Homework 7

9.12 1) Not really sure if we are supposed to solve it or not so let's just solve it

$$T = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \quad \begin{bmatrix} P(k \to k) & P(L \to K) \\ P(k \to L) & P(L \to L) \end{bmatrix}$$

a) What fraction @ Gen 1

$$[0.5 \,, 0.5]\begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} = [0.45 \,, 0.55]$$

b) WHAT Fraction @ Gen 2

$$[0.45 \quad 0.55]\begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} = [0.425 \,, 0.575]$$

c) Long Run Fraction

$$\begin{array}{c} \\ \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}^{100} \end{array} = \begin{array}{cc} k & L \\ \lbrack 0.4 \,, & 0.6 \rbrack \end{array}$$

Q.2) a) Fraction of recruits that eventually become a supervisor

$$\begin{array}{c|cccc} & R & T & S & Q \\ \hline R & 0.2 & 0.6 & 0 & 0.2 \\ T & 0 & 0.55 & 0.15 & 0.3 \\ S & 0 & 0 & 1 & 0 \\ Q & 0 & 0 & 0 & 1 \end{array}$$

Transient states: R, T ; absorbing S and Q

Prob of reaching S from R: $f_r = 0.2f_r + 0.6f_t + 0\times1 + 0.2\times0$

Prob of reaching S from T: $f_t = 0.55f_t + 0.15\times1 + 0.3\times0$

$\therefore f_r = \dfrac{0.6}{0.8} f_t$ $\quad 0.45f_t = 0.15 \quad f_t = \dfrac{1}{3}$

$\therefore f_r = 0.75 \times \dfrac{1}{3} = \boxed{0.25}$

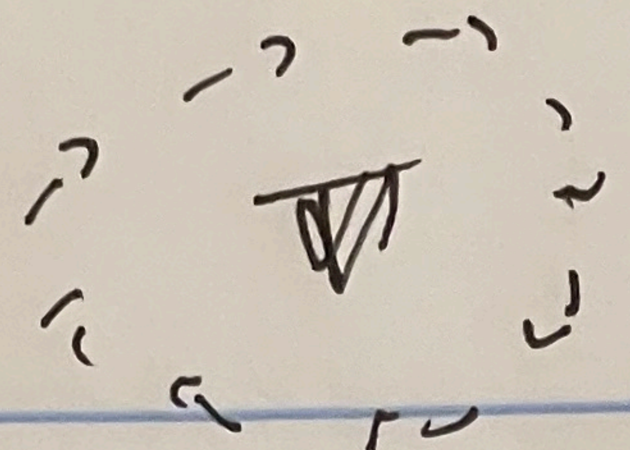
6.) E_r = expected time starting from R
E_T = expected time starting from T

$$\therefore E_r = 1 + 0.2E_R + 0.6E_T + 0\times0 + 0.2\times0$$
$$\Rightarrow 0.8E_R = 1 + 0.6E_T$$
$$\therefore E_R = \frac{1+0.6E_T}{0.8}$$

$$E_T = 1 + 0.55E_T + 0.15\times0 + 0.3\times0$$
$$\therefore 0.45E_T = 1 \quad \therefore E_T = \frac{1}{0.45}$$

$$\therefore E_R = \frac{1+0.6(1/0.45)}{0.8} = \boxed{2.9167}$$

That's the ~~expecthe~~ expected time in weeks till recruit quits or becomes supervisor



Q.3.) V is circular

1.) Argue that $T(i,j) = T(j,i)$

For any vertex i, we can either move to i+1 or i-1; and since it's simple and unbiased the probability of moving to either is 0.5.

∴ The graph is also undirected ∴ $(i,j) = (j,i)$ ∴ $T(i,j) = T(j,i)$

2.) A stationary distribution satisfies that

$$\pi T = \pi$$

Let $\pi(k) = \frac{1}{N}$ for all $k \in V$; i.e each vertex is equally likely.

$$\therefore (\pi T)(j) = \sum_{k \in V} \pi(k) T(k,j)$$

T is symmetric ∴ $\displaystyle\sum_{k \in \text{Neighbours of } j} \frac{1}{N} \cdot \frac{1}{2} = \frac{2 \cdot \frac{1}{2}}{N} = \frac{1}{N}$

∴ $\pi T(j) = \pi(j)$ for all $j \in V$. ∴ $\pi(k) = \frac{1}{N}$ is indeed stationary

3.) Code (✓)

$$m_L(k) = \frac{1}{L} \sum_{j=1}^{L} \mathbb{1}\{X_j = k\}$$

Let L= 100,000 in our sim

Rest done in Vs code :)