

ABCDEFGHIJKLMNOPQRSTUVWXYZ
 65
 97 (1) CNS

- Caesar cipher - Julius cipher
- shifting letters in plaintext by certain no. of positions
- known as shift/key
- substitution cipher

- limited keyspace

shift $\rightarrow 3$

A \rightarrow D

B \rightarrow E

easy to crack

shift $\rightarrow 3$

HELLO \rightarrow PT

KHQR

-3 -3 -3 -3

\rightarrow Decrypt

HELLO

eg: h e t 1 1 0

ch = h, offset = 'a' i.e. 97
 Int, ascii - h \rightarrow 104

$$(ch - offset + shift) \% 26 + offset$$

$$(104 - 97 + 3)$$

$$7 + 3$$

$$10$$

$$10 \% 26 + offset$$

$$10 + 97$$

107 \rightarrow i.e. K

h \rightarrow K

②

- Playfair cipher - 1854
- Charles Wheatstone
- substitution cipher
- encrypt pairs of alphabets instead of single

- key: monarchy
- plaintext: instruments

1. Generate key square (5x5)

- 5x5 grid of alphabets: acts as a key
- each 25 alphabets must be unique
- J is omitted, if J comes \rightarrow I replaced

2. Algo to encrypt plaintext

Plaintext: "instruments"

if odd count, 'z' is added at last

After split: (in) (st) (ru) (me) (nt) (sz)

2 2 2 2 2 2
digraph

1. Pair with same letters: x \rightarrow not allowed

he(11)o

he 1(x) o

↑

add bogus letter

2. If letter is standing alone, add bogus letter

helloe

he 1x 1o e(2)

x y z

	A	B
1	1	
2	2	
3	3	
4	4	
5	5	
6		

max=5

classmate
Date: _____
Page: _____

Decrypt

-shift back

encrypt(text, 26-shift)

(~~k~~, 23)
k

(h - offset + shift) * 26 + offset

107 - 97 + 23

10 + 23

33

* 26 + 97

~~24 + 97 = 121~~

8 + 97 = 105 = i.e h

(ascii)

Solving:

in

0	M	O	N	A	R
1	C	H	Y	B	D
2	E	F	G	I	K
3	L	P	Q	S	T
4	U	V	W	X	Z

M	O	N	A	R
C	H	Y	B	D
E	F	G	I	K
L	P	Q	S	T
U	V	W	X	Z

digraph: 'in' maps to

if same row, next char

S → E

E → I

SE → EI

su → mz

me

M	O	N	A	R
C	H	Y	B	D
E	F	G	I	K
L	P	Q	S	T
U	V	W	X	Z

if same col, next char
bottom

m → C

e → I

me → CI

nt → tq

SZ: EX

Diaprs:

same row, char before
same col, char upper
else

same letters on
horizontal opposite

PT: instruments

LT: g a t m z c l q t x

Decrypt:

Turn ciphertext back into a vector

1. take inverse of key matrix K^{-1}
2. multiply it by ciphertext matrix C
3. reduce each ele. of resulting vector by mod 26

- code:
1. processes 3 chunks of message at a time
 2. converts each char. to no.
 3. multiply key matrix with message vector (3)
 4. convert above resultant 3 chunk encrypted vector back to char and append to encrypted text.

A=0, B=1, ..., Z=25

(1)

Vigenere cipher - most designed to work with upper case

- method of encrypting alphabetic text

- uses simple form of polyalphabetic substitution

* - encryption of original text is done using
Vigenere square or table

26 alphabets, 26 times in diff, shifted cyclically each time

0	A	-	Z
1	B	-	Z A
2	C	-	Z A B
3	D	-	Z A B C
...
26	Z	-	A, B, ..., Y

example: Plaintext: GREEKS FOR GREEKS $n=13$

Key: AYUSH

key: AYUSHAYUSHAYU $n=13$

'AYUSH' repeated until length of the PT

G E E . . .

A Y U . . .

