Probability and Random Variables

Assignment 3

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Outline

Problem

Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

1. number greater than 4
2. six appears on at least one die

When a die is tossed two times, we obtain $6 \times 6 = 36$ number of observations.
Let X be the random variable, which represents the number of successes.

Number greater than 4

Here, success refers to the number greater than 4.
P(X= 0)= P(number less than or equal to 4 on both the tosses)

$$= \frac{4}{6} \times \frac{4}{6} = \frac{4}{9} \tag{1}$$

P(X=1) = P(number less than or equal to 4 on first toss and greater than 4 on second toss) + P(number greater than 4 on first toss and less than or equal to 4 on second toss)

$$= \frac{4}{6} \times \frac{2}{6} + \frac{4}{6} \times \frac{2}{6} = \frac{4}{9} \tag{2}$$

P(X=2) = P(number greater than 4 on both the tosses)

$$= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9} \tag{3}$$

Thus the probability distribution is as follows:

$$P(X_1 = k) = \begin{cases} \frac{4}{9}, & k = 0 \\ \frac{4}{9}, & k = 1 \\ \frac{1}{9}, & k = 2 \end{cases} \tag{4}$$

Six appears on at least one die

Here, success means six appears on at least one die.
P(Y=0) = P(six appears on none of the dice)

$$= \frac{5}{6} x \frac{5}{6} = \frac{25}{36} \tag{5}$$

P(Y=1) = P(six appears on at least one of the dice)

$$= \frac{11}{36} \tag{6}$$

Thus the probability distribution is as follows:

$$P(X_1 = k) = \begin{cases} \frac{25}{36}, & k = 0 \\ \frac{11}{36}, & k = 1 \end{cases} \tag{7}$$