Problem Statement

The pharmaceutical company Sun Pharma is manufacturing a new batch of painkiller drugs, which are due for testing. Around 80,000 new products are created and need to be tested for their time of effect (which is measured as the time taken for the drug to completely cure the pain), as well as the quality assurance (which tells you whether the drug was able to do a satisfactory job or not).

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

b.) Calculate the required probability.

Answer 1:

a) Binomial Distribution would accurately portray the above scenario.

The three conditions that this distribution follows:

1. Total number of trials is fixed at n, where n is no. of trials.
2. Each trial is binary, i.e., has only two possible outcomes - success or failure.
3. Probability of success is same in all trials, denoted by p.

b) The theoretical probability that at most, 3 drugs are not able to do a satisfactory job,

Binomial Distribution Formula

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

where
n = the number of trials (or the number being sampled)
x = the number of successes desired
p = probability of getting a success in one trial
$q = 1 - p$ = the probability of getting a failure in one trial

Here n=10

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not , so

p = probability that a drug is not able to produce a satisfactory result = 1/5
q = probability that a drug is able to produce a satisfactory result = 4/5

The required probability is,

p(ns≤3) = p(ns=0) + p(ns=1) + p(ns=2) + p(ns=3)

$$= {}^{10}C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10} + {}^{10}C_1 (1/5)^1 (4/5)^9 + {}^{10}C_2 (1/5)^2 (4/5)^8 + {}^{10}C_3 (1/5)^3 (4/5)^7$$

$$= 0.1074 + 0.26843 + 0.301984 + 0.2013$$

$$= 0.8791.$$

∴ Required probability is $\underline{0.8791}$ $\underline{(87.91\%)}$.

So for given a small sample of 10 drugs, the theoretical probability that at most, 3 drugs are not able to do a satisfactory job is 87.91 %.

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.)Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.
b.)Find the required range.

<u>Answer 2:</u>

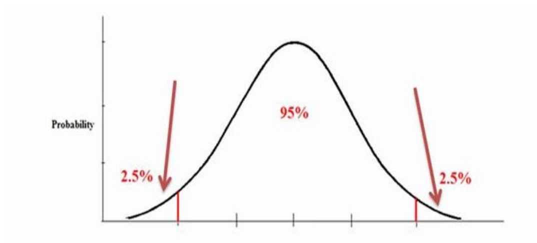
a) Here n = 100 which is greater than n > 30, the sampling distributions become normally distributed, the central limit theorem says that, for any kind of data, provided a high number of samples has been taken, the following properties hold true:

1. **Sampling distribution's mean** (μ(x)) = **Population mean** (μ)
2. Sampling distribution's standard deviation (**Standard error**) = S/√n
3. **For n > 30**, the sampling distribution becomes a **normal distribution**

b) The required range in which the population mean might lie — with a 95% confidence level is calculated by,

$$\text{Confidence interval} = (\bar{X} - \frac{Z^{*}S}{\sqrt{n}}, \bar{X} + \frac{Z^{*}S}{\sqrt{n}}),$$

Where mean $\bar{X}$ = 207, S = 65, n = 100 and Z* = Z-score associated with a y% confidence level = 1.96



$\alpha = 0.05$

$$\Rightarrow \text{confidence interval} : \left[207 - \left(\frac{1.96 \times 65}{\sqrt{100}}\right), 207 + \left(\frac{1.96 \times 65}{\sqrt{100}}\right)\right]$$

$$: (207 - 12.74, 207 + 12.74)$$

$$: (194.26, 219.74)$$

So the required range in which the population mean might lie — with a 95% confidence level is (194.26, 219.74)

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

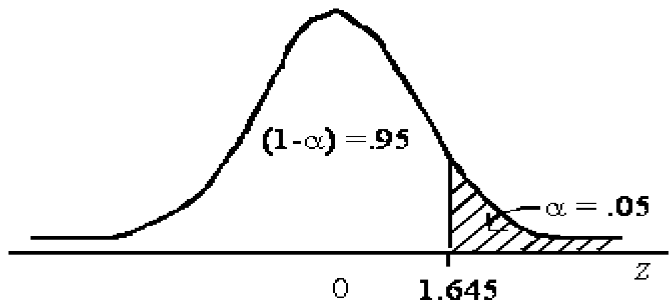
b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current hypothesis test conditions (sample size, mean, and standard deviation), the value of α and β come out to 0.05 and 0.45 respectively.

Now, a different sampling procedure is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions either method would be more preferred than the other.