$i=6$

$j=A$

\rightarrow search in table where row = 6
col = A

A

Y

6

G

E

C

G \rightarrow G

E \rightarrow C

Proportionally
also val. are
considered

$$\rightarrow (P_i + K_i) \cdot 1.26 + \text{offset}$$

if 'A' $\rightarrow 65$

$$E + Y$$

$$69 + 89 \cdot 1.26 + 65 \cdot 0 \rightarrow 97$$

Algebraically
observe

$$\frac{158 \cdot 1.26}{2} + 65 \rightarrow 67 \rightarrow C$$

(key)
(101)

Plaintext (E)
(Row)

(4)

(C)

\rightarrow cell

Encyphor:

$$\therefore (P_i + K_i) \cdot 1.26 = E_i$$

$$(4 + 24) \cdot 1.26$$

$$28 \cdot 1.26$$

$$2 = E_i$$

$$E \rightarrow 4$$

$$4 \rightarrow 24$$

$$C \rightarrow 2$$

$$\therefore E_i = C$$

Decyphor:

$$D_i = \frac{C - Y \cdot 1.26}{(E_i - K_i) \cdot 1.26}$$

$$= \frac{(2 - 24) \cdot 1.26}{(4 - 24) \cdot 1.26}$$

$$= -22 \cdot 1.26 = \underline{4}$$

$$4 \rightarrow E$$

\therefore (E) \rightarrow original PT

③

Hill Cipher

- polygraphic substitution cipher based on linear algebra
- each letter replaced by a number 1-26 represented

block cipher

$$A=0, B=1, \dots, Z=25$$

eg:

ACT

n=3

- process plaintext msg. in form of chunks

- key matrix dimensions are chosen based on the chunk size of the PT you want to encrypt

① Key: GYBNQKURP

len = $n \times n$ where n = text length

$$\begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix}$$

twice

② ACT is written as vector:

$$\begin{bmatrix} 0 \\ 2 \\ 19 \end{bmatrix}$$

③ Enciphered vector:

$$\begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix} \times \begin{bmatrix} 0 \\ 2 \\ 19 \end{bmatrix} = \begin{bmatrix} 67 \\ 222 \\ 319 \end{bmatrix} \begin{matrix} \div 26 \\ \div 26 \\ \div 26 \end{matrix} \begin{bmatrix} 15 \\ 14 \\ 267 \end{bmatrix} \begin{matrix} \rightarrow P \\ \rightarrow O \\ \rightarrow H \end{matrix}$$

⑤

Rail Fence

- zig-zag cipher i.e. transposition cyphre

eg: rails=3 , PT: GREEKS FOR GREEKS

no. of cols = len of plain text

1	G			S			G			S
2		E		K		F		R		
3			E				O		E	K

ET: Rowise Read

G S G S E K F R E K E O E

Decryption:

code wise:

HELIO THERE

rail vector of size 3 where 3 == key

	0	1	2	→ rows
rail:	H	E	L	
	O	L	H	
	R	T		
		E		
		E		

e: H O R E L T E E L H

Decryption:

pos(key, 0)

$s = 0 + 2 + 0 + 2 + 0 + 1$

$d = 1 + 1 + 1 + 1$

* 1 2 3 4 5 6 7 8 9

H O R E L T E E L H

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

$n = 10$

→ rows

→ Initially

pos:	0	1	2
	0	0	0
①	1	1	1
	1	1	1
	1	1	
		1	
		1	

3 5 2
a, 1, 2 i i

	0	1	2	ind = 0
tail:	H	E	L	row i = 0
	O	L	H	$j = 0 + 2$
	R	T		ind = 0 + 2
		E		
		E		row i = 1
				$j = 0 + 1$

Example:

primes: $p=17$ $q=11$

① $n=187$

→ find n

② Euler's totient

$$\begin{aligned}\phi(n) &= (p-1) * (q-1) \\ &= (17-1) * (11-1) \\ &= 16 * 10 \\ &= 160\end{aligned}$$

③ choose val. of e

$$1 < e < \phi n \quad \& \quad \gcd(\phi(n), e) = 1$$

considering $e=7$

$$1 < 7 < \phi n \quad \gcd(160, 7) = 1$$

co-prime

④ Determine d

$$d \equiv e^{-1} \pmod{\phi n}$$

$$de \equiv 1 \pmod{\phi n}$$

$$de \pmod{\phi n} = 1$$

$$(d * 7) \pmod{160} = 1$$

$$(23 * 7) \pmod{160} = 1$$

$$161 \pmod{160} = 1$$

$$\therefore \boxed{d=23}$$

MOD3

RSA algorithm

- Rivest Shamir Adleman
- asymmetric cryptography

En: \downarrow Plaintext
 $C = P^e \bmod n$
 \uparrow
 ciphertext

Imp

* If en. is done using public key, the de. must be performed using private key of same user.

