Numerical Analysis of Microscopic Traffic Flow Models

MA 203: Numerical Methods

Group 16

1 Introduction

Traffic congestion in growing urban areas leads to inefficiency, longer commutes and pollution.Microscopic traffic simulations model driver behavior, crucial for understanding urban traffic dynamics. These simulations aid in optimizing traffic signals to improve flow efficiency, considering individual driver responses.

2 Objectives

- Analyze OVM and an updated "follow the leader" model to see how well they predict and suit different traffic situations.
- Use runge-kutta methods to solve ordinary differential equations in traffic flow models.
- Create visualizations and graphs to explain simulation and model results clearly to a broader audience.

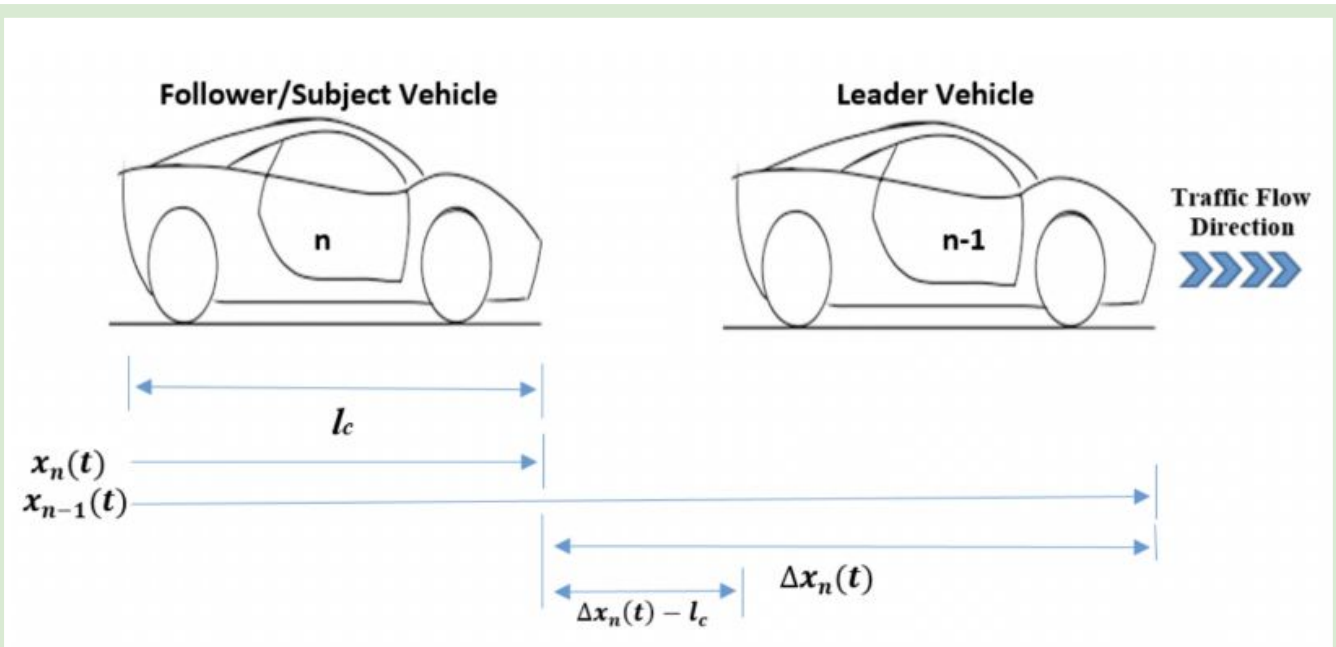
3 Mathematical Model

3.1 OVM

Optimal Velocity Model involves a set of differential equations that capture the behavior of individual vehicles in traffic. The core equations of OVM are:

$$\frac{d^2x_i(t)}{dt^2} = a(V(\Delta x_i(t)) - \frac{dx_i(t)}{dt})$$

$$V(\Delta x_i) = V_1 + V_2.\tanh[C_1(\Delta x - l_c) - C_2]$$



3.2 Our Model

Inspired by Newton's Mechanics, OVM and car-following theories, our model links response i.e vehicle acceleration to stimulus i.e velocity difference, modulated by sensitivity characterized by a sigmoid function.
We represent our model as the following equations:

