Assignment 2

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Question: Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$ Also, find the probability of obtaining at most one success.

Solution: For binomial distribution :
Given, Mean = 9 and Standard Deviation(S.D) = $\frac{3}{2}$

$$\text{Mean} = \text{np} = 9 \tag{1}$$

$$\text{Variance} = (\text{S.D.})^2 = \text{npq} = \frac{9}{4} \tag{2}$$

By substituting equation(1) in equation(2):

$$\text{q} = \frac{1}{4} \tag{3}$$

Since, p = $1-$q

$$\text{p} = 1 - \frac{1}{4} = \frac{3}{4} \tag{4}$$

Using equation (4) in equation (1):

$$\text{n} = \frac{9}{p} = \frac{4 \times 9}{3} = 12 \tag{5}$$

Thus distribution is:
Let X$\sim$ Bin(n,p) $\sim$ Bin(m,p).
Now let $0 \leq k \leq (n+m),$ then

$$\text{P(X+Y=k)} = \sum_{i=0}^{k} \text{P(X=i,Y=k}-\text{i)} \tag{6}$$

$$= \sum_{i=0}^{k} \text{P(X=i) P(Y=k-i)} \tag{7}$$

$$\sum_{i=0}^{k} \left(\begin{array}{c} n \\ i \end{array} \right) p^i (1-p)^{n-i} \left(\begin{array}{c} m \\ k-i \end{array} \right) p^{k-i} (1-p)^{m-k+i} \tag{8}$$

$$= p^k (1-p)^{n+m-k} \sum_{i=0}^{k} \left(\begin{array}{c} n \\ i \end{array} \right) \left(\begin{array}{c} m \\ k-i \end{array} \right) \tag{9}$$

$$= \left(\begin{array}{c} n+m \\ k \end{array} \right) \text{p}^k \, (1-\text{p})^{n+m-k} \tag{10}$$

$$\text{P(k=r)} =^{12} C_r (\text{p})^r (\text{q})^{12-r} \tag{11}$$

$$\text{P(k=r)} =^{12} C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{12-r} \tag{12}$$

$$\text{r} = 0, 1, 2, 3...$$

P(at most one success) = P(k=0) + P(k=1)

$$=^{12} C_0 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^{12} +^{12} C_1 \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^{11} \tag{13}$$

$$= \left(\frac{1}{4}\right)^{12} + 36 \left(\frac{1}{4}\right)^{12} = \frac{37}{4^{12}} \tag{14}$$