Algo:

- 1) Select 2 large prime nos p, q
- 2) Cal. $n = p \cdot q$
3. Cal. $\phi(n) = (p-1)(q-1) \rightarrow$ Euler's totient
4. choose value of e

$$1 < e < \phi(n) \text{ and } \gcd(\phi(n), e) = 1$$

co-prime

\Rightarrow congruent

5. calculate

$$d \equiv e^{-1} \bmod \phi(n) \rightarrow \text{Multiplicative inverse of } d$$

$$\text{i.e. } ed \equiv 1 \bmod \phi(n)$$

$$\rightarrow ed \bmod \phi(n) = 1$$

Specific for user

6. public key = $\{e, n\}$ \rightarrow used in encryption
- private key = $\{d, n\}$ \rightarrow private key used for decryption

Encryption

$$C = M^e \bmod n$$

\rightarrow publicity of user A

Decryption

$$M = C^d \bmod n$$

\rightarrow private key of user A

Numerical:

$$c = 8$$

$$e = 13$$

$$n = 33$$

$$M = ?$$

$$M = c^d \bmod n$$

$$\rightarrow 1. n = 33$$

$$\therefore 11 \times 3 = 33$$

$$\therefore p = 3, q = 11$$

2. Euler's totient:

$$\phi(n) = (p-1) * (q-1)$$

$$= (3-1) * (11-1)$$

$$= 2 * 10$$

$$= 20$$

3. determine d

d is mul. inv of $e \bmod \phi(n)$

$$d \equiv e^{-1} \bmod \phi(n)$$

$$d \cdot e \equiv 1 \bmod \phi(n)$$

$$d \cdot e \bmod \phi(n) = 1$$

$$(d * 13) \bmod 20 = 1$$

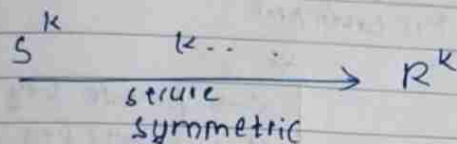
$$\therefore d = \underline{\underline{17}}$$

$$\therefore M = 8^{17} \bmod 33$$

$$\therefore M = 2$$

• Diffie-Hellman key exchange algo.

- not an encryption algo
- symmetric key ~~exchange~~ encryption: requires secure & reliable channel for key exchange shared



- public channel can be used to create a confidential shared key
- assy. en. is used to exchange the secret key

• Algorithm:

1. consider a prime no. ' q '
2. select α such that it must be the primitive root of q and $\alpha < q$

' α ' is a primitive root of q if

$$\alpha \bmod q$$

$$\alpha^2 \bmod q$$

$$\alpha^3 \bmod q \dots \alpha^{q-1} \bmod q$$

gives results $1, 2, 3, \dots, q-1$

if $q=7, \alpha=3$

$$3^1 \bmod 7 = 3$$

$$3^2 \bmod 7 = 2$$

$$3^3 \bmod 7 = 6$$

$$3^4 \bmod 7 = 4$$

$$3^5 \bmod 7 = 5$$

$$3^6 \bmod 7 = 1$$

$\therefore 3$ is PR of 7

$1, 3, 2, 6, 4, 5$

$$34 \bmod 26$$

ans - before decimal * 26

③ \therefore public key = $e, n \rightarrow 7, 1874$
private key = $d, n \rightarrow 23, 1874$

$$M = 88$$

$$M < N$$

Encryption:

$$C = M^e \bmod n$$
$$= 88^7 \bmod 187$$

$$\underline{\underline{C = 11}}$$

* see cal.

Decryption:

$$M = C^d \bmod n$$
$$= 11^{23} \bmod 187$$

$$\underline{\underline{M = 88}}$$

A: PK given, gen pub. key
B: PK given, gen pub. key

Example: ①

$$q=7 \rightarrow \text{prime}$$

step 2. $x < q$

$$x=3$$

or

$$x=5$$

↑ is taken here

x & q are global, known to everyone

step (3.)