Answer 3:

a) 1. Critical Value Method:

H_0: Mean Time of Effect ≤ 200 seconds and
H_1: Mean Time of Effect > 200 seconds



This is a one-tailed test. So, for 5% significance level, we would have only one critical region on the right side with a total area of 0.05. This means that the area till the critical point (the cumulative probability of that point) would be 1 - 0.050 = 0.950.The z-score for 0.950 in the z-table is 1.645.

The critical values can be calculated from the equation,

$$\text{critical value} = \mu + \frac{z_c \times \sigma}{\sqrt{n}}$$

where
$$\mu = 200$$
$$z_c = 1.645$$
$$n = 100$$
$$\sigma = 65$$

$$\therefore \quad C.V = 200 + \frac{1.645 \times 65}{\sqrt{100}}$$

$$C.V = 210.69$$

We have to use the + sign since the critical value is on the right-hand side (upper-tailed test). Since the sample mean 207 seconds is less than the critical value (210.69 seconds), we fail to reject the null hypothesis and the painkiller drug have a time of effect of at most 200 seconds and can be considered as having done a satisfactory job.

2. P Value method :

$$z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

where $\bar{x}$ = sample mean = 207
μ = population mean = 200
σ = sample standard deviation (Population SD)
n = sample size = 100

$$\therefore \quad z = \frac{207 - 200}{65/\sqrt{100}}$$

$$\therefore \quad z = 1.076.$$

Since the sample mean lies on the right side of the hypothesized mean of 200 seconds, the z-score comes out to be positive. Cumulative probability of sample point = 0.8599.

For one-tailed test → p = 1 - 0.8599 = 0.1401

Since the p-value is more than the significance level (0.1401> 0.05), we fail to reject the null hypothesis and the painkiller drug have a time of effect of at most 200 seconds and can be considered as having done a satisfactory job.

b) **H_0: Mean Time of Effect ≤ 200 seconds and**
H_1: Mean Time of Effect > 200 seconds
A type I-error represented by α occurs when we reject a true null hypothesis.
A type-II error represented by β occurs when we fail to reject a false null hypothesis.

1st condition: alpha = 0.05 and beta = 0.45

Here Value of alpha is very less (0.05) means the investigator has set 5% as the maximum chance of incorrectly rejecting the null hypothesis but beta value is very high (0.45) means we fail to reject the a false null hypothesis which will lead to **wrong conclusion**. For given drug case beta value can't be as high as 0.45, this error is potentially life-threatening if the unsatisfactory painkillers drugs will serve to market. It can be used when committing type II error will not cause significant result on conclusion like if we are evaluating effectiveness of medicine for some uncured disease at initial phase before its mass production

2st condition: alpha = 0.15 and beta = 0.15

This condition is suitable for given drug case because patients' needs the pain killers on time to relieve pain (which works effectively), beta value is also very less compare to condition 1 means less chances of failing to reject the false null hypothesis which would be effective for given scenario

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign for its existing subscribers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use. Explain why and how A/B testing can be used to decide which option is more effective. Give a step wise procedure for the test that needs to be conducted.

Why A/B Testing can be used?

A/B testing helps companies to make careful changes to user experiences while collecting data on the results. This allows them to construct hypotheses, and to learn better why certain elements of their experiences impact user behavior. AB testing can be used consistently to continually improve a given experience, improving a single goal like conversion rate over time.

Since the team is currently divided on which option to use out of Two taglines were proposed for the campaign, to resolve such conflicts, we can use A/B testing. **A/B testing** provides a way to test two different versions of the same element and see which one performs better

Step wise procedure for the test that needs to be conducted

Here A/B testing needs to be done to decide which option is more effective out of two proposed taglines for the effective online Ad campaign for existing subscribers.

1) Collect the Data: Which parts of the site subscribers like too visit and frequency of the subscribers, timing etc.

2) Identify the goal: Out of two variations which variation has more conversion rate (more clicks on the ad)

3) Generate the Hypothesis: Once we have identify the goal we can form hypothesis.

> **H_0:** variation A (tagline 1) **is better or as good as the** variation B (tagline 2)
> **H_1:** variation B (tagline 2) **is better than** variation A (tagline 1).

4) Create Variations: Using A/B testing, software makes the desired changes to incorporate both tagline and create two variation A with tagline1 and variation B with tagline2 for experimentation.

5) Run the experiment: All visitor of the platform are divided in two categories one experiencing the variation A (tagline 1) whiles others experiencing the variation B (tagline 2) , Wait for sufficient time to collect reliable observation and participation from subscribers. Their interactions are counted and measured.

6) Analyze the result: We have divided the population in two categories out of which we can find proportion of population turns positively for company for each category, by performing the two proportion test we need to find out whether there is a statically significant difference between two variations or not depending upon the significant level.