

Q.1.	Group of People	Balopenidol		
		X	Y	Z
	A	25	7	13
		27	8	18
	B	21	16	19
		24	11	14
	C	29	19	30
		31	21	27

Step 1 :

total no. of obs. (N) = 18

grand sum (Σx) = 360

grand mean (\bar{x}) = $\frac{360}{18} = 20$

Correction Factor (CF) = $(\Sigma x^2)/N = (360)^2/18 = 7200$

$\therefore CF = 7200.$

Sum of squares of all obs. (Σx^2) = $25^2 + 27^2 + 7^2 + 8^2 + 13^2 + 18^2 + 21^2 + 24^2 + 16^2 + 11^2 + 19^2 + 14^2 + 29^2 + 31^2 + 19^2 + 21^2 + 30^2 + 27^2$

$\therefore \Sigma x^2 = 8144.$

Step 2 :

① total sum of squares (SST) = $\Sigma x^2 - CF$
= $8144 - 7200$

$\therefore SST = 944.$

② Sum of squares between rows (SSR)

$$\left\{ \therefore SSR = \frac{\Sigma R_i^2}{b \times r} - CF \right\}$$

$$R_A^2 = (25+27+7+8+13+18)^2 = (98)^2$$

$$\therefore R_A^2 = 9604 \quad \text{--- (1)}$$

$$R_B^2 = (21+24+16+11+19+14)^2 = (104)^2$$

$$= 10816 \quad \text{--- (2)}$$

$$R_C^2 = (29+31+19+21+30+27)^2 = (157)^2$$

$$= 24649 \quad \text{--- (3)}$$

Now, from (1) (2) & (3)

$$\Sigma R_i^2 = R_A^2 + R_B^2 + R_C^2$$

$$= 9604 + 10816 + 24649$$

$$= 45069$$

$$SSR = \frac{45069}{3 \times 2} - CF = \frac{45069}{6} - 7200$$

$$= 346.33$$

(3) Sum of squares btw columns (SSC)

$$\left\{ SSC = \frac{\Sigma C_i^2}{a \times r} - CF \right\}$$

$$C_x^2 = (25+27+21+24+29+31)^2 = (157)^2$$

$$= 24649 \quad \text{--- (i)}$$

$$C_y^2 = (7+8+16+11+19+21)^2 = (82)^2$$

$$= 6724 \quad \text{--- (ii)}$$

$$C_z^2 = (13+18+19+14+30+27)^2 = (121)^2$$

$$= 14641 \quad \text{--- (iii)}$$

$$\begin{aligned}\text{Now, } \sum C_i^2 &= C_x^2 + C_y^2 + C_z^2 \\ &= 24649 + 6724 + 14641 \\ &= 46014.\end{aligned}$$

$$SSC = \frac{46014}{3 \times 2} - CF = \frac{46014}{6} - 7200$$

$$\therefore SSC = 469.$$

(4) Sum of square within cells (SSE)

first compute sum of squares betw cells (SSB-cells).

$$1^{st} \text{ cell} = 25 + 27 = 52$$

$$2^{nd} \text{ cell} = 7 + 8 = 15$$

$$3^{rd} \text{ cell} = 13 + 18 = 31$$

$$4^{th} \text{ cell} = 21 + 24 = 45$$

$$5^{th} \text{ cell} = 16 + 11 = 27$$

$$6^{th} \text{ cell} = 19 + 14 = 33$$

$$7^{th} \text{ cell} = 29 + 31 = 60$$

$$8^{th} \text{ cell} = 19 + 21 = 40$$

$$9^{th} \text{ cell} = 30 + 27 = 57$$

\therefore Squares of cells.

$$= (52)^2, (15)^2, (31)^2, (45)^2, (27)^2, (33)^2, (60)^2, (40)^2, (57)^2.$$

$$\Rightarrow 2704, 225, 961, 2025, 729$$

$$1089, 3600, 1600, 3249$$

$$\therefore \text{Sum} = 2704 + 225 + 961 + 2025$$

$$+ 729 + 1089 + 3600 +$$

$$1600 + 3249$$

$$SSB_{\text{cell}} = \frac{16182}{8} - CF$$

$$= 16182.$$

$$= \frac{16182}{2} - 7200$$

$$= 891.$$

Now, calculating SSE

$$\therefore SSE = SSI - SSB_{\text{cell}}$$

$$= 944 - 891$$

$$= 53.$$

⑤ Sum of square for interaction (SSI).

$$\begin{aligned} SSI &= SSB_{cell} - SSR - SSC \\ &= 891 - 346.33 - 469 \\ &= 75.67 \end{aligned}$$

Step 3: [ANOVA Table.]

Source of Variation.	SS	df	MS	F ratio.
Rows.	346.33	(r-1) (3-1)=2	$346.33/2$ $= 173.165$	$F_r = 173.165/4.417$ $= 39.20$
Columns.	469	2	$469/2$ $= 234.5$	$F_c = 234.5/4.417$ $= 53.09$
Interactions	75.67	4	$75.67/4$ $= 18.917$	$F_t = 18.917/4.417$ $= 4.28.$
Error	53	9	4.417	-
total.	944	17	-	-

(i) Do the balopexidol acts differently?
Yes ($F = 53.09$, $P < 0.05$)

$$\begin{aligned} &F_{\alpha} \\ &F(2,9) \\ &= 4.29. \end{aligned}$$

(ii) Are the differently group of people affect differently?
Yes ($F = 39.20$, $P < 0.05$)

$$\begin{aligned} &F(4,9) \\ &= 3.63 \end{aligned}$$

(iii) Is the interaction term significant?
Yes ($F = 4.28$, $P < 0.05$)

Q.2.