$x \rightarrow$ private key of user
 $y \rightarrow$ public key of user

theory [assume x_A (private key) and $x_A < q$
of A (user)

$$\text{calculate } y_A = x^{x_A} \bmod q$$

APPⁿ [key gen. of person)
Assume private key $x_A=3$ $\therefore (x_A < q$
 $3 < 7 \text{ yes})$
 \therefore calculating public key $y_A = x^{x_A} \bmod q$
 $= 5^3 \bmod 7$
 $\therefore y_A = 6$

step 4: assume $x_B \rightarrow$ PK of user B $x_B < q$

$$\text{Calculate public key: } y_B = x^{x_B} \bmod q$$

key generation of person 2

Let private key $x_B = 4$

$(x_B < q)$

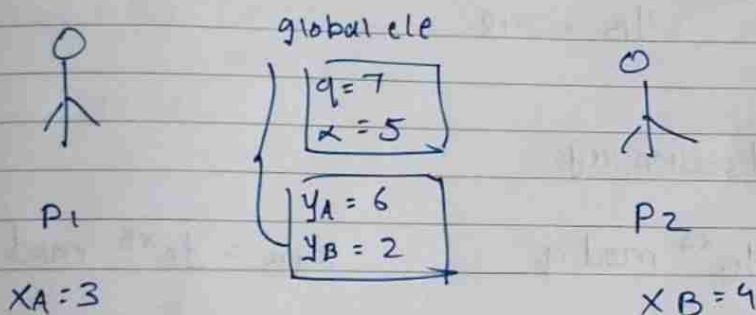
$(4 < 7)$

$$\therefore \text{Public key} \rightarrow y_B = x^x \pmod q$$

$$= 5^4 \pmod 7$$

$$\therefore y_B = 2$$

visualization:



(Shared Session Key)

- 5. Now we'll calculate secret key
- For this both sender & receiver uses public keys

Person 1

$$K_1 = (y_B)^{x_A} \pmod q$$

user 2 pub. key

$$= 2^3 \pmod 7$$

$$K_1 = 1$$

Person 2

$$K_2 = (y_A)^{x_B} \pmod q$$

user 1 pub. key

$$= 6^4 \pmod 7$$

$$K_2 = 1$$

As $K_1 = K_2$
Thus, keys are exchanged.

Eg: 2

$$q = 353 \quad \alpha = 3$$

$$\therefore x_A = 97 \quad x_B = 233$$

$$\begin{aligned} \therefore \text{Public key of A: } \alpha^{x_A} \bmod q \\ = 3^{97} \bmod 353 \\ y_A = 40 \end{aligned}$$

$$\begin{aligned} \text{Public key of B: } \alpha^{x_B} \bmod q \\ = 3^{233} \bmod 353 \\ y_B = 248 \end{aligned}$$

\therefore shared session key:

$$\begin{aligned} K_A &= y_B^{x_A} \bmod q \\ &= 248^{97} \bmod 353 \\ &= \underline{160} \end{aligned}$$

$$\begin{aligned} K_{AB} &= y_A^{x_B} \bmod q \\ &= 40^{233} \bmod 353 \\ &= \underline{160} \end{aligned}$$

Eg: 3

$$q = 17$$

$$\alpha = 5$$

$$x_A = 4$$

$$x_B = 6$$

$$\begin{aligned} \therefore y_A &= \alpha^{x_A} \bmod q \\ &= 5^4 \bmod 17 \end{aligned}$$

$$y_A = 13$$

$$\begin{aligned} \therefore K_{AB} &= y_B^{x_A} \bmod q \\ &= 2^4 \bmod 17 \\ &= 16 \end{aligned}$$

283929

$$7^1 \bmod 13 = 7$$

$$7^2 \bmod 13 = 10$$

$$7^3 \bmod 13 = 5$$

⋮

⋮

$$7^{q-1} \bmod q$$

$$\begin{aligned} y_B &= \alpha^{x_B} \bmod q \\ &= 5^6 \bmod 17 \end{aligned}$$

$$y_B = 2$$

$$\begin{aligned} K_{AB} &= y_A^{x_B} \bmod q \\ &= 13^6 \bmod 17 \\ &= 15.99 \dots \\ &\approx 16 \end{aligned}$$