Assignment 2

Yash R Ramteke
bt21btech11006

Question: Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$ Also, find the probability of obtaining at most one success.

Solution: For binomial distribution :
Given, Mean = 9 and Standard Deviation(S.D) = $\frac{3}{2}$

$$\text{Mean = np = 9} \tag{1}$$

$$\text{Variance = (S.D.)}^2 = npq = \frac{9}{4} \tag{2}$$

By substituting equation(1) in equation(2):

$$q = \frac{1}{4} \tag{3}$$

Since, p = 1−q

$$p = 1 - \frac{1}{4} = \frac{3}{4} \tag{4}$$

Using equation (4) in equation (1):

$$n = \frac{9}{p} = \frac{4 \times 9}{3} = 12 \tag{5}$$

Thus distribution is:
Let $X \in \{0, 1\}$
$X \sim \text{Bin(n,p)} \sim \text{Bin(m,p)}$
Now let $0 \le k \le (n + m)$, then

$$P(X) = \sum_{i=0}^{k} P(i, k - i) \tag{6}$$

$$\sum_{i=0}^{k} \binom{n}{i} p^i(1-p)^{n-i} \binom{m}{k-i} p^{k-i}(1-p)^{m-k+i} \tag{7}$$

$$= p^k(1-p)^{n+m-k} \sum_{i=0}^{k} \binom{n}{i}\binom{m}{k-i} \tag{8}$$

$$= \binom{n+m}{k} p^k (1-p)^{n+m-k} \tag{9}$$

$$P(k=r) =^{12} C_r(p)^r(q)^{12-r} \tag{10}$$

$$P(k=r) =^{12} C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{12-r} \tag{11}$$

$$r = 0, 1, 2, 3...$$

P(at most one success) = P(k=0) + P(k=1)

$$=^{12} C_0 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^{12} +^{12} C_1 \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^{11} \tag{12}$$

$$= \left(\frac{1}{4}\right)^{12} + 36\left(\frac{1}{4}\right)^{12} = \frac{37}{4^{12}} \tag{13}$$