$$\frac{dv_n}{dt} = -\frac{v_n - v_{n-1}}{1 + e^{v_{n-1} - v_n}} \quad -(1)$$

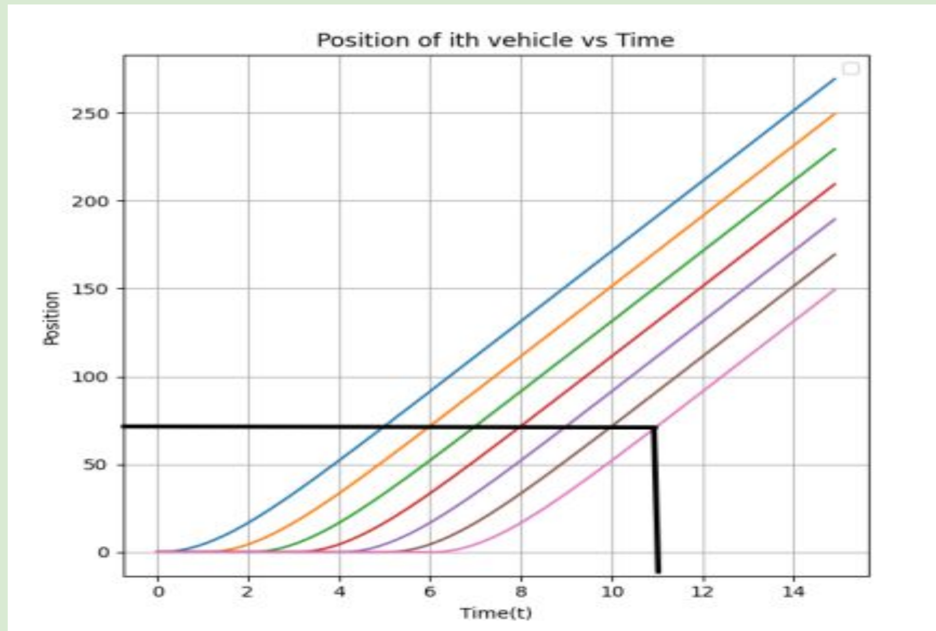
$$x_{n+1} = x_n + h.(v_n) \quad -(2)$$

4 Methodology and Numerical Solution

We implemented a fourth-order Runge Kutta (RK4) method to calculate the velocities and Euler's method to get new positions based on the new velocities.

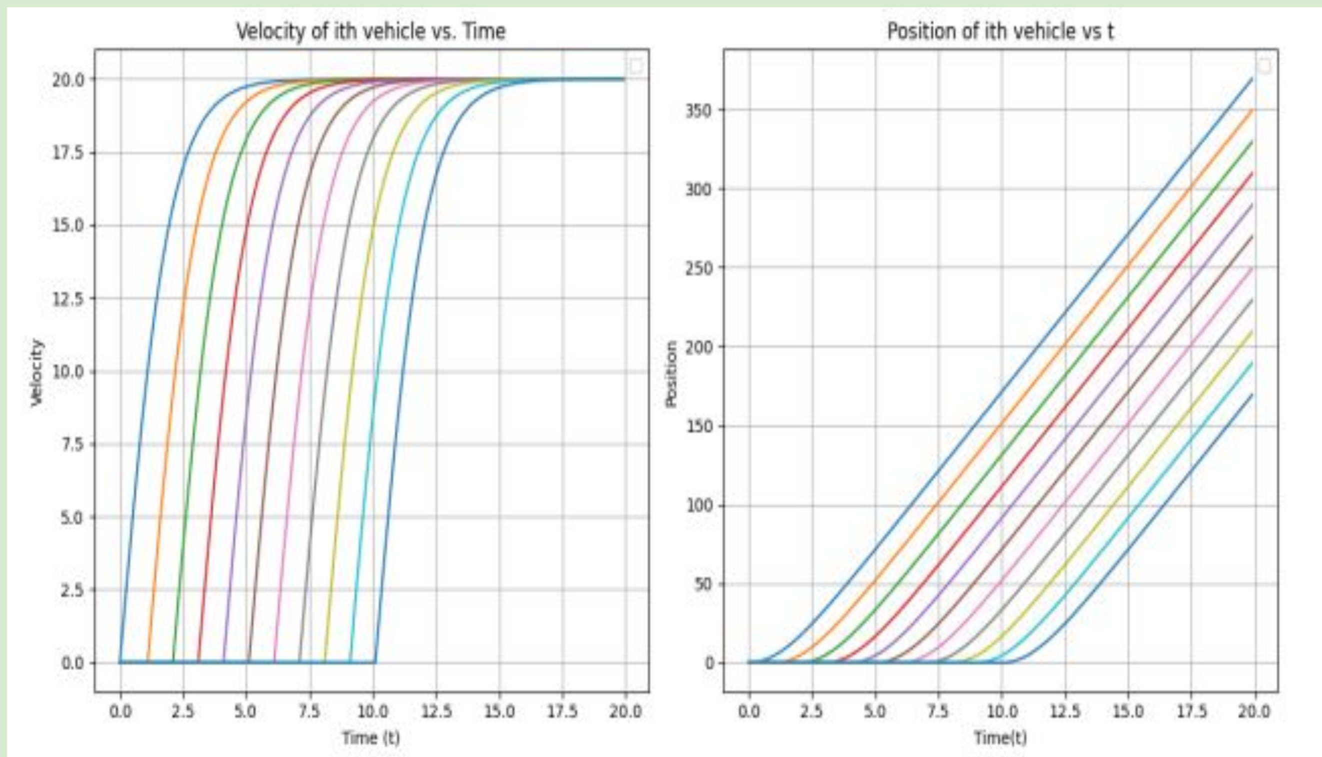
5 Real Life Application

Our model can be helpful in fixing the traffic signal controlling mechanism.From the plot, we can get the time corresponding to the distance that the last vehicle must traverse so as to cross the signal.



6 Results and Discussion

The vehicles start moving with a slight delay between starting time of two consecutive vehicles. Here we observe that with time, the velocity of all the vehicles converges to the maximum speed of 1st vehicle.



After all the vehicles attain the maximum speed, it is seen that the distance between any two consecutive vehicles becomes constant.
This model can play a significant role in setting the traffic signal parameters.

Our model very well describes the motion of multiple vehicles on a highway starting from a traffic signal. But it only accounts for single lane linear motion with no overtaking.
This model can be further extended to develop another model that incorporates overtaking, multi lane motion where sudden change can happen in the density of the traffic on that particular lane due to the entry of one more vehicle to make it more realistic.

References

[1] R. Jiang, Q. Wu, and Z. Zhu, "Full velocity difference model for a car-following theory," Physical Review E (Online). Available: https://link.aps.org/doi/10.1103/PhysRevE.64.017101

[2] IIT Bombay (NPTEL), Car following models (Online).Available: https://www.civil.iitb.ac.in/~vmtom/nptel/533_CarFol/web/web.html

[3] T. Nagatani, "The physics of traffic jams," Reports on Progress in Physics, vol. 65, no. 9, pp. 1331–1386, 2002.