Bernoulli Distribution

In probability theory and statistics the Bernoulli distribution, named after swiss mathematician Jacob Bernoulli is the discreate probability distribution of a random variable which takes the value 1 with probability p and the value 0 with probability $q=1-p$. Less formally it can be thought of as a model for the set of possible outcomes of any single experiment that asks a yes-no question

1. Discreate Random Variable
2. Outcomes are Binary
   Eg:- Tossing a coin {H, T}
   $Pr(H) = 0.5 = p$
   $Pr(T) = 0.5 = 1-p = q$
3. Whether the Person will Pass/Fail
   $Pr(Pass) = 0.7 = p$
   $Pr(Fail) = 1-0.7 = 0.3 = q$

1. PMF

$PMF = p^K * (1-p)^{1-K}$ $K \in \{0, 1\}$

if $K=1$

$Pr(K=1) = p^1 (1-p)^{1-1}$
$= p$

Simplifide

$$Pmf \begin{cases} q=1-p & \text{if } K=0 \\ p & \text{if } K=1 \end{cases}$$

if $(K=0)$

$Pr(K=0) = p^0 * (1-p)^{1-0}$
$= 1-p = q$

2. Mean of Bernolli Distribution

$F(K) = \sum_{i=1}^{K} K \cdot P(K)$

$P_r(K=1) = 0.6 = p$

$= [0 * 0.4 + 1 * 0.6]$ $\quad P_r(K=0) = 0.4 = q$

$= 0.6 = p$ //

3. Median of Bernolli Distribution

$$\text{Median} \begin{cases} 0 & \text{if } p < 1/2 \\ [0, 1] & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$$

4. Variance

$Var = p * (1 - p)$

$= pq$

Std

$Std = \sqrt{pq}$

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Binomial Distribution

In Probability theory and statistics the binomal distribution with parameters n and p is the discrete probability distribution of the number of sucess in a sequence of n independent experiments each asking a yes-no question and each with its own Boolean-valued outcome: success or failure $q=1-p$. A single success failure experiment is also called a Bernoulli trail or Bernouli experiment, and a sequence of outcomes is called a Bernoulli process; for a single trial, i.e. $n=1$ the binomal distribution is a Bernoulli distribution.

1. Disrcreate random variable
2. Every experiment outcome is binary
3. There experiment is performed for n times
   Eg:- Tossing a coin 10 times

Notation: $B(n,p)$

Parameter: $n \in \{0,1,2,3 ---\}$ → no of trials
$p \in [0,1]$ → success probability for each trial
$q = 1-p$

Suppoce: $k \in \{0,1,2,3---\}$ → no of succes

PMF

$$Pr(k,n,p) = {}^{n}C_{k}\, p^{k} (1-p)^{n-k}$$
for $k = 0,1,2,3 --- n$

Mean

Mean $= np$

Variance

Var $= npq$

Std

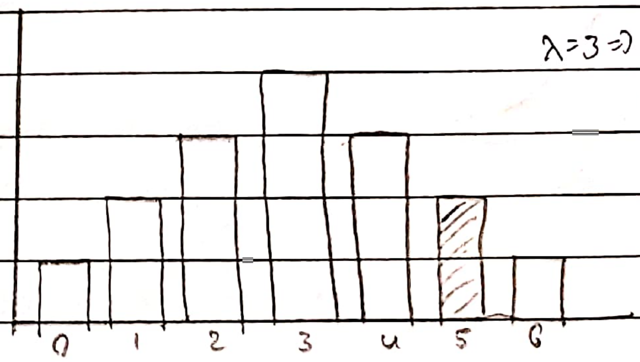
std $= \sqrt{npq}$

Poission Distribution

1. Discreate Randome Variable
2. Discribes the number od events occuring in a dixed time interval

Eg:

