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Subject: DAA

Experiment number: 04

Aim: To implement dynamic algorithms

Theory:

Details – Dynamic Programming is a technique in computer programming that helps to efficiently solve a class of problems that have overlapping subproblems and optimal substructure property. If any problem can be divided into sub-problems, which in turn are divided into smaller sub-problems, and if there are overlapping among these sub-problems, then the solutions to these sub-problems can be saved for future reference. The approach of solving problems using dynamic programming algorithm has following steps:

1. Characterize the structure of an optimal solution.
2. Recursively define the value of an optimal solution.
3. Compute the value of an optimal solution, typically in a bottom-up fashion.
4. Construct an optimal solution from computed information.

Algorithm:

MATRIX-CHAIN-ORDER (p)

1.  $n \leftarrow \text{length}[p] - 1$
2. for  $i \leftarrow 1$  to  $n$
3. do  $m[i, i] \leftarrow 0$
4. for  $l \leftarrow 2$  to  $n$  //  $l$  is the chain length
5. do for  $i \leftarrow 1$  to  $n - l + 1$
6. do  $j \leftarrow i + l - 1$
7.  $m[i, j] \leftarrow \infty$
8. for  $k \leftarrow i$  to  $j - 1$
9. do  $q \leftarrow m[i, k] + m[k + 1, j] + p_{i-1} p_k p_j$
10. If  $q < m[i, j]$
11. then  $m[i, j] \leftarrow q$
12.  $s[i, j] \leftarrow k$
13. return  $m$  and  $s$ .

**PRINT-OPTIMAL-PARENS (s, i, j)**

1. if  $i = j$
2. then print "A"
3. else print "("
4. PRINT-OPTIMAL-PARENS (s, i, s[i, j])
5. PRINT-OPTIMAL-PARENS (s, s[i, j] + 1, j)
6. print ")"

Program:

```

#include <iostream>
#include <climits>
#include <random>
#include <ctime>
using namespace std;
int **matrix;

//determining cost of multiplication
void matrixChainOrder(int p[], int n, int m[][100], int s[][100]) {
    for(int i=1; i<=n; i++)
        m[i][i] = 0;

    for(int l=2; l<=n; l++) {
        for(int i=1; i<=n-l+1; i++) {
            int j = i+l-1;
            m[i][j] = INT_MAX;

            //finding cost i.e., number of Multiplications
            for(int k=i; k<=j-1; k++) {
                int q = m[i][k] + m[k+1][j] + p[i-1]*p[k]*p[j];

                //finding minimum number of scalar Multiplications
                if(q < m[i][j]) {
                    m[i][j] = q;
                    s[i][j] = k;
                }
            }
        }
    }
}

```

```

        s[i][j] = k;
    }
}

}

//printing the parenthesis
void printOptimalParenthesis(int s[][100], int i, int j) {
    if(i == j)
        cout << "A" << i;
    else {
        cout << "(";
        printOptimalParenthesis(s, i, s[i][j]);
        printOptimalParenthesis(s, s[i][j]+1, j);
        cout << ")";
    }
}

int main() {
    int p[10];
    srand ( time(NULL) );
    random_device rd;
    mt19937 gen(rd());
    uniform_int_distribution<> distr(15, 46);

    for(int i=0; i<10; ++i)
        p[i] = distr(gen);

    int n = sizeof(p)/sizeof(p[0]) - 1;

    int m[100][100];

```

```

int m[100][100];
int s[100][100];

matrixChainOrder(p, n, m, s);

cout << "Minimum Number of Scalar Multiplications: " << m[1][n] << endl;

//printing cost table
cout << "m table:";

for(int a = 0; a < 10; a++)
{
    for(int b = 0; b < 10; b++)
    {
        if(m[a][b] == 0){continue; }
        cout << m[a][b] << " ";
    }
    cout << endl;
}

//printing parenthesization table
cout << "s table:";

for(int a = 0; a < 10; a++)
{
    for(int b = 0; b < 10; b++)
    {
        if(s[a][b] == 0){continue;}

```

```

        cout << s[a][b] << " ";
    }
    cout << endl;
}

cout << "Optimal Parenthesization: ";
printOptimalParenthesis(s, 1, n);
cout << endl;

return 0;
}

```

output:

```
Minimum Number of Scalar Multiplications: 159315
m table:
36800 78200 122705 93825 107625 124065 134475 159315
72000 146200 80025 104025 118425 127305 157245
77400 56025 80025 94425 103305 133245
29025 56025 69825 78255 109695
25800 39840 48450 79290
19200 31680 49620
33280 81120
38272

s table:
1 2 3 1 5 5 5 5
2 2 2 5 5 5 5
3 3 5 5 5 5
4 5 5 5 5
5 5 5 5
6 7 8
7 8
8

Optimal Parenthesization: ((A1(A2(A3(A4A5))))((A6A7)A8)A9))

...Program finished with exit code 0
Press ENTER to exit console.
```

Time complexity Analysis:

There are three nested loops. Each loop executes a maximum  $n$  times.

1.  $l$ , length,  $O(n)$  iterations.
2.  $i$ , start,  $O(n)$  iterations.
3.  $k$ , split point,  $O(n)$  iterations

So, Body of loop constant complexity

**Total Complexity is:  $O(n^3)$**

**Inference:**

While we perform matrix multiplication, the number of multiplications increases (as multiplication takes more time than addition and subtraction). But if we optimally parenthesize the matrices then the number of multiplications may decrease. For that we use a dynamic programming approach and perform the matrix chain multiplication and give the optimal parenthesis. In the matrix chain multiplication problem, the minimum number of multiplication steps required to multiply a chain of matrices has been calculated.

**Conclusion:**

**In this Practical, I performed the program of matrix chain multiplication and found the minimum number of multiplication steps required to multiply the chain of metrics and accordingly give the optimal parenthesization.**