

Given

$$T = \left( \frac{-\log(u)}{\lambda} \right)^{1/\alpha} = \frac{-\log(u)^{1/\alpha}}{\lambda^{1/\alpha}}$$

given that  $u$  is uniform(0,1) we can show that  
this dist is weibull( $\lambda, \alpha$ )

$$e^{-\lambda u^{\alpha}} - 1 = e^{\lambda T} - e^{\lambda T}$$

Since  $u$  is uniform(0,1) we know  $\log(u)$  follows  
an exponential(1)  $\therefore$  to CDF  $F_T(t) = e^{-\lambda t}$

$$\therefore \text{CDF of } T: F_T(t) = P(U \leq e^{-\lambda t}) = (1 - e^{-\lambda t})^{\alpha}$$

$$P(T \leq t) = P\left(\left(\frac{-\log(u)}{\lambda}\right)^{1/\alpha} \leq t\right)$$

$$= P(u > e^{-\lambda t}) = 1 - e^{-\lambda t}$$

$$e^{-\lambda t} = e^{-\lambda t} \cdot b \quad \text{PDF of } T$$

$$f_T(t) = \frac{d}{dt}(1 - e^{-\lambda t}) = \lambda e^{-\lambda t}$$

That is weibull PDF ergo they are  
the same.

8.2.) change of variables  $\rightarrow$  weibull

weibull: it's great for predicting wind speeds

$$f(t) = \lambda t^{\alpha-1} e^{-\lambda t^\alpha}$$

$T^\alpha$  has an exponential family dist.

$$T^\alpha = \int_0^t \lambda s^{\alpha-1} e^{-\lambda s^\alpha} ds = 1 - e^{-\lambda t^\alpha}$$

$y = T^\alpha$  CDF of  $Y$ :

$$F_{Y_\alpha}(y) = P(Y \leq y) = P(T^\alpha \leq y) = P(T \leq y^{1/\alpha})$$

$$(+) \rightarrow (1 - e^{-\lambda(y-\alpha)})^{\alpha} = (1 - e^{-\lambda y})^{\alpha}$$

PDF of  $Y$

$$f_{Y_\alpha}(y) = \frac{d}{dy} F_{Y_\alpha}(y) = \frac{d}{dy} (1 - e^{-\lambda y})^{\alpha} = \lambda \alpha e^{-\lambda y}$$

$b + k - g - 1 = (b + k - g - 1)$  that's exponential distribution!

so  $b + k - g - 1 = 0$  that's exponential distribution!

(2 ε, 1)

(1.8)

$$8.5.) \quad \frac{1}{z^2} \quad \text{Let } Y = \frac{1}{x} \quad \therefore \frac{dy}{dx} = -\frac{1}{x^2} \quad \text{P.F.E.} = \left| \frac{dy}{dx} \right|$$

$$Z = Z + 0 \cdot \varepsilon = -J + (S) \exists P = (X) \exists \quad \therefore$$

$$f_X(x) = f(y) \left( \frac{1}{x} \right) \frac{dy}{dx} \quad (S) \text{ INV} \leftarrow P = (X) \text{ INV} \quad \therefore$$

$$\left( = \left( \frac{(1-x)}{\sqrt{2\pi x^3}} e^{-\frac{1}{2}(x)} \right) \frac{1}{x^2} \right) = (P, Z) \text{ INV} \sim X \quad \therefore$$

$$= \frac{\sqrt{x}}{\sqrt{2\pi x^3}} e^{-\frac{1}{2}x} = \frac{1}{2\pi} x^{-3/2} e^{-\frac{1}{2}x} = X \quad (S.8)$$

$$(X-Z) \stackrel{?}{=} (X) \stackrel{?}{=} \text{since}$$

$$(X_S) \stackrel{?}{=} (X-Z) \stackrel{?}{=} (X) \stackrel{?}{=} (X)X \quad \therefore$$

$$\text{Gesuchte WO: } \left( \begin{matrix} S \\ S \end{matrix} \right) \text{ Q(X)} \frac{1}{\sqrt{\pi x^3}} (-x) \quad 0 \leq x \leq 1 \quad \therefore$$

$$\frac{1}{x^3} = \frac{S}{x^3} \quad \therefore \quad x^3 \pm = S \quad \therefore \quad S = X \quad (S.8)$$

$$\frac{S}{x^3} \cdot g \stackrel{?}{=} (X^3) \stackrel{?}{=} \sqrt[3]{X^3}$$

$$\frac{1}{x^3} \cdot ((X^3)_S) + (X^3)_S = X^3 \quad \therefore$$

$$\frac{g}{x^3} = \frac{S}{x^3} \cdot g = \frac{1}{x^3} \cdot g \frac{S}{\sqrt[3]{x^3}} =$$

(1,3,5)

Q.1.)

$$X = 3Z + 5 \quad \text{since } Z \sim N(0,1) \quad \therefore X \sim N(5, 9) \quad (\text{?})$$

$$\therefore E(X) = aE(Z) + b = 3 \cdot 0 + 5 = 5$$

$$\therefore \text{Var}(X) = a^2 \text{Var}(Z) = 9$$

$$\therefore X \sim N(5, 9) = \frac{1}{\sqrt{2\pi} \cdot 3} \exp\left(-\frac{(x-5)^2}{18}\right)$$

Q.2.)  $X = |Z|$  since  $Z \sim N(0, 1)$   $\therefore X \sim \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

$$\text{since } f(x) = f(-x)$$

$$\therefore f_X(x) = f_2(x) + f_2(-x) = 2f_2(x)$$

$$\therefore \text{for } x \geq 0 \quad f_X = \frac{2}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right); 0 \text{ otherwise}$$

Q.3.)  $X = Z^2 \quad \therefore Z = \pm \sqrt{x} \quad \therefore \frac{dz}{dx} = \frac{1}{2\sqrt{x}}$

$$f_Z(\sqrt{x}) = \frac{1}{\sqrt{2\pi}} e^{-x/2}$$

$$\therefore f_X(x) = (f_{\sqrt{x}}) + f_{-\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{2}{\sqrt{2\pi}} e^{-x/2} \cdot \frac{1}{2\sqrt{x}} = \frac{e^{-x/2}}{\sqrt{x} \sqrt{2\pi}} = \frac{1}{\sqrt{2\pi x}} e^{-x/2}$$