You are conducting a study to compare the performance of three different type of enhancers [enhancer A, enhancer B, and enhancer C] in promoting the growth of a specific plant species. You have collected data on the height of the plants in each group after six weeks. The dataset continue the following measurement

(a) Enhancer A: [18, 20, 22, 25, 21]

(b) Enhancer B: [15, 16, 14, 17, 18]

(c) Enhancer C: [20, 24, 22, 21, 25]

Do the baloperidol set differently?

are the different groups of people affected differently?

is the interaction term significant?

Step 1.

(N) total no. of obs = 18

grand sum ($\sum n$) = 360

grand mean (\bar{x}) = $360/18 = 20$

Correction factor (CF) = $(\sum x^2)/N = \frac{(360)^2}{18} = 7200$.

Sum of squares of all

$$\begin{aligned} \text{obs. } (\sum x^2) &= 25^2 + 27^2 + 7^2 + 8^2 + 13^2 + 18^2 + 21^2 + 24^2 \\ &+ 16^2 + 11^2 + 19^2 + 14^2 + 29^2 + 31^2 + 19^2 + 21^2 \\ &+ 30^2 + 27^2 \end{aligned}$$

NA

a) Explain the basic concept of the Kruskal-Wallis test and when it is appropriate to use it.

- The Kruskal-Wallis test is a non-parametric method used to compare three or more independent groups when the dependent variable is either ordinal or continuous, but not normally distributed. It is essentially an extension of the Mann-Whitney test to more than two groups.
- The test ranks all the data from the groups together, then compares the avg. ranks between groups. If the same groups are from the same population, their ranks should be similarly distributed.
- A significant result indicates that at least one group tends to have larger or smaller values than the others.

b) Calculate the Kruskal-Wallis statistic for the given datasets.

→ data :- enhancer : 18, 20, 22, 25, 21
(A)

enhancer (B) : 15, 16, 14, 17, 18

enhancer (C) : 20, 24, 22, 21, 25

Step 1: combine and rank all data.

values	group	ranks.
14	B	1
15	B	2
16	B	3
17	B	4
18	A	5.5
18	B	5.5
20	A	7.5
20	C	7.5
21	A	9.5
21	C	9.5
22	A	11.5
22	C	11.5
24	C	13
25	A	14.5
25	C	14.5

Step 2 : Sum of ranks for each group.

- enhancer A : $5.5 + 7.5 + 9.5 + 11.5 + 14.5 = 48.5$
- enhancer B : $1 + 2 + 3 + 4 + 5.5 = 15.5$
- enhancer C : $7.5 + 9.5 + 11.5 + 13 + 14.5 = 56$

Step 3: Kruskal-Wallis formula.

$$H = \frac{12}{N(N+1)} \sum_{i=1}^k \frac{R_i^2}{n_i} - 3(N+1)$$

where,

$$N = 15$$

$$k = 3$$

$$n_i = 5 \text{ each}$$

$$R_1 = 48.5$$

$$R_2 = 15.5$$

$$R_3 = 56$$

$$\sum \frac{R_i^2}{n_i} = \frac{(48.5)^2}{5} + \frac{(15.5)^2}{5} + \frac{(56)^2}{5}$$

$$= 1145.7$$

$$\therefore H = \frac{12}{15 \times 16} \times 1145.7 - 3 \times 16$$

$$= 9.285$$

Step 4: Adjust for ties.

$$18 (2 \text{ time}) - t = 2$$

$$20 (2 \text{ time}) - t = 2$$

$$21 (2 \text{ time}) - t = 2$$

$$22 (2 \text{ time}) - t = 2$$

$$25 (2 \text{ time}) - t = 2$$

$$C = 1 - \frac{\sum (t^3 - t)}{N^3 - N}$$

$$\sum (t^3 - t) = 5 \times (8 - 2)$$

$$= 5 \times 6$$

$$= 30$$

$$\therefore C = \frac{1 - 30}{3375 - 15} = 0.99107$$

$$\therefore H_{adj} = \frac{H}{C} = \frac{9.285}{0.99107} = 9.37$$

$$\therefore H_{adj} = 9.37$$

2.) State the null and alternative hypothesis for the Kruskal-Wallis test in this context

→ Null hypothesis (H_0).

- the population median of plant heights for all the three enhancers are the same.

Alternative hypothesis (H_1).

- at least one enhancer tends to yield different plant heights than the others.

d.) Interpret the results of the Kruskal-Wallis test and provide a conclusion regarding the effect of different enhancers on plant height.

→ From chi-square table,

for $df = k - 1 = 2$, critical value at $\alpha = 0.05$ is about 5.991. Our $H \approx 9.37 > 5.991$,

so we reject H_0 .

Conclusion:

there is a statistically significant difference in plant heights among at least one pair of the three enhancers at the 5% significance level.