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ASSIGNMENT 8 — Combinatorics

1 Password Generator Configuration

The OS constraint for having at least two characters of each set (upper case, lower case numbers, special characters, numbers) makes sense since humans are susceptible to creating passwords that are easy to remember. We define *easy to remember* passwords as those comprised of only english words (upper and lower case characters in orders that make up real words and not just arbitrary arrangements). These passwords are easy to brute force since the permutations that make up real english words is a small number.

Having a similar constraint for password generators makes the generators less secure as the constraint reduces the number of possible passwords. The constraint does not help the generators in any meaningful way since the permutation of alphabets resulting in real words is very small, hence unlikely to be generated by the password generators.

2 Co-op Team

Since the order of picking students for the team does not matter and we cannot pick the same student twice, we shall use combinations.

$$= \binom{7}{6} \cdot \binom{3}{1}^6$$
$$= 7 \cdot 3^6$$
$$= 5103 \text{ways of forming the coop team.}$$

Since we can only pick one student from each school, we use $\binom{3}{1}$ six times to pick six students. And we also have to pick the 6 school that will supply the students, $\binom{7}{6}$.

3 Routes to Starbucks

Since I can only move $\uparrow$ 6 times and $\longrightarrow$ 10 times to get to the Starbucks store, we want to know the number of arrangements of $\uparrow$ and $\longrightarrow$ that get me to the store.

$$= \binom{16}{10, 6}$$
$$= \frac{16!}{6! \cdot 10!}$$
$$= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot)(10!)}$$
$$= 4 \cdot 14 \cdot 13 \cdot 11$$
$$= 8008$$

4 Menchies

Since we are may choose multiple scoops of the same topping, and the order of choosing the toppings does not matter, we shall use multichoose to calculate the number of toppings required to create 90 masterpieces.

$$\left(\!\!\binom{n}{3}\!\!\right) = 90$$
$$90 = \binom{n-1+3}{3}$$
$$90 = \frac{(n+2)!}{(n-1)! \cdot 3!}$$
$$90 = \frac{(n+2)(n+1)(n)}{3!}$$
$$540 = (n)(n+1)(n+2)$$
$$n = 7.1842$$

Rounding up, we shall need 8 toppings to be able to create 90 unique masterpieces.

5 Robot Testing

If we select the recipe for the first robot as the first 10 bolts listed in order and the recipe for the second robot is the next 10 bolts listed in order, the given problem transforms into calculating the number of permutations of 20 bolts.

$= 20!$

$= 2432902008176640000$ must be tested.

6 Elective Courses

Since the order of choice does not matter here, we use combinations. Here we assume no student can take more one elective, hence the number of students left in the pool after each round will reduce.

$$= \binom{150}{30} \cdot \binom{120}{30} \cdot \binom{90}{45} \cdot \binom{45}{45}$$
$$= \frac{150!}{30! \cdot 30! \cdot 45! \cdot 45!}$$
$$= 567478665606028146690670587016504399052244296653987897520170713436347384878680826880 00$$

Submitted by Yashashwin Karthikeyan, Roozbeh Ali on April 1, 2024.