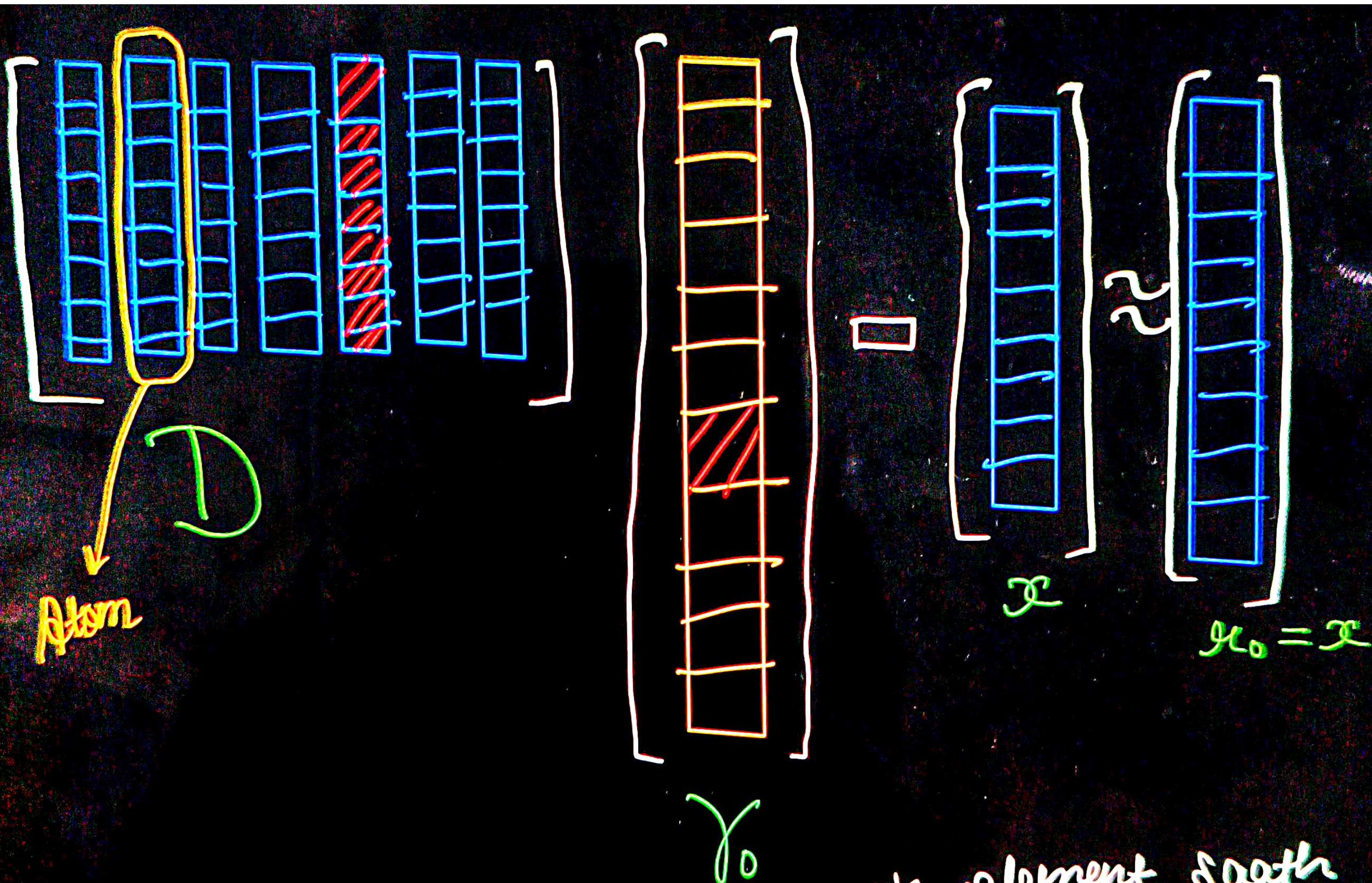
column signal or signal

Dictionary or measuring matrix or sensing matrix

Paper notation → $x = D\gamma$

Sparse approximation or sparse representation

Sci-Kit learn notation → $y \approx Xw$



→ ek atom (column) aur γ_0 ka ek element saath my lena hai

Input: Dictionary D, signal x, target sparsity K or target error ϵ

Output: Sparse representation γ such that $x \approx D\gamma$

Initialize: Set $I := ( )$, $r := x$, $\gamma := 0$

while (stopping criterion not met) **do**

$\hat{k} := \underset{k}{\operatorname{argmax}} \, |d_k^T r|$ ← maximize correlation of residual vector r with columns of dictionary D.

$x \approx D_I \gamma_I$

$\gamma_I \approx D_I^+ x$

$I := (I, \hat{k})$ ← Indices of selected atoms or columns in D that maximize k above.

$\gamma_I := (D_I)^+ x$ ← γ_I sparse representation of x

$\underset{\gamma}{\operatorname{argmin}} \, \|x - D\gamma\|_2^2$

$r := x - D_I \gamma_I$

end while

Target fixed number of non-zero elements end while

Target fixed number of non-zero elements

$$\underset{\underset{\gamma}{w}}{\operatorname{argmin}} \; \| \underset{x}{y} - \underset{D\gamma}{Xw} \|_2^2$$

Subject to $\| \underset{\gamma}{w} \|_0 \leq n_{nonzero\_coeffs}$

Target error or tolerance (tol)

$$\underset{\underset{\gamma}{w}}{\operatorname{argmin}} \; \| \underset{\gamma}{w} \|_0$$

Subject to $\| \underset{x}{y} - \underset{D\gamma}{Xw} \|_2^2 \leq tol$