No od people visiting banks every hour



$\lambda = 3 \Rightarrow$ Expect no od event occur at every time interval

PMF

$$P(x=5) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

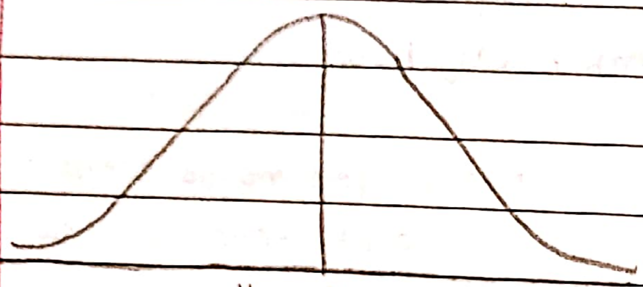
$$= \frac{e^{3} 3^{5}}{5!} = 0.101 = 10.1\%$$

Mean

$$\text{Mean} = E(x) = \mu = \lambda * t$$

Normal / Gaussion Distribution

In Statistics a normal distribution or Gaussion distri- is a type of continuous probablity distribution for a real- random variable



μ = median = mode

Notation : $N(\mu, \sigma^2)$

Parameters : $\mu \in R$ = mean

$\sigma^2 \in R > 0$ = Variance

$x \in R$

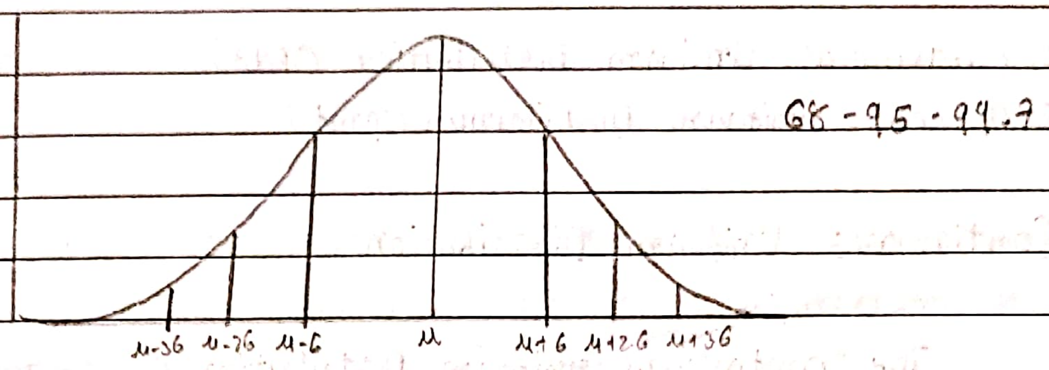
PDf $= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Mean : Average

Variance

Std

Empirical Rule of Normal Distribution



Probability

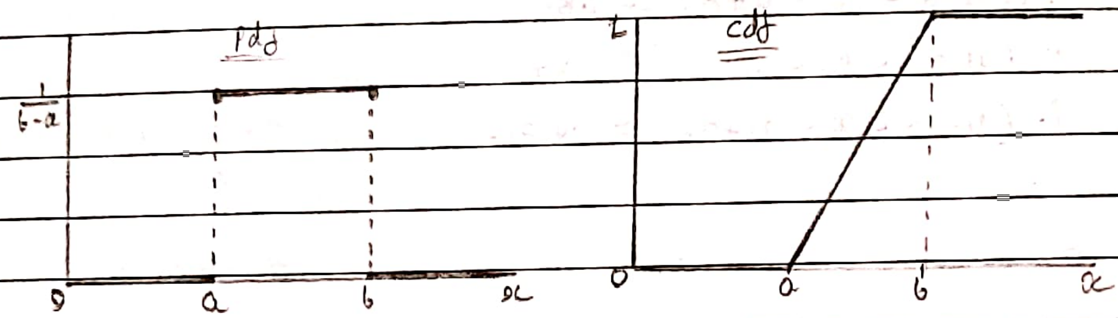
$$Pr(\mu-\sigma \le x \le \mu+\sigma) \approx 68\%$$
$$Pr(\mu-2\sigma \le x \le \mu+2\sigma) \approx 95\%$$
$$Pr(\mu-3\sigma \le x \le \mu+3\sigma) \approx 99.7\%$$

Uniform Distribution

1. Continuous Uniform Distribution (pdf)
2. Discrete uniform Distribution (pmf)

1. Continuous Uniform Distribution

The continuous uniform Distribution or rectangular distribution is a family of symmetric distributions. The distribution describes an experiment where there is an arbitrary outcome that lies between certain bounds. The bounds are defined by the parameters a and b which are the minimum and maximum values



