2) Given

$$T = \left(\frac{-\log(u)}{\lambda}\right)^{1/\alpha} = \frac{-\log(u)}{}^{1/\alpha}$$

given that u is uniform on (0,1) show that this dist is weibull(λ, α)

since u is uniform(0,1) we know $\log(u)$ follows an exponenenential (1) $\therefore$

$\therefore$ CDF of T:

$$P(T \leq t) = P\left(\left(\frac{-\log(u)}{\lambda}\right)^{1/\alpha} \leq t\right)$$

$$= P(u > e^{-\lambda t^{\alpha}}) = 1 - e^{-\lambda t^{\alpha}}$$

PDF of T

$$f_T(t) = \frac{d}{dt}(1 - e^{-\lambda t^{\alpha}}) = \lambda \alpha t^{\alpha - 1} e^{-\lambda t^{\alpha}}$$

that is weibull PDF ergo they are the same.

$u =$

Q.2.) change of variables → weibull

weibull: great for predicting wind speeds

$$f(t) = \lambda\alpha t^{\alpha-1} e^{-\lambda t^{\alpha}}$$

1.) T^{α} has an exponential family dist.

$$T^{\alpha} = \int_0^t \lambda\alpha t^{\alpha-1} e^{-\lambda t^{\alpha}} dt = 1 - e^{-\lambda t^{\alpha}}$$

$y = T^{\alpha}$ CDF of Y:

$$F_Y(y) = P(Y \leq y) = P(T^{\alpha} \leq y) = P(T \leq y^{1/\alpha})$$

$$= 1 - e^{-\lambda (y^{1/\alpha})^{\alpha}} = 1 - e^{-\lambda y}$$

PDF of Y

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy}(1 - e^{-\lambda y}) = \lambda e^{-\lambda y}$$

that's exponential distribution!

9.5.) $\frac{1}{z^2}$ ∴ let $Y = \frac{1}{x}$ ∴ $\frac{dy}{dx} = -\frac{1}{x^2} = \left|\frac{dy}{dx}\right|$

$$f_X(x) = f(y)\left(\frac{1}{x}\right)\frac{dy}{dx}$$

$$= \left(\frac{1}{\sqrt{2\pi(1/x)}}\, e^{-\frac{1}{2}\left(\frac{1}{x}\right)}\right)\frac{1}{x^2}$$

$$= \frac{\sqrt{x}}{\sqrt{2\pi}\,x^2}\, e^{-\frac{1}{2x}} = \frac{1}{2\pi}\, x^{-3/2} e^{-\frac{1}{2x}}$$

(1.3.5)

Q.1.)

$X = 3Z + 5$

$\therefore E(X) = aE(Z) + b = 3 \cdot 0 + 5 = 5$

$\therefore Var(X) = a^2 \, var(Z) = 9$

$\therefore X \sim N(5, 9) = \frac{1}{\sqrt{2\pi \cdot 9}} \exp\left(-\frac{(x-5)^2}{18}\right)$

Q.2.) $X = |Z|$

since $f(x) = f(-x)$

$\therefore f_X(x) = f_Z(x) + f_Z(-x) = 2 f_Z(x)$

$\therefore$ for $x \geq 0$ $f_X = 2 \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$; 0 otherwise

Q.3.) $X = Z^2$ $\therefore$ $z = \pm\sqrt{x}$ $\therefore$ $\frac{dz}{dx} = \frac{1}{2\sqrt{x}}$

$f_Z(\sqrt{x}) = \frac{1}{\sqrt{2\pi}} e^{-x/2}$

$\therefore f_X x = (f_Z\sqrt{x}) + f_Z(-\sqrt{x})) \cdot \frac{1}{2\sqrt{x}}$

$= \frac{2}{\sqrt{2\pi}} e^{-x/2} \cdot \frac{1}{2\sqrt{x}} = \frac{e^{-x/2}}{\sqrt{x}\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi x}} e^{-x/2}$