Notation: $U(a,b)$

Parameters: $-\infty < a < b < \infty$

$$pdf = \begin{cases} \frac{1}{b-a} & \text{for } x \in [a,b] \\ 0 \end{cases} \qquad cdf = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a,b] \\ 1 & \text{for } x > b \end{cases}$$

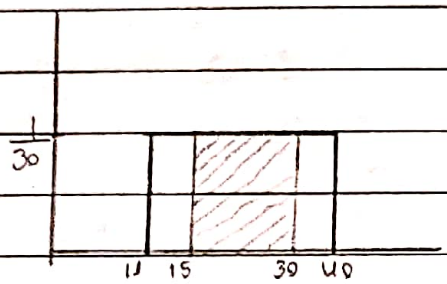
Mean $= \frac{1}{2}(a+b)$ Median $= \frac{1}{2}(a+b)$

Variance $= \frac{1}{12}(b-a)^2$

Eg

The number of candice solid daily at a shop is uniformly distributed with a maximum of 40 and minimum of 10

Q Probability of daily sale between 15 and 30



$x_1 = 15$

$x_2 = 30$

$$Pr(15 \leq x \leq 30) = (x_2 - x_1) * \frac{1}{b-a}$$

$$= 15 * \frac{1}{30}$$

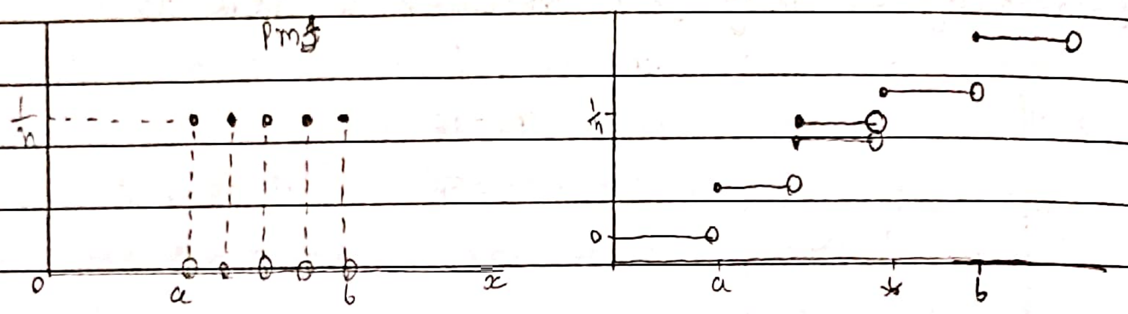
$$= 0.5$$

$$Pr(x \geq 20) = (40-20) * \frac{1}{30}$$

$$= 0.66$$

② Discrete Uniform Distribution

The Discrete Uniform Distribution is a symmetric probability distribution wherein a finite number of values are equally likely to be observed every one of n values has equal probability 1/n. Another way of saying "discrete uniform distribution" would be "a known finite number of outcomes equally likely to happen".



Eg:- Rolling a dice

$\{1, 2, 3, 4, 5, 6\}$ $\quad P_r(1) = 1/6$

$P_r(2) = 1/6$

$a = 1 \quad b = 6 \quad P_r(3) = 1/6$

$\frac{1}{n} \Rightarrow n = b - a + 1$

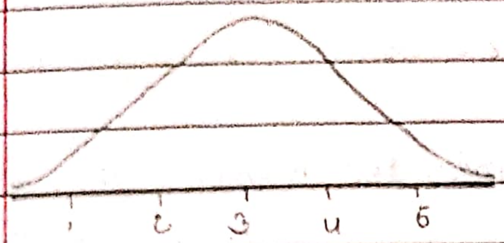
Notation :- $U(a, b)$

Parameters : a, b with $b \geq a$

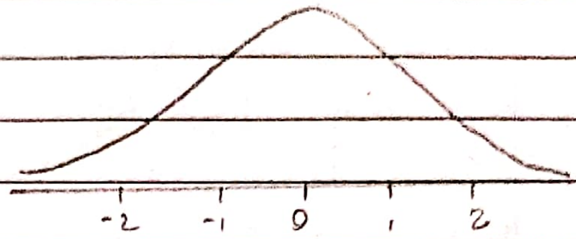
Pmf : $1/n$

Mean : $\frac{a+b}{2}$ $\qquad$ Median :- $\frac{a+b}{2}$

Standard Normal Distribution And Z-score



Normal Distribution



Standard Normal Distribution

$$Z\ Score = \frac{x_i - \mu}{\sigma}$$

$x = \{1, 2, 3, 4, 5\}$ $= \mu = 3$

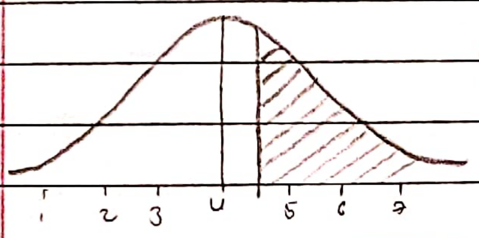
$\sigma = 1.414 \approx 1$

$$Z\ Score = \frac{x_i - \mu}{\sigma}$$

$= \frac{1-3}{1} = -2$ $\frac{4-3}{1} = 1$

$= \frac{2-3}{1} = -1$ $\frac{5-3}{1} = 2$

$= \frac{3-3}{1} = 0$



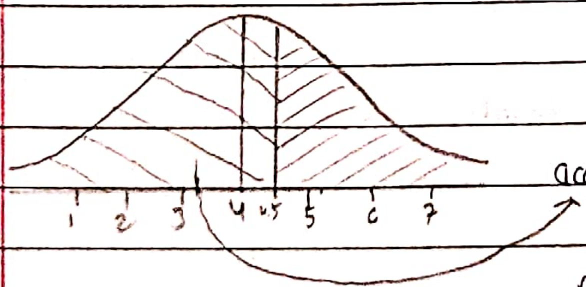
$\mu = 4$
$\sigma = 1$

How many standard deviation 4.5 is away from mean?

$x_i = 4.5$

$$Z\text{-}Score = \frac{4.5-4}{1} = 0.5$$

$\mu = 4$ $\sigma = 1$ What percentage of data is falling above 4.5?



$$Z\text{-}Score = \frac{4.5-4}{1} = 0.5$$

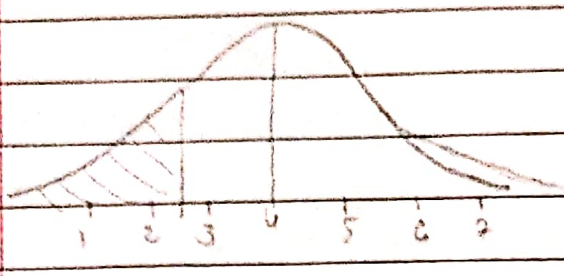
according to Z-table = 0.69146

Area under the curve ($Z > 4.5$) $= 1 - 0.69146$

$= 0.30854$

$\boxed{= 30.85\%}$

What Percentage of data is falling below 2.5?



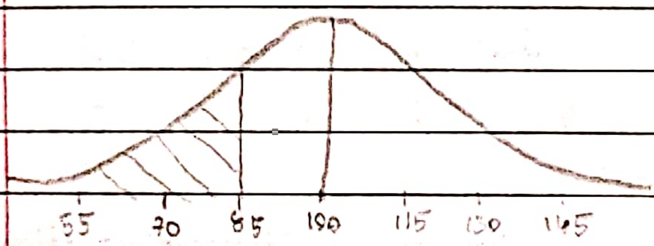
$z\text{-score} = \frac{2.5-4}{1} = -1.5$

Area under the curve $(\leq 2.5) = 0.06681$

$= 6.6\%$

In India the average IQ is 100, with a standard deviation of 15 What is the persentang of the population would you expect to have an IQ lower than 85%.

$\mu = 100, \sigma = 15, x_i = 85$



$z\text{-score} = \frac{x_i - \mu}{\sigma}$

$= \frac{85-100}{15} = -1$

Area under the curve $= 0.15866$

$= 15.866$

Area under the curve $(\geq 85) = 1 - 0.15866$

$\approx 84\%$